Supplemental Material

Modified Tributary Area and Pressure Arch Theories for Mine Pillar Stress Estimation in Mountainous Areas

Yang Yu [1], Jin Ma [1], Shenen Chen [2], Kazhong Deng [3], Bingqian Chen [1], Fenfen Hua [1] and Jianrong Kang [1,*]

[1] School of Geography, Geomatics and Planning, Jiangsu Normal University, Xuzhou 221116, China; yuyang1989cumt@gmail.com (Y.Y.); 15736750986@163.com (J.M.); bingqian@jsnu.edu.cn (B.C.); huafenfen_hff@163.com (F.H.)

[2] Department of Civil and Environment Engineering, University of North Carolina at Charlotte, Charlotte, NC 28223-001, USA; schen12@uncc.edu

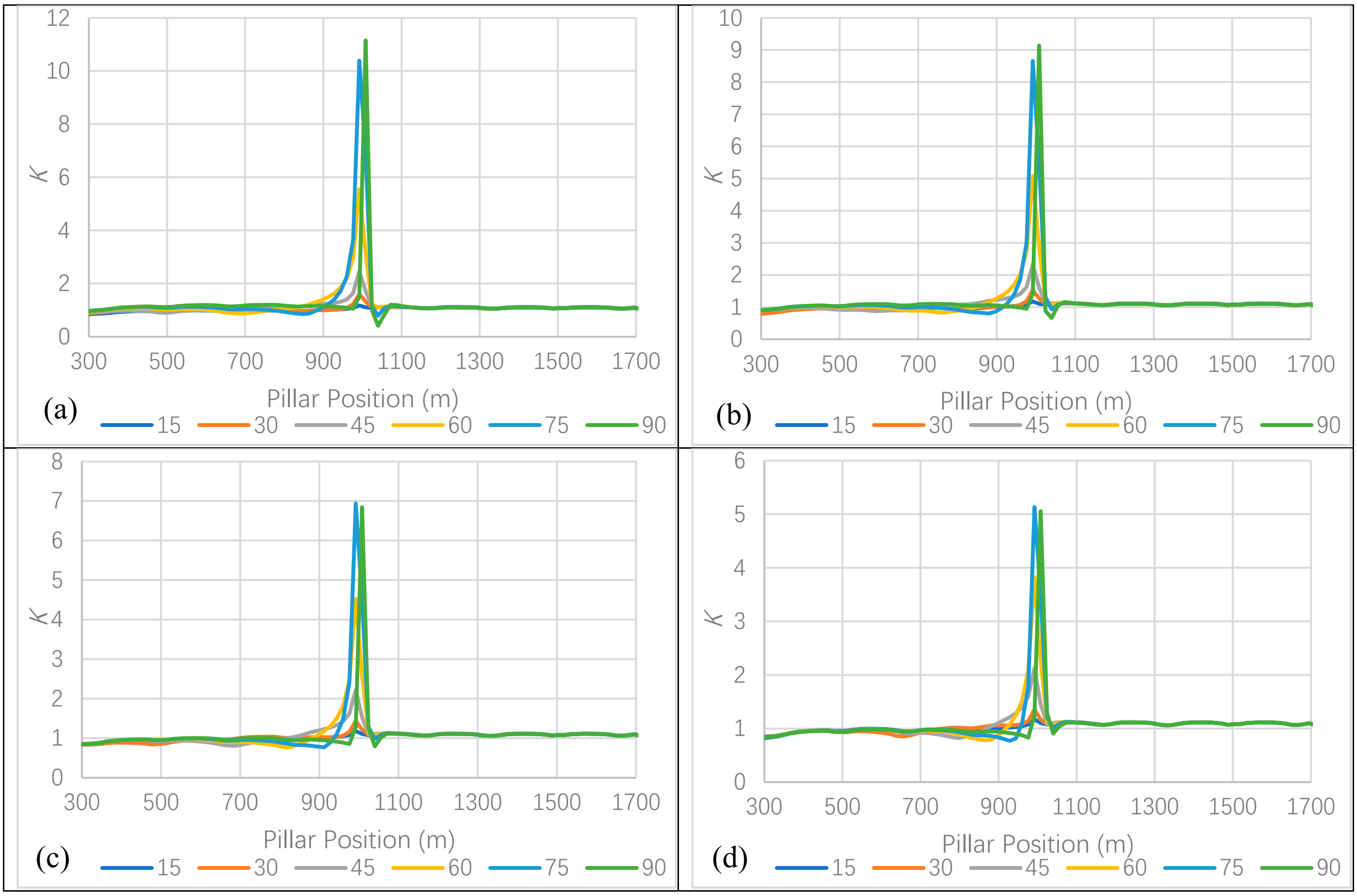
[3] School of Environment Science and Spatial Information, China University of Mining and Technology, Xuzhou 221116, China; kzdeng@cumt.edu.cn

* Correspondence: kangjianrong@jsnu.edu.cn

It is not necessary to view the content of this material unless further details or raw data are needed. All of the raw data and *K* curves of 1200 models were listed in this supplemental material. It also contains some study details.

1. The K of the base models

Figure 1.1 is the simulation results used in Section 3.1. The single slope models with H_B of 10 m, w_P of 8 m, r_m of 1, *E* of 15GPa, and υ of 0.15 were analyzed. The H_D of the models varied from 100 – 500 m and the α was 15° – 90°. The distribution law of *K* has been discussed in Section 3.1.



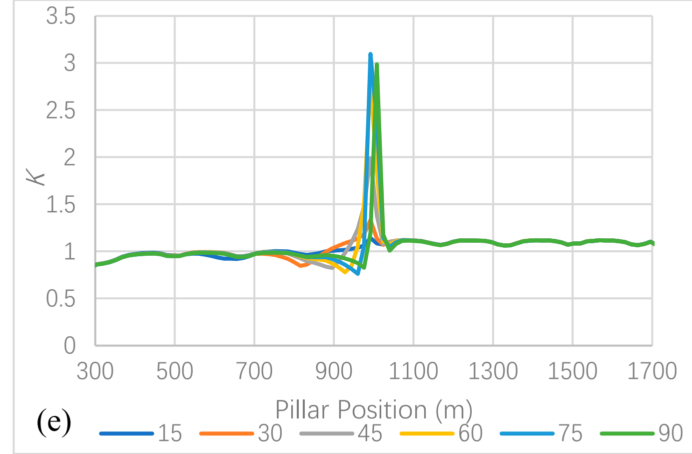
	Figure 1.1 The K of the base model: (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.

Figure 1.2 shows the effect of topography on the maximum K. The maximum K increases non-linearly (conforms to cubic polynomial curve) with the increase of α, and it increases linearly with the increase of H_{D}. The superimposed effects of α and H_{D} can be expressed by an empirical function (there may exist other curved surface functions that can fit the data well, the primary principal to choose the surface in this paper is selecting a surface with the largest R^2 and least RMSE and reducing the coefficients as much as possible):

$$K_{\mathrm{m}} = (m_1\alpha^3 + m_2\alpha^2 + m_3\alpha + m_4)H_{\mathrm{D}} + m_5 \quad (1\text{-}1)$$

where K_{m} is the maximum value of K curve, m_1 to m_5 are coefficients.

It is found that although the fitting results of Eq. (1-1) are good (Figure 1.3a), there still exists a defect (the void area A marked by a red rectangle in Figure 1.3a) on the curved surface, which will make the calculation results of Eq. (1-1) underestimated around the defect, because that part of the curved surface is below the axis limits. Further analysis shows that defect A in Figure 1.3(a) can be eliminated by changing Eq. (1-1) into Eq. (1-2):

$$K_{\mathrm{m}} = (m_1\alpha^3 + m_2\alpha^2 + m_3\alpha + m_4)H_{\mathrm{D}} + m_5\, H_{\mathrm{D}}/\alpha + m_6 \quad (1\text{-}2)$$

The fitting results of Eq. (1-2) are shown in Figure 1.3(b). The defect A disappears, and defect C (the area C marked by a red rectangle in Figure 1.3b) appears when $\alpha < 15°$. This is acceptable as the topographical effects will not be obvious and can be neglected when α is small. The curved surface also bends upward when $\alpha < 20°$ (area B marked by red rectangle in Figure 1.3b). This indicates that the K_{m} can be overestimated by Eq. (1.2) in area B. It will also be fine as overestimation of pillar stress usually leads to a more conservative analysis, which is beneficial to pillar safety. Therefore, Eq. (1-2) can be used to predict K_{m}.

The coefficients in Eq. (1-2) are acquired by curve fitting toolbox in MATLAB, and the accuracy of curve fitting is affected by the initial values of coefficients. The initial values of coefficients were first generated randomly by default, and then were set as the former fitting results, repetitively, until a stable fitting result was reached. The final results of the coefficients in Eq. (1-2) are listed in Table 1-1.

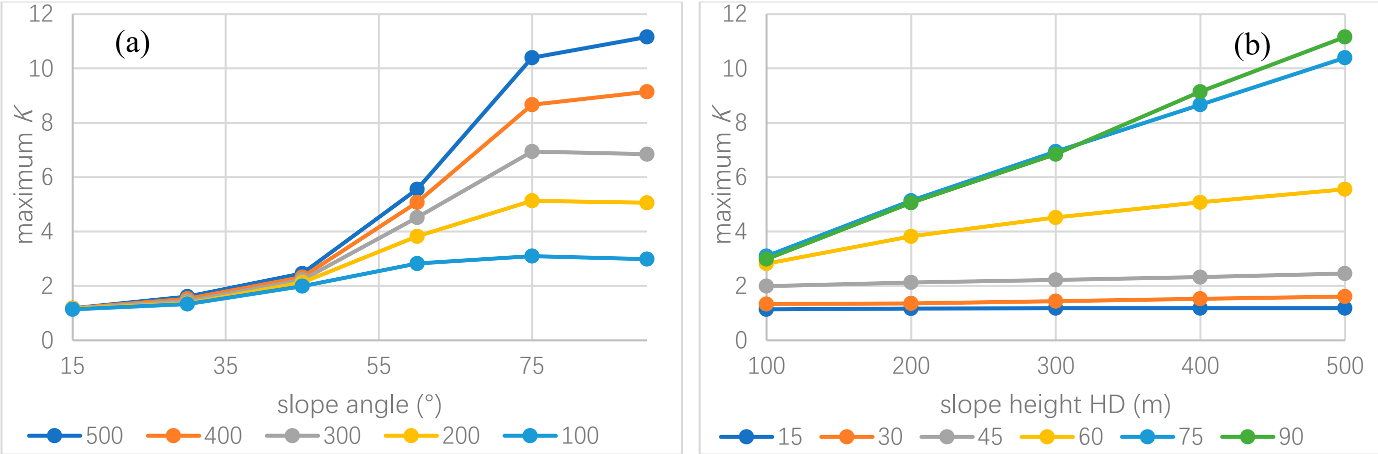


Figure 1.2 Topographical effects on maximum K: (a) effect of α when H_D = 100-500 m; (b) effect of H_D when α =15-90°

Table 1-1 Fitting results of Eq. (1-2)

m_1	m_2	m_3	m_4	m_5	m_6	Adjusted R-square	RMSE
-4.024e-07	7.741e-05	-0.004565	0.1051	-0.8025	1.488	0.9918	0.2674

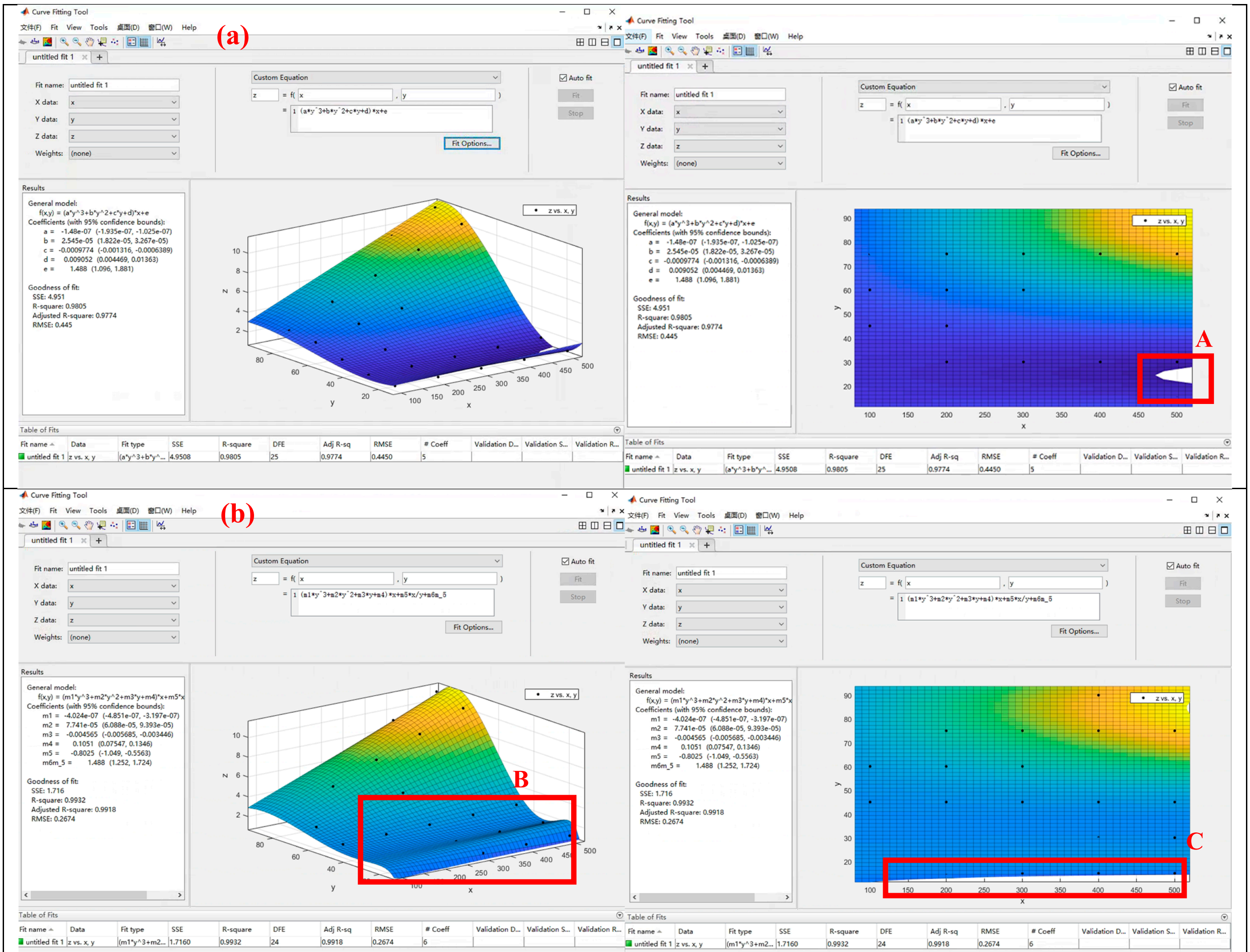


Figure 1.3 Fitting results of Equations (1-1) and (1-2): (a) Fitting results of Eq. (1-1); (b) Fitting results of Eq. (1-2)

2. The effect of υ on K

Figures 2.1-2.5 show the effect of υ. The H_B was set to 10 m. υ varied from 0.1 to 0.49 and E was 15 GPa. The other parameters are kept fixed as mentioned in Section 3.1. The distribution law of K has been discussed in Section 3.2.

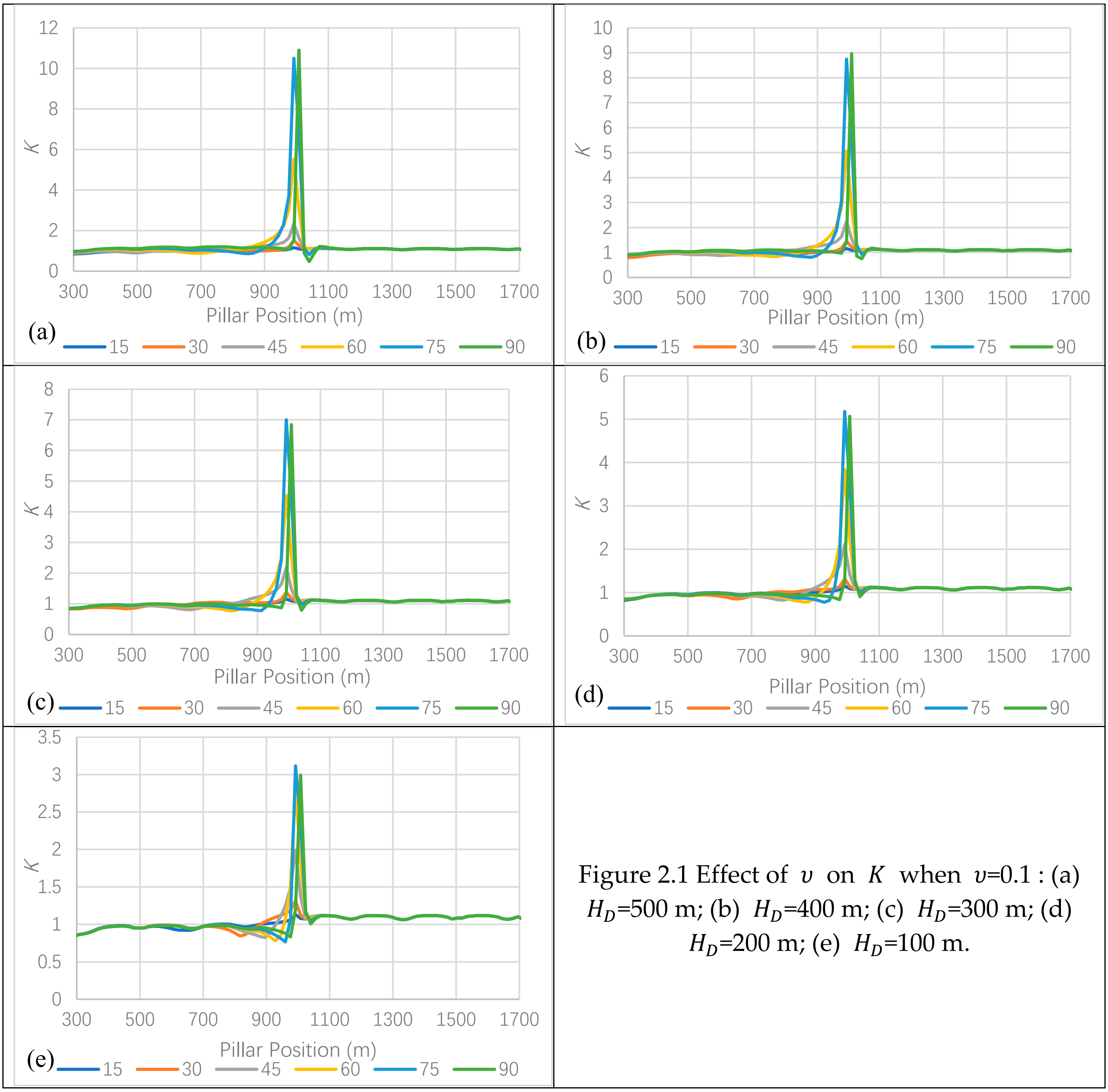


Figure 2.1 Effect of υ on K when υ=0.1 : (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.

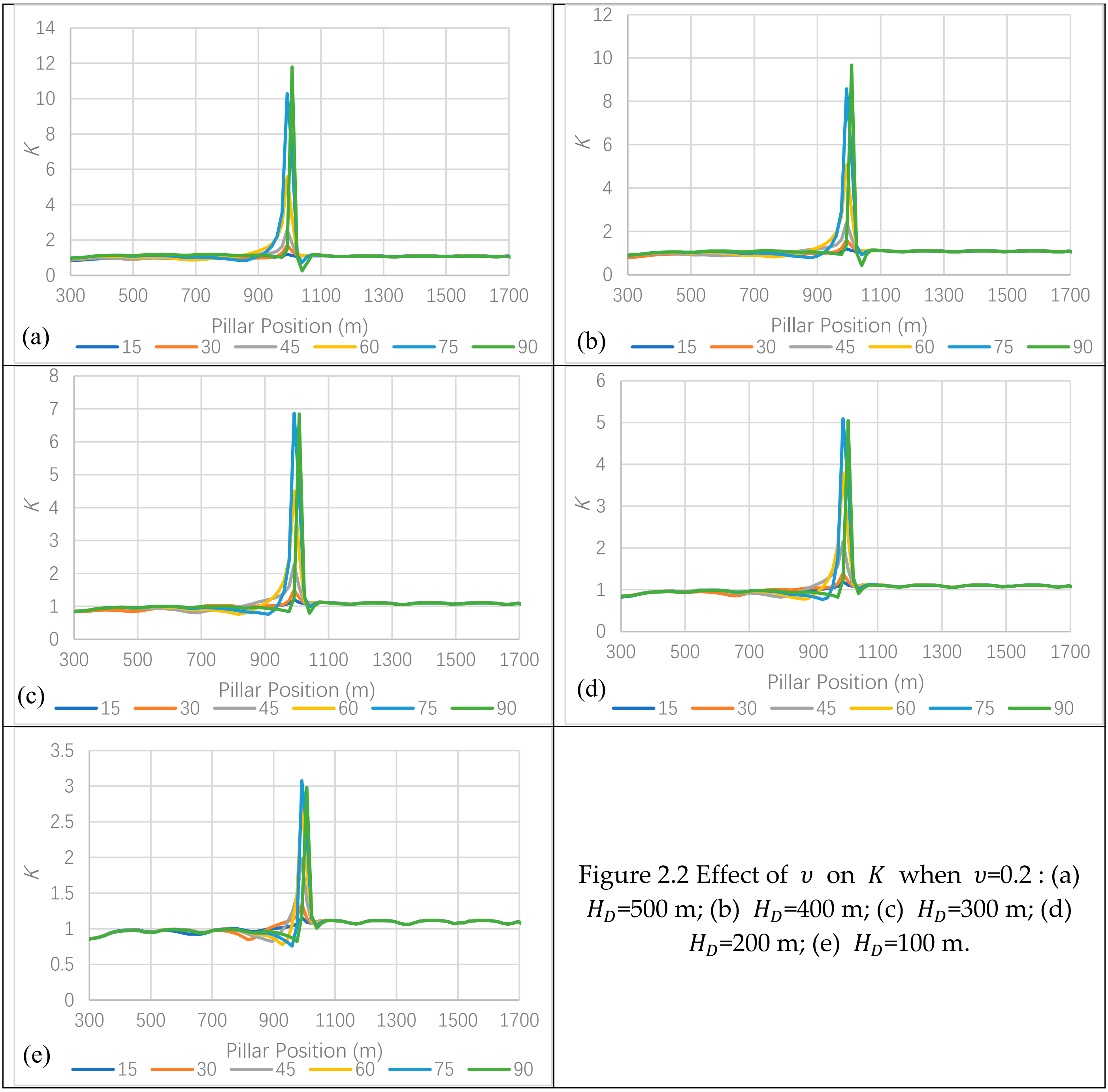
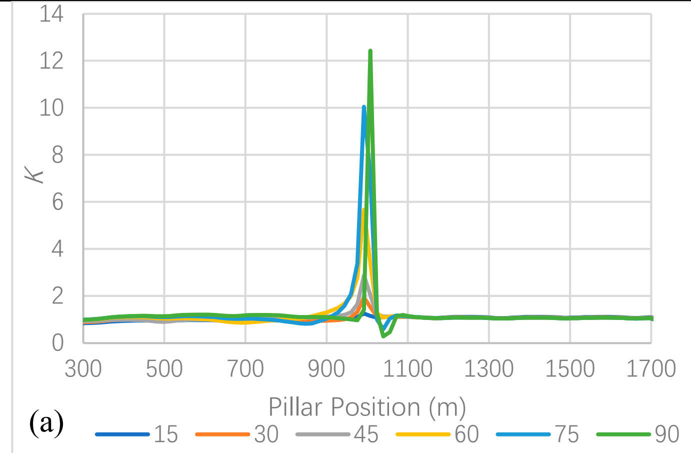
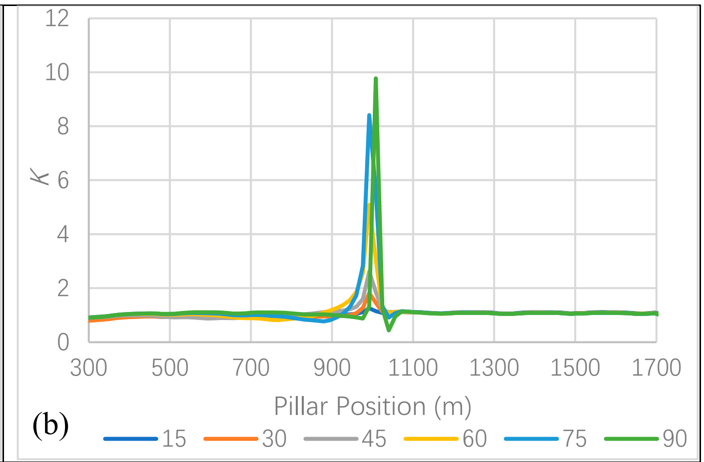
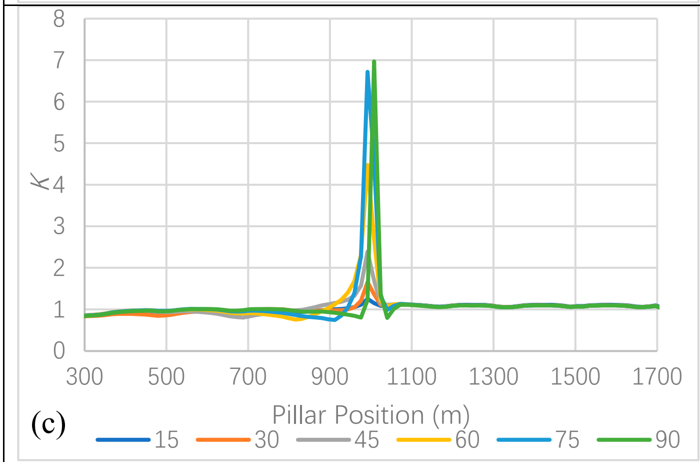
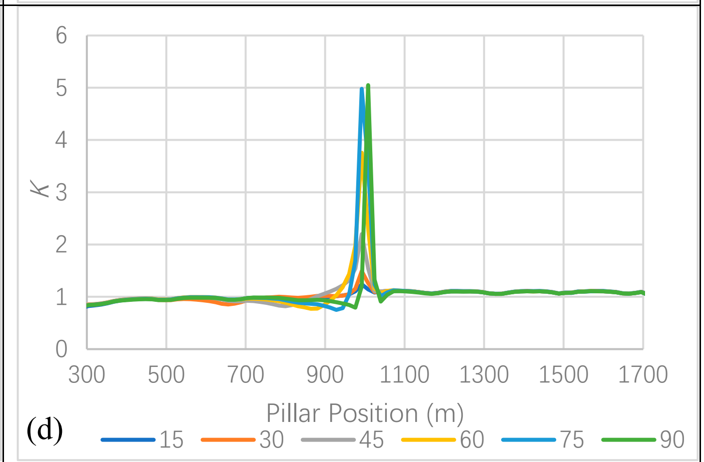


Figure 2.2 Effect of υ on K when υ=0.2 : (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.









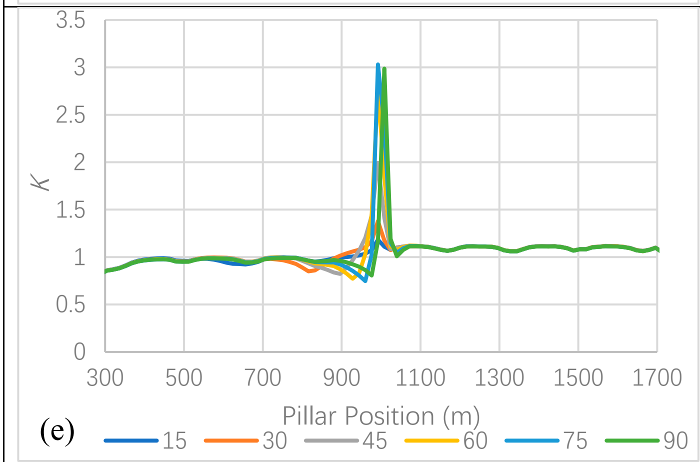
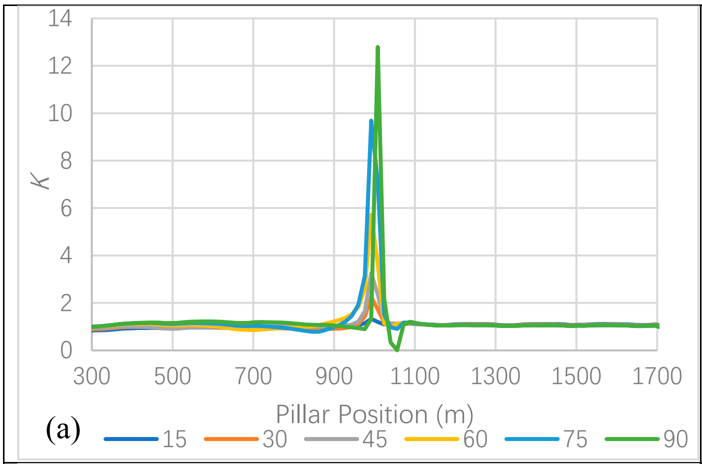
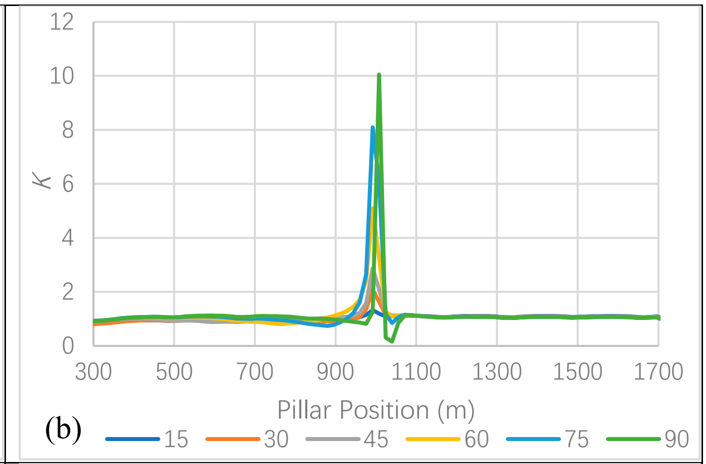
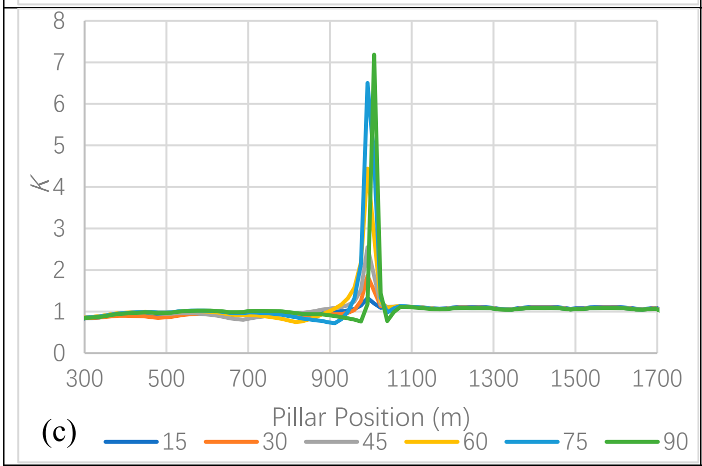
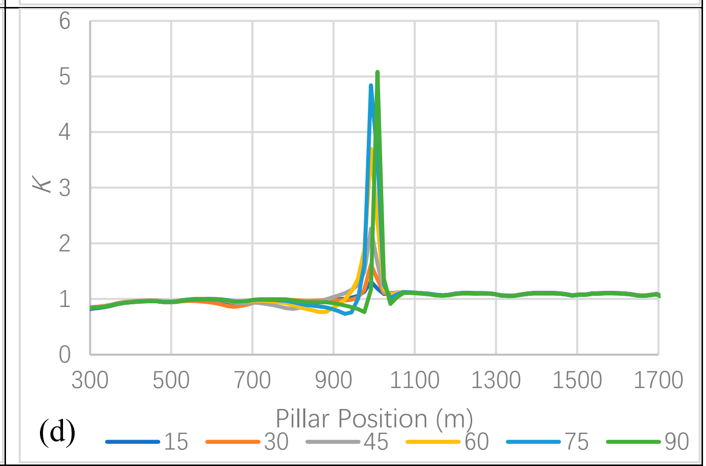


Figure 2.3 Effect of υ on K when υ=0.3 : (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.









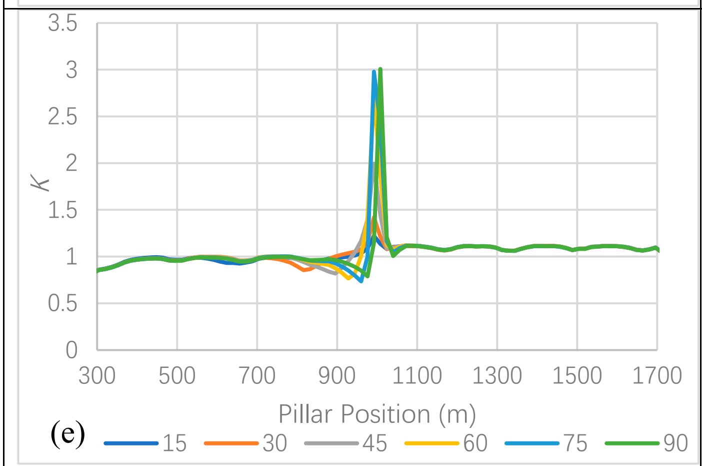


Figure 2.4 Effect of υ on K when υ=0.4 : (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.

