

## Article

# A New Shear Constitutive Model Characterized by the Pre-Peak Nonlinear Stage

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**Abstract:** The pre-peak shear stress-displacement curve is an important part of the study of the shear mechanical behavior of rock joints. Underpinned by the Haldane distribution, a new semi-analytical model for the pre-peak shear deformation of rock joints was established in this paper, the validity of which was verified by laboratory and in situ experimental data. Other existing models were employed to make comparisons. The comparison results show that the model has superior adaptability and is more suitable for convex-type shear constitutive curves than existing models. Besides, only one parameter was introduced to the model, which is more convenient for application. All of these imply that the proposed model is an effective tool to evaluate the pre-peak shear constitutive curves of different rock joints. The research results can provide a reference for further understanding of the shear fracture characteristics of rock materials.

**Keywords:** fracture characteristics; rock joint; pre-peak shear constitutive curve; shear deformation evolution



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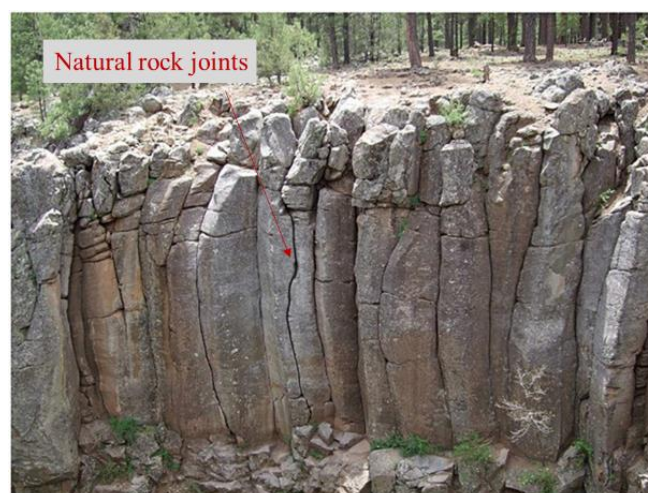
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## 1. Introduction

The intact rock masses in nature are cut by rock joints into rock blocks, forming jointed rock masses as shown in Figure 1 [1,2]. The failure of rock joints dominates more than that of rock blocks during the instability of jointed rock masses [3,4]. Most geotechnical engineering projects are usually constructed in jointed rock masses [5], and the shear mechanical properties of rock joints are one of the main factors controlling the mechanical stability of underground excavations [6]. For example, it has been reported that the collapse of the São Paulo metro station in Brazil may be related to an over-simplified geomechanical model used in the engineering design process [7]. Therefore, understanding the shear deformation behavior of rock joints is very important for the safety assessment of geoengineering structures (such as rock slopes, tunnels, dam foundations, chambers, and waste repositories) [8,9].

Rock materials are a complex mineral aggregate formed under physical and chemical actions after a long geological process [10–12], which makes the link between stress and strain (or displacement) of rock materials has been one of the most pressing issues over the past decades [13]. Extensive theoretical analysis, experimental tests, and numerical simulation studies have been implemented at the laboratory scale on the shear deformation behavior of rock joints, and fruitful results have been obtained [14–20]. It mainly includes the research on shear strength [21,22], shear constitutive models [23,24], surface failure characteristics [25–28], as well as the influencing laws of rock type [29], boundary conditions [30] and loading mode [31], and other factors. Jing [32] and Muralha et al. [33] provide an admirable summary of the suggested method for laboratory shear tests of rock joints and the importance of rock joints in understanding rock mechanics for a variety of

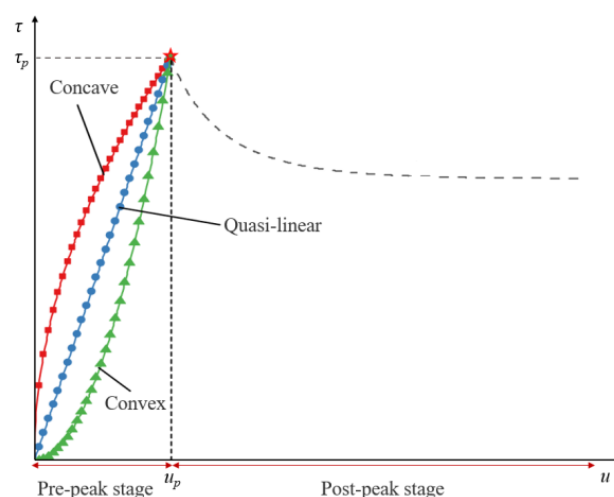
engineering applications. The shear stress/shear displacement curve is one of the most important considerations to evaluate the shear mechanical properties of rock joints [34]. Nonetheless, it is still difficult to establish a constitutive model that can fully reflect the shear stress/shear displacement curve behavior of joint materials, especially considering the complex nonlinear mechanical properties of joint materials [35,36]. Among the aforementioned pioneering studies, it is fully recognized that the shear stress/shear displacement curve of the rock sample subjected to shear tests can be divided by peak points into two critical stages, the pre-peak stage and post-peak stage. Because the shear test requires specific test conditions, the test results themselves have a certain discreteness [37]. In practice, it is difficult to select the most suitable test results to evaluate the shear constitutive relationship [38], and the shear stress/shear displacement curve is essential to describe the shear fracture characteristics of rock joints [39]. According to the classical failure curve of rock joints [40,41], except for peak stress, the stress thresholds of crack compaction, crack initiation and rock damage are all located in the pre-peak stage, which is closely associated with crack development. However, there are rare reports on the pre-peak stress and deformation characteristics [42]. Therefore, it is still necessary to study the theoretical modeling of pre-peak shear stress/shear displacement curves of rock joints.