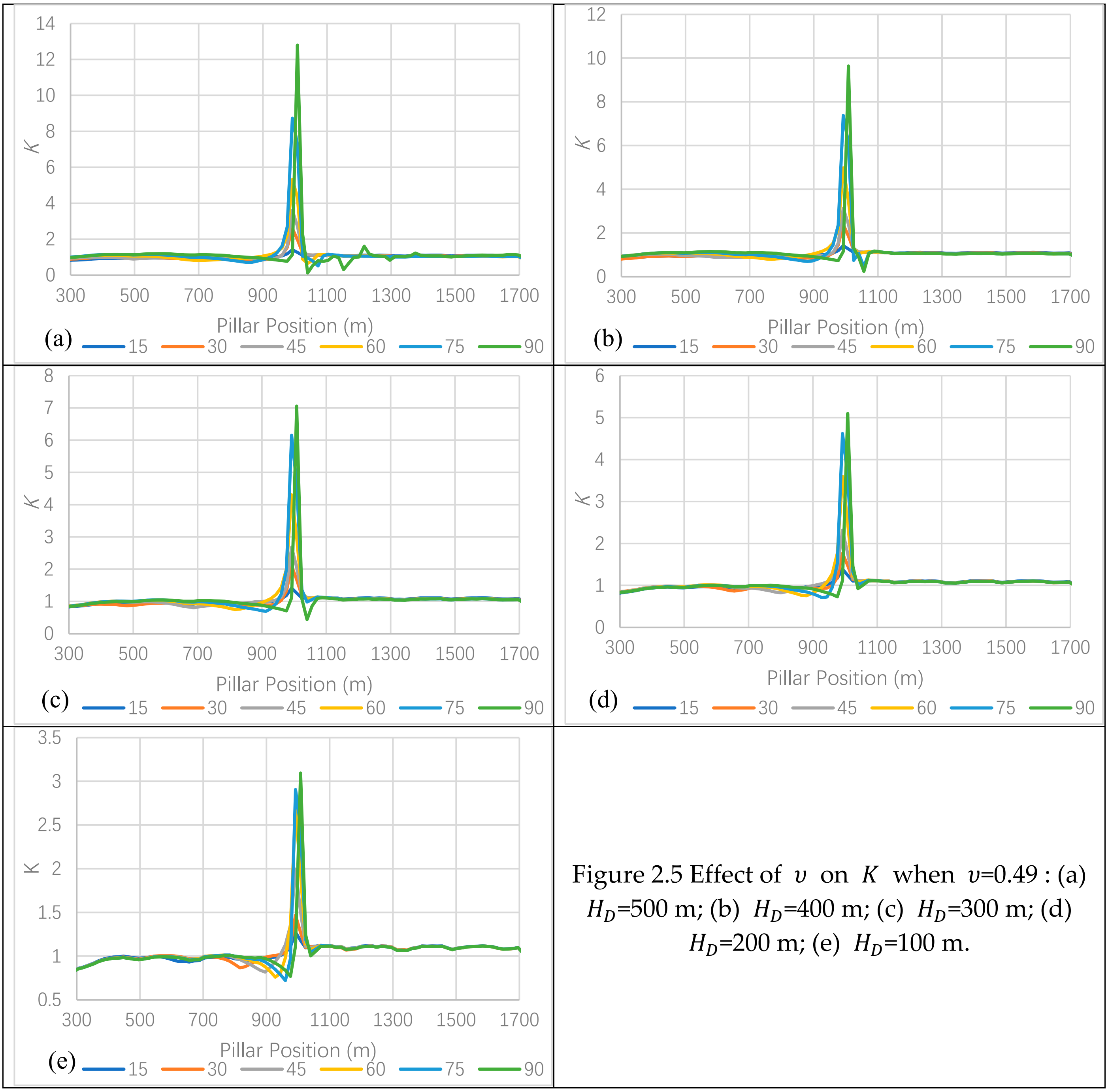


Figure 2.5 Effect of υ on K when υ=0.49 : (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.

Figures 2.6-2.7 show the topographical effects on the maximum K. The maximum K increases non-linearly with the increase of α (Figure 2.6). The maximum K increases linearly with the increase of H_D (Figure 2.7). The distribution laws are exactly like the laws that are previously mentioned (Figure 1.2). In fact, such laws exist in the rest of the models, and the curved surface equations are similar to Eq. (1-2) in pattern; the only difference is that the coefficients are affected by the mining conditions such as υ, E, and w_p, etc. However, another method to predict the maximum K is proposed so that the models of different mining conditions can be linked together.

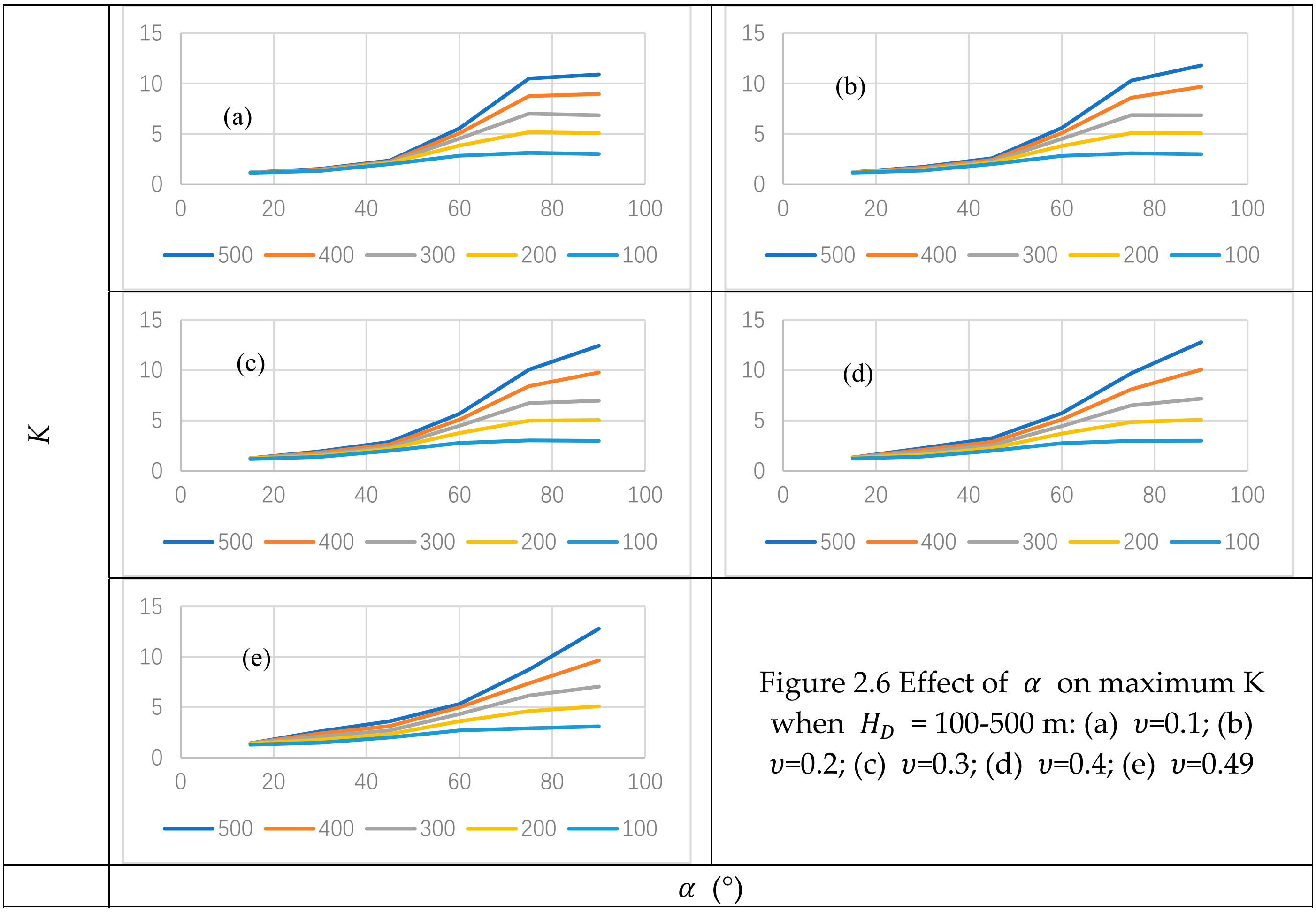


Figure 2.6 Effect of α on maximum K when H_D = 100-500 m: (a) υ=0.1; (b) υ=0.2; (c) υ=0.3; (d) υ=0.4; (e) υ=0.49

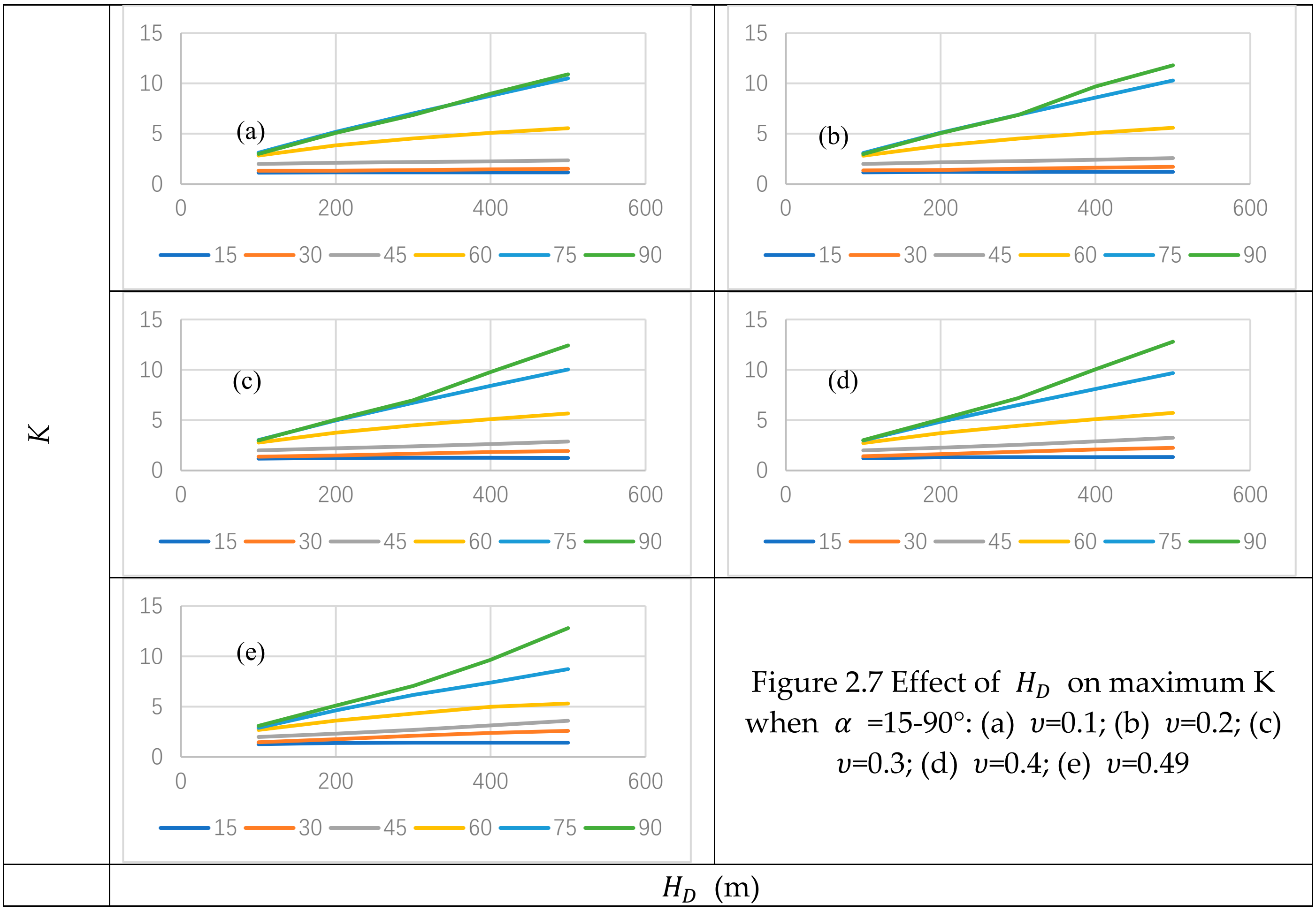


Figure 2.7 Effect of H_D on maximum K when α =15-90°: (a) υ=0.1; (b) υ=0.2; (c) υ=0.3; (d) υ=0.4; (e) υ=0.49

As the variation of υ will increase or decrease the K_{m}, and the growth rate of K_{m} can be calculated by Eq. (2-1):

$$\Delta = K_{\mathrm{mb}}/K_{\mathrm{ma}} - 1 \quad (2\text{-}1)$$

where Δ is growth rate of K_{m}, K_{ma} is the simulated K_{m} for the base models that are described in Section 1, and K_{mb} is the simulated K_{m} after the mining condition is changed.

Table 2-1 shows the Δ after the mining condition is changed. The value of Δ will vary significantly if α and υ are changed, while the variation of Δ is not large if H_{D} is changed (such as the values with a gray background). The effect of H_{D} on Δ can be neglected by selecting the maximum Δ to represent the Δ of different H_{D}. Table 2-2 shows an example of Δ in which the effect of H_{D} is neglected. It can be found that the relationship between α and Δ conforms to Fourier function, and the relationship between υ and Δ conforms to power function (Figure 2.8). The superimposed effects of α and υ on Δ can be expressed by an empirical function:

$$\Delta_{\upsilon} = m_1 \upsilon^{m_2} (m_3 + m_4 \sin m_6 \alpha + m_5 \cos m_6 \alpha)/\alpha \quad (2\text{-}2)$$

where Δ_{υ} is the growth rate of K_{m} when υ is changed, m_1 to m_6 are coefficients, which are listed in Table 2-3.

The calculation results of Eq. (2-2) should be conservative enough to provide a safer stress estimation because the maximum Δ is used.

Table 2-1 The Δ for different mining conditions after υ is changed.

H_D (m)	υ	α (°)						H_D (m)	υ	α (°)					
		15	30	45	60	75	90			15	30	45	60	75	90
500	0.1	-2%	-5%	-5%	0%	1%	-2%	200	0.1	-2%	-4%	-2%	0%	1%	0%
	0.15	0%	0%	0%	0%	0%	0%		0.15	0%	0%	0%	0%	0%	0%
	0.2	2%	6%	5%	1%	-1%	6%		0.2	2%	5%	2%	0%	-1%	0%
	0.3	7%	21%	17%	2%	-3%	11%		0.3	7%	15%	8%	-1%	-3%	2%
	0.4	13%	40%	32%	3%	-7%	15%		0.4	13%	29%	15%	-2%	-6%	5%
	0.49	20%	61%	46%	-4%	-16%	15%		0.49	20%	46%	21%	-5%	-11%	3%
400	0.1	-2%	-5%	-3%	0%	1%	-2%	100	0.1	-2%	-2%	-1%	0%	1%	0%
	0.15	0%	0%	0%	0%	0%	0%		0.15	0%	0%	0%	0%	0%	0%
	0.2	2%	6%	4%	0%	-1%	6%		0.2	2%	3%	1%	-1%	-1%	0%
	0.3	7%	19%	13%	0%	-3%	7%		0.3	6%	10%	4%	-2%	-3%	0%
	0.4	13%	36%	24%	0%	-7%	10%		0.4	12%	19%	7%	-3%	-6%	1%
	0.49	20%	56%	35%	-2%	-15%	5%		0.49	19%	30%	9%	-6%	-10%	1%
300	0.1	-1%	-1%	0%	0%	1%	0%								
	0.15	0%	0%	0%	0%	0%	0%								
	0.2	1%	1%	0%	-1%	-1%	0%								
	0.3	4%	3%	0%	-2%	-2%	0%								
	0.4	7%	7%	0%	-3%	-4%	1%								
	0.49	11%	10%	0%	-5%	-6%	4%								

Table 2-2 The maximum Δ for different mining conditions after υ is changed.

υ	α (°)					
	15	30	45	60	75	90
0.1	-1%	-1%	0%	0%	1%	0%
0.15	0%	0%	0%	0%	0%	0%
0.2	2%	6%	5%	1%	-1%	6%
0.3	7%	21%	17%	2%	-2%	11%
0.4	13%	40%	32%	3%	-4%	15%
0.49	20%	61%	46%	-2%	-6%	15%

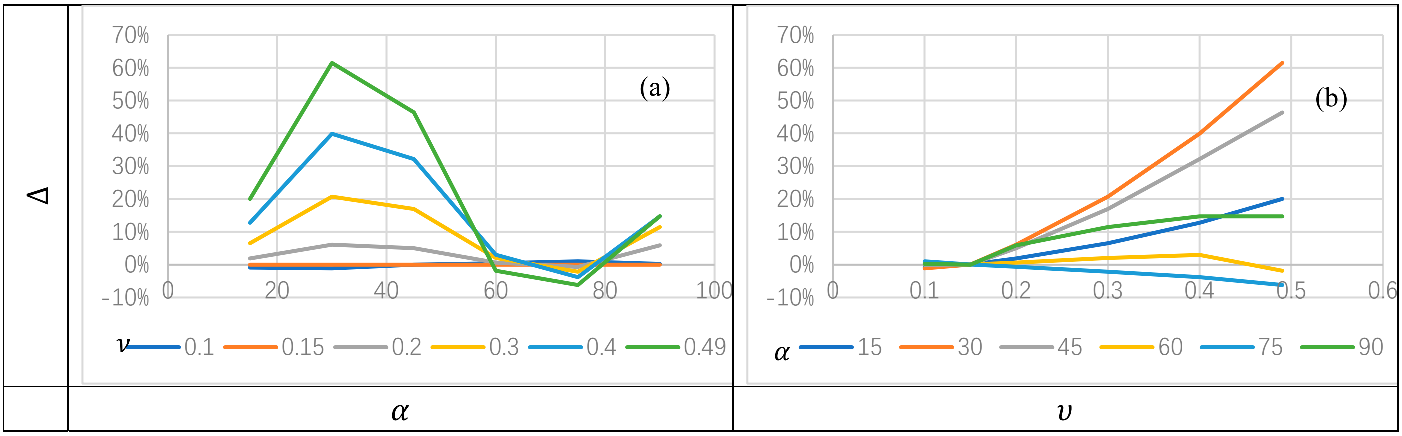


Figure 2.8 The effects of α and υ on Δ: (a) effect of α; (b) effect of υ

Table 2-3 Fitting results of Eq. (2-1)

m_1	m_2	m_3	m_4	m_5	m_6	Adjusted R-square	RMSE
3.304	2.297	15.28	-8.601	-18.46	5.488	0.963	0.02908

Figure 2.9 shows that defect A appears because the curved surface is above the axis limit around that area, indicating the curve may overestimate the pillar stress around A. This should be acceptable as overestimation of pillar stress usually produces a conservative pillar design and enhances pillar safety.

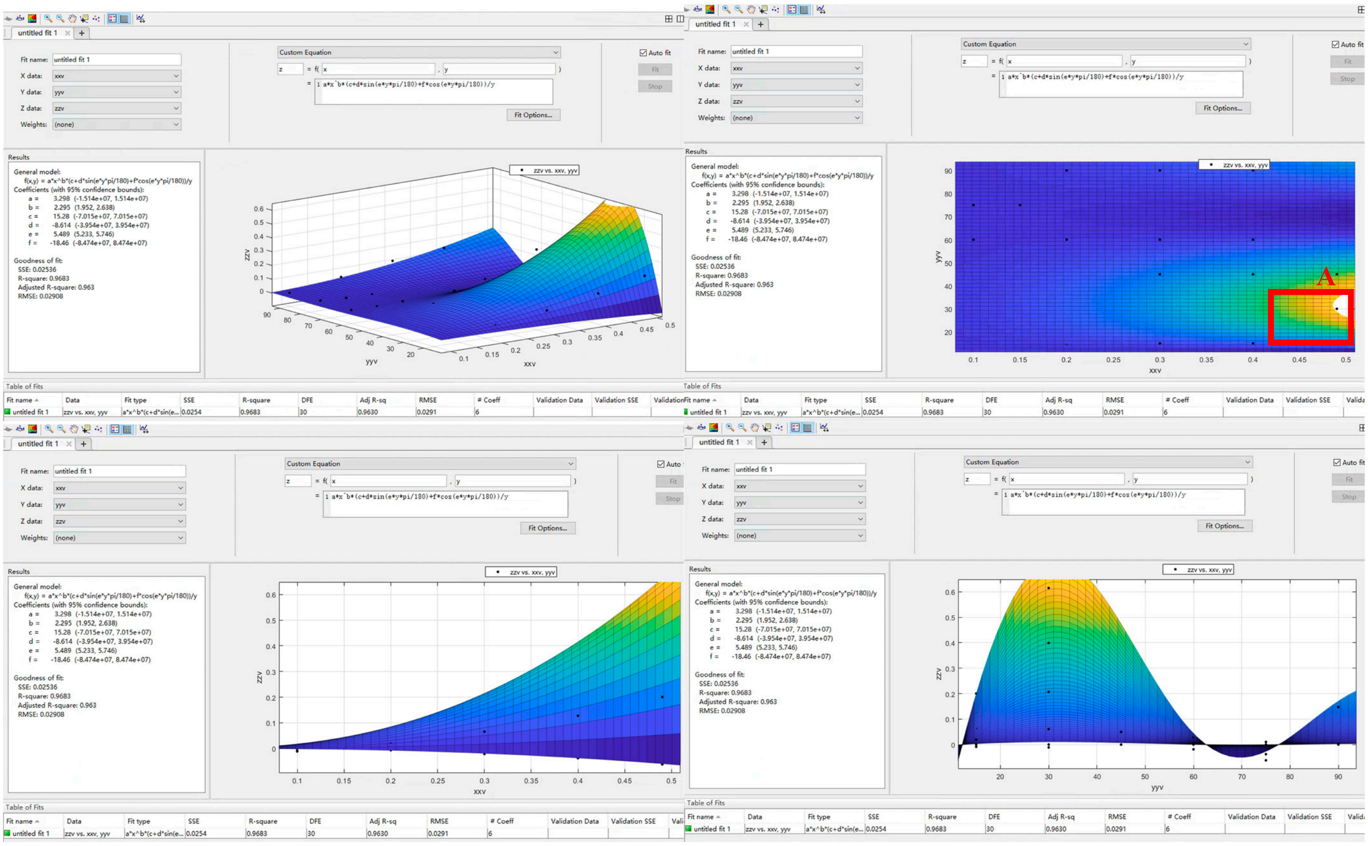


Figure 2.9 Fitting results of Eq. (2-2) by MATLAB

3. The effect of E on K

Figures 3.1-3.9 show the effect of E. The H_B was fixed at 10 m. E varied from 2 GPa to 100 GPa and υ was 0.15. The other parameters were kept fixed as mentioned in Section 3.1. The distribution law of K has been discussed in Section 3.2.

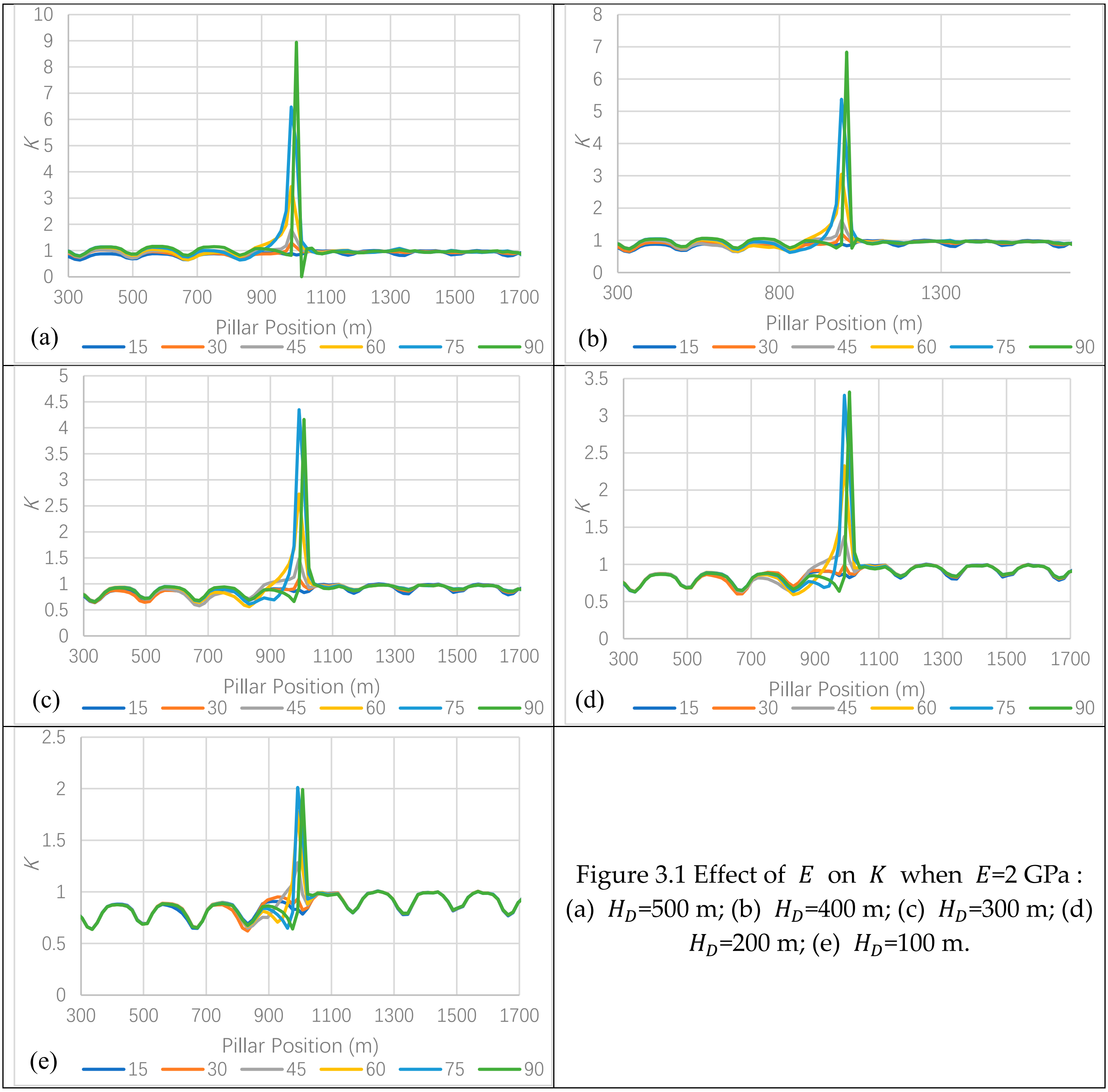
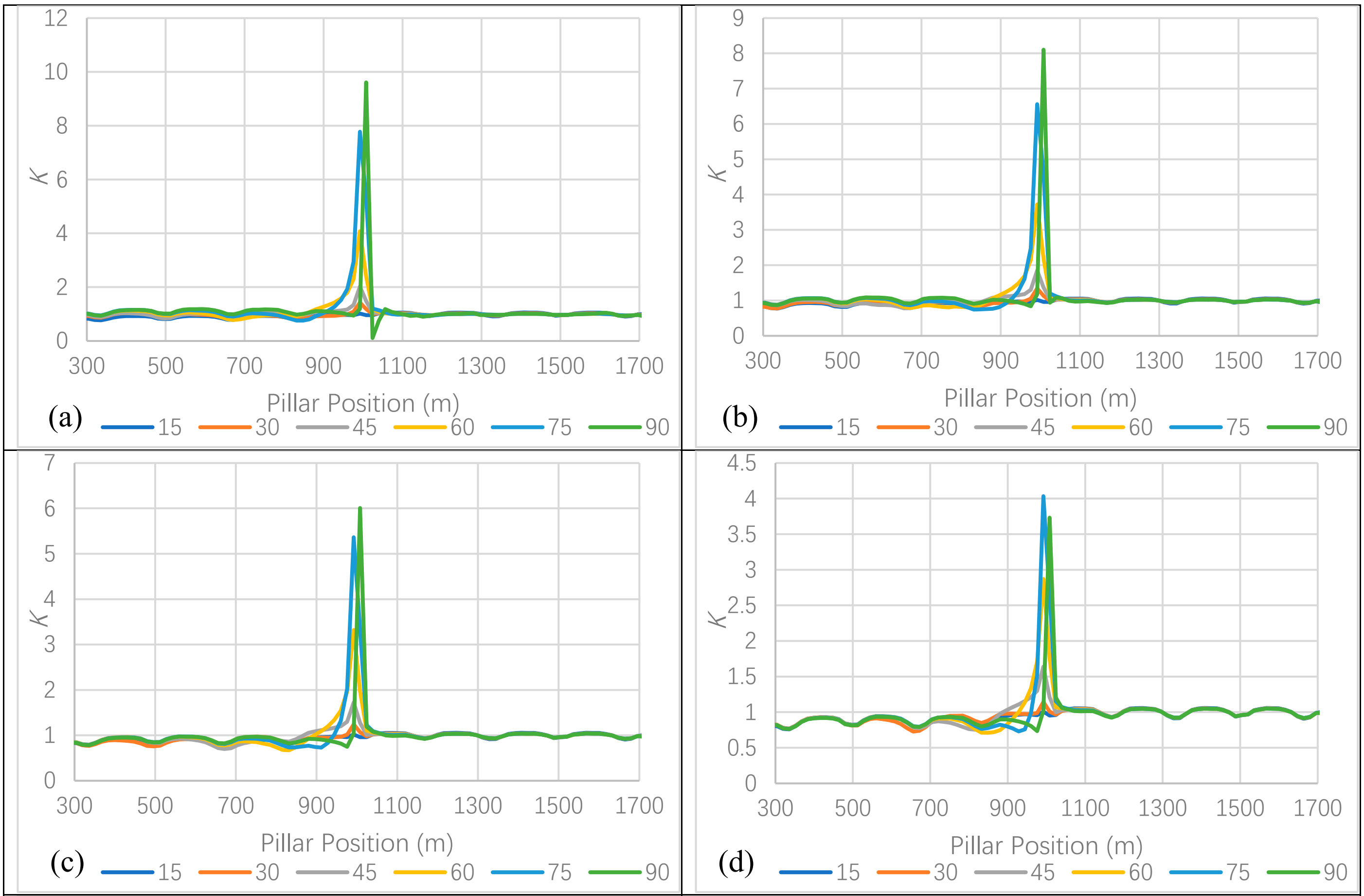


Figure 3.1 Effect of E on K when E=2 GPa : (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.



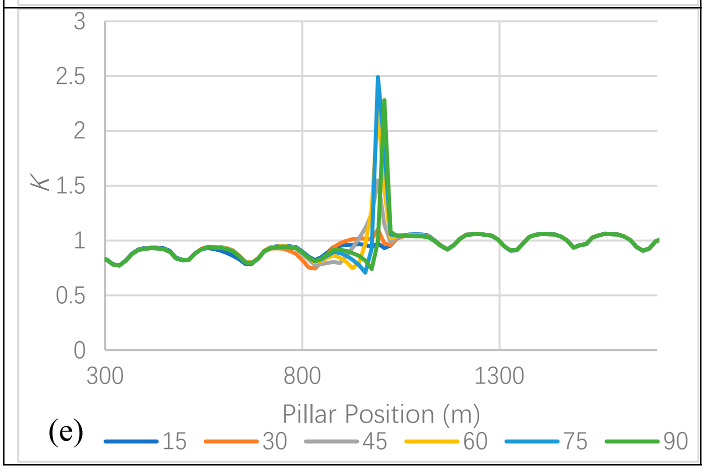


Figure 3.2 Effect of E on K when E=4 GPa : (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.

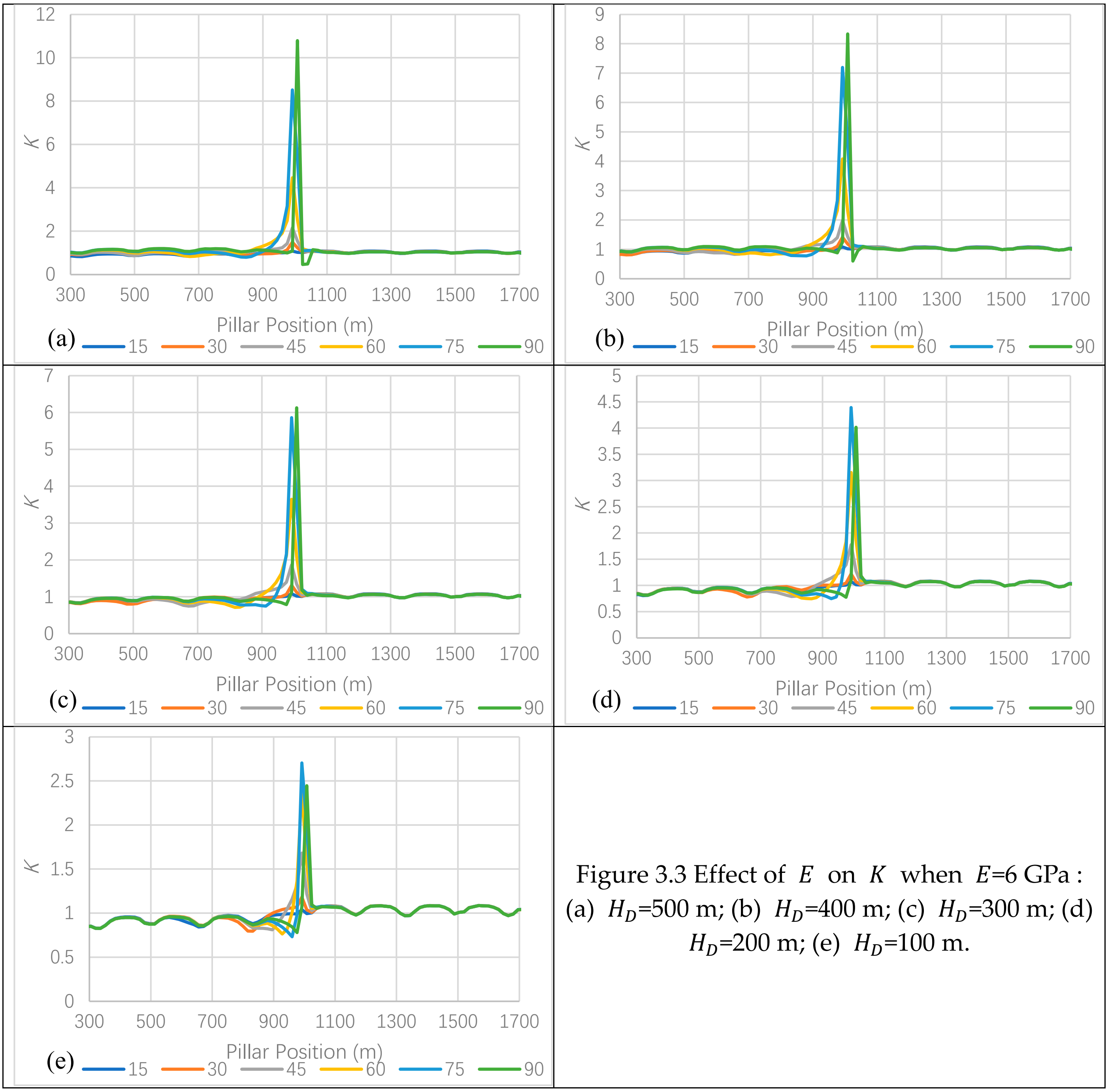
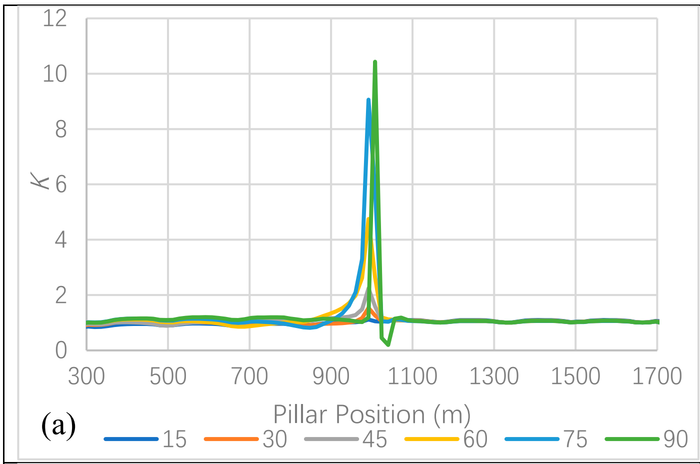
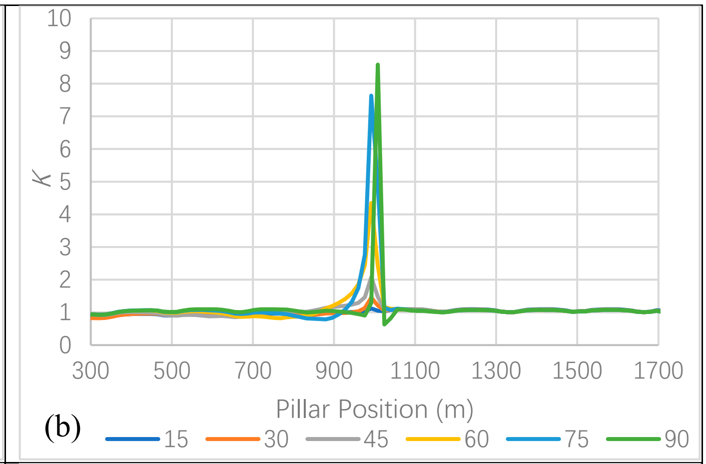
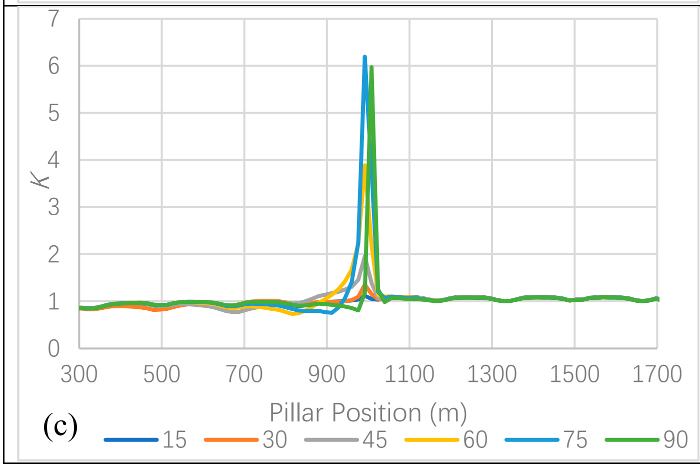
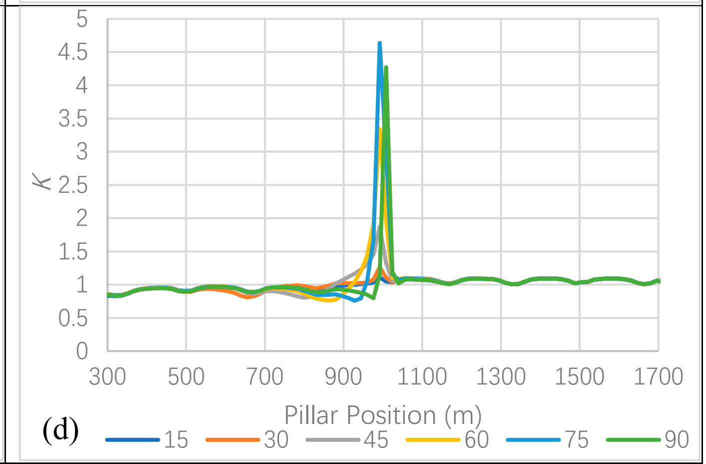


Figure 3.3 Effect of E on K when E=6 GPa : (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.









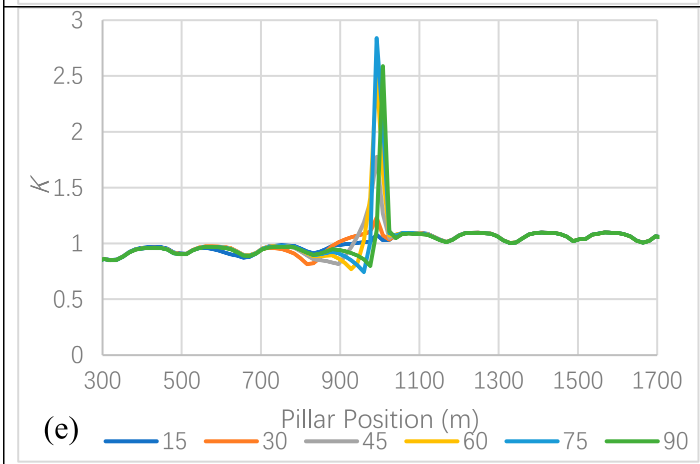


Figure 3.4 Effect of E on K when E=8 GPa : (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.

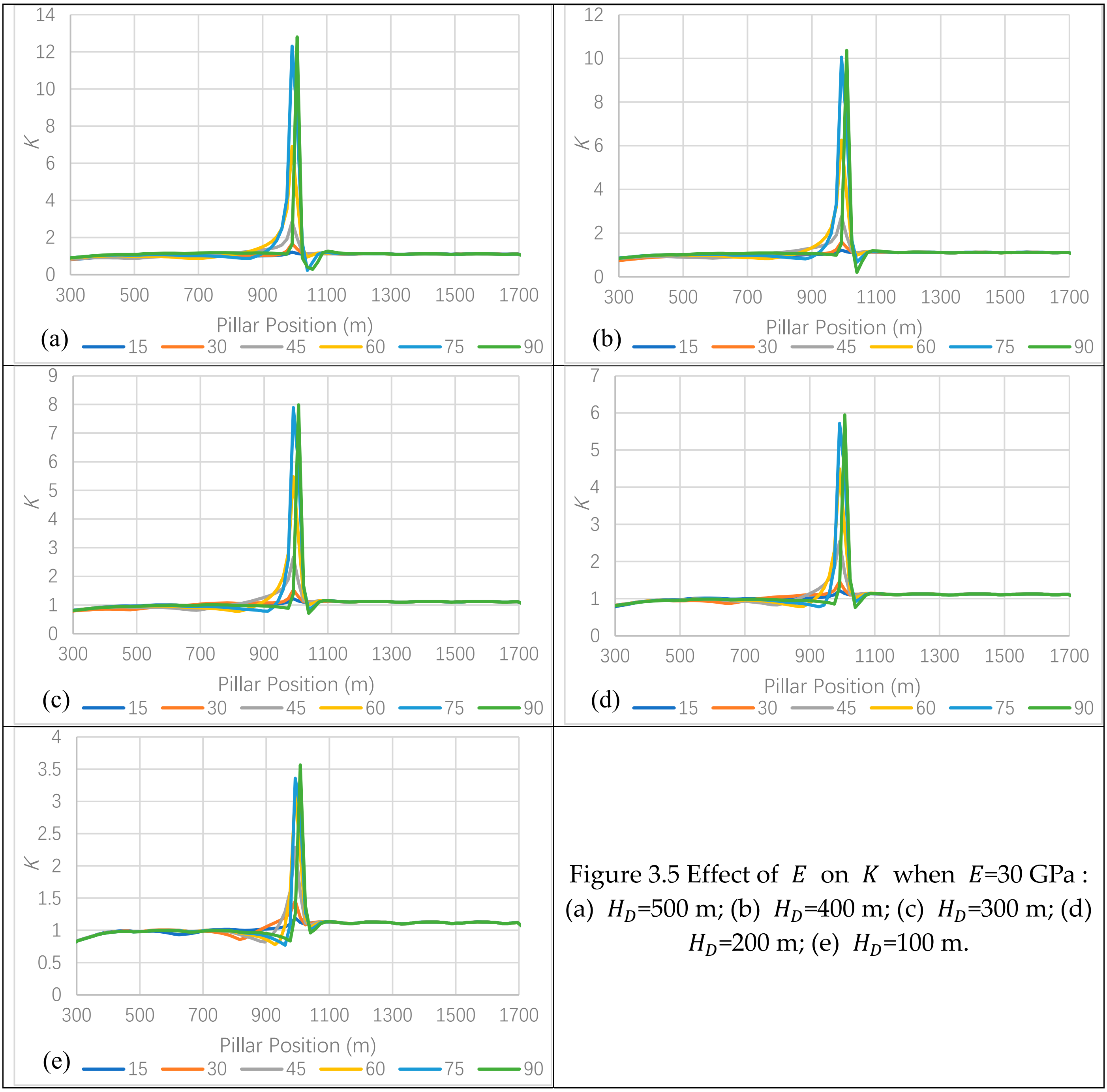


Figure 3.5 Effect of E on K when E=30 GPa : (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.

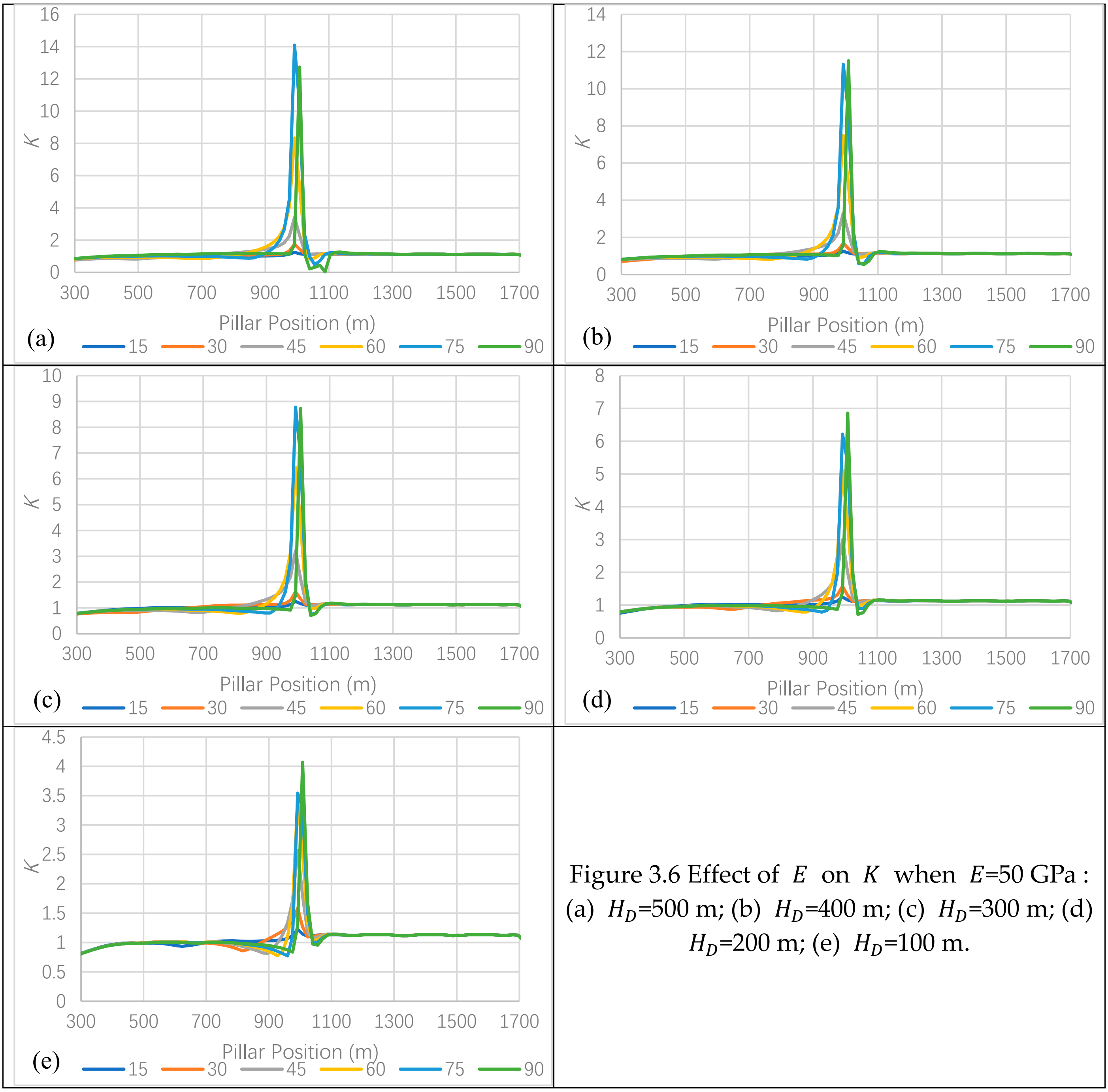
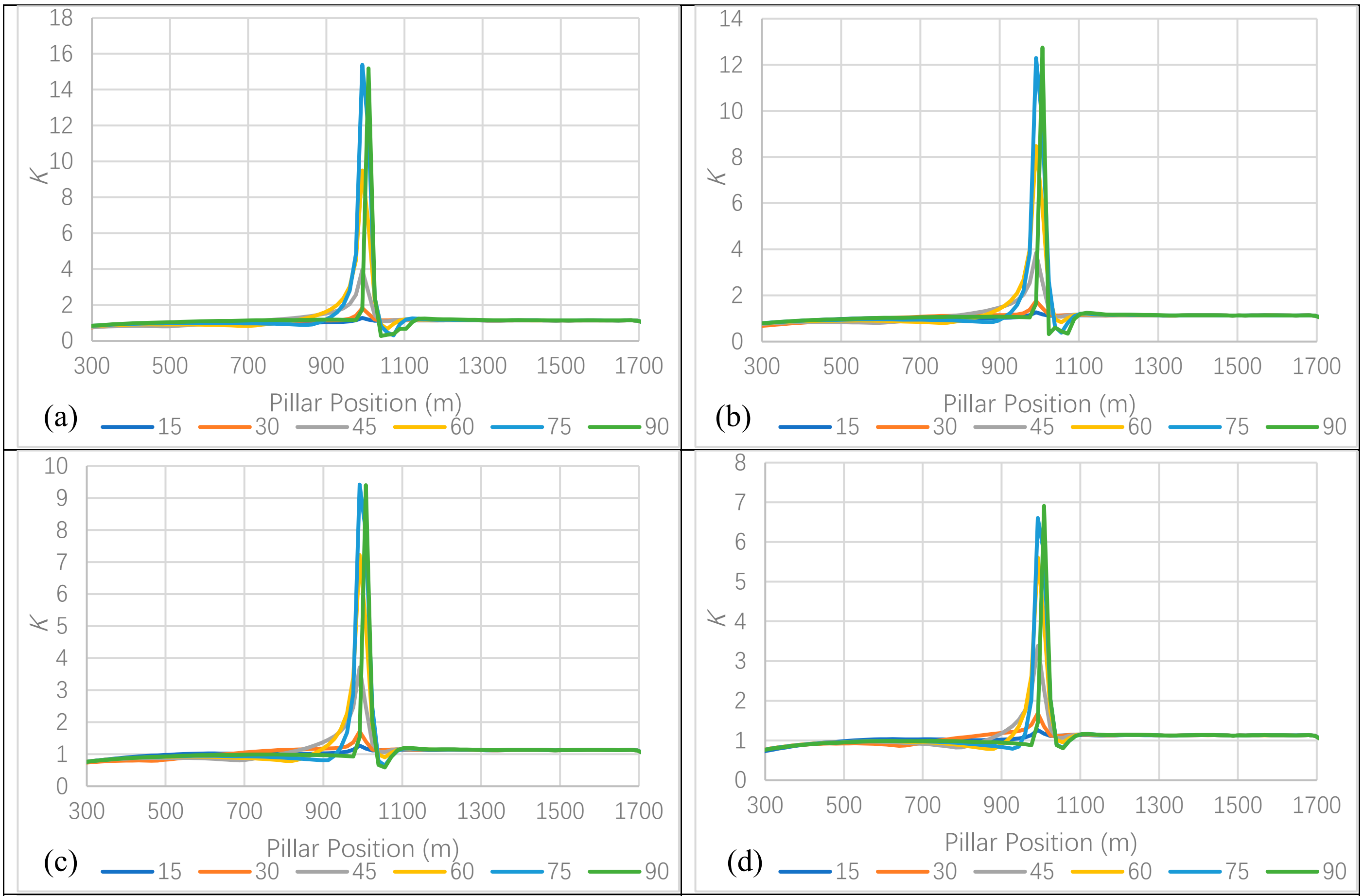


Figure 3.6 Effect of E on K when E=50 GPa : (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.



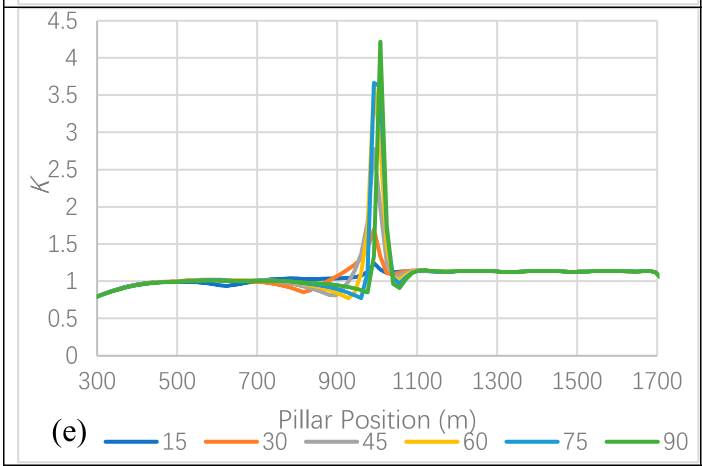


Figure 3.7 Effect of E on K when E=70 GPa : (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.

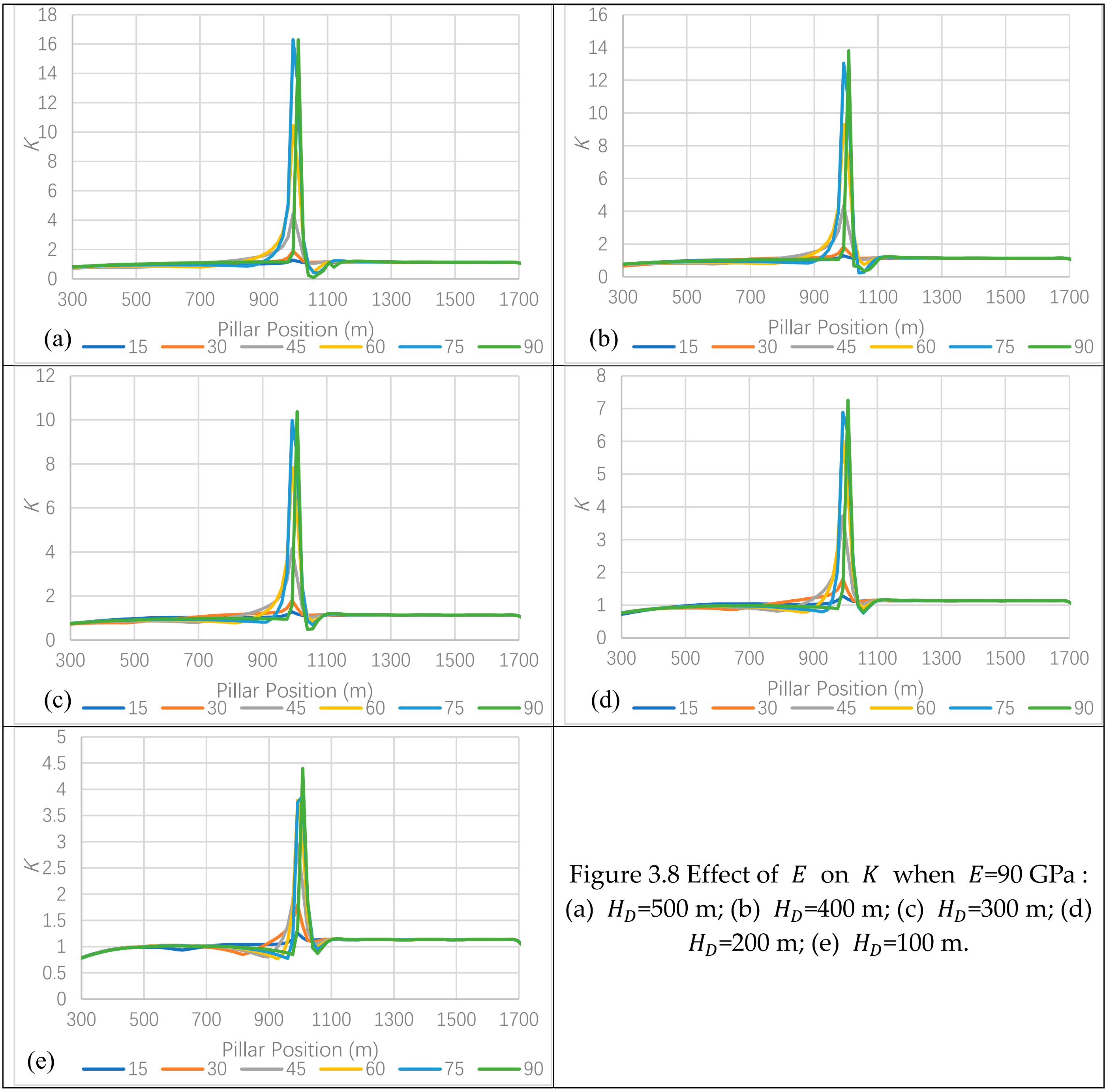


Figure 3.8 Effect of E on K when E=90 GPa : (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.

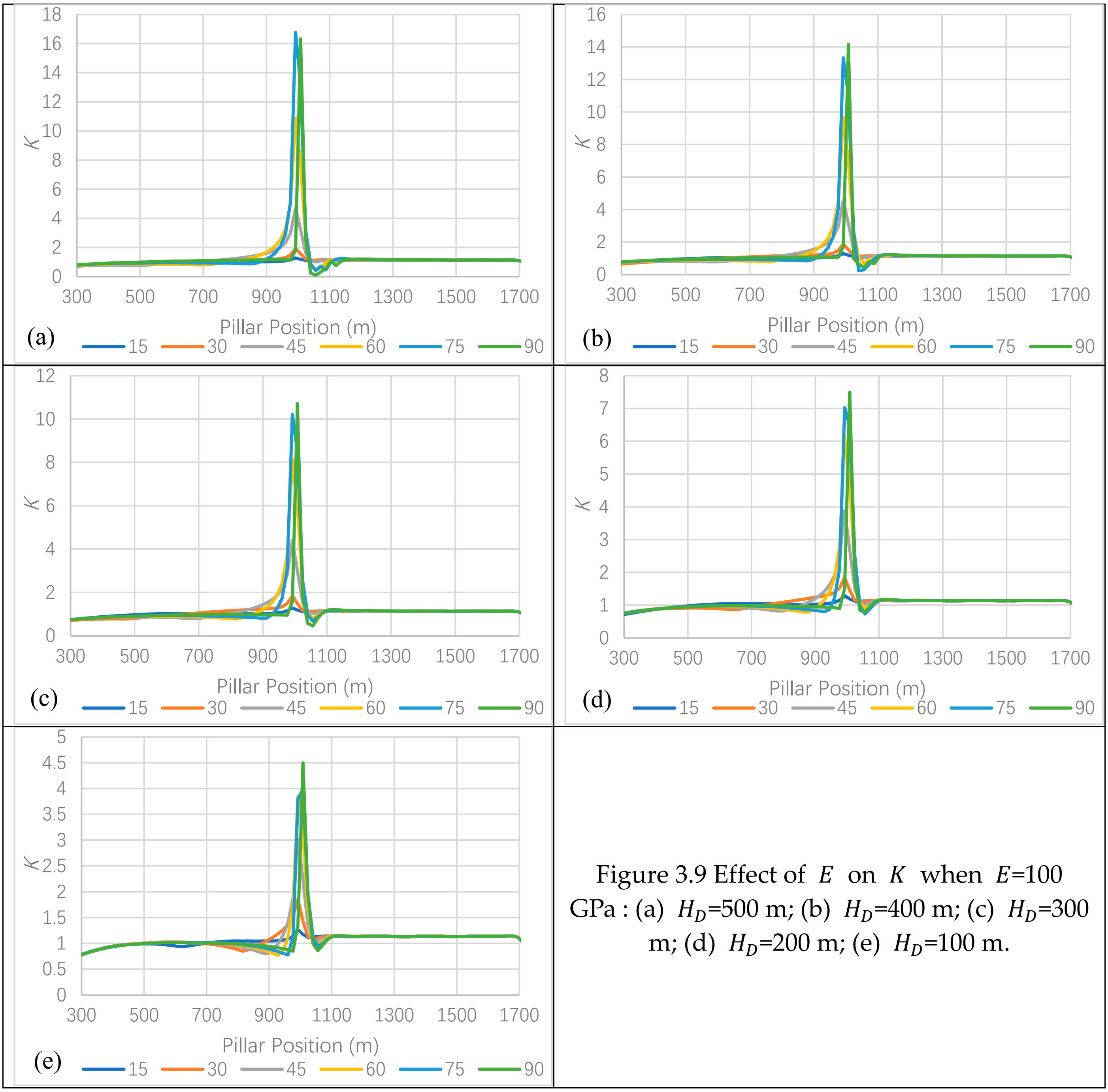


Figure 3.9 Effect of E on K when E=100 GPa : (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.

As the variation of E also increases or decreases the K_m as discussed in the above section, the Δ after E is changed, is calculated by Eq. (2-1), and the results are listed in Table 3-1. As mentioned before, the variation of H_D will not lead to a significant variation of Δ, the effect of H_D is neglected by selecting the maximum Δ to represent the Δ of different H_D. Table 3-1 is changed to Table 3-2. Regression analysis on Table 3-2 shows that the effect of E on Δ can be expressed by:

$$\Delta_E = (0.03852E^{0.5337} - 0.1552)[0.7676\sin(6.097\alpha - 3.921) + 2.846\sin 1.359\alpha] \quad (3\text{-}1)$$

where Δ_E is the growth rate of K_m when E is changed; the RMSE is 0.0367 and the R^2 is 0.988.

Figure 3.10 shows the fitting results.

Table 3-1 The Δ for different mining conditions after E is changed.

H_D (m)	E GPa	α (°)						H_D (m)	E GPa	α (°)					
		15	30	45	60	75	90			15	30	45	60	75	90
500	2	-15%	-19%	-27%	-38%	-38%	-20%	200	2	-14%	-26%	-35%	-39%	-36%	-34%
	4	-10%	-11%	-18%	-27%	-25%	-14%		4	-9%	-16%	-23%	-25%	-21%	-26%
	6	-8%	-7%	-12%	-20%	-18%	-3%		6	-7%	-10%	-16%	-18%	-14%	-21%
	8	-5%	-4%	-9%	-15%	-13%	-6%		8	-5%	-7%	-12%	-13%	-10%	-16%
	15	0%	0%	0%	0%	0%	0%		15	0%	0%	0%	0%	0%	0%
	30	4%	4%	17%	24%	18%	15%		30	4%	8%	19%	18%	11%	18%
	50	6%	9%	39%	50%	36%	14%		50	6%	16%	41%	34%	21%	36%
	70	8%	13%	60%	71%	48%	36%		70	8%	24%	59%	47%	29%	37%
	90	9%	18%	81%	88%	57%	46%		90	9%	32%	75%	57%	34%	44%
	100	9%	20%	91%	96%	62%	46%		100	10%	36%	83%	62%	37%	48%
400	2	-15%	-21%	-30%	-40%	-38%	-25%	100	2	-12%	-24%	-36%	-37%	-35%	-33%
	4	-10%	-12%	-20%	-27%	-24%	-11%		4	-7%	-17%	-22%	-22%	-20%	-24%
	6	-8%	-7%	-15%	-20%	-17%	-9%		6	-5%	-11%	-15%	-15%	-13%	-18%
	8	-5%	-5%	-11%	-14%	-12%	-6%		8	-4%	-8%	-11%	-10%	-8%	-13%
	15	0%	0%	0%	0%	0%	0%		15	0%	0%	0%	0%	0%	0%
	30	4%	5%	19%	23%	16%	13%		30	4%	9%	15%	12%	8%	19%
	50	6%	10%	43%	48%	31%	26%		50	7%	19%	29%	22%	14%	36%
	70	8%	15%	65%	67%	42%	39%		70	9%	28%	39%	29%	18%	41%
	90	9%	20%	87%	83%	51%	51%		90	10%	35%	48%	34%	25%	47%
	100	9%	23%	97%	91%	54%	55%		100	11%	39%	52%	37%	29%	51%
300	2	-15%	-24%	-33%	-40%	-37%	-39%								
	4	-10%	-14%	-22%	-26%	-23%	-12%								
	6	-8%	-9%	-16%	-19%	-16%	-10%								
	8	-5%	-6%	-12%	-14%	-11%	-13%								
	15	0%	0%	0%	0%	0%	0%								
	30	4%	6%	20%	21%	14%	17%								
	50	6%	13%	45%	43%	27%	28%								
	70	8%	19%	67%	60%	36%	37%								
	90	9%	25%	87%	74%	44%	52%								
	100	9%	29%	97%	80%	47%	57%								

Table 3-2 The maximum Δ for different mining conditions after E is changed.