**Figure 1.** Example of jointed rock mass(modified after [1]).

Modeling studies to investigate the pre-peak shear stress- displacement curve of rock joints began in the late 1960s and early 1970s. Goodman [43] and Saeb et al. [44] regarded the pre-peak shear stress/shear displacement curve as quasi-linear and used linear functions to characterize the pre-peak shear stress- shear displacement curve of rock joints. Obviously, this oversimplification cannot describe the nonlinear deformation characteristics in the pre-peak stage. Based on a series of direct shear tests, Kulhawy [45] first proposed a hyperbolic function to describe the nonlinear pre-peak shear stress- displacement relationship. Subsequently, referring to the work of Kulhawy [45], Bandis et al. [46] proposed a hyperbolic model to fit the pre-peak shear stress-shear displacement curve and verified the applicability of this model by comparing it with some existing test results. Under the framework of plasticity theory, Desai and Fishman [47] established a pre-peak constitutive model to simulate joints under monotonic loading, unloading and reverse loading. Nassir et al. [48] further generalized the Bandis model [46] by introducing the dilation initiation coefficient to characterize the fraction of peak shear displacement at which dilation initiates, and then calibrated the model based on experimental results. Inspired by the idea of normalization, Ban et al. [49] made the shear stiffness dimensionless and established a hyperbolic shear constitutive considering shear stiffness softening to describe the pre-peak shear stress/shear displacement curve. Kou et al. [50] also studied the pre-peak shear behavior of rock joints contain the triangular-shaped primary and subordinated asperities through pre-peak cyclic shear test. Despite those models inspiring the research on the

characterization of pre-peak shear constitutive curves, they only accord well with curves with specific morphology (i.e., the concave-type curve in Figure 2). Shen et al. [51] pointed out that the pre-peak shear stress curves of many joints presents a convex-type shear constitutive curve (i.e., convex-type curve). However, due to the limitation of hyperbolic function itself, the hyperbolic models mentioned above cannot represent this convex-type curve. In fact, there is currently no model that can simultaneously represent the three types of pre-peak shear curves shown in Figure 2. Therefore, it is necessary to carry out in-depth research.



**Figure 2.** Examples of the pre-peak shear stress-shear displacement curve (modified after [51]).

To remedy this limitation, the semi-analytical model based on the Haldane distribution and exponential correction coefficient was built in this paper, which can represent the shear deformation of three kinds of pre-peak curves. Then the shear test data of different types of rock joints were selected for validity verification. Comprehensive comparisons between the proposed model and other existing models were launched to illustrate the superiority of this model.

## 2. Model Development

The statistical damage-based approach is widely used to deal with the constitutive response to study the deformation process and failure mechanism of rock materials [52,53]. According to the statistical damage theory, it is assumed that the damage variable obeys a certain distribution function, such as Weibull distribution [54], improved Harris distribution [55], normal (Gaussian) distribution [56], Haldane distribution [57], etc. A comprehensive review of these distribution functions and corresponding constitutive models is provided by Lin et al. [58]. These distribution functions are widely used in many fields, including geophysics, biology, population growth, and economics [59].

The Weibull distribution function is the most commonly used distribution function to numerically model the rock damage, the expression of which  $F(x)$  is shown in Equation (1) [60,61].

$$F(x) = 1 - e^{-\left(\frac{x}{x_0}\right)^m} \quad (1)$$

where  $x$  is a variable parameter,  $e$  is Euler's number,  $x_0$  and  $m$  are Weibull distribution parameters without clear physical meanings and mathematical definitions [62].

Although previous studies have proved that the results obtained by the Weibull distribution are statistically acceptable, only focusing on some statistical indicators such as coefficient of determination ( $R^2$ ) and root mean square error (RMSE) (see Equation (A1) in Appendix A) may mask the shortcomings of this distribution, and blind use of this method may cause large errors [63]. Beyond that, it has also been documented [64–67] that there

is not enough evidence showing that the Weibull distribution always takes precedence over other distributions and whether the Weibull distribution is the most appropriate statistical distribution function for quasi-brittle materials (such as rock, ceramics, and concrete) is questionable. As Weibull emphasized in his pioneering paper [54], the Weibull distribution is an empirical distribution function, similar to other distribution functions. The establishment of a constitutive model using the Weibull distribution is also a complicated process in which three unknown variables need to be solved according to at least three sets of data [57]. It can be found that the Weibull function for possible prediction of the stress–strain ( $\sigma$ – $\epsilon$ ) relationship may have three unknown constants ( $\lambda$ ,  $m$  and  $\epsilon_0$ ):

$$\sigma = \lambda \left[ 1 - \frac{1}{e^{(\frac{\epsilon}{\epsilon_0})^m}} \right] \quad (2)$$

It is best to use distribution functions with fewer parameters in modeling. Referring to Palchik [68], the Haldane distribution was used to establish the constitutive model of rock joints. As shown in Equation (3), the Haldane distribution contains only two known parameters (2 and 1/2):

$$F(x) = \frac{1}{2} (1 - e^{-2x}) \quad (3)$$

It can be deduced that  $F(x) = 0$  when  $x = 0$ , which is consistent with the fact that the shear stress/shear displacement curve initiates from the origin. Additionally, the graph of Equation (3) is in the first quadrant since the shear displacement is nonnegative, which makes the physical significance of Equation (3) more explicit. Therefore, Equation (3) can be applied to establish a joint shear constitutive curve, namely,

$$\tau = \frac{\alpha}{2} (1 - e^{-2u}) \quad (4)$$

Since the pre-peak curve must pass the peak point ( $u_p, \tau_p$ ), substitute  $u_p$  and  $\tau_p$  into Equation (4) to obtain the expression of  $\alpha$ :

$$\alpha = \frac{2\tau_p}{1 - e^{-2u_p}} \quad (5)$$

Substitute Equation (5) into Equation (4):

$$\tau = \tau_p \left( \frac{1 - e^{-2u}}{1 - e^{-2u_p}} \right) \quad (6)$$

To obtain a model with wider applicability, the correction coefficient  $\left( \frac{1 - e^{-2u}}{1 - e^{-2u_p}} \right)^{\gamma-1}$  is adapted to construct an improved model suitable for describing different pre-peak curves:

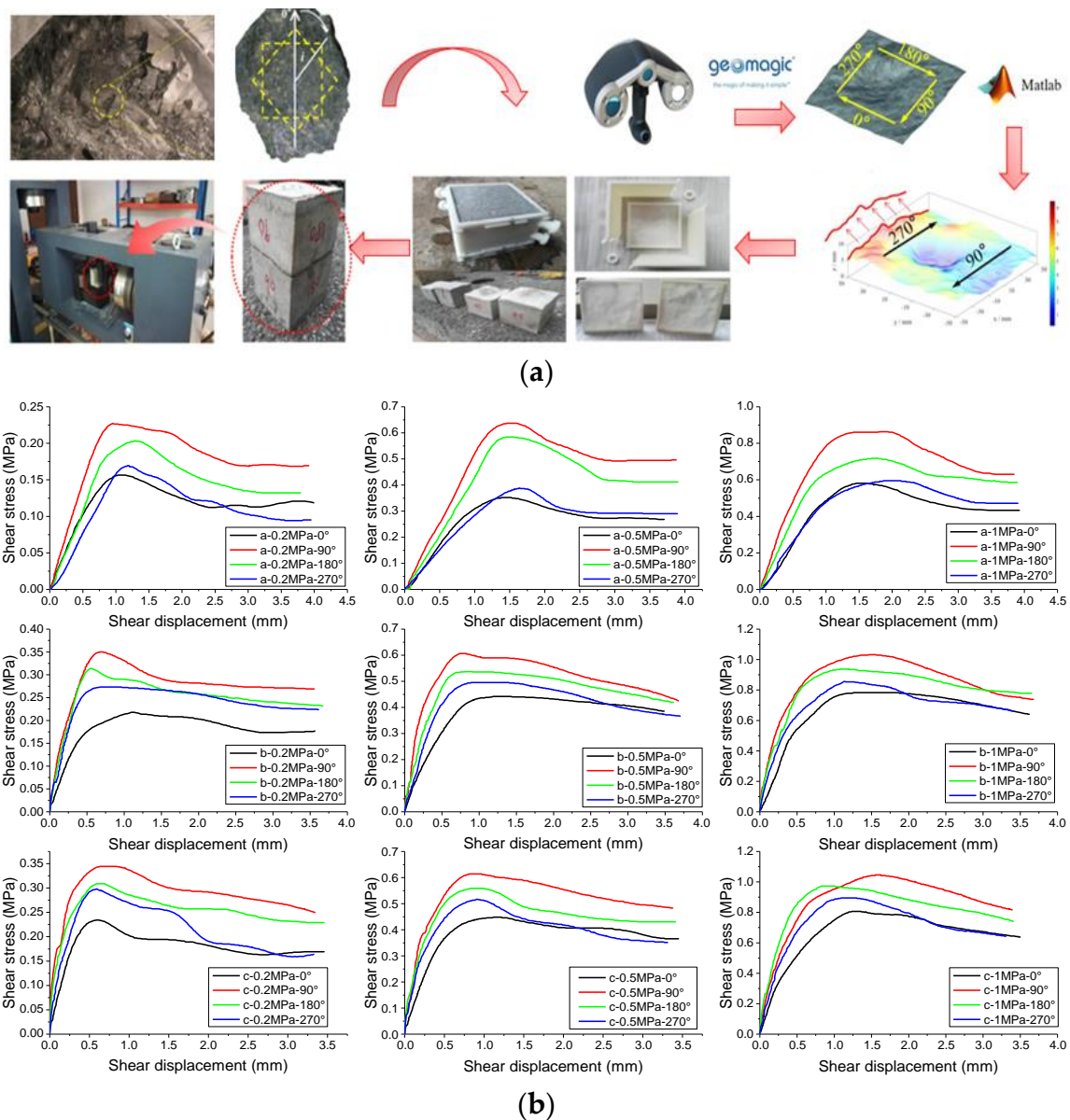
$$\tau = \tau_p \left( \frac{1 - e^{-2u}}{1 - e^{-2u_p}} \right) \left( \frac{1 - e^{-2u}}{1 - e^{-2u_p}} \right)^{\gamma-1} = \tau_p \left( \frac{1 - e^{-2u}}{1 - e^{-2u_p}} \right)^{\gamma} \quad (7)$$

Equation (7) is the semi-analytical model based on the Haldane distribution function, which is used to describe the pre-peak stage of the shear stress/shear displacement curve. The model parameter  $\gamma$  can be determined using the statistical method of successive approximations. Whether Equation (7) can predict the pre-peak shear constitutive relationship requires a further comparison between the observed and predicted shear stress/shear displacement curves, which is discussed in Section 3.

### 3. Model Validation

In the first series of verification tests, the shear test results of artificial joints conducted by Bao et al. [69] were adopted. As shown in Figure 3a, the target joints are taken from the face of the Guanshan tunnel in Gansu Province, China, and the lithology of the rock joints

is diorite. The three-dimensional point cloud of target joints was obtained by scanning the collected samples with a handheld 3D laser scanner, and the resin mold was made. Then, three groups of artificial joint samples with different strengths were made according to the mixing ratio shown in Table 1 (numbered as group a, group b and group c). During the shear test, the normal stresses were set as 0.2 MPa, 0.5 MPa, and 1 MPa, respectively. Jointed samples are numbered X-Y-Z, where X represents group number (a, b, c), Y represents the normal stress, and Z represents the shear direction ( $0^\circ$ ,  $90^\circ$ ,  $180^\circ$ , and  $270^\circ$ ). The anisotropic shear test results of each joint sample under different normal stresses are shown in Figure 3b.



**Figure 3.** Preparation process and shear test results of joint samples, (a) Sample-making process, (b) Shear stress-shear displacement curves [69].