E GPa	α (°)					
	15	30	45	60	75	90
2	-12%	-19%	-27%	-37%	-35%	-20%
4	-7%	-11%	-18%	-22%	-20%	-11%
6	-5%	-7%	-12%	-15%	-13%	-3%
8	-4%	-4%	-9%	-10%	-8%	-6%
15	0%	0%	0%	0%	0%	0%
30	4%	9%	20%	24%	18%	19%
50	7%	19%	45%	50%	36%	36%
70	9%	28%	67%	71%	48%	41%
90	10%	35%	87%	88%	57%	52%
100	11%	39%	97%	96%	62%	57%

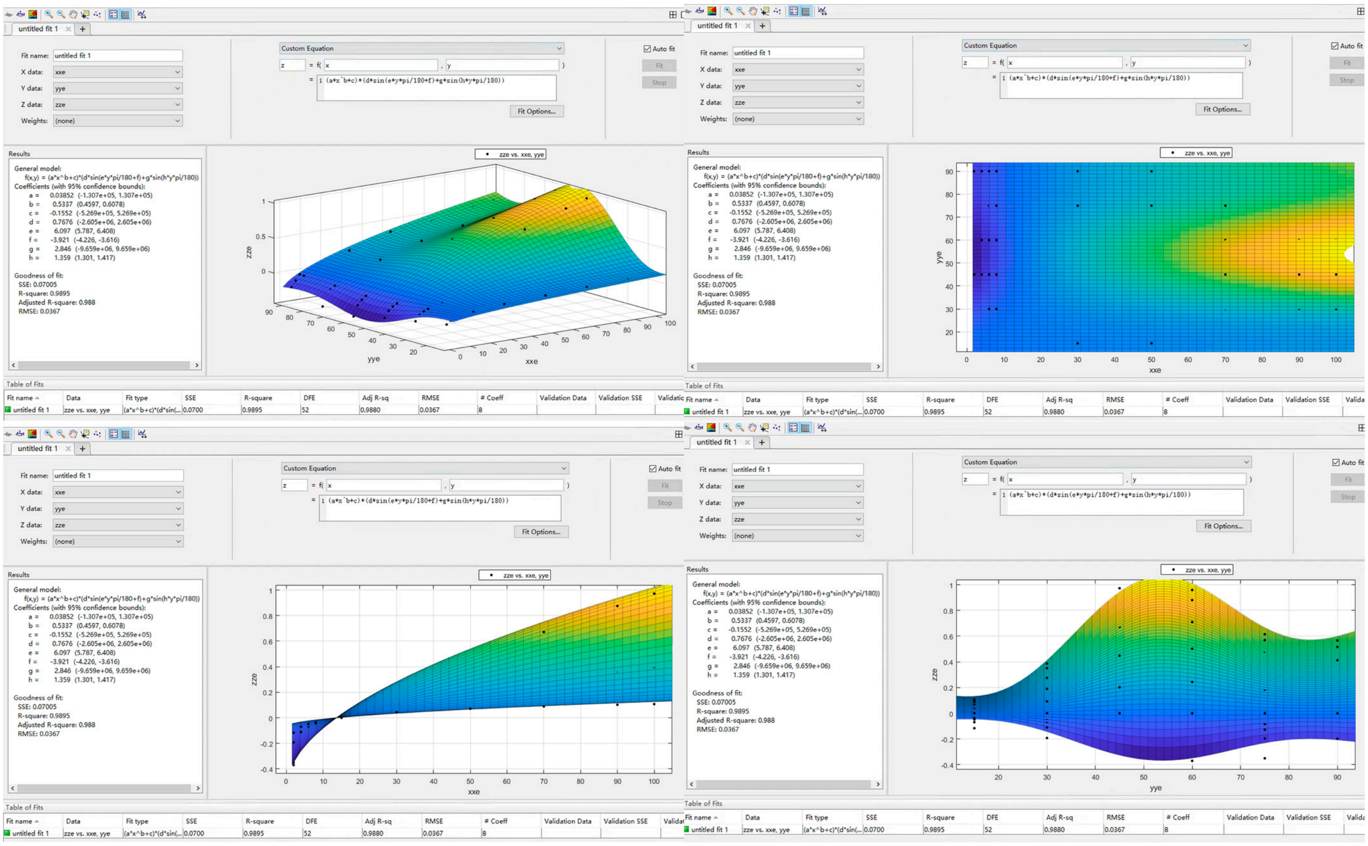


Figure 3.10 Fitting results of Eq. (3-1) by MATLAB

4. The effect of w_p on K

Figures 4.1-4.5 show the effect of w_p. w_P varies from 6 m to 20 m, and the other parameters were kept fixed as mentioned in Section 3.1. The distribution law of K has been discussed in Section 3.4.

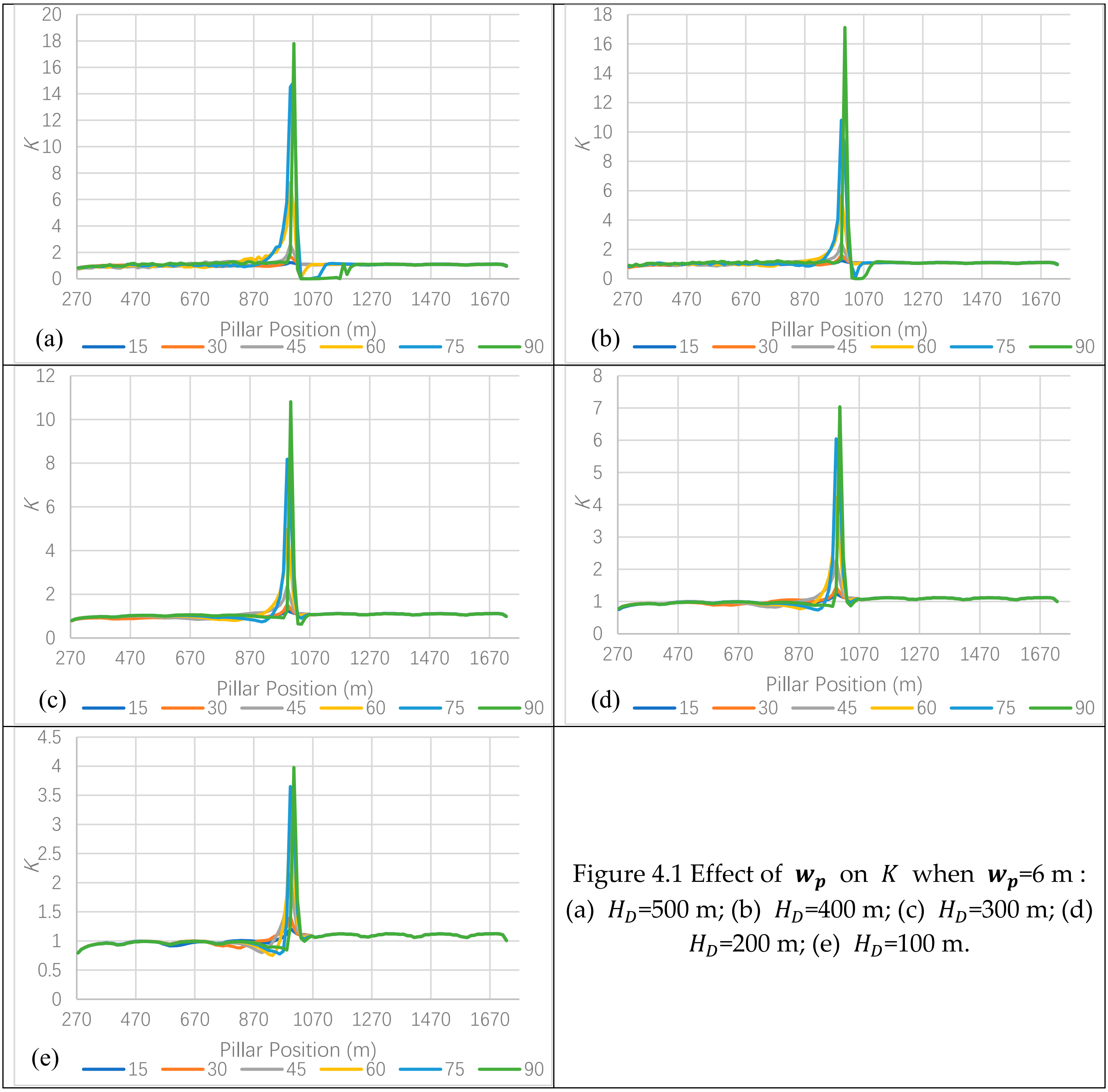
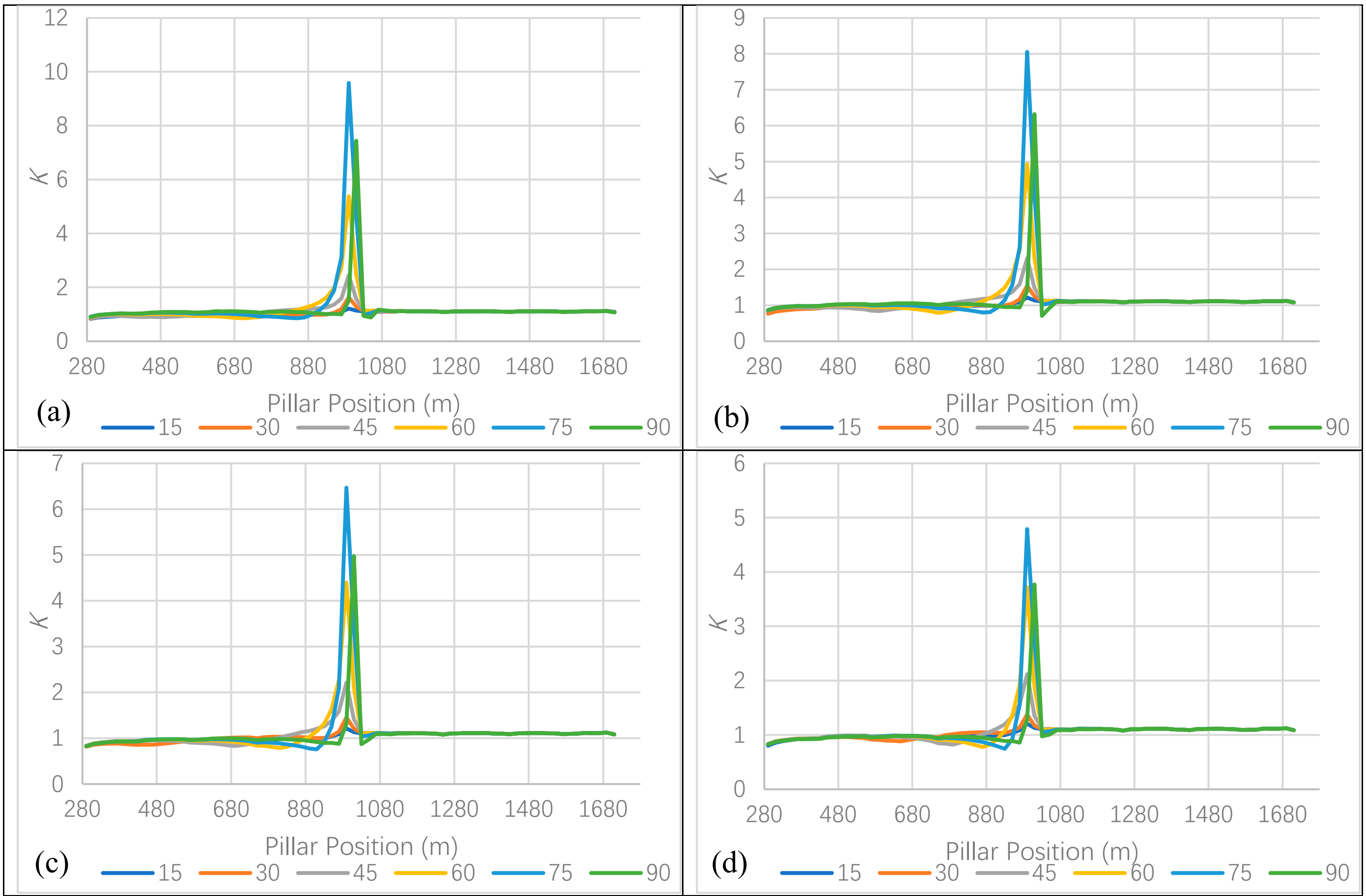


Figure 4.1 Effect of $\boldsymbol{w_p}$ on K when $\boldsymbol{w_p}$=6 m : (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.



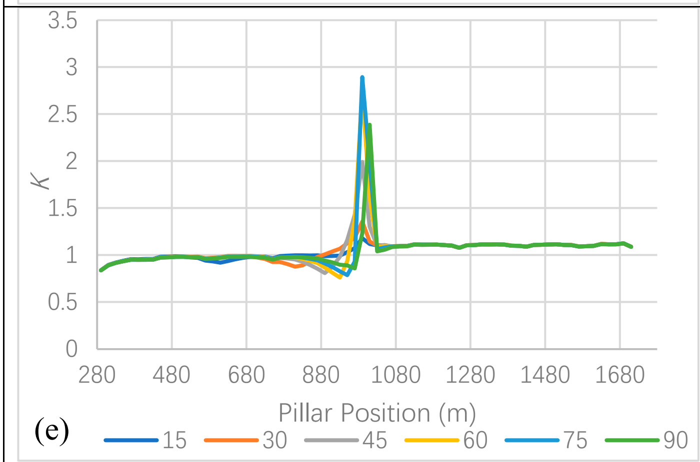
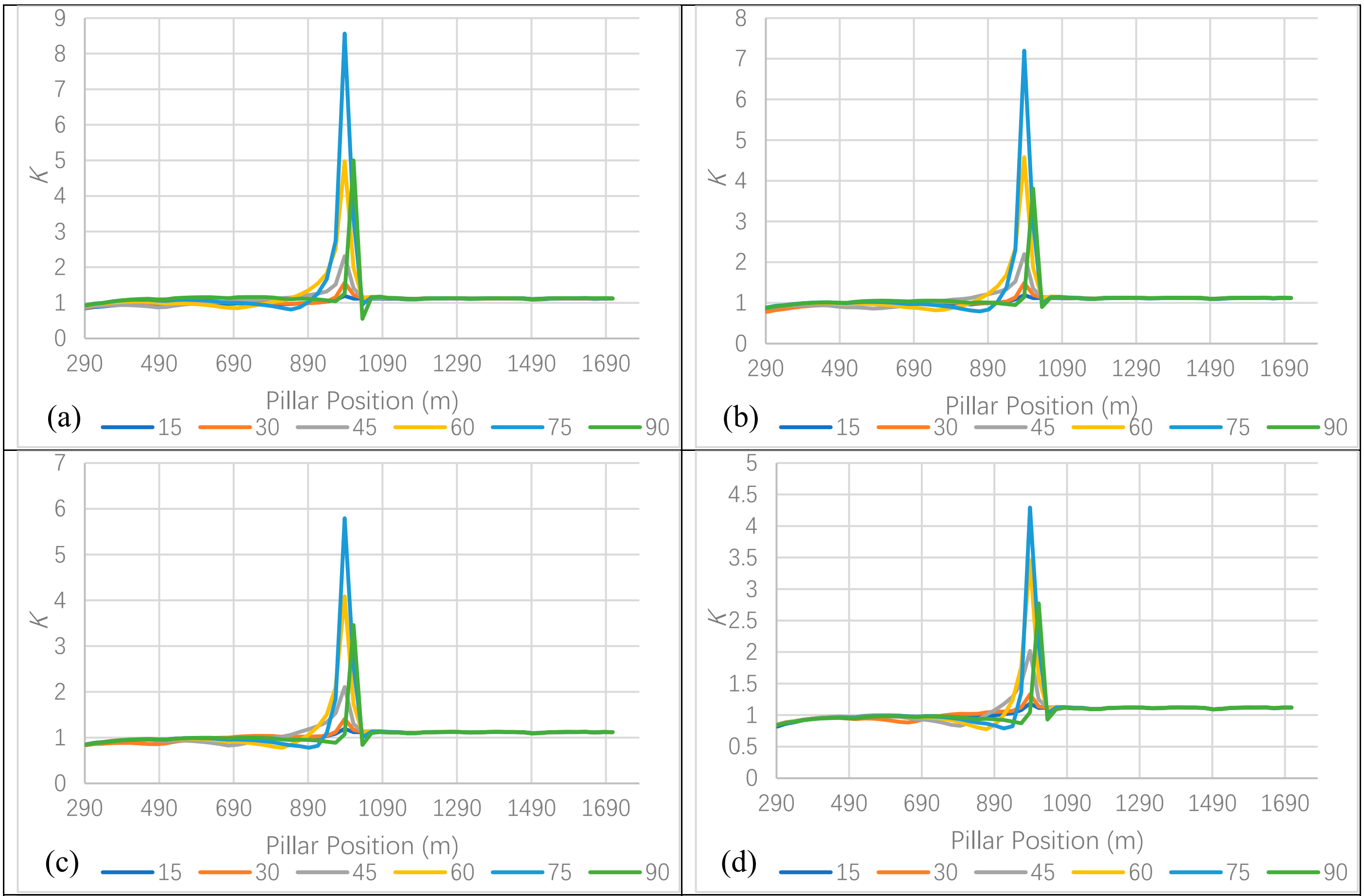


Figure 4.2 Effect of $\boldsymbol{w_p}$ on K when $\boldsymbol{w_p}$=10 m : (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.



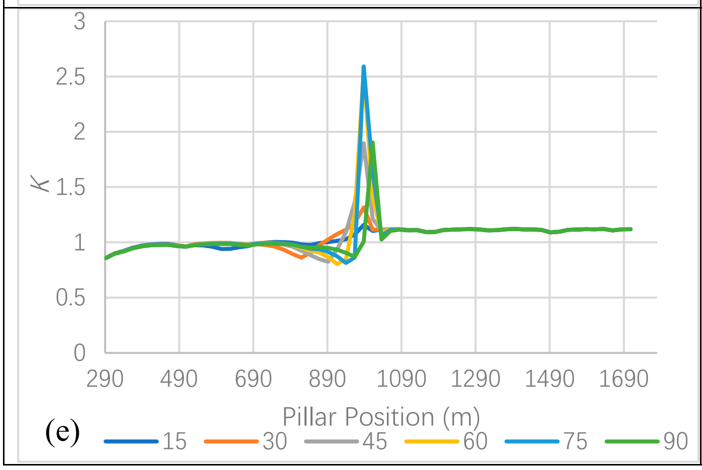
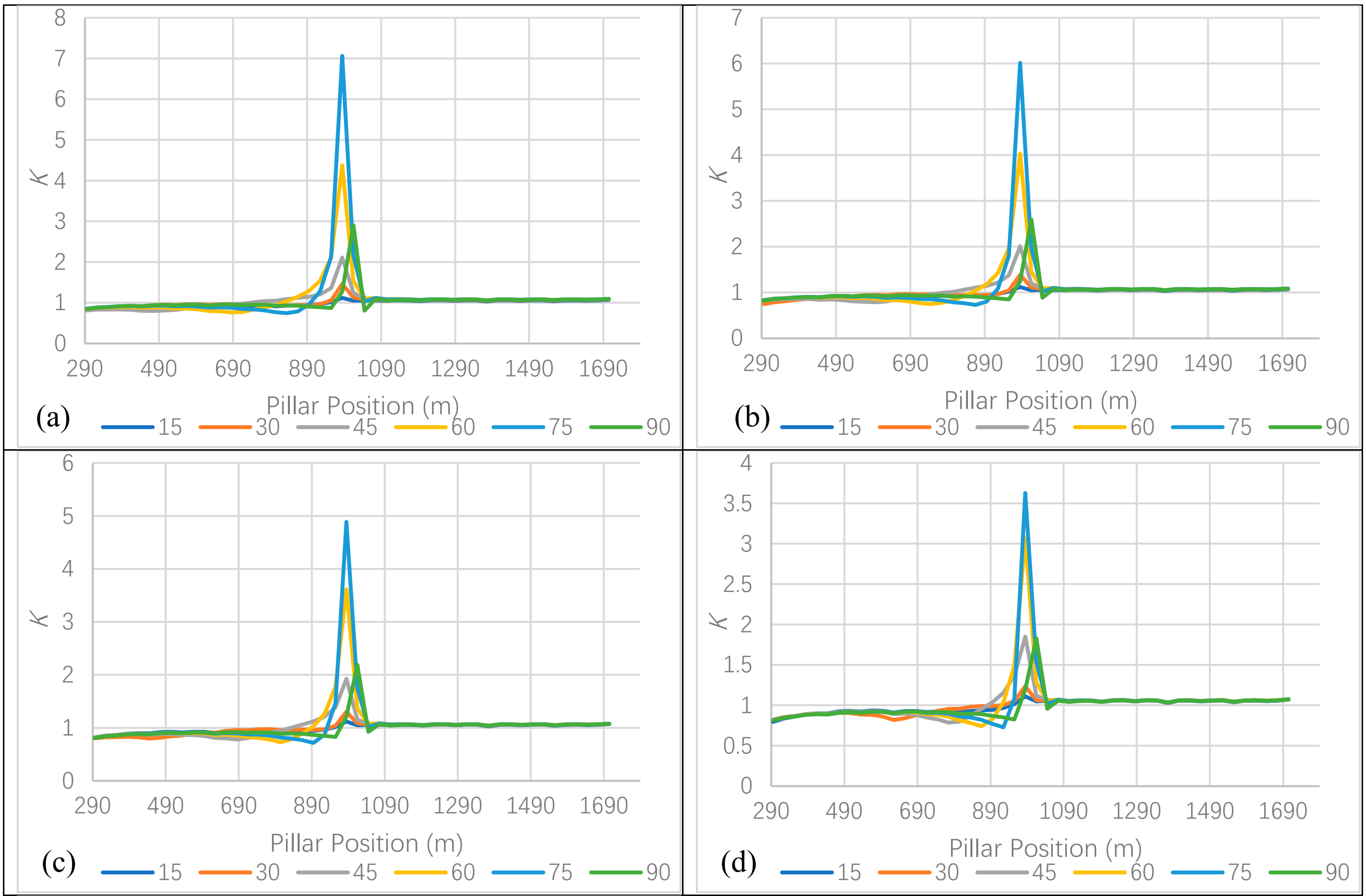


Figure 4.3 Effect of $\boldsymbol{w_p}$ on K when $\boldsymbol{w_p}$=12 m : (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.



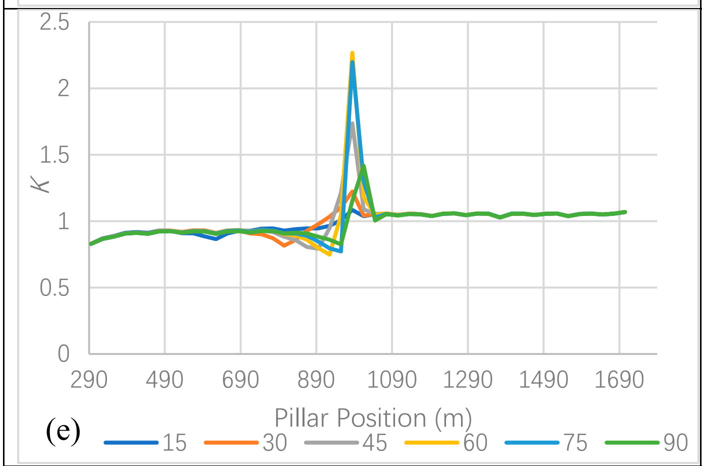
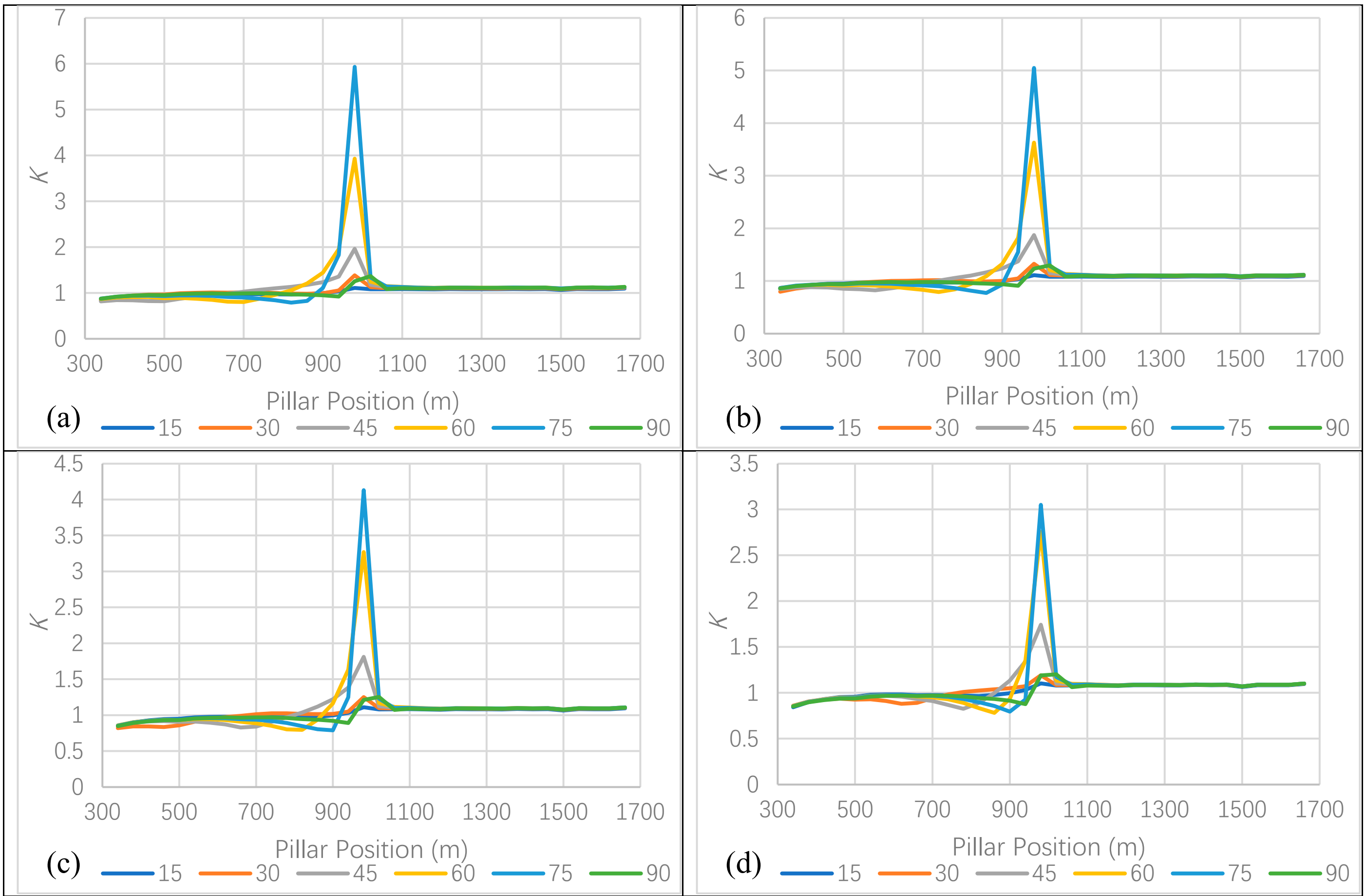


Figure 4.4 Effect of $\boldsymbol{w_p}$ on K when $\boldsymbol{w_p}$=15 m : (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.



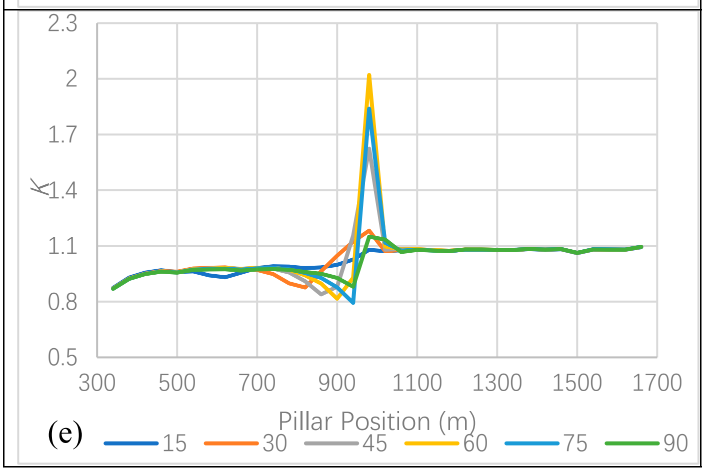
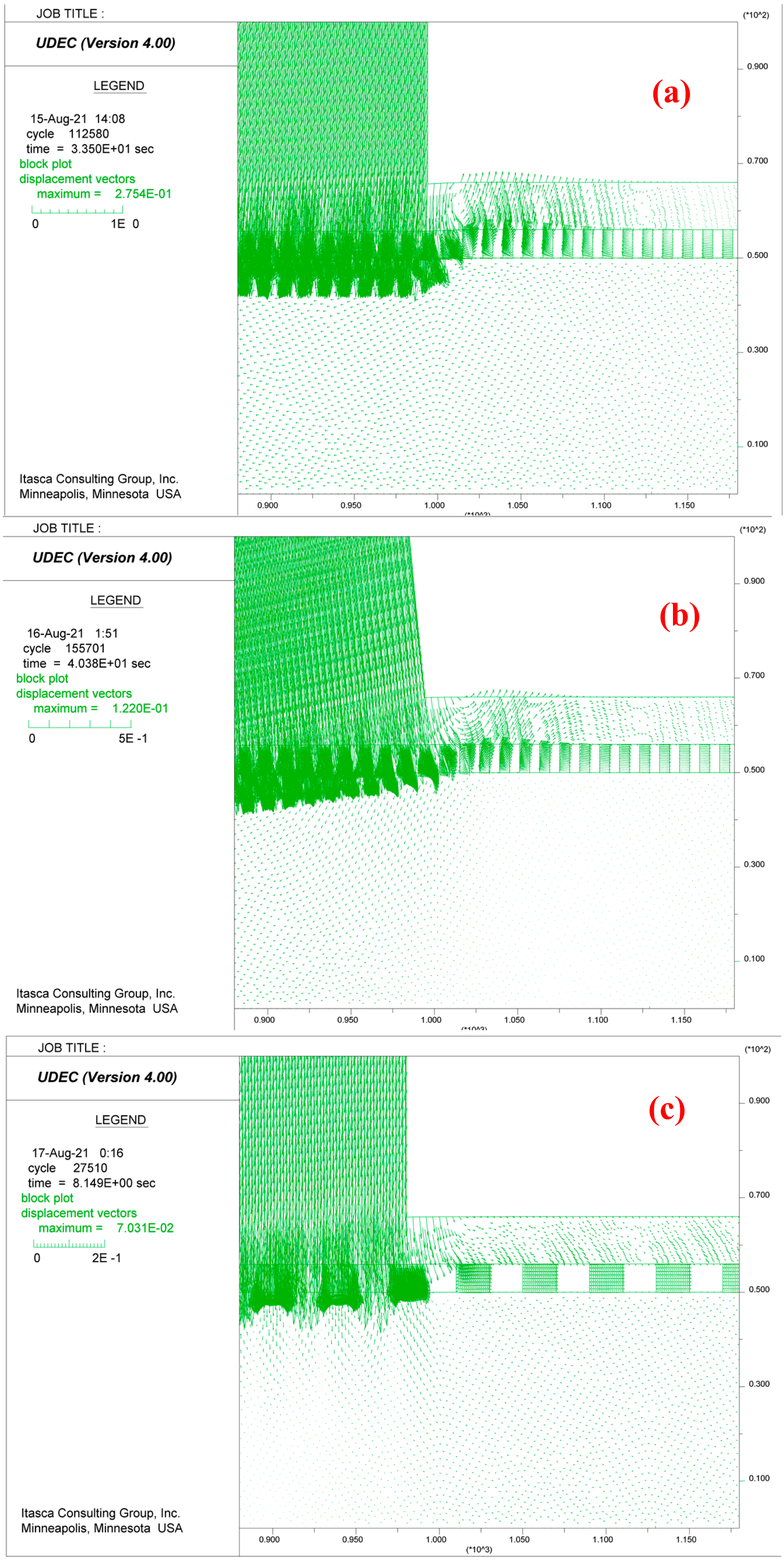


Figure 4.5 Effect of $\boldsymbol{w_p}$ on K when $\boldsymbol{w_p}$=20 m : (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.

Figure 4.6 shows the displacements of the models. The ground surface uplifts significantly when w_p is small, which makes the pillar stress under the slope bottom very small.