**Table 1.** Ingredient proportions of artificial joints samples [69].

| Group | Water | Cement | Sand | Silicon | Water Reducer |
|-------|-------|--------|------|---------|---------------|
| a     | 1     | 2      | 2    | 0       | 0             |
| b     | 1     | 2      | 3    | 0.1     | 0.1           |
| c     | 1     | 2      | 3    | 0.2     | 0.2           |



To facilitate a comparison of model results, the proposed model was compared with Ban et al. [49] model, Bandis et al. [46] model, and Nassir et al. [48] model. The expressions of each model are as follows:

Ban et al. [49] model:

$$\tau = k_i \left[ \left( 1 - \frac{a}{b} \right) u + \frac{au_p}{b^2} \ln \left( \frac{b^2 u + bu_p}{bu_p} \right) \right] \quad (8)$$

where parameters  $a$  and  $b$  are model parameters, and  $k_i$  is the initial shear stiffness. All three parameters can be obtained by fitting test data.

Bandis et al. [46] model:

$$\tau = \frac{u}{m + nu} \quad (9)$$

where  $m$  represents the reciprocal of the initial shear stiffness and  $n$  is the reciprocal of the horizontal asymptote to the hyperbolic curve. Both  $m$  and  $n$  are positive.

Nassir et al. [48] model:

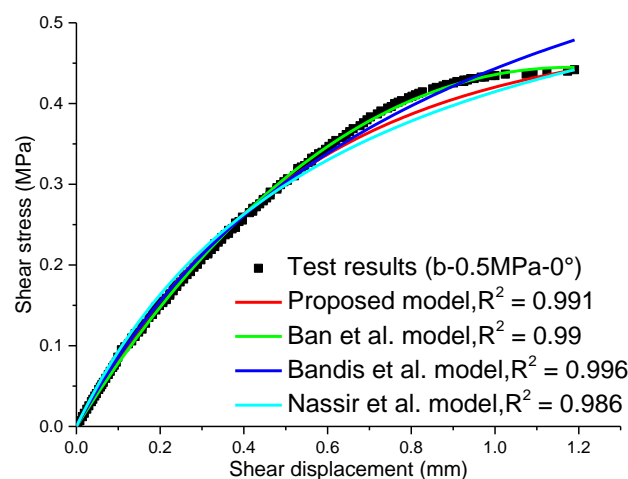
$$\tau = \frac{k_i u}{1 + \delta u} \quad (10)$$

where parameters  $\delta$  can be obtained by Equation (11):

$$\delta = \frac{1 - \frac{\sigma_n \tan(\varphi_b)}{\tau_p \eta}}{u_p \left( \frac{\sigma_n \tan(\varphi_b)}{\tau_p} - 1 \right)} \quad (11)$$

where  $\eta$  and  $\varphi_b$  are dilation initiation coefficient (ranging from 0 to 1) and basic friction angle, respectively.

According to Refs [70,71], to calibrate the above four models, the expressions of each constitutive model were implemented in Origin software so that we may utilize the Levenberg–Marquardt (L-M) algorithm. The error of each model between the measured stress and the theoretical value is measured by  $R^2$ . As shown in Figure 4, the above four models are successively substituted into the experimental results in Figure 3a to solve. Owing to the limited paper length, only the comparison results of experiment b-0.5MPa-0° are given. The  $R^2$  of each theoretical curve fitted by the proposed model and other models was listed in Table 2, which indicates that the theoretical curves agree well with the experimental results. However, it should be noted that the proposed model only contains one unknown parameter  $\gamma$ , while other models contain multiple unknown parameters. Fewer parameters are the advantage of the proposed model.