JOB TITLE :
UDEC (Version 4.00)
LEGEND
15-Aug-21 14:08
cycle 112580
time = 3.350E+01 sec
block plot
displacement vectors
maximum = 2.754E-01
0 1E 0
Itasca Consulting Group, Inc.
Minneapolis, Minnesota USA
(a)
(*10^2)
0.900
0.700
0.500
0.300
0.100
0.900 0.950 1.000 1.050 1.100 1.150
JOB TITLE :
UDEC (Version 4.00)
LEGEND
16-Aug-21 1:51
cycle 155701
time = 4.038E+01 sec
block plot
displacement vectors
maximum = 1.220E-01
0 5E -1
Itasca Consulting Group, Inc.
Minneapolis, Minnesota USA
(b)
(*10^2)
0.900
0.700
0.500
0.300
0.100
0.900 0.950 1.000 1.050 1.100 1.150
JOB TITLE :
UDEC (Version 4.00)
LEGEND
17-Aug-21 0:16
cycle 27510
time = 8.149E+00 sec
block plot
displacement vectors
maximum = 7.031E-02
0 2E -1
Itasca Consulting Group, Inc.
Minneapolis, Minnesota USA
(c)
(*10^2)
0.900
0.700
0.500
0.300
0.100
0.900 0.950 1.000 1.050 1.100 1.150
(*10^3)

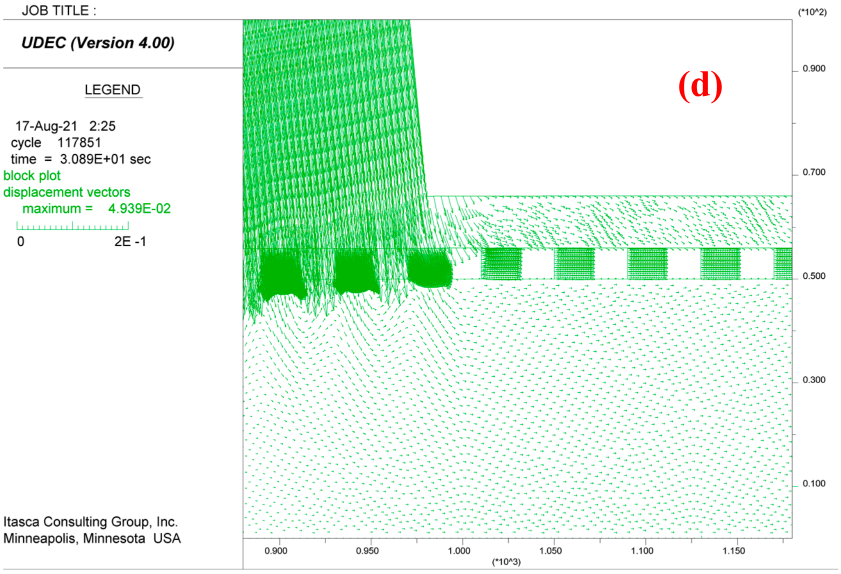


Figure 4.6 The displacement of the model: (a) w_p = 6 m, H_D = 500 m and α = 90°; (b) w_p = 6 m, H_D = 500 m and α = 75°; (c) w_p = 20 m, H_D = 500 m and α = 90°; (d) w_p = 20 m, H_D = 500 m and α = 75°;

The Δ after w_p is changed, is calculated by Eq. (2-1), and the results are listed in Table 4-1. As mentioned before, the effect of H_D can be neglected by selecting the maximum Δ to represent the Δ of different H_D. Table 4-1 is changed to Table 4-2. Regression analysis on Table 4-2 shows that the effect of w_p on Δ can be expressed by:

$$\Delta_{\mathrm{p}} = 0.6814e^{0.03723\alpha}(0.5075e^{-0.3228w_p} - 0.03182) \quad (4\text{-}1)$$

where Δ_{p} is the growth rate of K_{m} when H_B is changed; the RMSE is 0.05177 and the R^2 is 0.9584.

Table 4-1 The Δ for different mining conditions after w_p is changed.

H_{D} (m)	w_p (m)	α (°)						H_{D} (m)	w_p (m)	α (°)					
		15	30	45	60	75	90			15	30	45	60	75	90
500	6	6%	6%	5%	31%	43%	60%	200	6	6%	7%	7%	11%	18%	39%
	8	0%	0%	0%	0%	0%	0%		8	0%	0%	0%	0%	0%	0%
	10	3%	1%	0%	-3%	-8%	-33%		10	3%	2%	0%	-3%	-7%	-25%
	12	1%	-2%	-6%	-10%	-18%	-55%		12	1%	-2%	-5%	-10%	-16%	-45%
	15	-5%	-9%	-14%	-21%	-32%	-74%		15	-5%	-9%	-13%	-20%	-29%	-64%
	20	-6%	-14%	-20%	-29%	-43%	-88%		20	-6%	-12%	-18%	-28%	-41%	-76%
400	6	6%	6%	5%	12%	25%	87%	100	6	6%	6%	8%	11%	18%	33%
	8	0%	0%	0%	0%	0%	0%		8	0%	0%	0%	0%	0%	0%
	10	3%	1%	0%	-3%	-7%	-31%		10	3%	2%	0%	-3%	-7%	-20%
	12	1%	-2%	-5%	-10%	-17%	-58%		12	1%	-1%	-5%	-9%	-16%	-36%
	15	-5%	-9%	-14%	-20%	-31%	-72%		15	-5%	-8%	-13%	-20%	-29%	-53%
	20	-6%	-13%	-19%	-28%	-42%	-86%		20	-6%	-11%	-18%	-28%	-41%	-61%
300	6	6%	6%	6%	10%	18%	58%								
	8	0%	0%	0%	0%	0%	0%								
	10	3%	1%	0%	-3%	-7%	-27%								
	12	1%	-2%	-5%	-10%	-17%	-49%								
	15	-5%	-9%	-13%	-20%	-30%	-68%								
	20	-6%	-13%	-18%	-28%	-40%	-82%								

Table 4-2 The maximum Δ for different mining conditions after w_p is changed.

H_D (m)	α (°)					
	15	30	45	60	75	90
6	6%	7%	8%	31%	43%	87%
8	0%	0%	0%	0%	0%	0%
10	3%	2%	0%	-3%	-7%	-20%
12	1%	-1%	-5%	-9%	-16%	-36%
15	-5%	-8%	-13%	-20%	-29%	-53%
20	-6%	-11%	-18%	-28%	-40%	-61%

The fitting result is acceptable as shown in Figure 4.7.

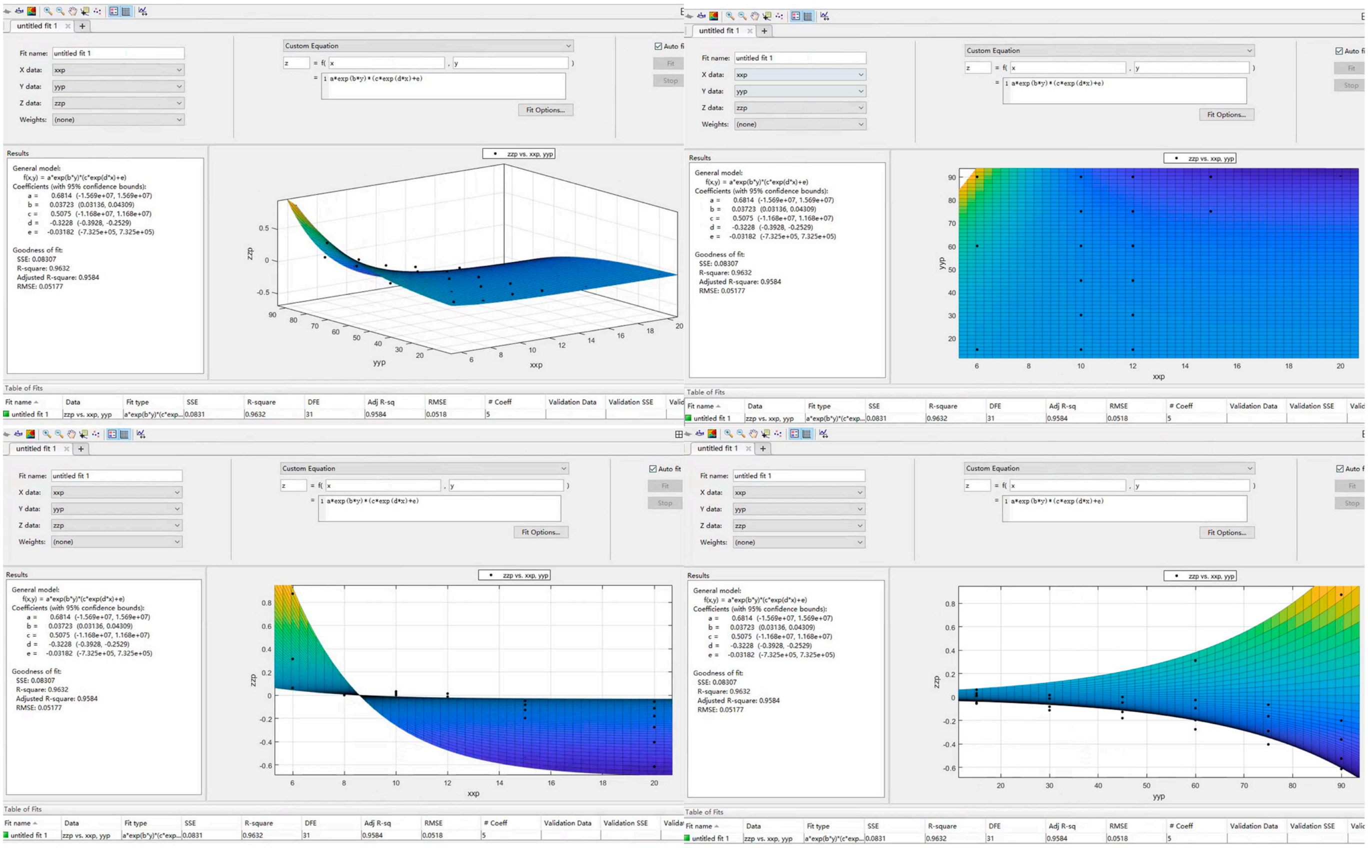
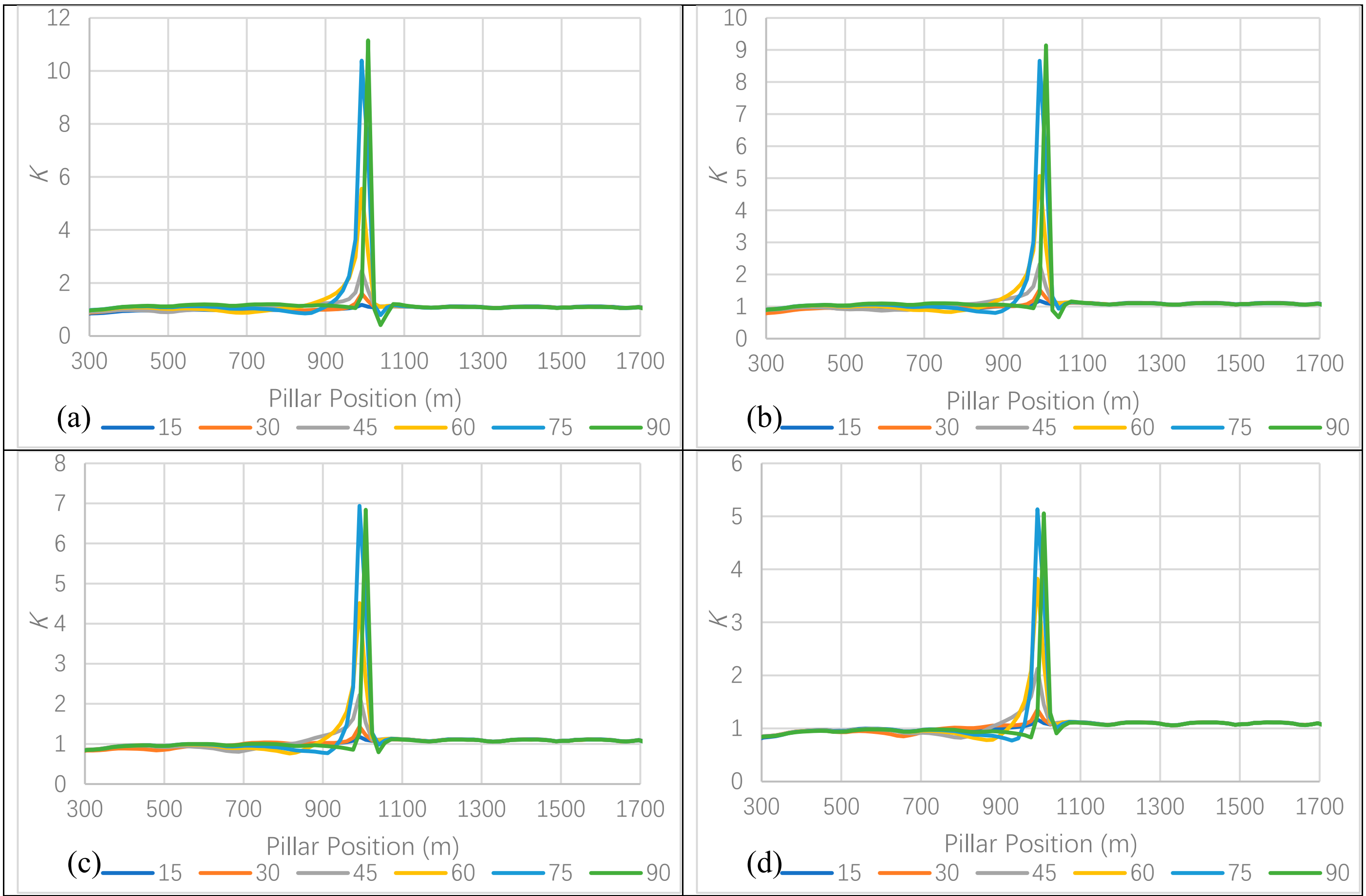


Figure 4.7 Fitting results of Eq. (4-1) by MATLAB

5. The effect of H_B on K

Figures 4.1-4.5 show the effect of H_B. H_B was set to 30 m, 50 m, 80 m, 100 m, 150 m, respectively, and the other parameters were kept fixed as mentioned in Section 3. 1. The distribution law of K has been discussed in Section 3.3.



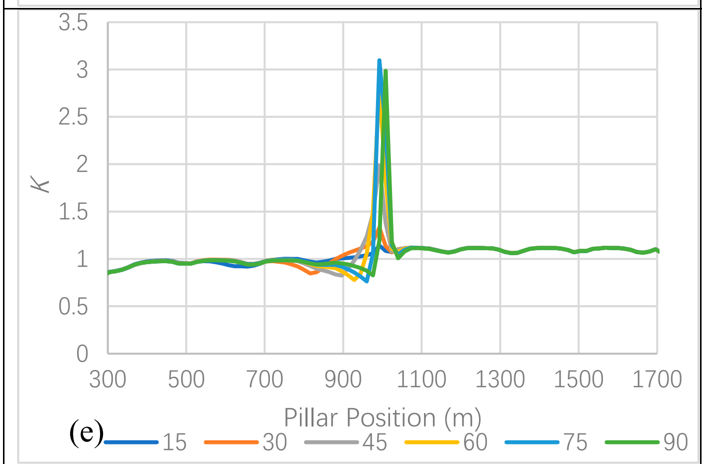
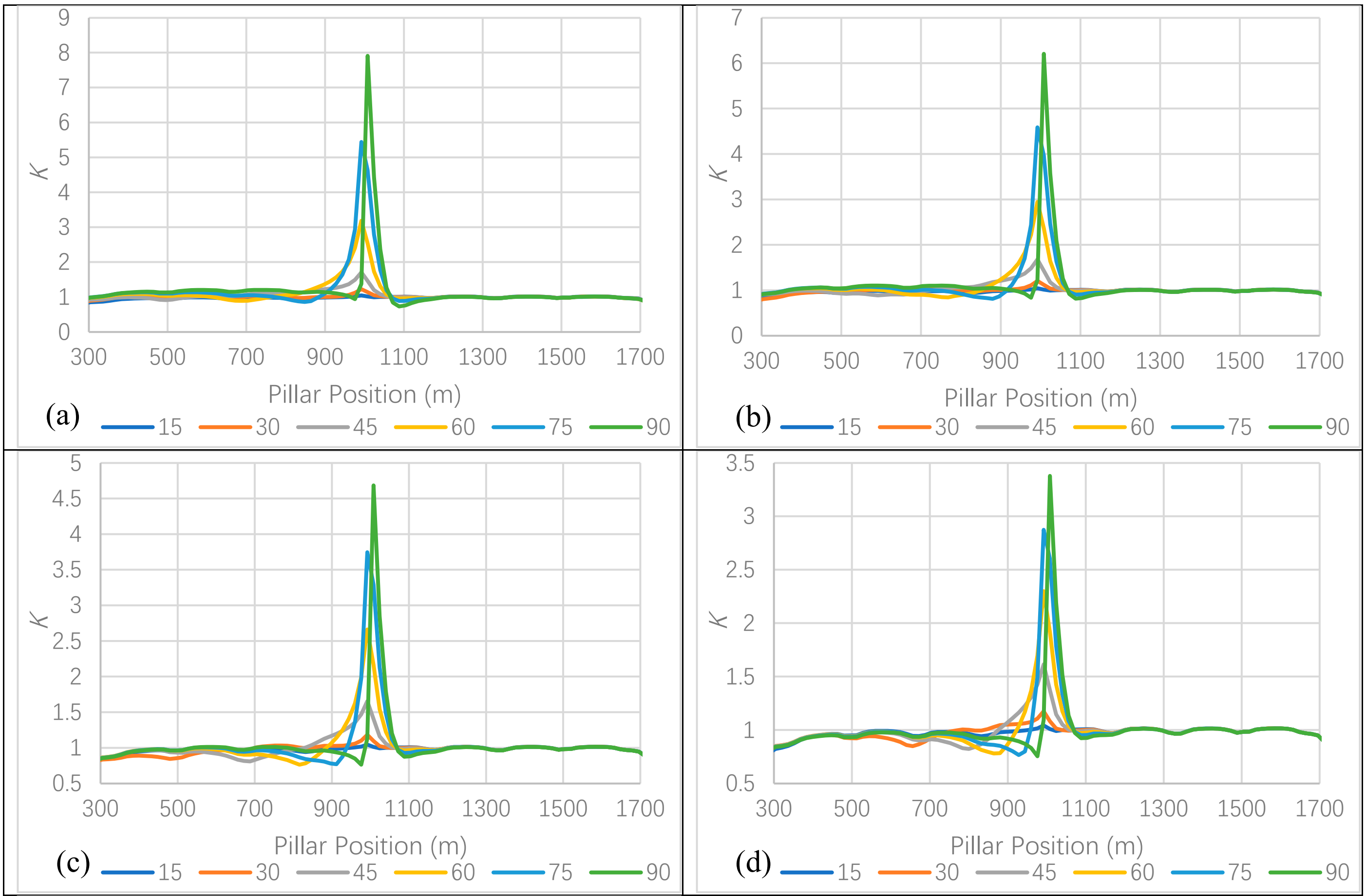


Figure 5.1 Effect of H_b on K when H_b=10 m : (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.



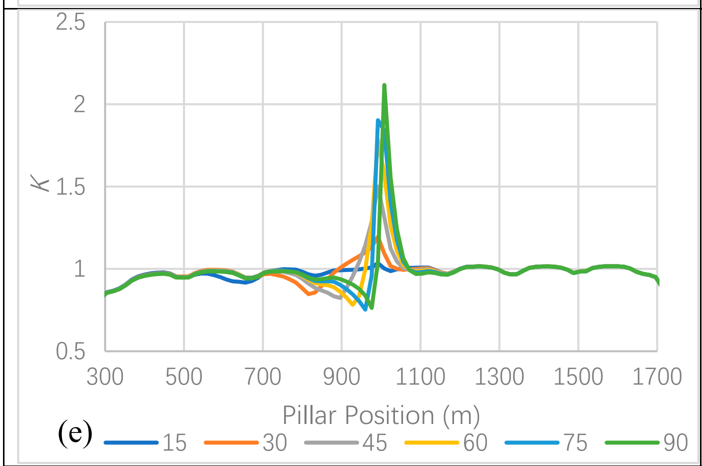


Figure 5.2 Effect of H_b on K when H_b=30 m : (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.

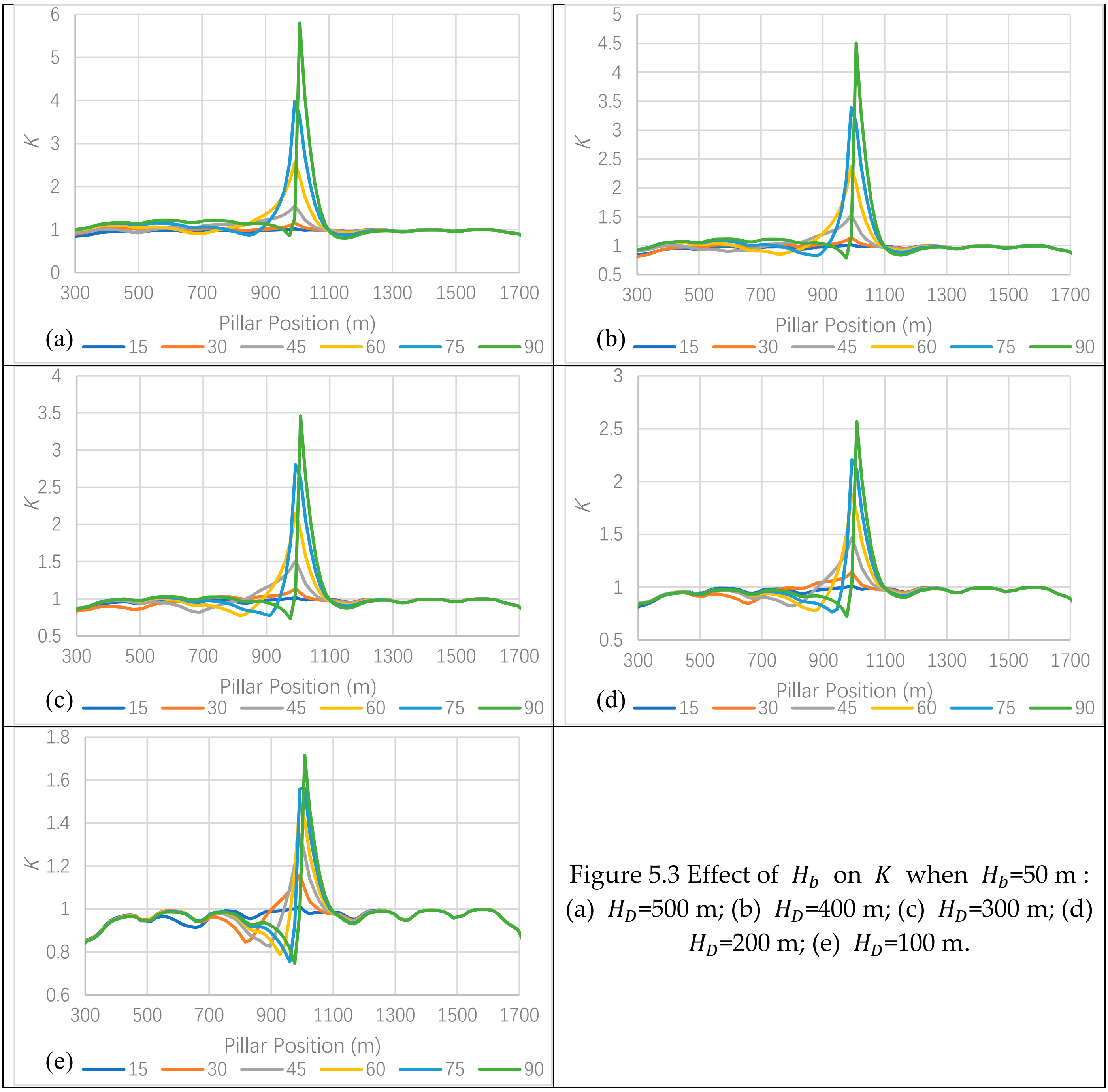


Figure 5.3 Effect of H_b on K when H_b=50 m : (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.

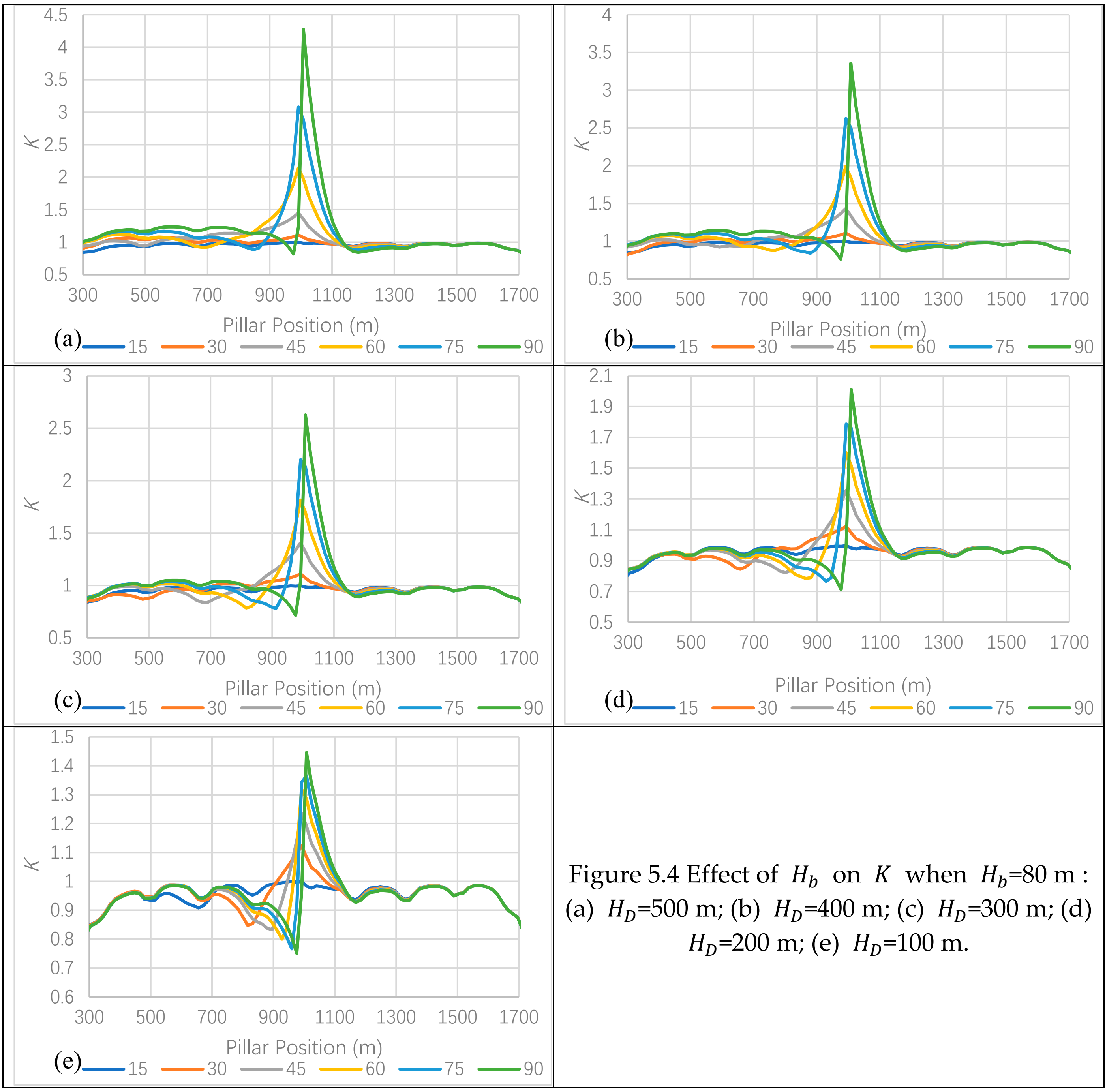


Figure 5.4 Effect of H_b on K when H_b=80 m : (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.

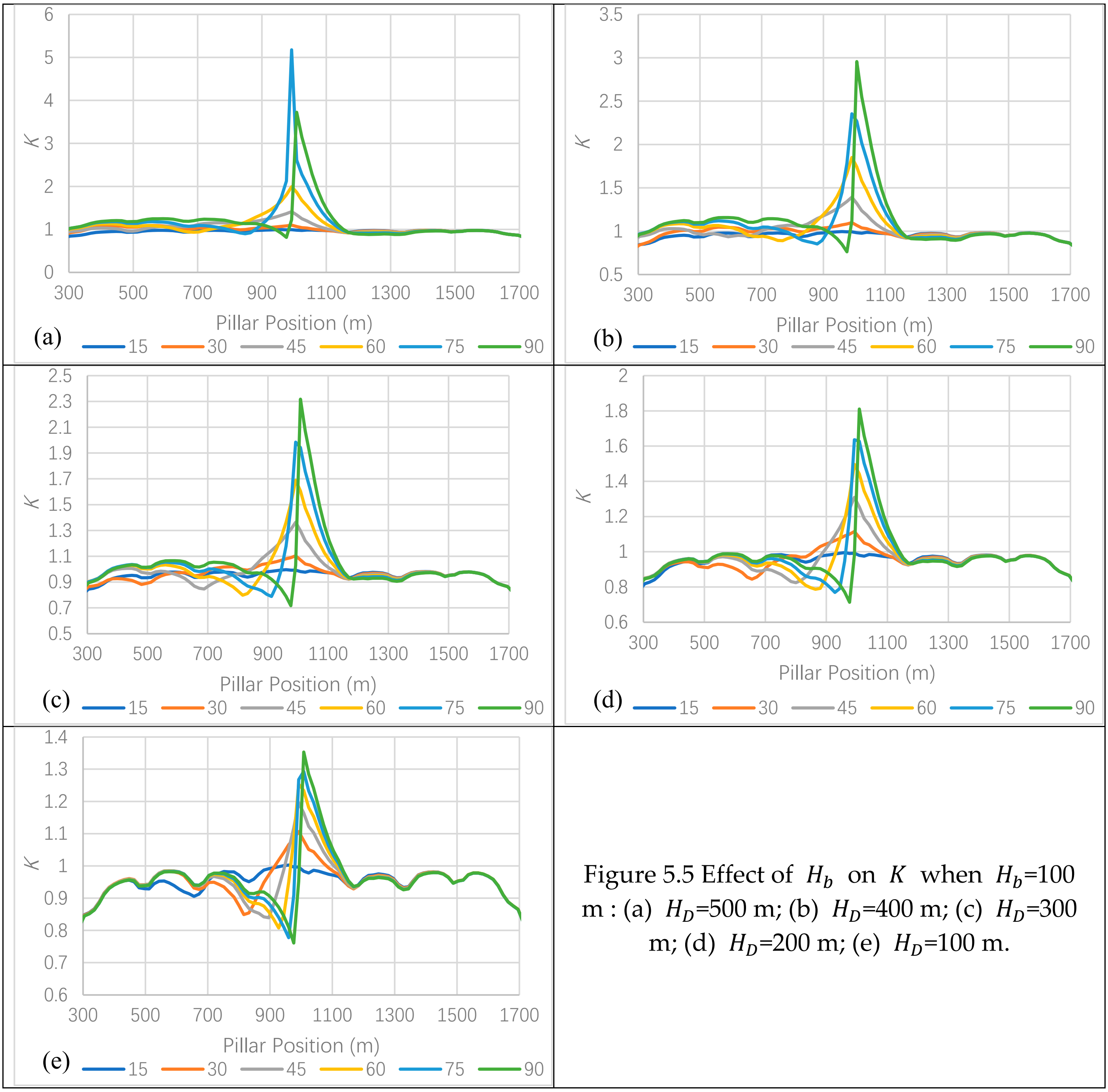
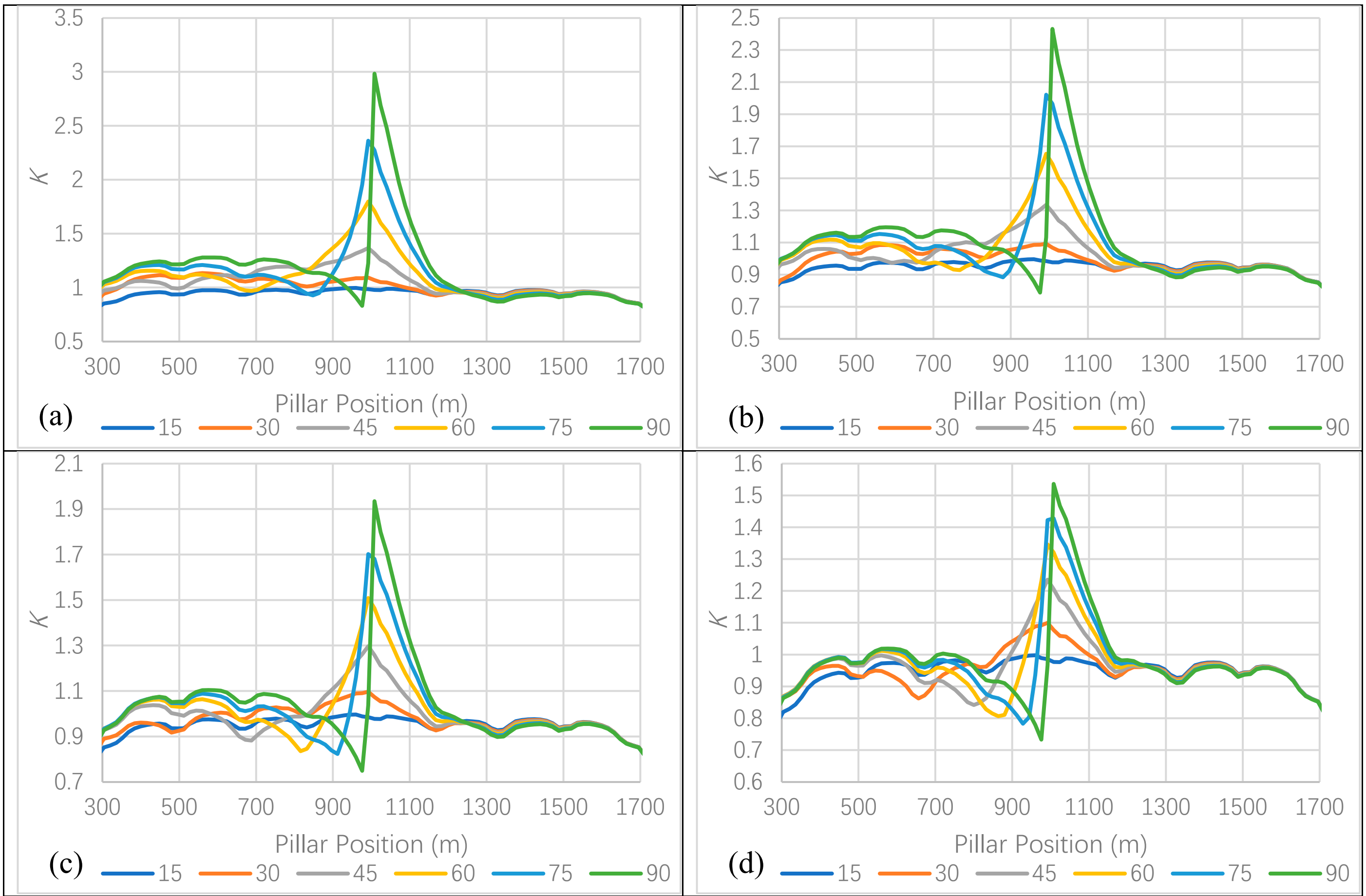


Figure 5.5 Effect of H_b on K when H_b=100 m : (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.



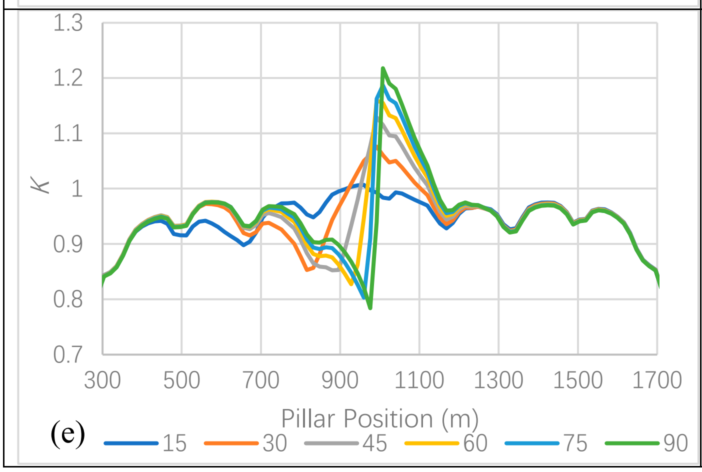


Figure 5.6 Effect of H_b on K when H_b=150 m : (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.

The Δ after H_B is changed, is calculated by Eq. (2-1), and the results are listed in Table 5-1. As mentioned before, the effect of H_D can be neglected by selecting the maximum Δ to represent the Δ of different H_D. Table 5-1 is changed to Table 5-2. Regression analysis on Table 5-2 shows that the effect of H_B on Δ can be expressed by:

$$\Delta_{\mathrm{B}} = (0.57e^{-0.04379H_B} - 0.3705)[1 - 0.6733\sin(2.353 - 3.044\alpha)] \quad (5\text{-}1)$$

where Δ_B is the growth rate of K_m when H_B is changed; the RMSE is 0.02992 and the R^2 is 0.9798.

Table 5-1 The Δ for different mining conditions after H_B is changed.

H_D (m)	H_B (m)	α (°)						H_D (m)	H_B (m)	α (°)					
		15	30	45	60	75	90			15	30	45	60	75	90
500	10	0%	0%	0%	0%	0%	0%	200	10	0%	0%	0%	0%	0%	0%
	30	-11%	-24%	-30%	-43%	-48%	-29%		30	-11%	-13%	-24%	-40%	-44%	-33%
	50	-14%	-29%	-37%	-54%	-62%	-48%		50	-13%	-16%	-31%	-51%	-57%	-49%
	80	-15%	-31%	-41%	-61%	-70%	-62%		80	-15%	-17%	-36%	-58%	-65%	-60%
	100	-15%	-32%	-42%	-64%	-50%	-67%		100	-15%	-18%	-38%	-61%	-68%	-64%
	150	-15%	-30%	-44%	-68%	-77%	-73%		150	-15%	-19%	-42%	-65%	-72%	-70%
400	10	0%	0%	0%	0%	0%	0%	100	10	0%	0%	0%	0%	0%	0%
	30	-11%	-21%	-28%	-42%	-47%	-32%		30	-9%	-10%	-25%	-37%	-39%	-29%
	50	-14%	-25%	-34%	-53%	-61%	-51%		50	-11%	-13%	-32%	-47%	-50%	-43%
	80	-15%	-28%	-38%	-61%	-70%	-63%		80	-12%	-16%	-38%	-53%	-56%	-52%
	100	-15%	-28%	-40%	-64%	-73%	-68%		100	-12%	-17%	-40%	-56%	-58%	-55%
	150	-15%	-29%	-43%	-67%	-77%	-73%		150	-12%	-19%	-43%	-59%	-62%	-59%
300	10	0%	0%	0%	0%	0%	0%								
	30	-11%	-18%	-25%	-41%	-46%	-32%								
	50	-14%	-21%	-32%	-52%	-60%	-49%								
	80	-15%	-23%	-37%	-60%	-68%	-62%								
	100	-15%	-23%	-39%	-63%	-71%	-66%								
	150	-15%	-24%	-42%	-67%	-75%	-72%								

Table 5-2 The maximum Δ for different mining conditions after H_B is changed.

H_B (m)	α (°)					
	15	30	45	60	75	90
10	0%	0%	0%	0%	0%	0%
30	-9%	-10%	-24%	-37%	-39%	-29%
50	-11%	-13%	-31%	-47%	-50%	-43%
80	-12%	-16%	-36%	-53%	-56%	-52%
100	-12%	-17%	-38%	-56%	-50%	-55%
150	-12%	-19%	-42%	-59%	-62%	-59%

The fitting result is acceptable as shown in Figure 5.7.

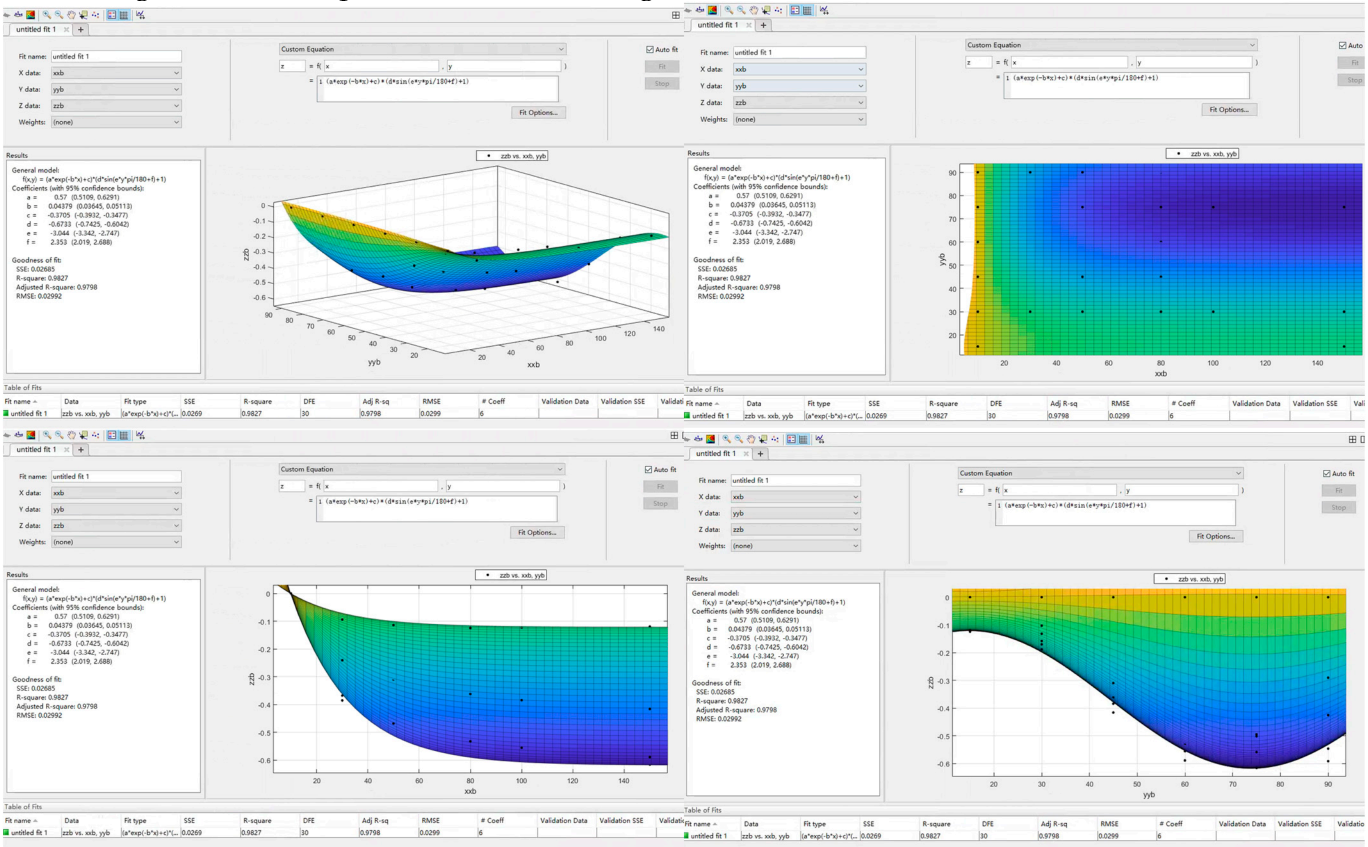
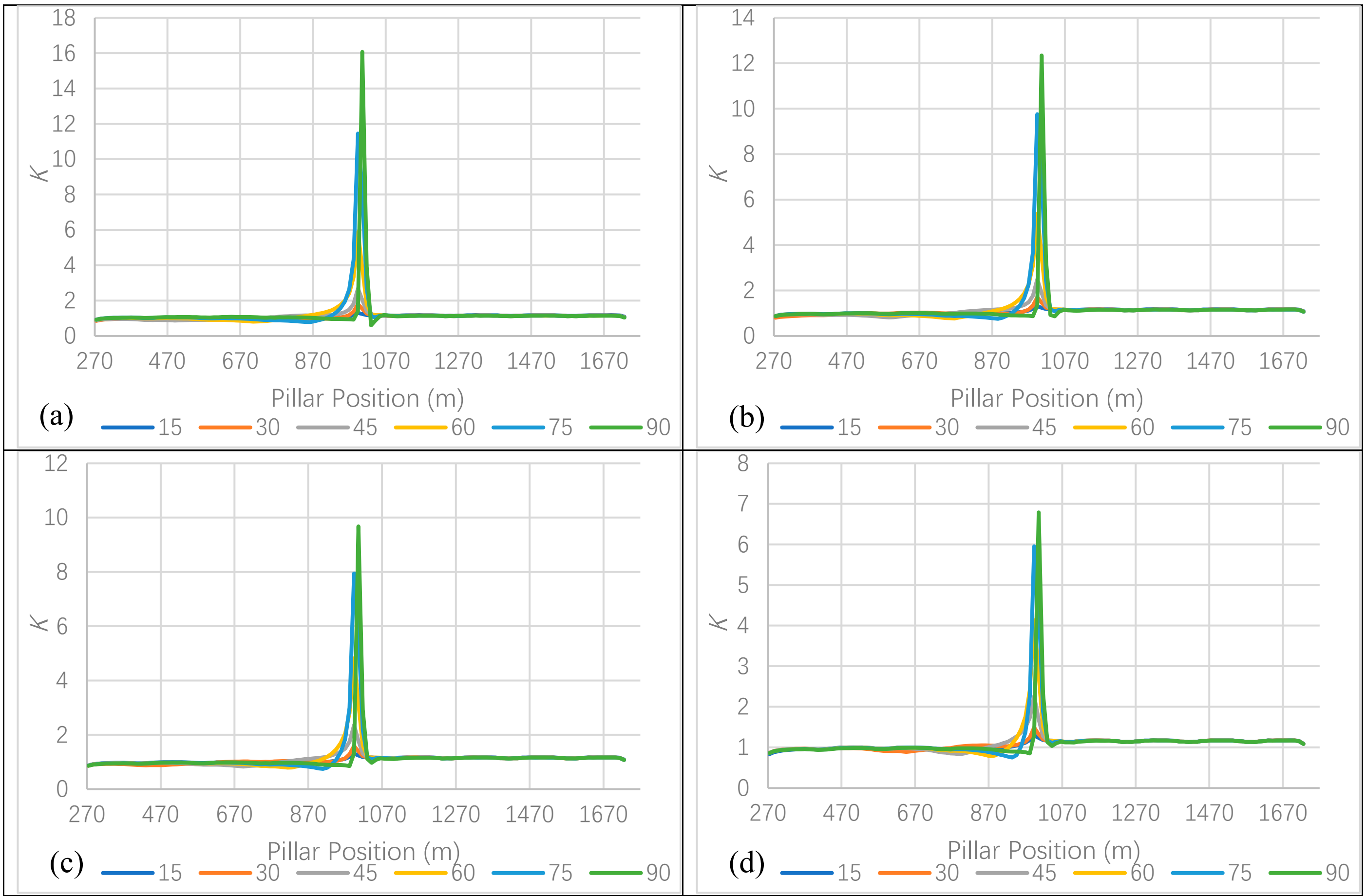


Figure 5.7 Fitting results of Eq. (5-1) by MATLAB

6. The effect of r_m on K

Figures 6.1-6.5 show the effect of r_m. r_m varies from 0.5 to 1.75, and the other parameters were kept fixed as mentioned in Section 3.1. The distribution law of K has been discussed in Section 3.5.



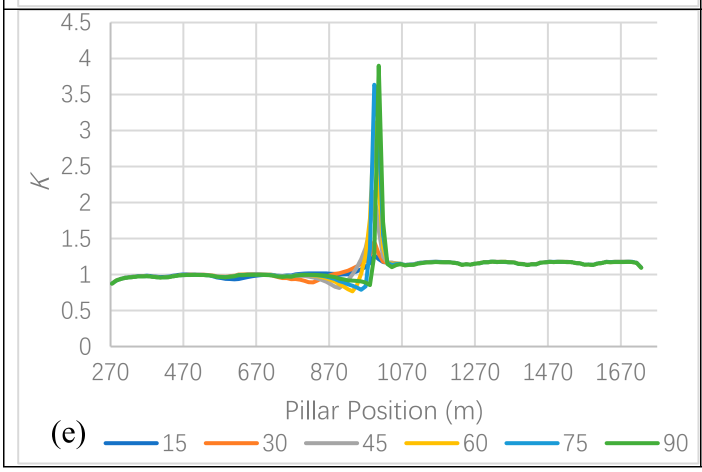


Figure 6.1 Effect of r_m on K when r_m=0.5: (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.

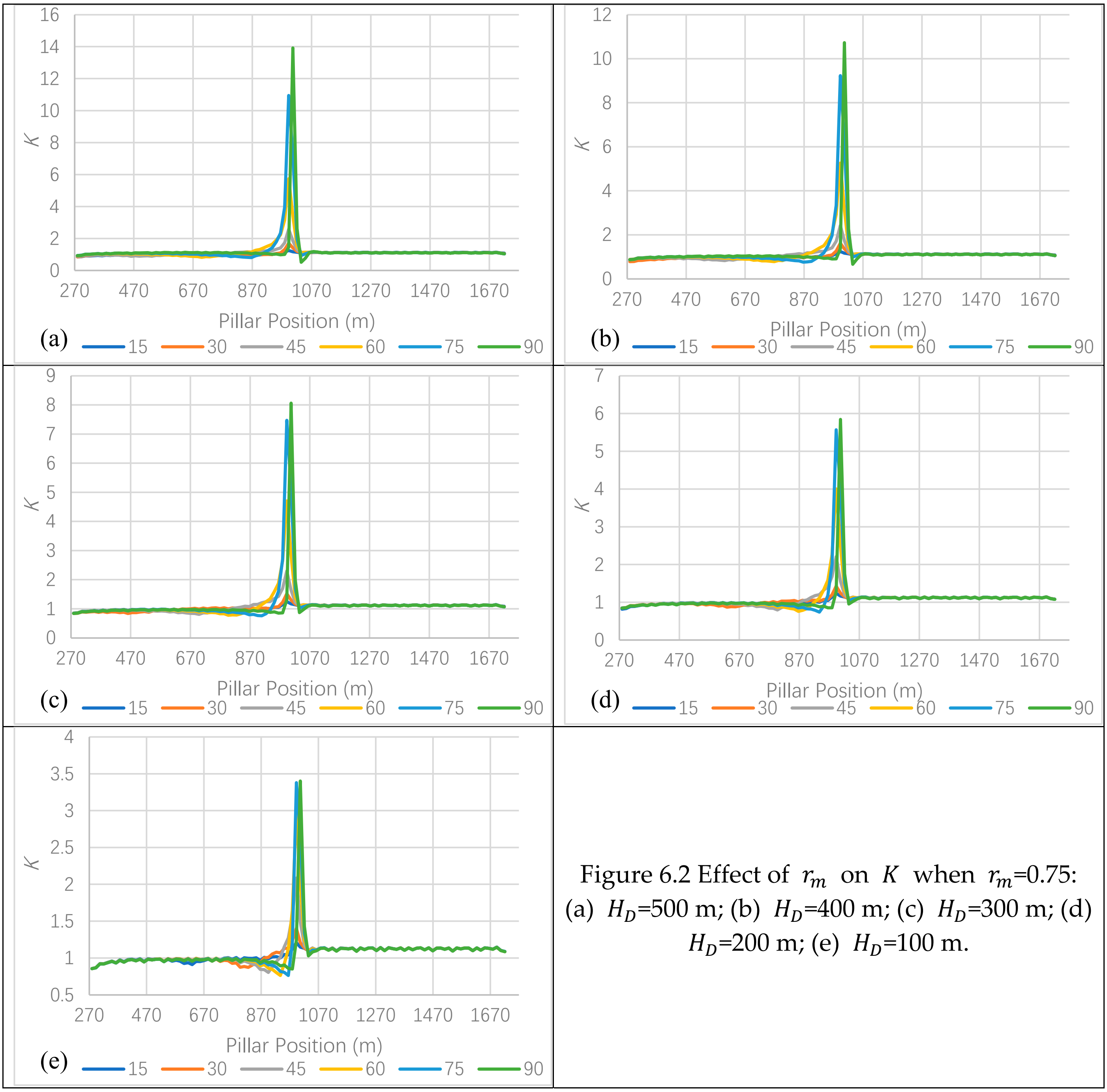


Figure 6.2 Effect of r_m on K when r_m=0.75: (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.

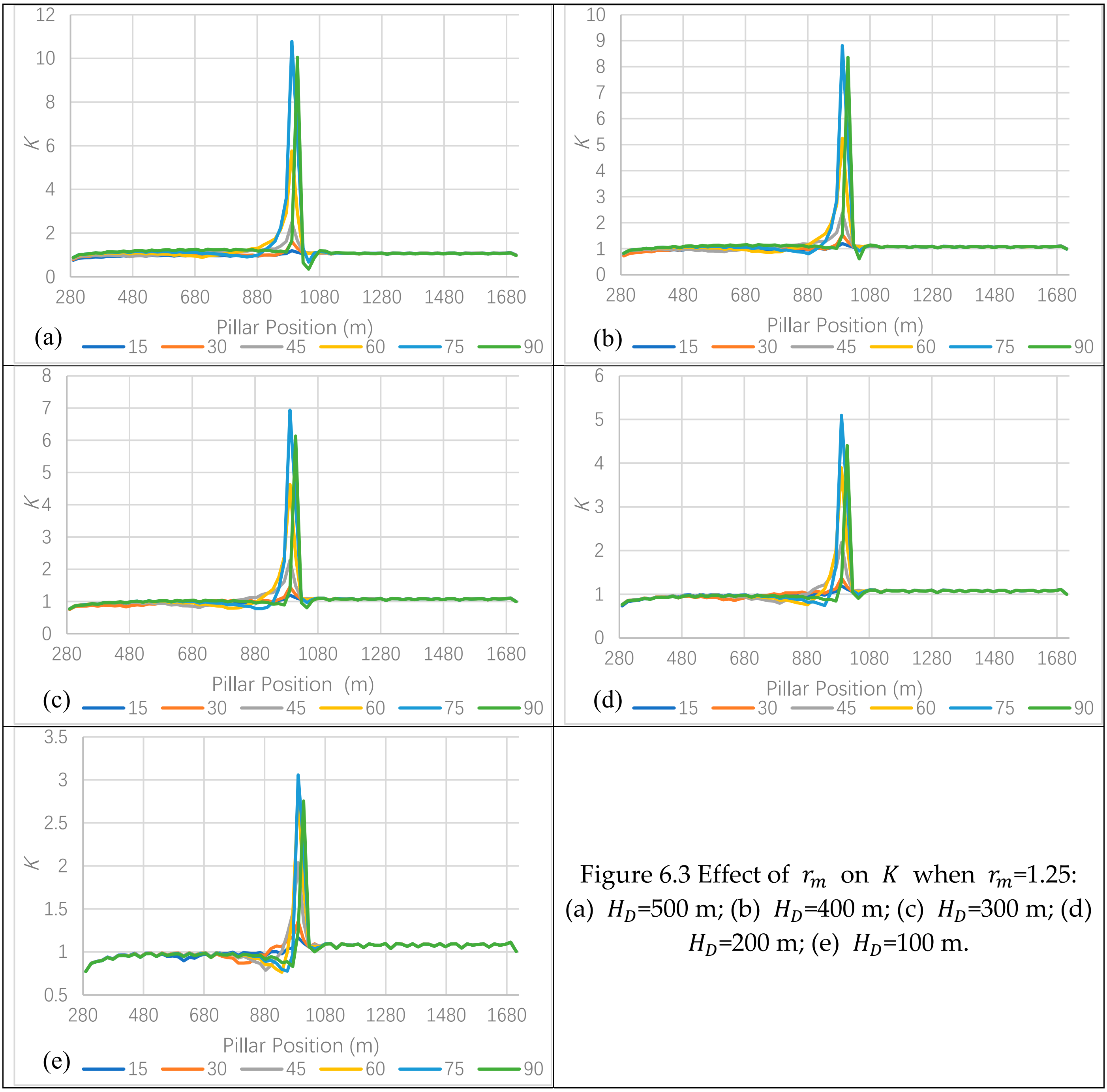


Figure 6.3 Effect of r_m on K when r_m=1.25: (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.

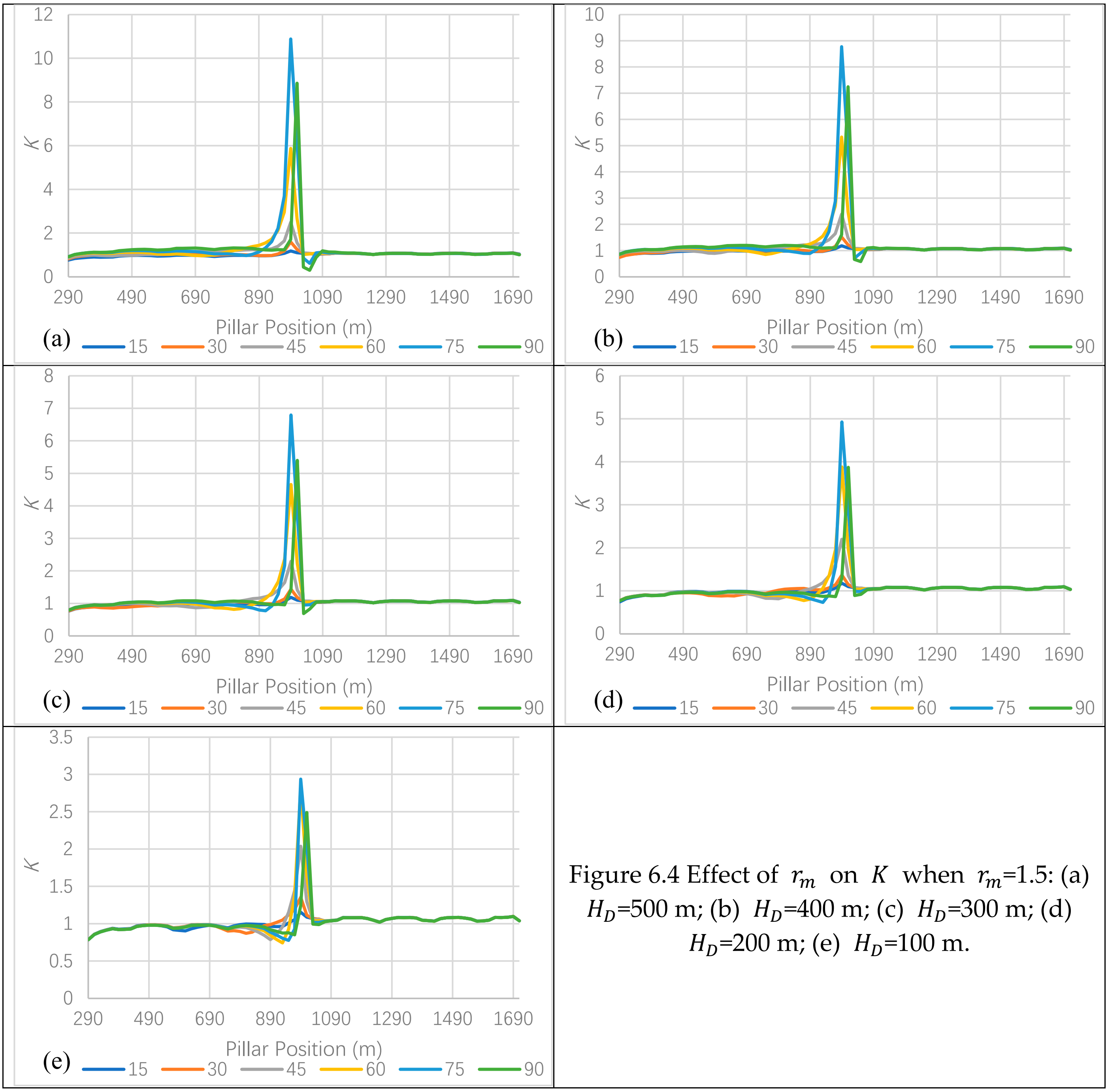
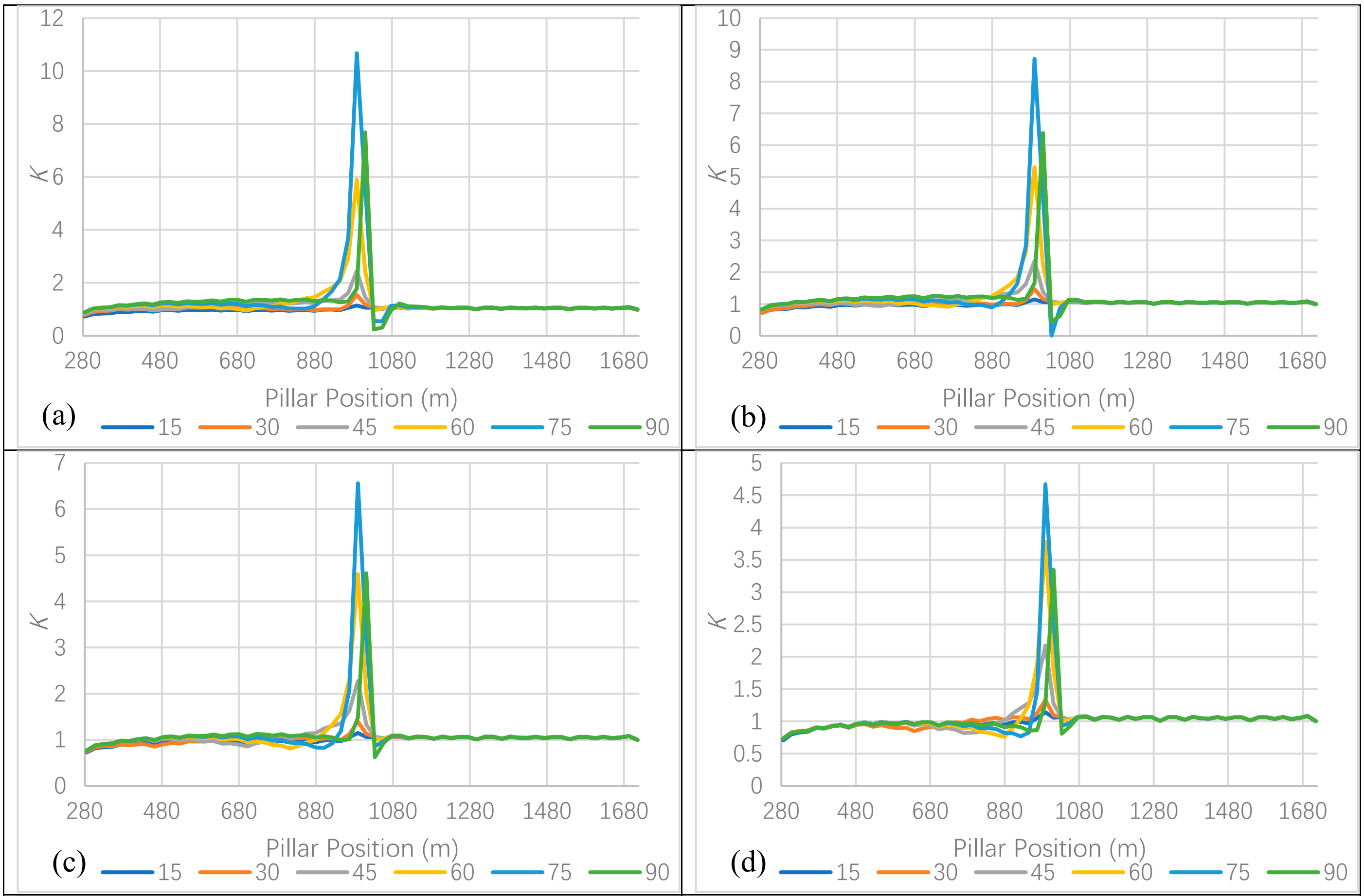


Figure 6.4 Effect of r_m on K when r_m=1.5: (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.



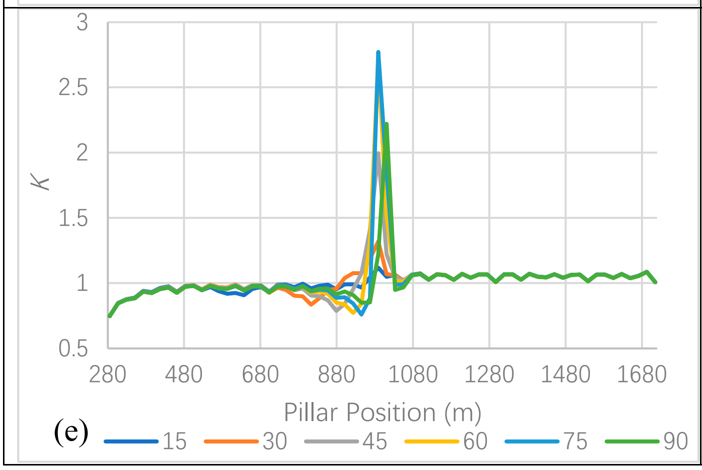


Figure 6.5 Effect of r_m on K when r_m=1.75: (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.

The Δ after r_m is changed, is calculated by Eq. (2-1), and the results are listed in Table 6-1. As mentioned before, the effect of H_D can be neglected by selecting the maximum Δ to represent the Δ of different H_D. Table 6-1 is changed to Table 6-2. Regression analysis in Table 6-2 shows that the effect of r_m on Δ can be expressed by:

$$\Delta_r = (0.066\mathrm{e}^{-1.265r_\mathrm{m}} - 0.01883)(1.664 \times 10^{-6}\mathrm{e}^{0.1822\alpha} + 1) + 0.04187/r_\mathrm{m} \quad (6\text{-}1)$$

where Δ_r is the growth rate of K_m when r_m is changed; the RMSE is 0.02892 and the R^2 is 0.9281.

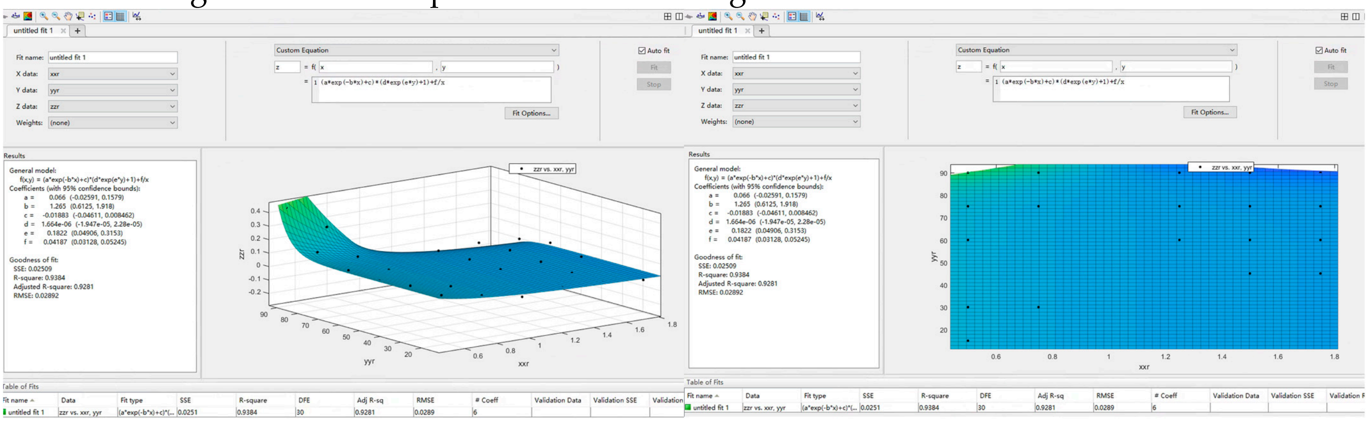
Table 6-1 The Δ for different mining conditions after r_m is changed.