**Figure 4.** Comparison between model and experimental results of b-0.5MPa-0° [46,48,49].

**Table 2.** Fitting effect of model and experimental results.

| Joint Samples | Model Expression of the Proposed Model   | Proposed Model | Ban et al. [49] Model | $R^2$<br>Bandis et al. [46] Model | Nassir et al. [48] Model |
|---------------|--|----------------|-----------------------|-----------------------------------|--------------------------|
| a-0.2MPa-0°   | $\tau = 0.1564 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 1.03361}} \right)^{1.28}$  | 0.999          | 0.991                 | 0.997                             | 0.994                    |
| a-0.2MPa-90°  | $\tau = 0.2271 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 0.9496}} \right)^{1.425}$  | 0.998          | 0.99                  | 0.98                              | 0.98                     |
| a-0.2MPa-180° | $\tau = 0.2036 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 1.2914}} \right)^{1.559}$  | 0.983          | 0.982                 | 0.98                              | 0.98                     |
| a-0.2MPa-270° | $\tau = 0.1693 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 1.174}} \right)^{2.184}$   | 0.996          | /                     | 0.97                              | 0.97                     |
| a-0.5MPa-0°   | $\tau = 0.352 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 1.388}} \right)^{1.792}$    | 0.999          | 0.994                 | 0.987                             | 0.98                     |
| a-0.5MPa-90°  | $\tau = 0.6369 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 1.462}} \right)^{2.208}$   | 0.982          | 0.98                  | 0.981                             | 0.972                    |
| a-0.5MPa-180° | $\tau = 0.584 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 1.4486}} \right)^{2.809}$   | 0.977          | /                     | 0.96                              | 0.97                     |
| a-0.5MPa-270° | $\tau = 0.3887 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 1.6379}} \right)^{2.355}$  | 0.983          | 0.991                 | 0.983                             | 0.983                    |
| a-1.0MPa-0°   | $\tau = 0.5807 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 1.462}} \right)^{2.0407}$  | 0.999          | 0.99                  | 0.983                             | 0.974                    |
| a-1.0MPa-90°  | $\tau = 0.8603 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 1.421}} \right)^{1.3905}$  | 0.998          | 0.992                 | 0.98                              | 0.984                    |
| a-1.0MPa-180° | $\tau = 0.7179 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 1.6919}} \right)^{1.427}$  | 0.995          | 0.993                 | 0.985                             | 0.993                    |
| a-1.0MPa-270° | $\tau = 0.5947 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 1.8727}} \right)^{1.846}$  | 0.996          | 0.995                 | 0.986                             | 0.99                     |
| b-0.2MPa-0°   | $\tau = 0.2146 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 1.0227}} \right)^{0.808}$  | 0.985          | 0.99                  | 0.981                             | 0.983                    |
| b-0.2MPa-90°  | $\tau = 0.35 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 0.668}} \right)^{0.82}$      | 0.994          | 0.989                 | 0.984                             | 0.985                    |
| b-0.2MPa-180° | $\tau = 0.3142 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 0.5537}} \right)^{0.921}$  | 0.998          | 0.998                 | 0.988                             | 0.98                     |
| b-0.2MPa-270° | $\tau = 0.2731 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 0.6777}} \right)^{0.7503}$ | 0.945          | 0.993                 | 0.975                             | 0.973                    |
| b-0.5MPa-0°   | $\tau = 0.4419 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 1.1901}} \right)^{1.0361}$ | 0.991          | 0.99                  | 0.996                             | 0.986                    |
| b-0.5MPa-90°  | $\tau = 0.6062 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 0.7438}} \right)^{0.72}$   | 0.979          | 0.99                  | 0.989                             | 0.988                    |
| b-0.5MPa-180° | $\tau = 0.5371 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 0.8265}} \right)^{0.80}$   | 0.983          | 0.979                 | 0.994                             | 0.984                    |
| b-0.5MPa-270° | $\tau = 0.4952 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 0.8926}} \right)^{0.906}$  | 0.987          | 0.99                  | 0.991                             | 0.984                    |
| b-1.0MPa-0°   | $\tau = 0.782 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 1.2619}} \right)^{1.118}$   | 0.992          | 0.99                  | 0.992                             | 0.99                     |
| b-1.0MPa-90°  | $\tau = 1.0315 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 1.4763}} \right)^{0.711}$  | 0.996          | 0.99                  | 0.99                              | 0.987                    |
| b-1.0MPa-180° | $\tau = 0.9371 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 1.0909}} \right)^{0.764}$  | 0.976          | 0.992                 | 0.981                             | 0.979                    |
| b-1.0MPa-270° | $\tau = 0.8562 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 1.1405}} \right)^{0.862}$  | 0.997          | 0.99                  | 0.991                             | 0.99                     |
| c-0.2MPa-0°   | $\tau = 0.2341 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 0.5905}} \right)^{0.759}$  | 0.976          | 0.998                 | 0.976                             | 0.998                    |
| c-0.2MPa-90°  | $\tau = 0.344 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 0.6338}} \right)^{0.467}$   | 0.935          | 0.96                  | 0.951                             | 0.94                     |
| c-0.2MPa-180° | $\tau = 0.3086 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 0.5918}} \right)^{0.523}$  | 0.98           | 0.996                 | 0.97                              | 0.96                     |
| c-0.2MPa-270° | $\tau = 0.2981 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 0.5917}} \right)^{0.683}$  | 0.985          | 0.995                 | 0.98                              | 0.982                    |
| c-0.5MPa-0°   | $\tau = 0.4476 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 1.128}} \right)^{0.803}$   | 0.975          | 0.99                  | 0.971                             | 0.973                    |

Table 2. Cont.

| Joint Samples | Model Expression of the Proposed Model   | Proposed Model | Ban et al. [49] Model | $R^2$<br>Bandis et al. [46] Model | Nassir et al. [48] Model |
|---------------|--|----------------|-----------------------|-----------------------------------|--------------------------|
| c-0.5MPa-90°  | $\tau = 0.6132 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 0.8099}} \right)^{0.6673}$ | 0.988          | 0.99                  | 0.964                             | 0.983                    |
| c-0.5MPa-180° | $\tau = 0.558 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 0.8316}} \right)^{0.622}$   | 0.983          | 0.985                 | 0.979                             | 0.976                    |
| c-0.5MPa-270° | $\tau = 0.517 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 0.911}} \right)^{0.661}$    | 0.99           | 0.99                  | 0.981                             | 0.98                     |
| c-1.0MPa-0°   | $\tau = 0.806 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 1.239}} \right)^{1.099}$    | 0.995          | 0.99                  | 0.982                             | 0.981                    |
| c-1.0MPa-90°  | $\tau = 1.0449 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 1.5556}} \right)^{0.828}$  | 0.997          | 0.99                  | 0.977                             | 0.983                    |
| c-1.0MPa-180° | $\tau = 0.9714 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 0.8282}} \right)^{0.7175}$ | 0.984          | 0.996                 | 0.973                             | 0.981                    |
| c-1.0MPa-270° | $\tau = 0.8942 \left( \frac{1 - e^{-2u}}{1 - e^{-2 \times 1.0947}} \right)^{0.8825}$ | 0.998          | 0.99                  | 0.981                             | 0.983                    |

Interestingly, in the process of solving, it was found that Ban et al. [49] model, Bandis et al. [46] model, and Nassir et al. [48] model performed poorly for some test results at the initial loading stage, i.e., the prediction results were significantly higher than the test results. On the contrary, the proposed model performed well in the entire loading stage. Taking the test results of a-0.5MPa-0° and a-1MPa-0° as an example, the application of the above four models is presented in Figure 5. It can be found that the Ban et al. [49] model, Bandis et al. [46] model, and Nassir et al. [48] model cannot describe the nonlinear deformation characteristics at the initial loading stage. To facilitate understanding, the shear stress/shear displacement curves of the initial loading stage are excerpted and magnified (i.e., the enlarged view in Figure 5). The  $R^2$  of the above four models in the enlarged view is shown in Figure 6. It can be seen that the fitting effect of the proposed model is still good ( $R^2 > 0.9$ ), while the fitting effect of the other three models is relatively poor (0.539–0.834). In addition, the proposed model guarantees that the model curve passes the peak point, whereas the Ban et al. [49] model and Bandis et al. [46] model do not. All these demonstrate the superiority of the proposed model.