H_D (m)	r_m	α (°)						H_D (m)	r_m	α (°)					
		15	30	45	60	75	90			15	30	45	60	75	90
500	0.5	11%	12%	9%	6%	10%	44%	200	0.5	11%	10%	6%	8%	16%	34%
	0.75	6%	6%	5%	3%	5%	25%		0.75	6%	6%	4%	5%	9%	16%
	1	0%	0%	0%	0%	0%	0%		1	0%	0%	0%	0%	0%	0%
	1.25	2%	1%	1%	4%	4%	-10%		1.25	2%	2%	3%	2%	-1%	-13%
	1.5	1%	-1%	2%	6%	5%	-21%		1.5	1%	1%	3%	2%	-4%	-23%
	1.75	-3%	-4%	0%	6%	3%	-31%		1.75	-2%	-2%	2%	-1%	-9%	-34%
400	0.5	11%	11%	7%	6%	13%	35%	100	0.5	11%	9%	8%	10%	17%	31%
	0.75	6%	6%	4%	4%	7%	17%		0.75	6%	5%	5%	6%	9%	14%
	1	0%	0%	0%	0%	0%	0%		1	0%	0%	0%	0%	0%	0%
	1.25	2%	1%	2%	3%	2%	-9%		1.25	2%	2%	2%	1%	-1%	-8%
	1.5	1%	-1%	3%	5%	1%	-21%		1.5	1%	1%	2%	0%	-5%	-17%
	1.75	-3%	-4%	1%	5%	1%	-30%		1.75	-2%	-1%	0%	-4%	-10%	-26%
300	0.5	11%	11%	6%	7%	14%	41%								
	0.75	6%	6%	4%	4%	8%	18%								
	1	0%	0%	0%	0%	0%	0%								
	1.25	2%	1%	3%	3%	0%	-10%								
	1.5	1%	0%	3%	3%	-2%	-21%								
	1.75	-3%	-3%	2%	2%	-5%	-33%								

Table 6-2 The maximum Δ for different mining conditions after r_m is changed.

	15	30	45	60	75	90
0.5	11%	12%	9%	10%	17%	44%
0.75	6%	6%	5%	6%	9%	25%
1	0%	0%	0%	0%	0%	0%
1.25	2%	2%	3%	4%	4%	-8%
1.5	1%	1%	3%	6%	5%	-17%
1.75	-2%	-1%	2%	6%	3%	-26%

The fitting results are acceptable as shown in Figure 6.6.



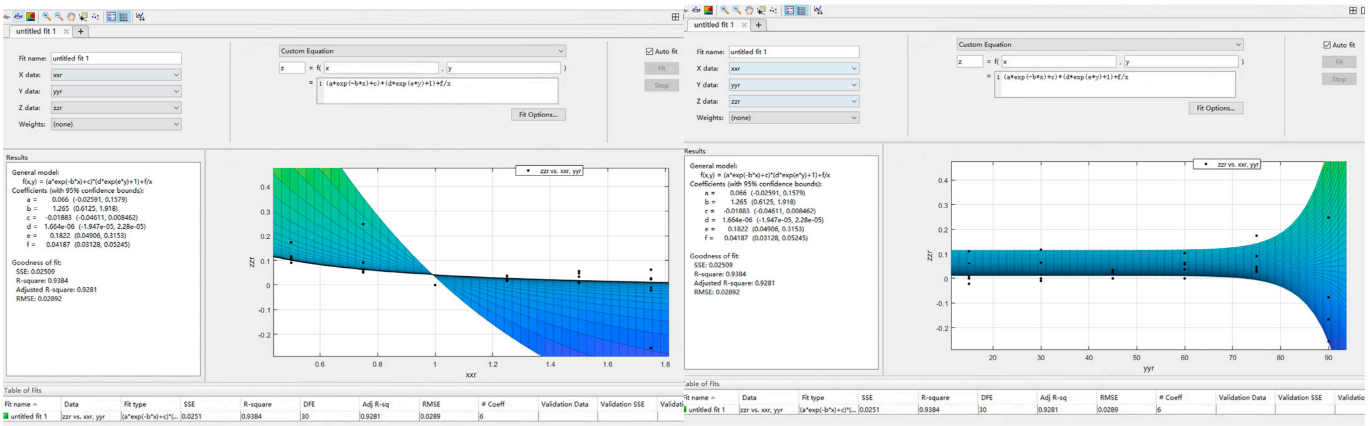
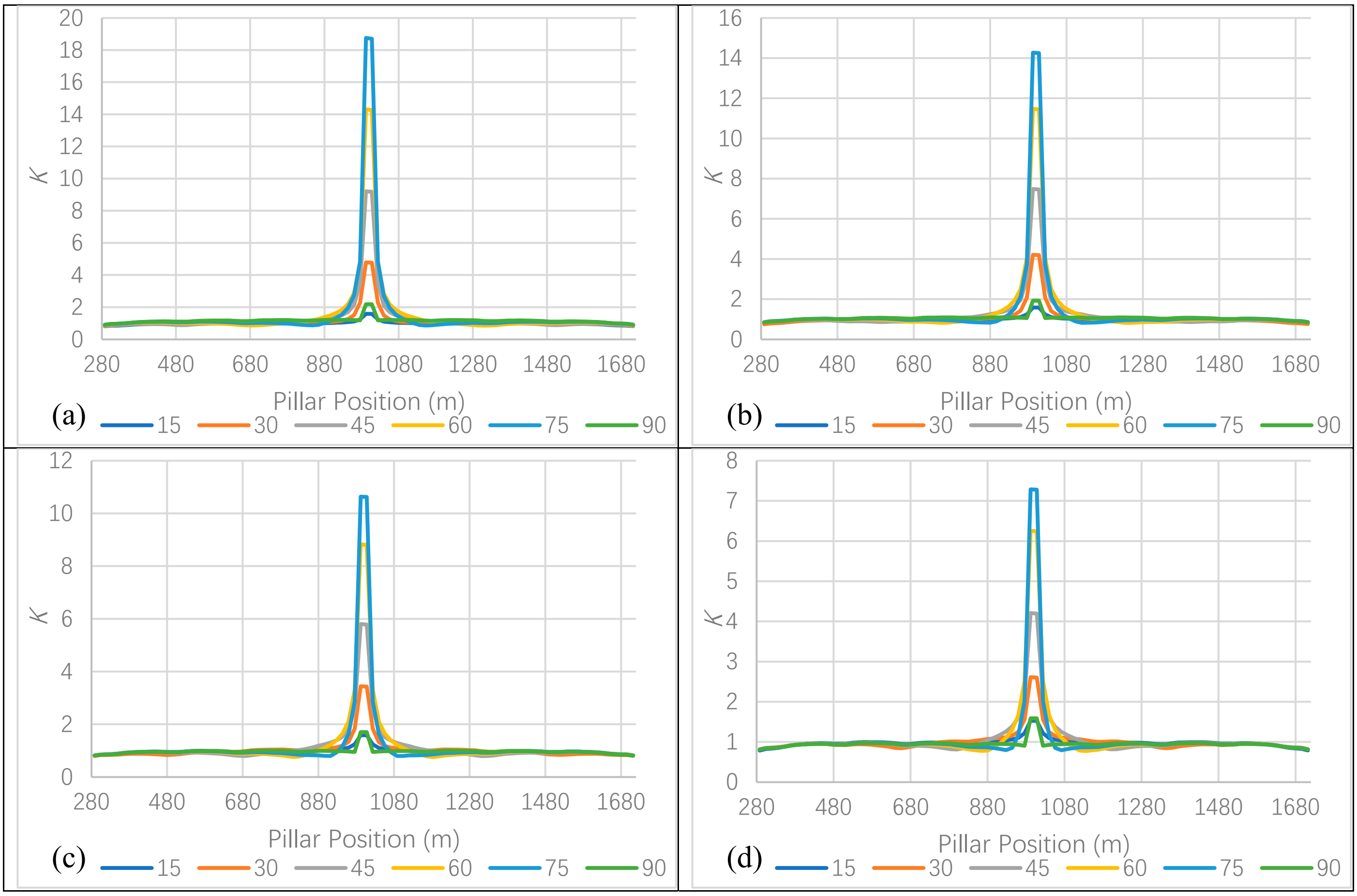


Figure 6.6 Fitting results of Eq. (6-1) by MATLAB

7. The effect of L_v on K

A valley's ground surface is the combination of two single slopes facing each other. There should also exist stress concentrations for a valley terrain. K_{m} has been used to represent the maximum K for a single slope terrain to make an analysis. K_{vm} is used to represent the maximum K for a valley terrain as K_{m} is very large when H_{B} is very small for a single slope. The H_{B} is fixed at 10 m to study the maximum K_{vm} for valley terrain. The other parameters are kept fixed as mentioned in Section 3.1. The valley width L_{v} varies from 16 m to 256 m.



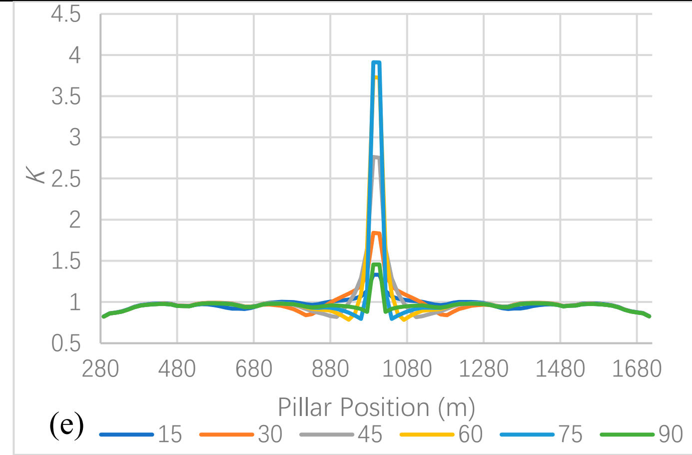
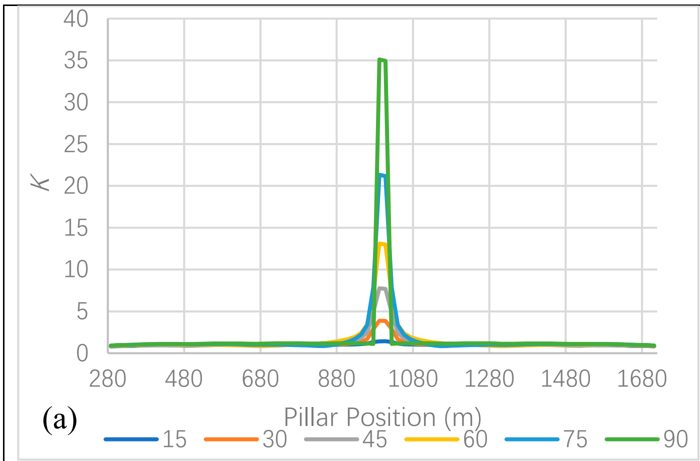
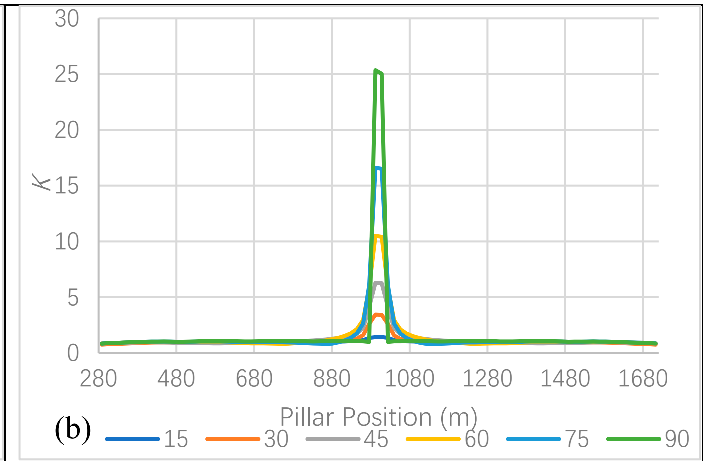
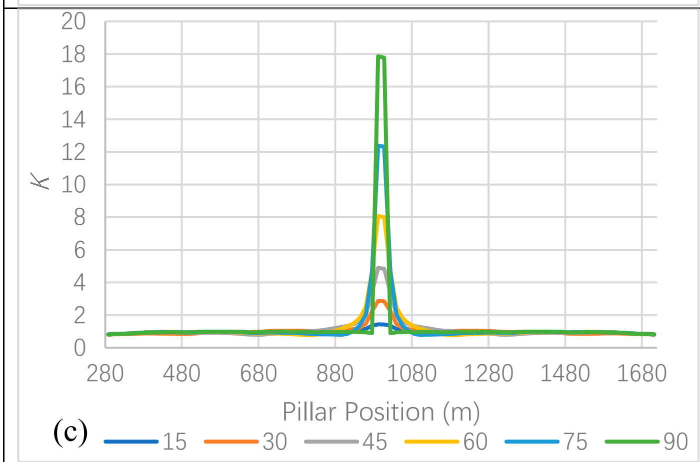
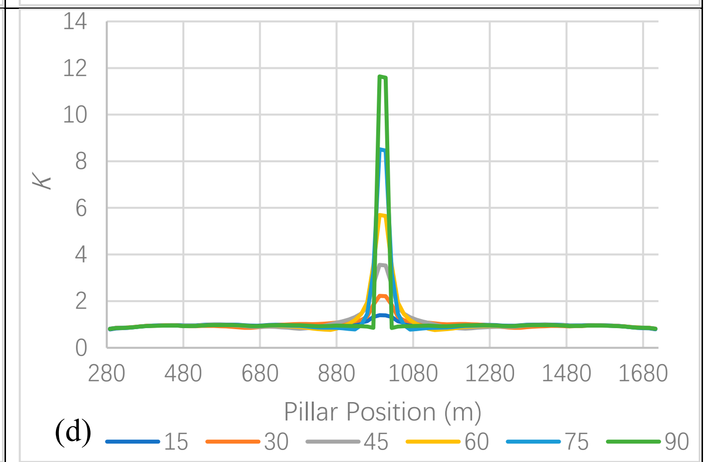


Figure 7.1 Effect of L_v on K when L_v=16 m: (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.









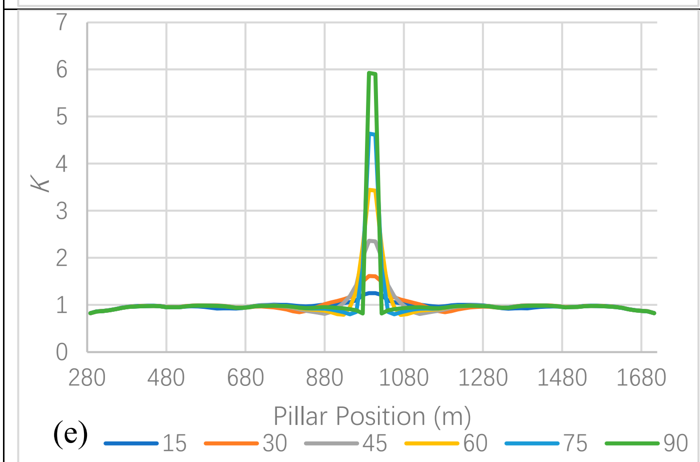
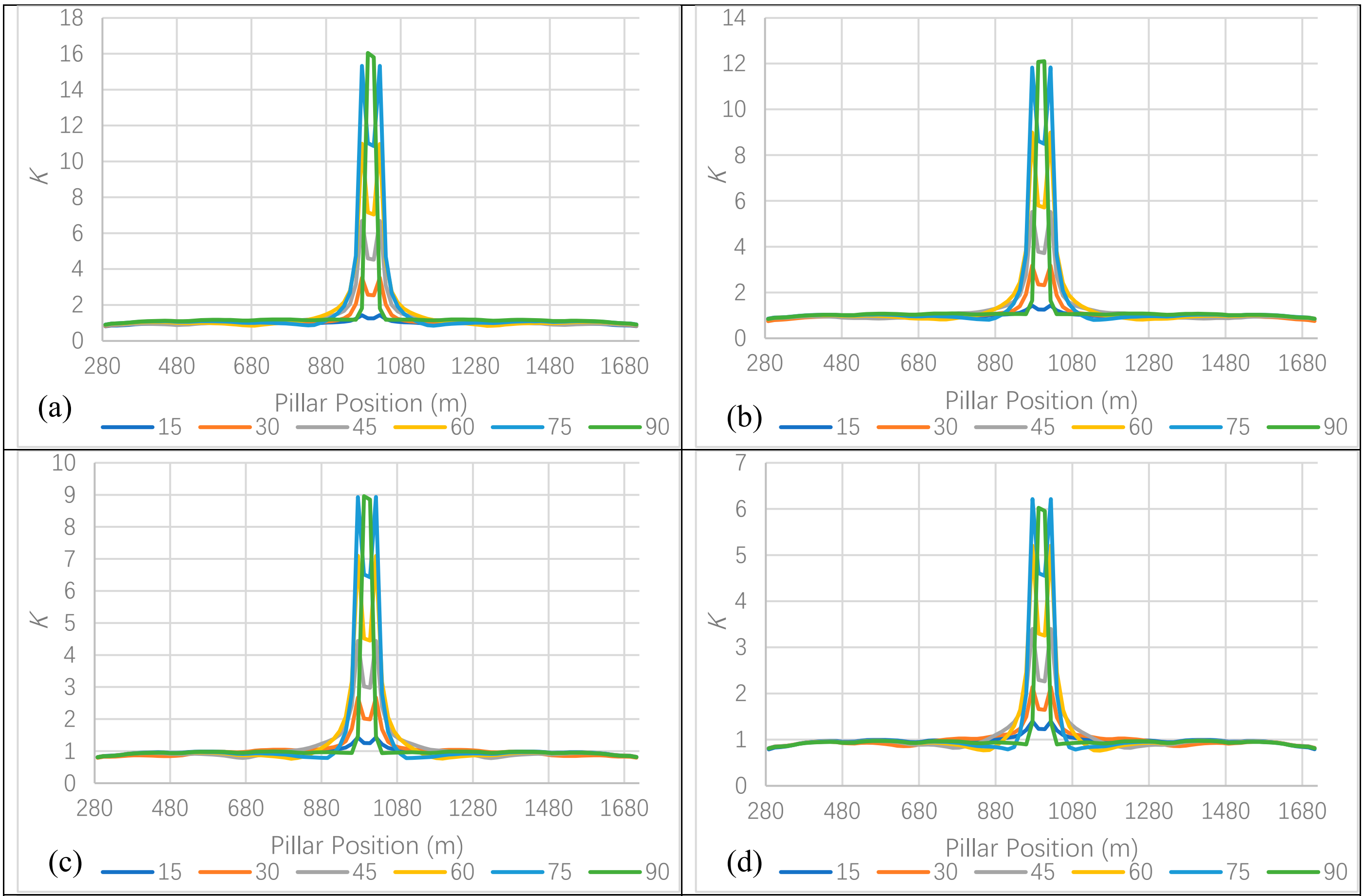


Figure 7.2 Effect of L_v on K when L_v=32 m: (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.



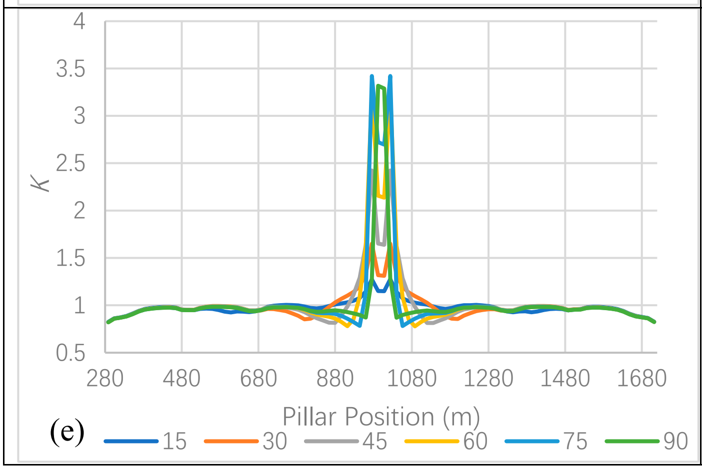
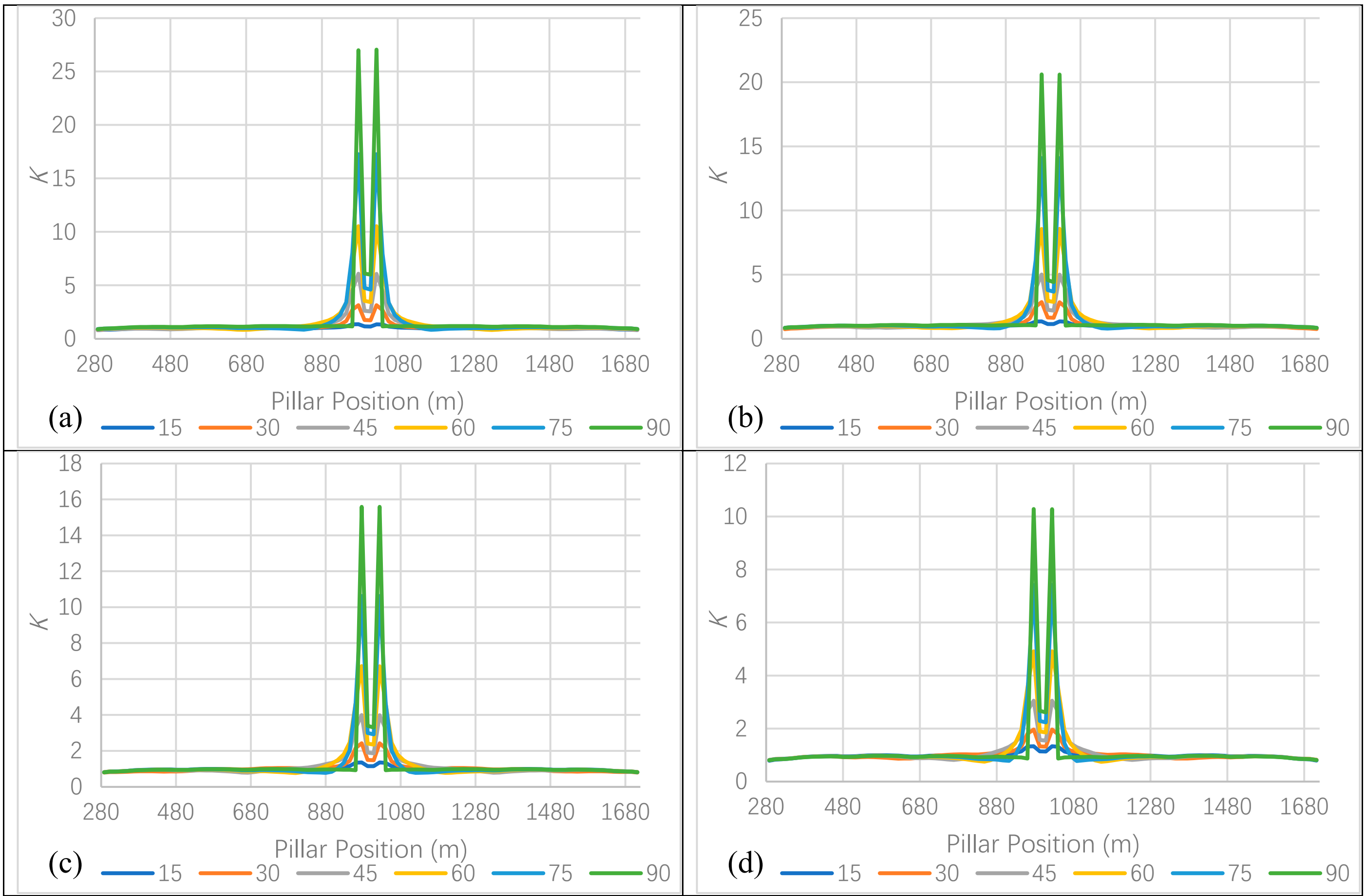


Figure 7.3 Effect of L_v on K when L_v=48 m: (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.



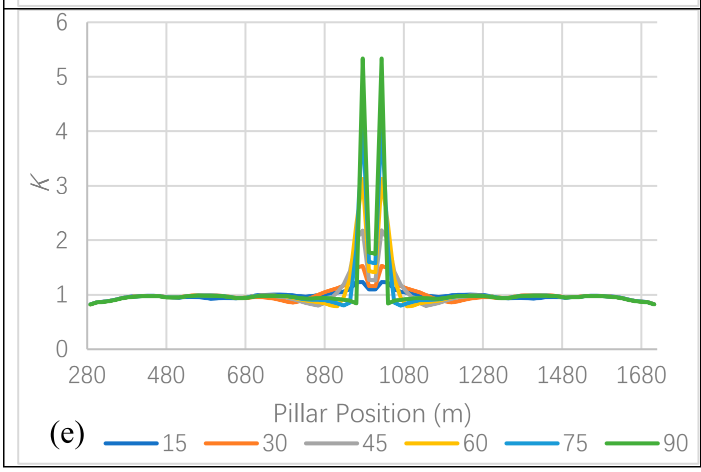


Figure 7.4 Effect of L_v on K when L_v=64 m: (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.

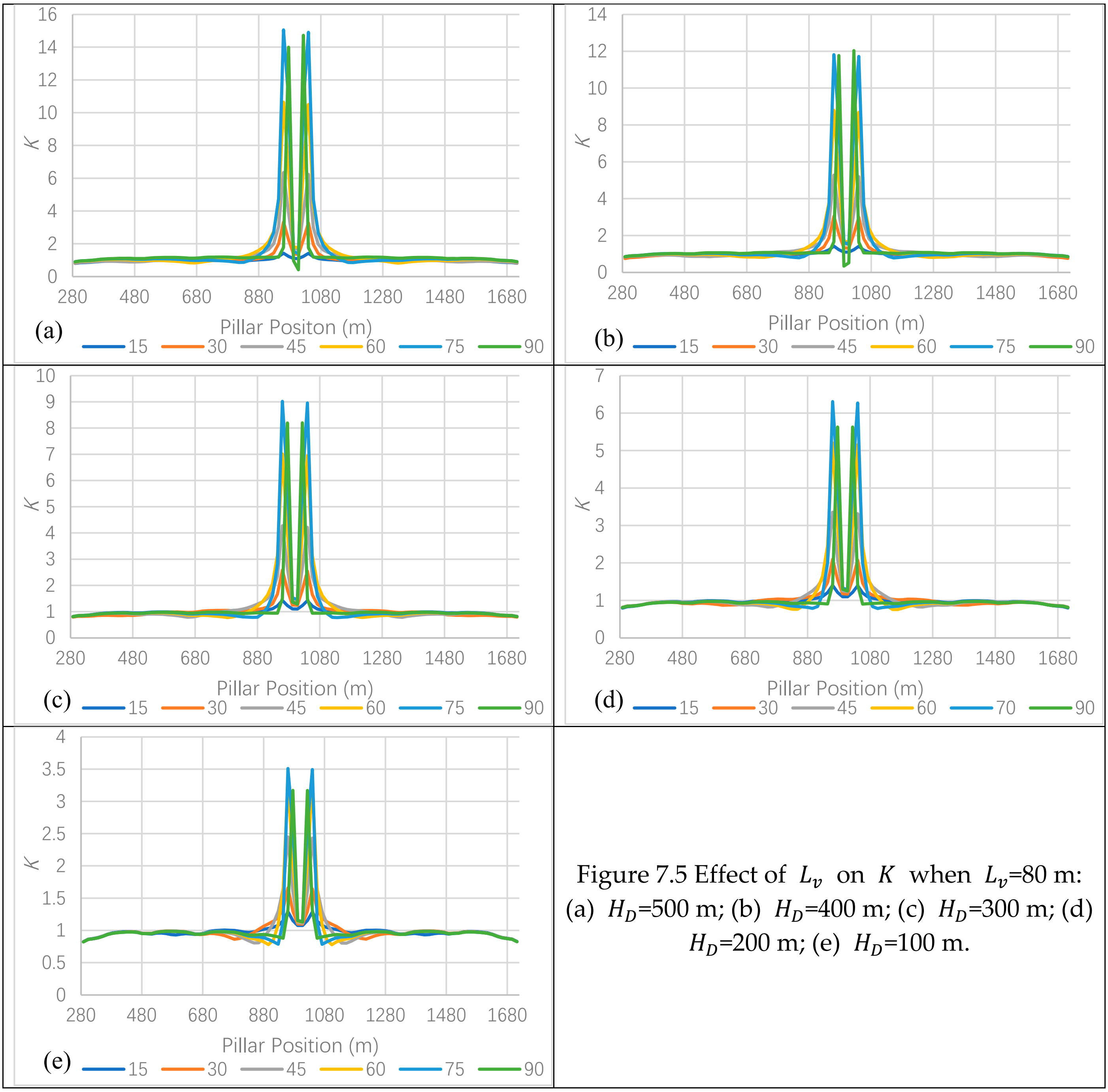
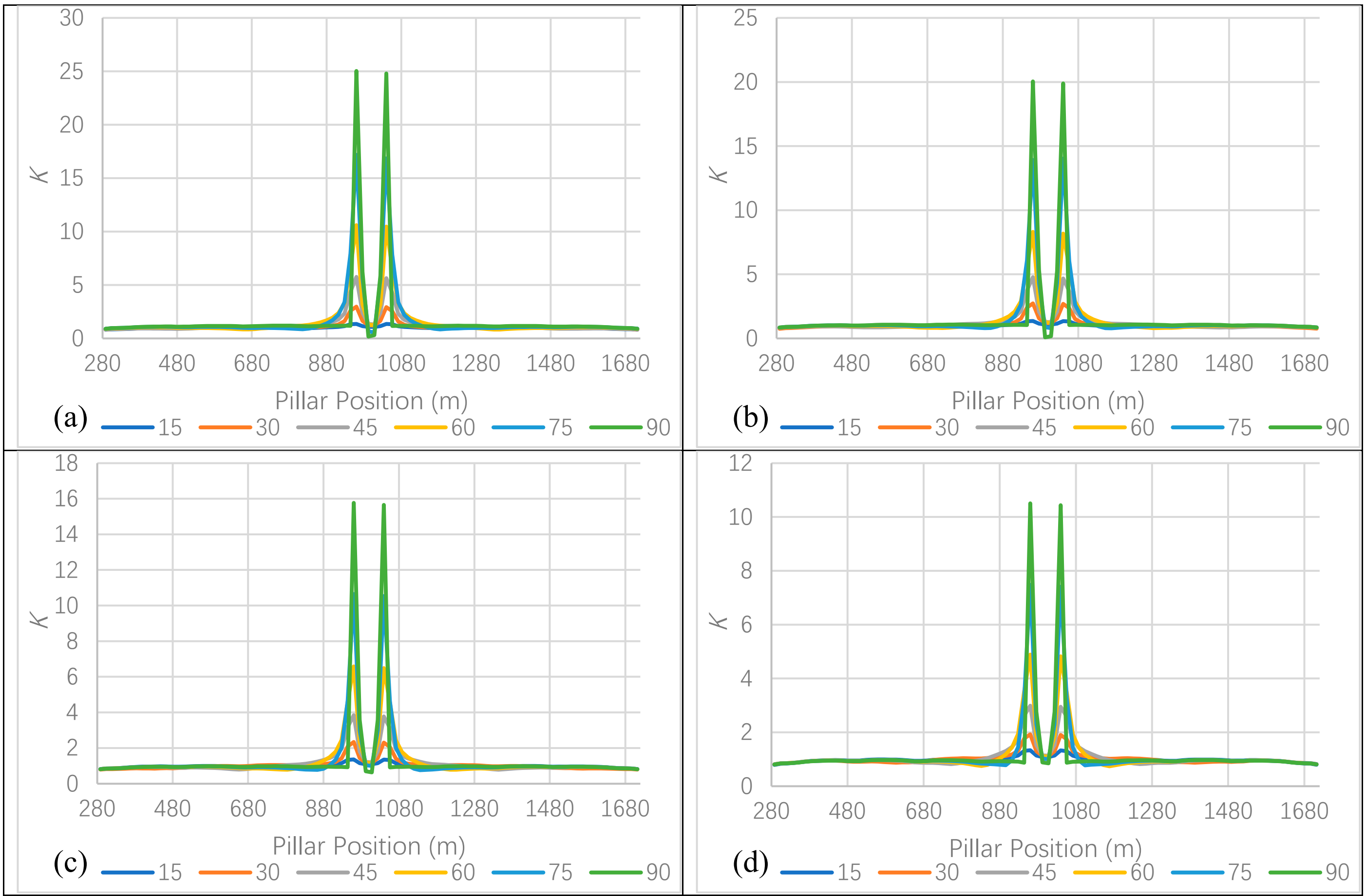


Figure 7.5 Effect of L_v on K when L_v=80 m: (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.