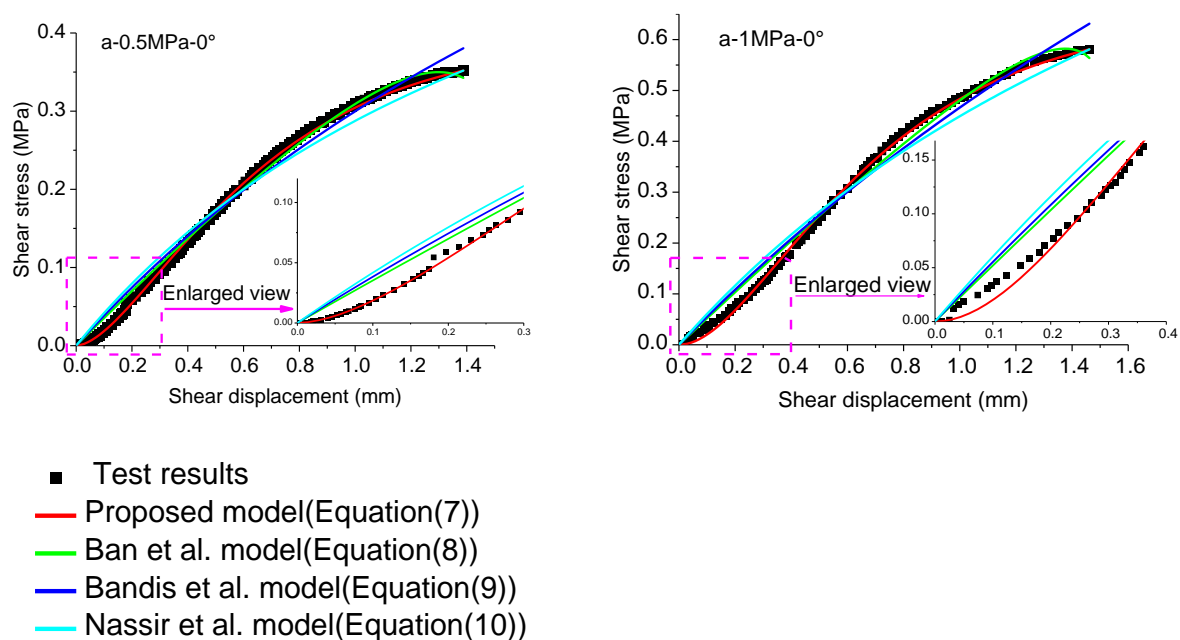


Figure 5. The comparison between each model and experimental results [46,48,49].



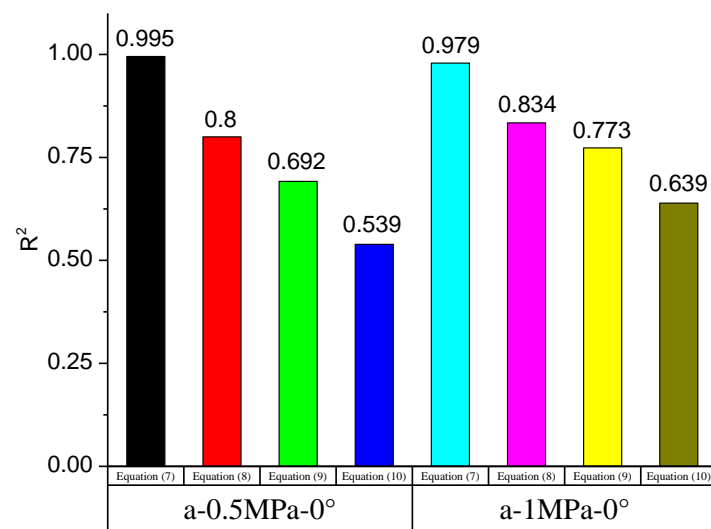


Figure 6.  $R^2$  of each model.

#### 4. Discussion

The shape of the pre-peak shear curve of rock joints depends on roughness, mechanical properties, geological environment, and other perturbation factors. Subjected to certain environmental factors, such as freeze–thaw and immersion, the pre-peak constitutive curve of rock joints is reflected in a convex-type curve. During the initial loading stage, the hangingwall and footwall of rock joints are compressed, and the internal pores and cracks are compacted. This results in a downward concave shear stress/shear displacement curve, and the tangent slope of the curve increases as the shear displacement is applied [72]. There would be a significant error induced by the use of Equations (8)–(10) when describing the pre-peak mechanical behavior of such joints.

The typical convex-type test results in Refs. [73,74] are selected for verification, as shown in Figure 7a,b, which can still be well characterized by the proposed model. However, applying hyperbolic models such as the Ban et al. [49] model, Bandis et al. [46] model, and Nassir et al. [48] model to this validation will result in the parameter fitting being negative (contradictory with each model parameter being greater than 0) or not converging (Figure 8). This is due to the functional nature of the hyperbolic model itself. Under limited conditions (such as model parameters greater than 0), they cannot effectively describe the convex-type curve.

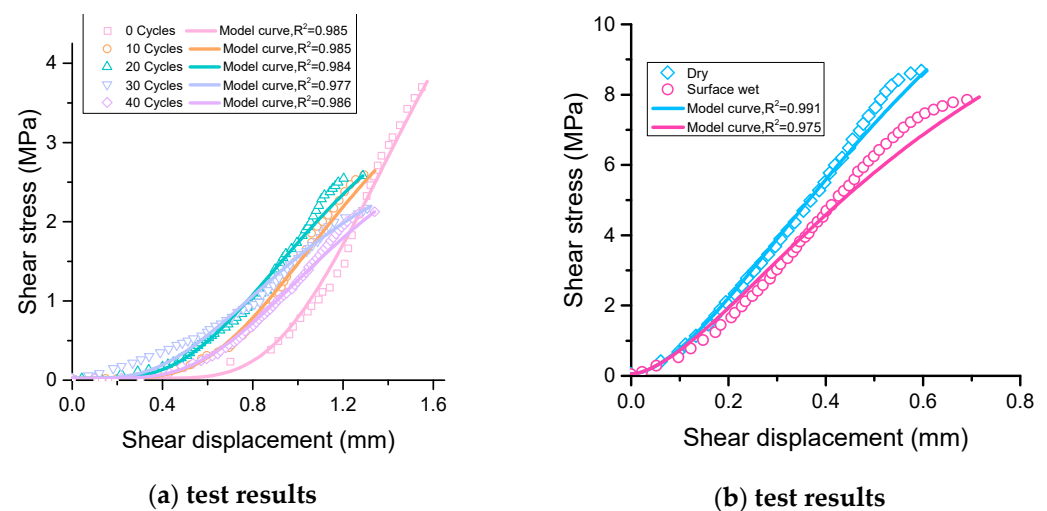
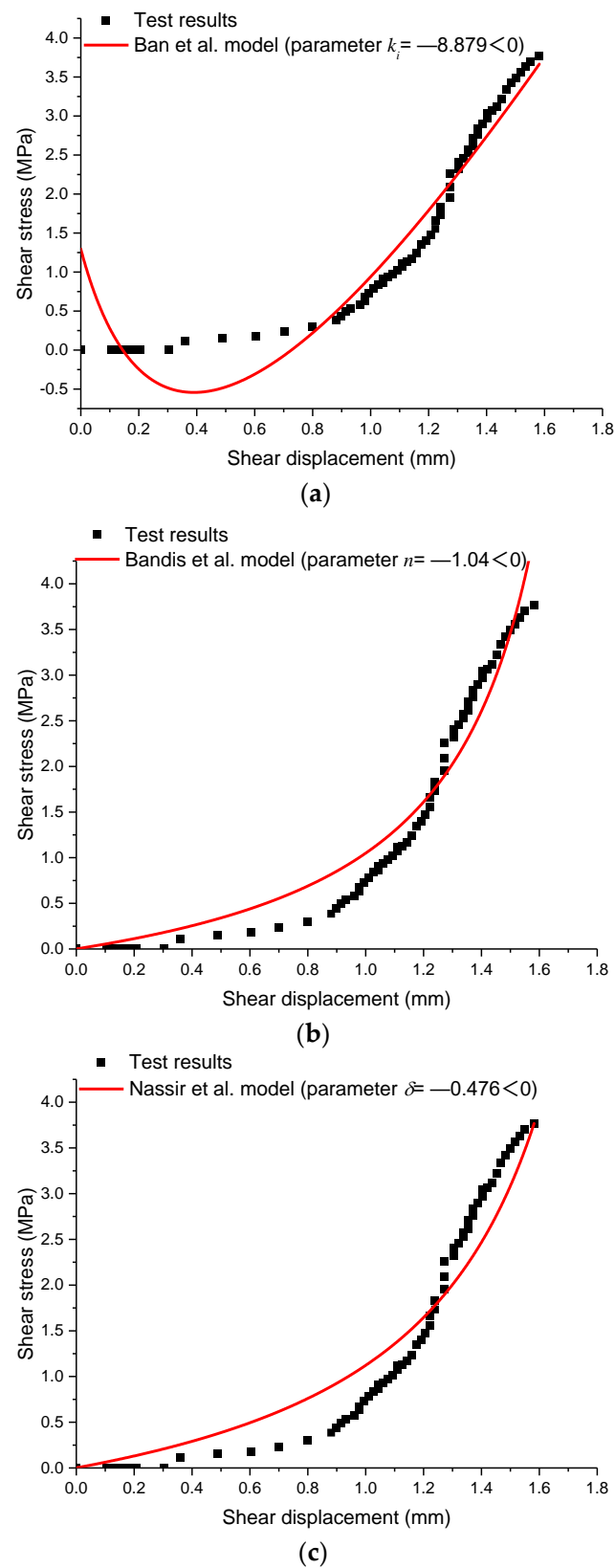


Figure 7. Comparison of test results [73,74] with model curves.

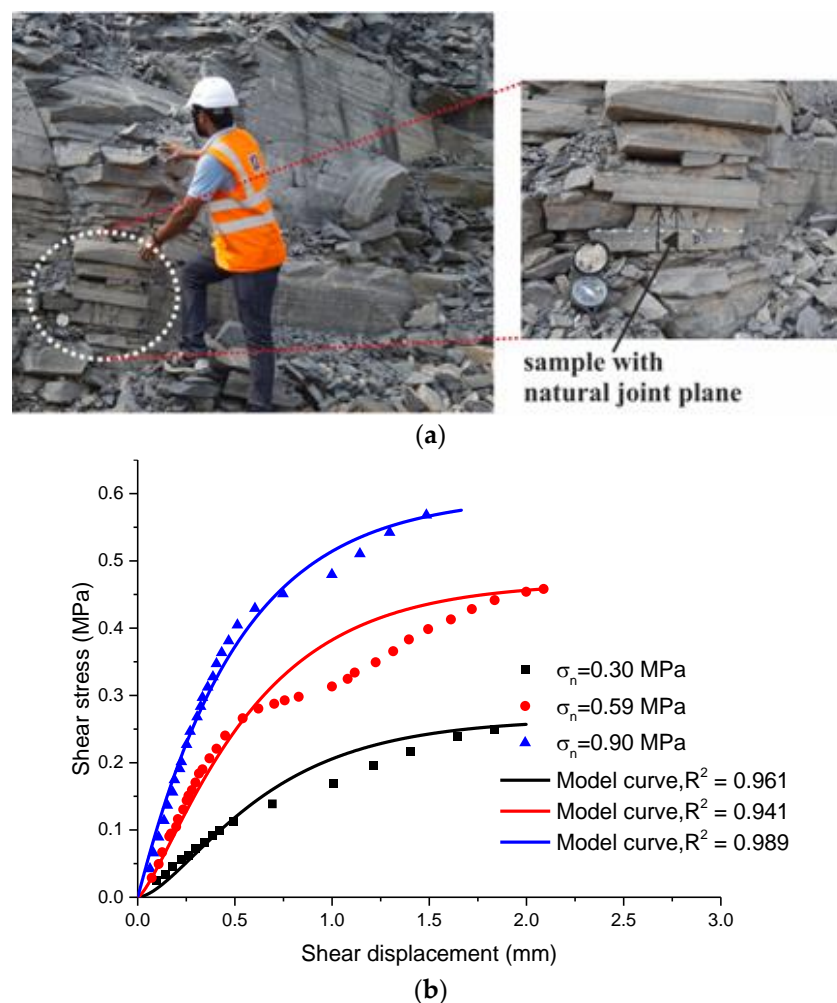


**Figure 8.** Comparison of test results (0 cycle) [73] with existing models, (a) Ban et al. [49] model, (b) Bandis et al. [46] model, (c) Nassir et al. [48] model.

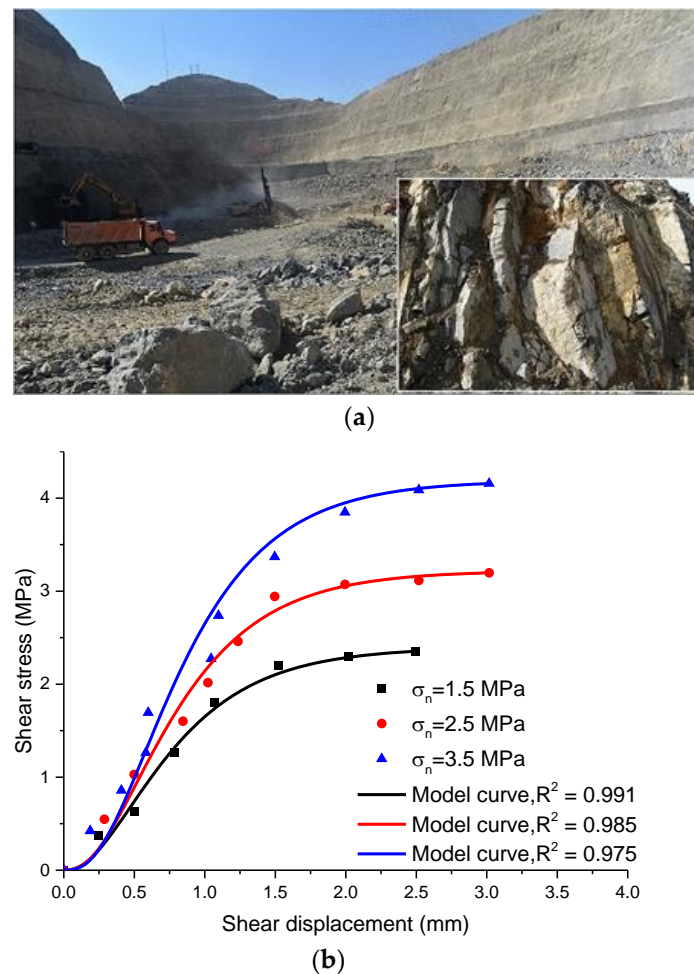
The experiment is one of the most fundamental and effective research methods. In Section 3, this paper only focuses on the performance of the proposed model in describing

the shear deformation of rock joints in laboratory experiments. Considering that there are many limitations of laboratory experiments, such as difficult field sampling, disturbance to the original state of samples, and defects in laboratory test instruments, the results may not be consistent with reality [75]. Therefore, the in situ shear test, which can obtain the mechanical parameters consistent with engineering practice, was employed.

Taking the test results [76,77] under in-situ conditions as an example, the performance of the model in reproducing in situ rock joint behavior is studied. Figure 9 shows the comparison of shear test data with limestone joints collected from the Lanjiberna limestone mine (located in Odisha, India) with model curves. Figure 10 shows the comparison of shear test results and model curves for sandstone joint samples collected from the upper reservoir of the Azad pump storage power plant project in western Iran. The corresponding fitting curves from the proposed model are well matched with the experimental data in Figures 9 and 10. Obviously, from Figures 9 and 10 (all  $R^2$  values are greater than 0.94), it is not difficult to find that the proposed model can also represent the pre-peak shear deformation behavior of in-situ rock joints to a certain extent. Of course, it is still a complex task to estimate the shear behavior of rock joints under in situ conditions due to various influencing factors presented in the field. We will study the performance of this model in reproducing in situ rock joint behavior in geotechnical engineering features such as rock slopes and tunnels in the future.



**Figure 9.** Comparison of in situ test results [76] with model curves, (a) sample collection, (b) shear stress-shear displacement curve.



**Figure 10.** Comparison of in situ test results [77] with model curves, (a) sample collection, (b) shear stress-shear displacement curve.

## 5. Conclusions

Based on Haldane distribution, a new semi-analytical model for the pre-peak shear deformation of rock joints is established. The effectiveness of the model was verified by laboratory and in situ experimental data. To better demonstrate the advantages of this model, it is compared with existing models. The results show that the results of the proposed model are similar to those of the existing models for quasilinear-type and concave-type curves. For convex-type curves, the effect of the proposed model is much better than that of the existing models. In addition, compared with existing models that have at least two or more parameters to be fitted, the model in this paper contains only one parameter, which is convenient for application.

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## Appendix A

The coefficient of determination ( $R^2$ ) and root mean square error (RMSE) can be used to assess the matching effect of the proposed model against the experimental data, which can be calculated by Equation (A1).

$$\begin{cases} R^2 = 1 - \frac{N-1}{N-2} \times \frac{\sum_{i=1}^N (y_{test}^i - y_{cal}^i)^2}{\sum_{i=1}^N (y_{test}^i - y_{ave})^2} \\ RMSE = \sqrt{\frac{\sum_{i=1}^N (y_{test}^i - y_{cal}^i)^2}{N}} \end{cases} \quad (A1)$$

where  $N$  is the number of measured points;  $y_{test}$  and  $y_{cal}$  are, respectively, the measured stress and the theoretical stress; and  $y_{ave}$  is the average of the  $y_{test}$ .

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