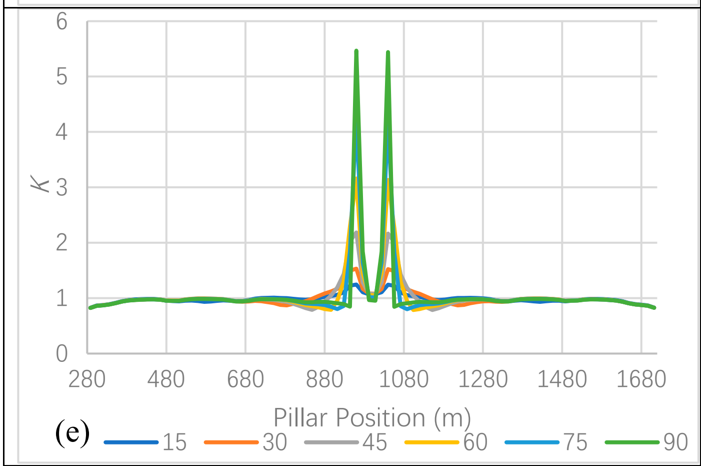
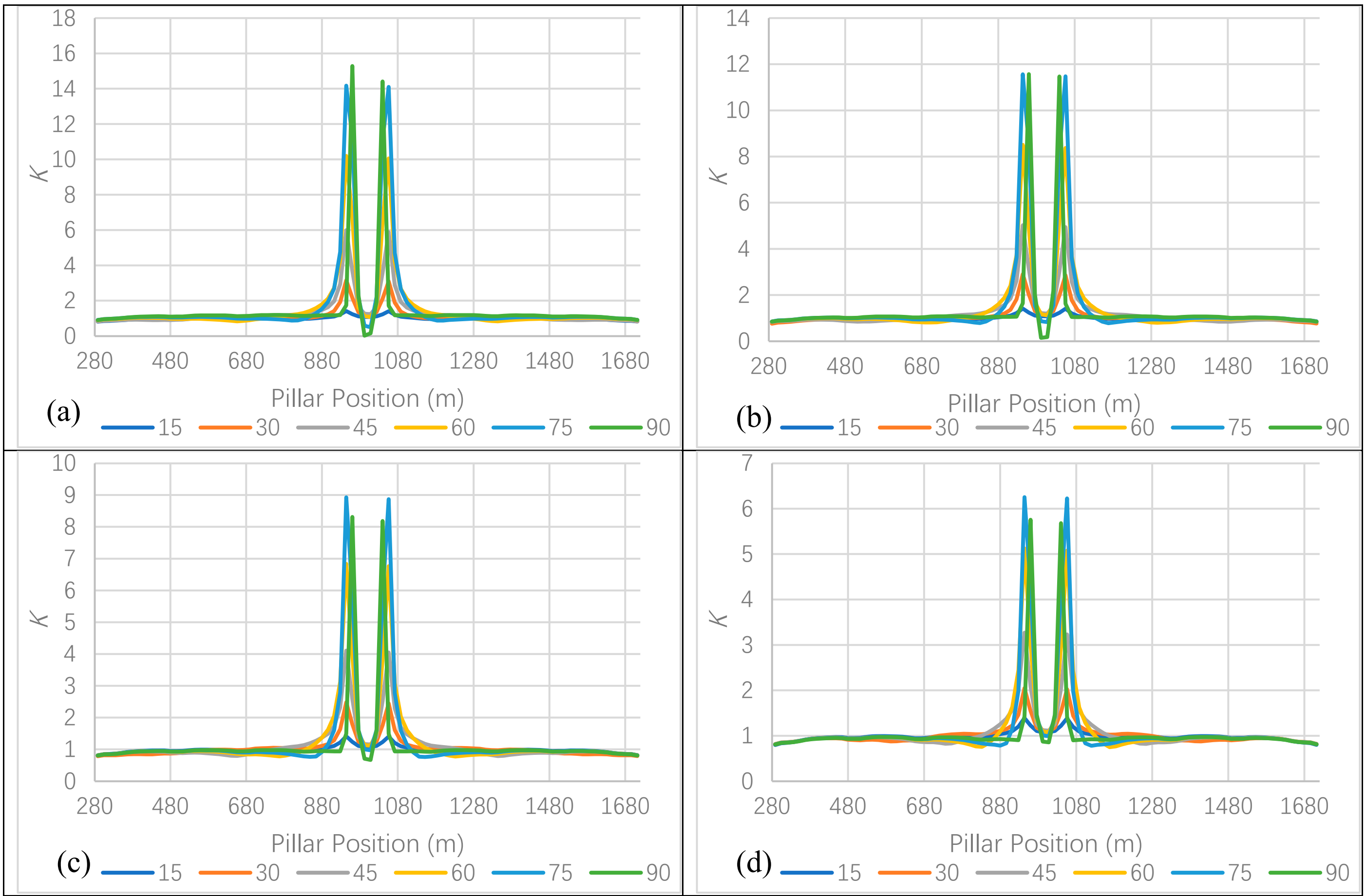


Figure 7.6 Effect of L_v on K when L_v=96 m: (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.



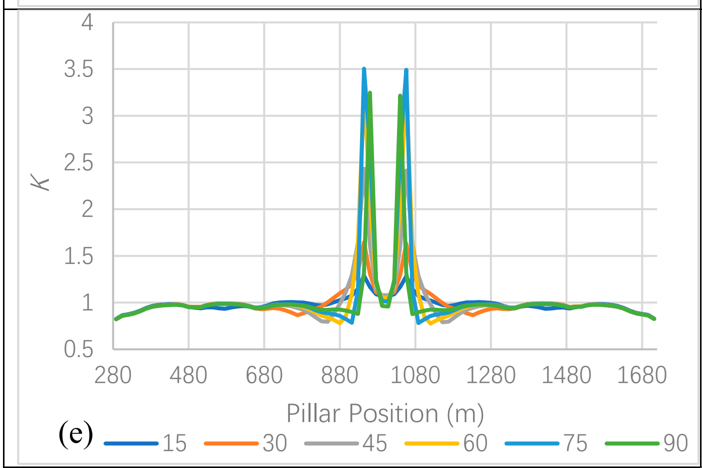
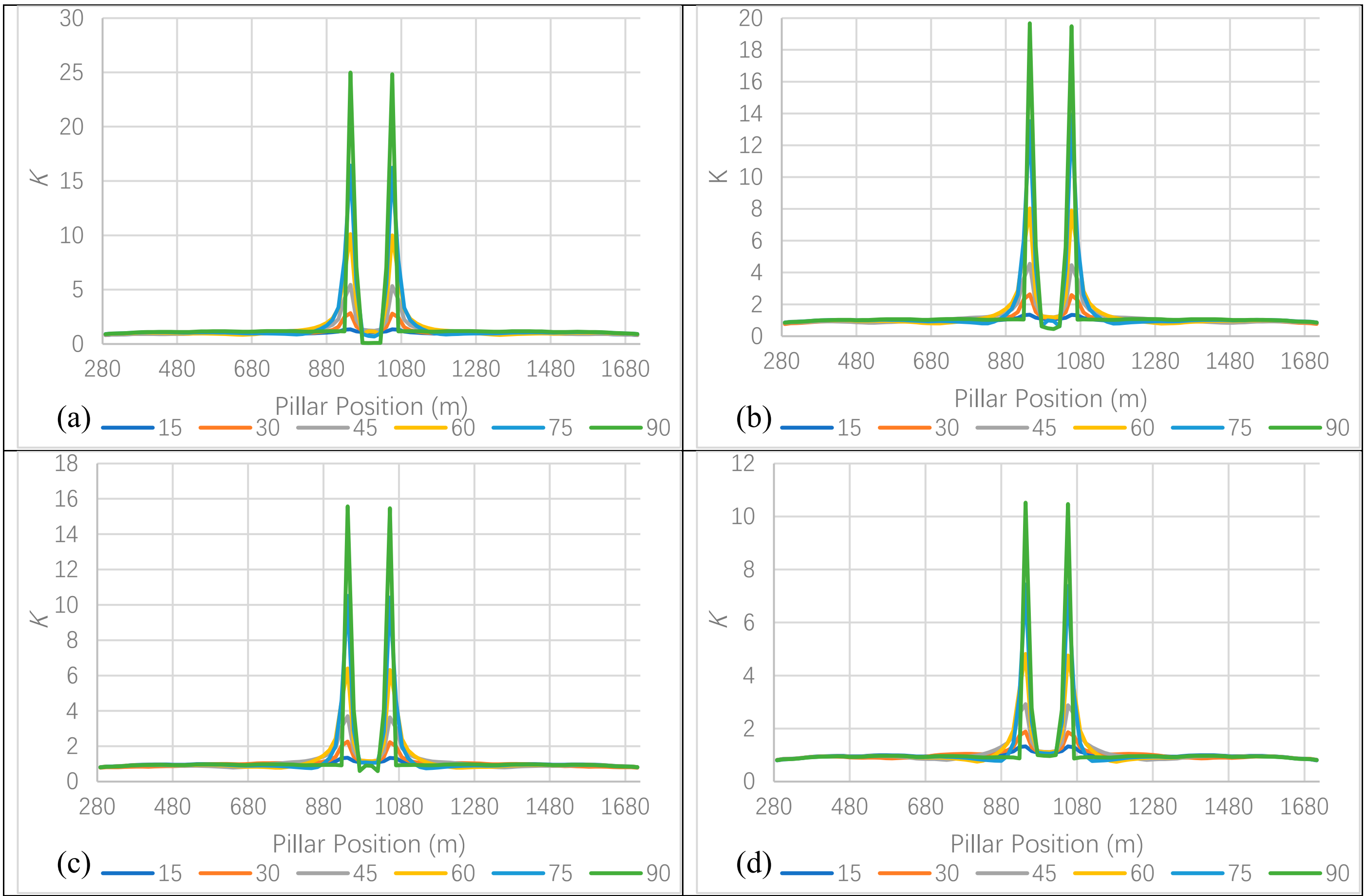


Figure 7.7 Effect of L_v on K when L_v=112 m: (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.



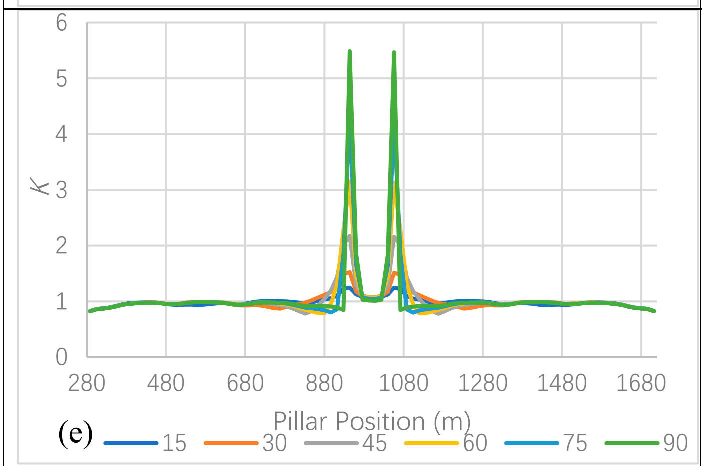
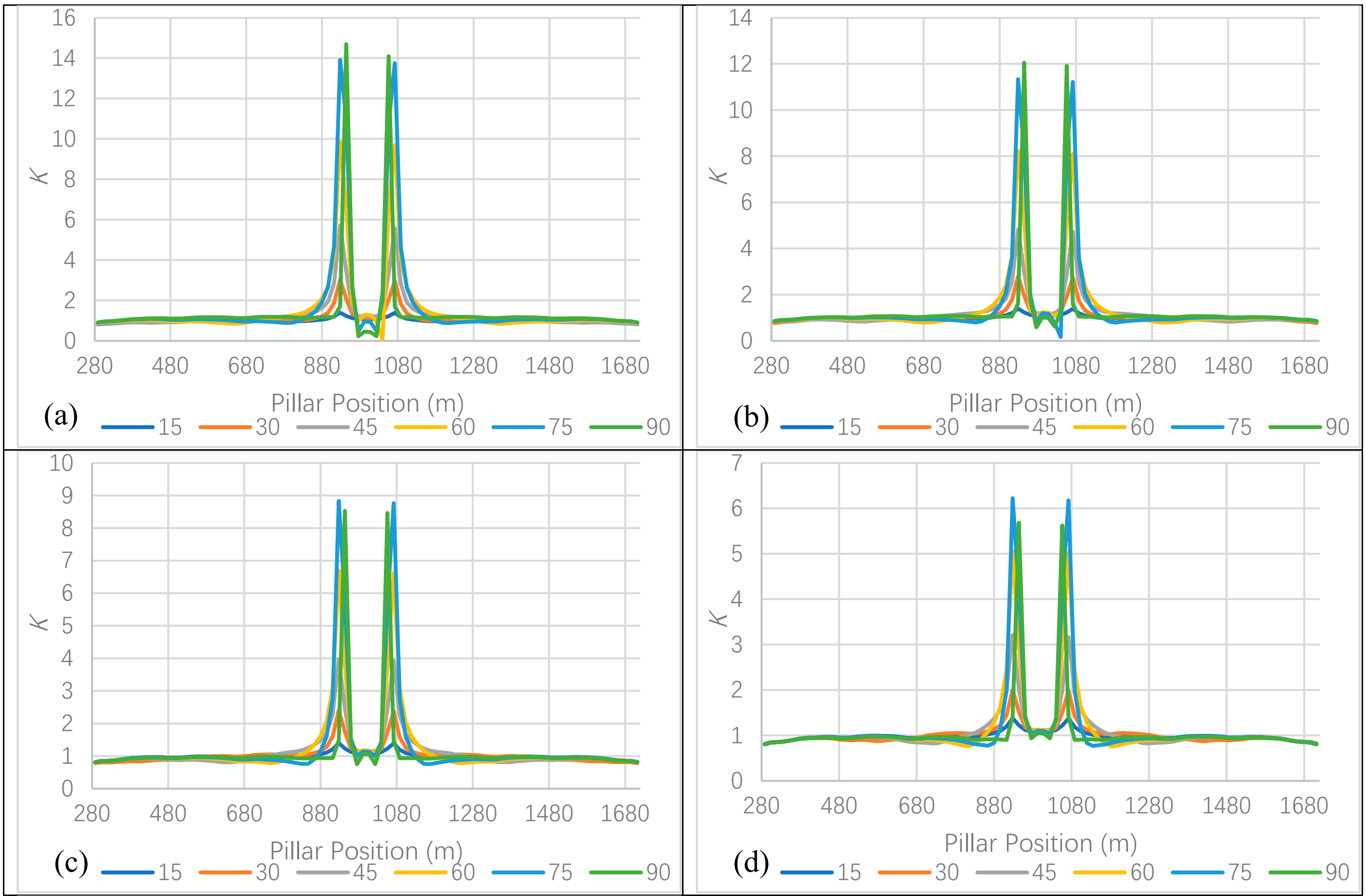


Figure 7.8 Effect of L_v on K when L_v=128 m: (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.



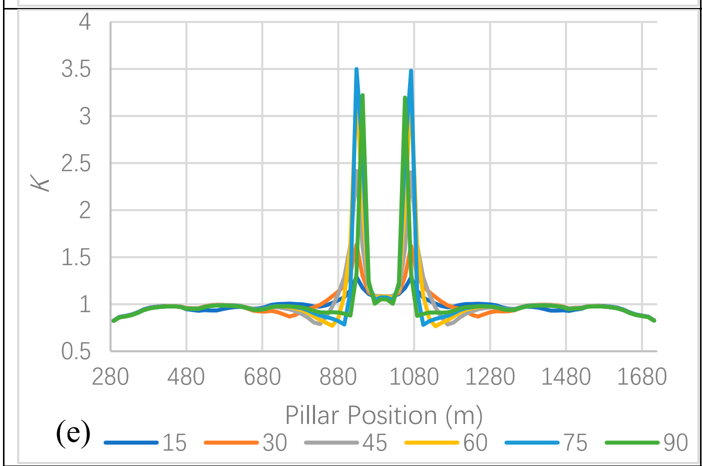
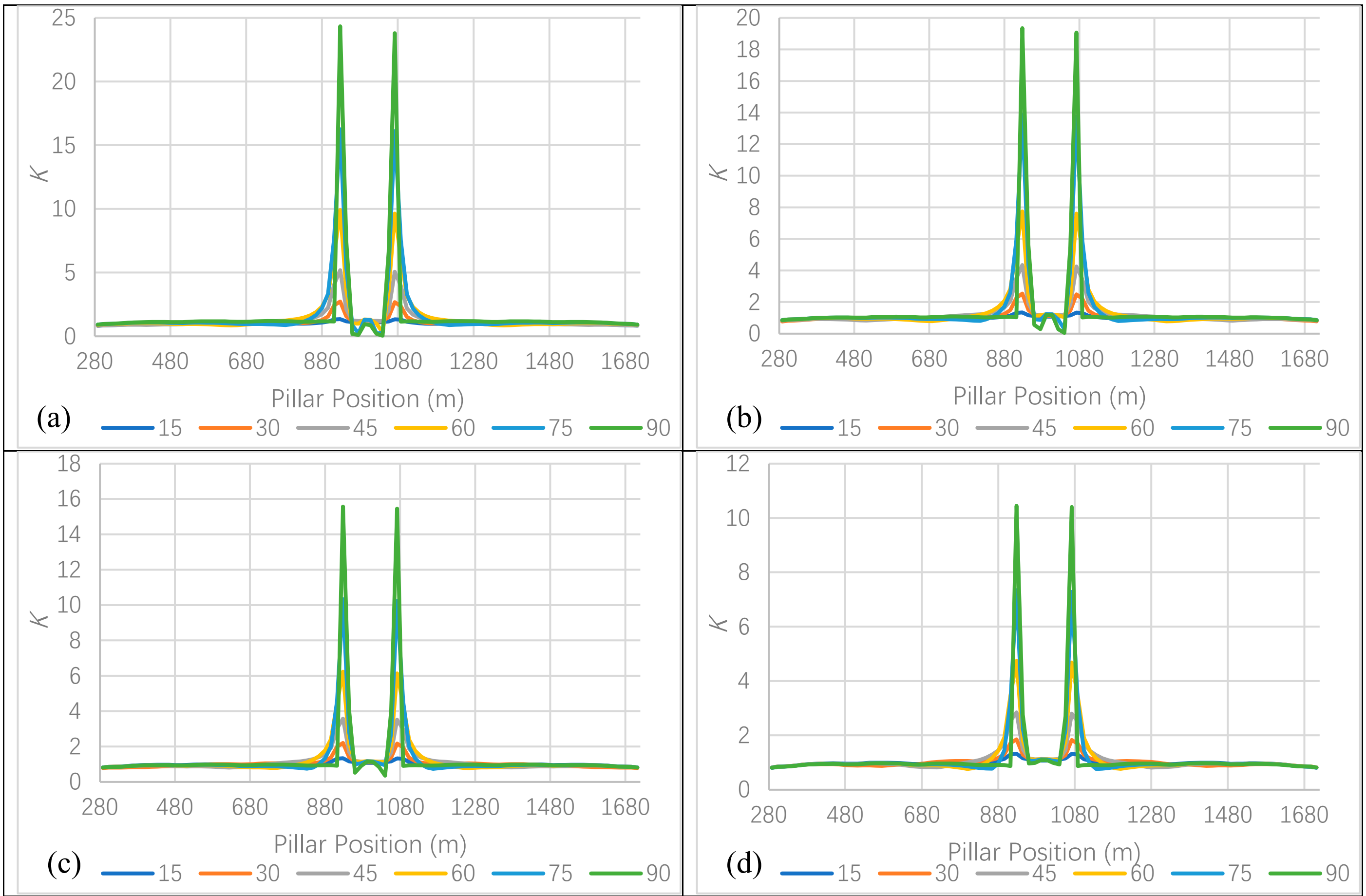


Figure 7.9 Effect of L_v on K when L_v=144 m: (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.



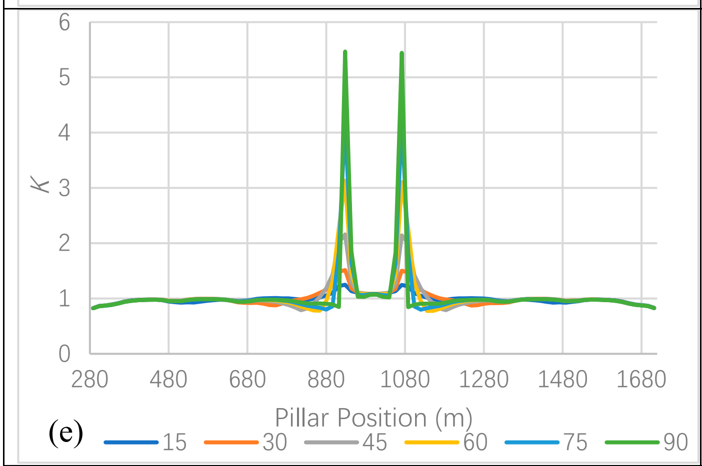


Figure 7.10 Effect of L_v on K when L_v=160 m: (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.

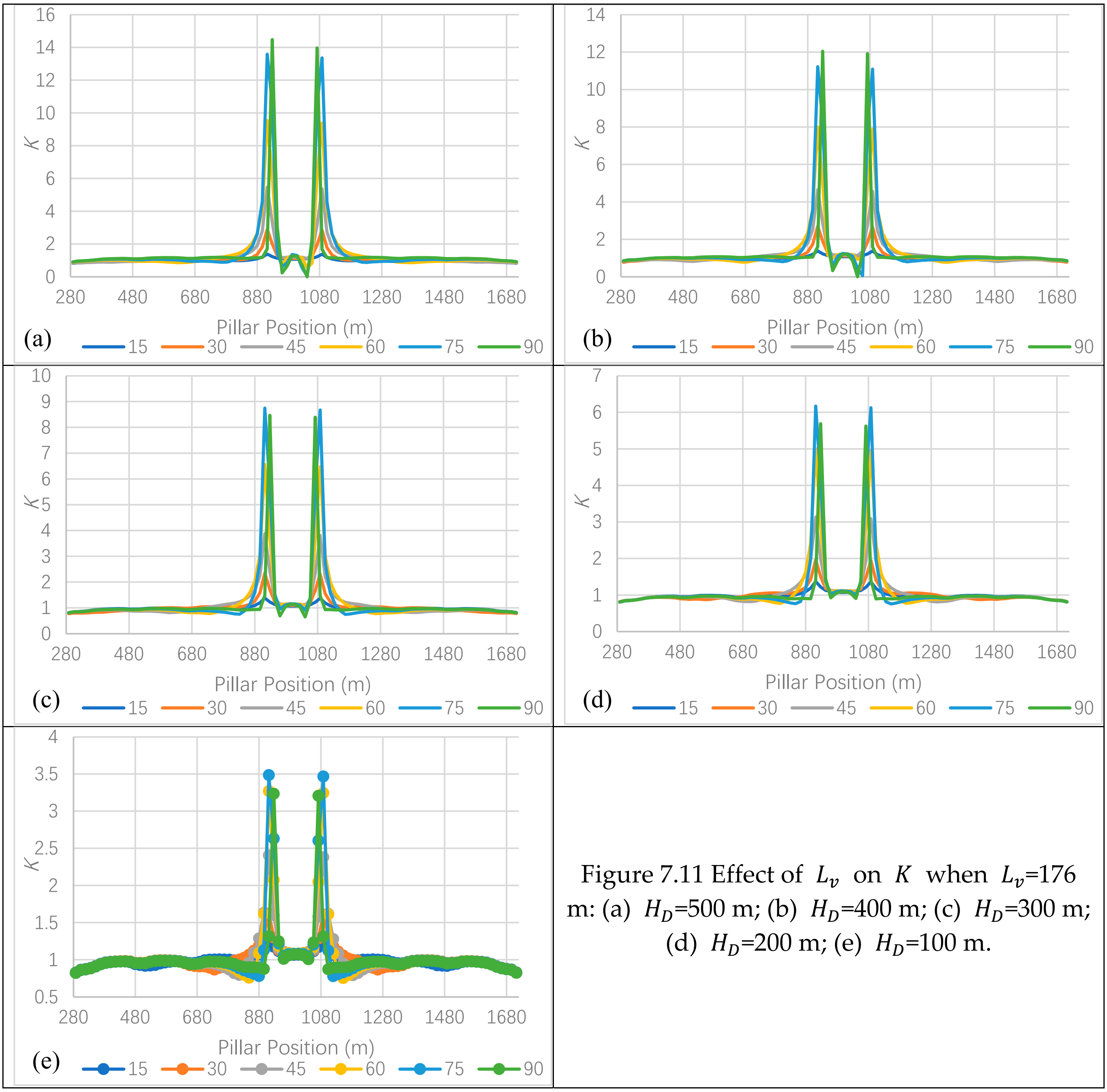
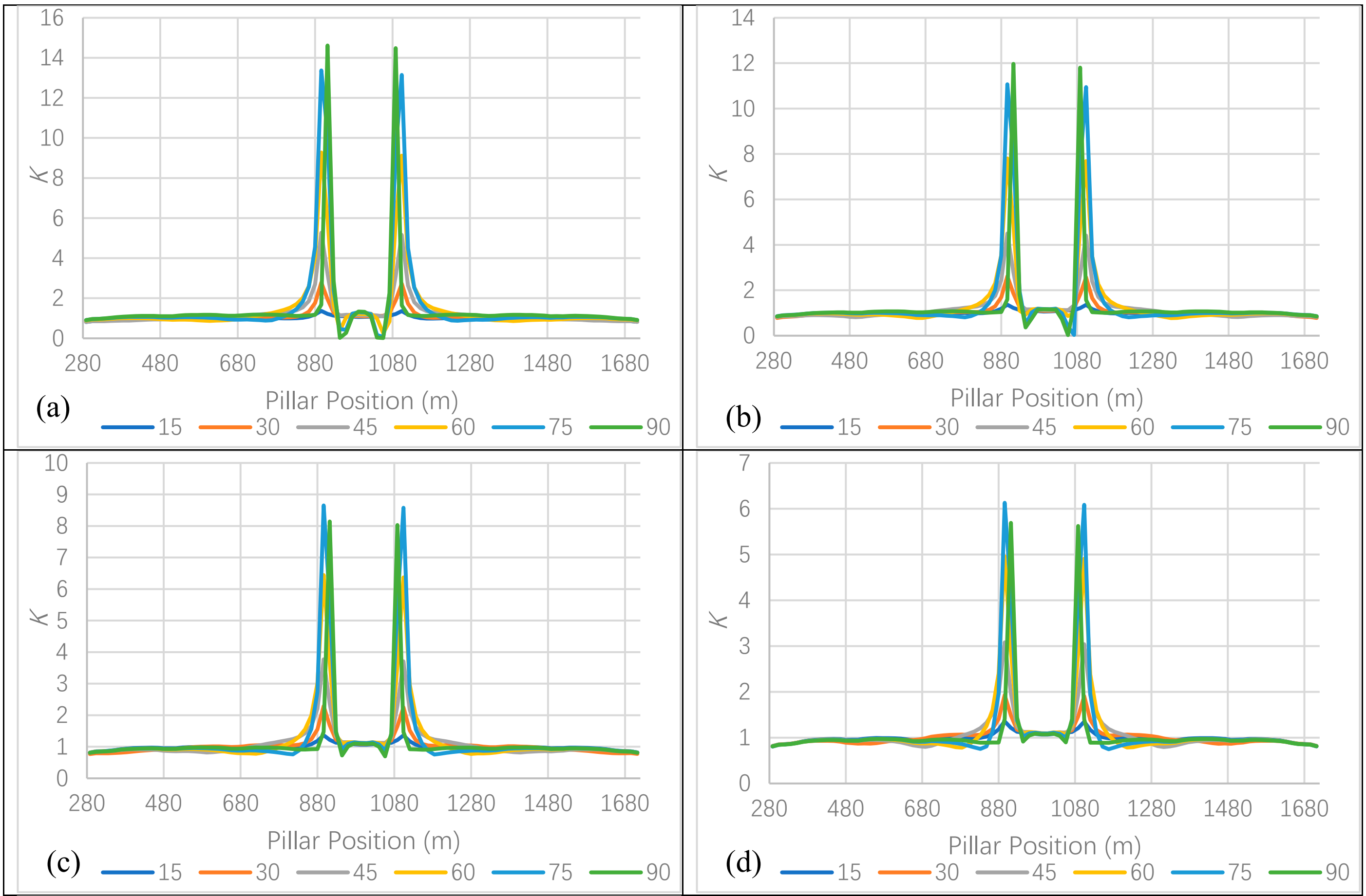


Figure 7.11 Effect of L_v on K when L_v=176 m: (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.



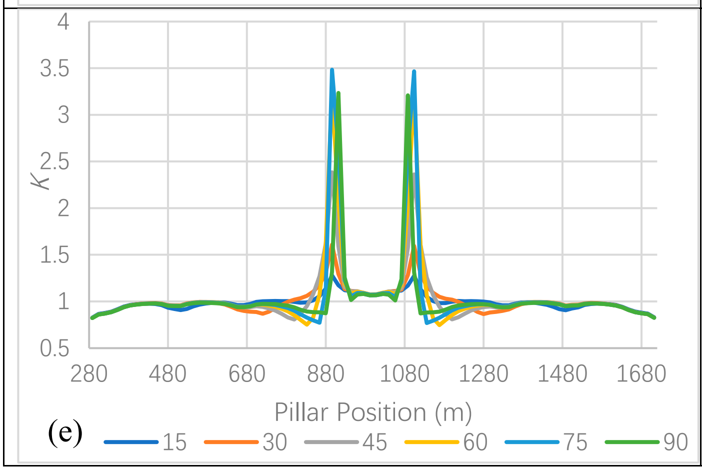
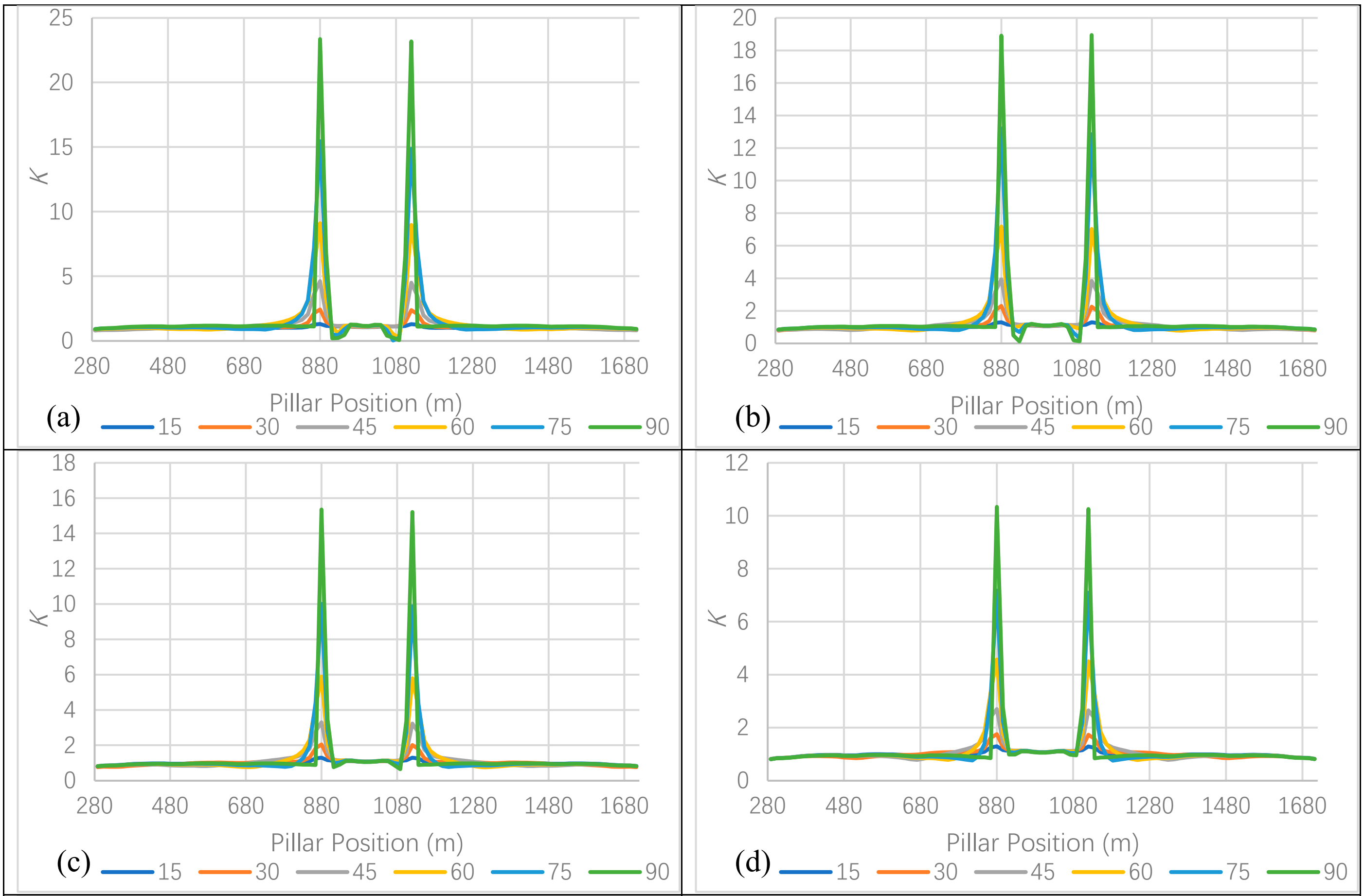


Figure 7.12 Effect of L_v on K when L_v=208 m: (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.



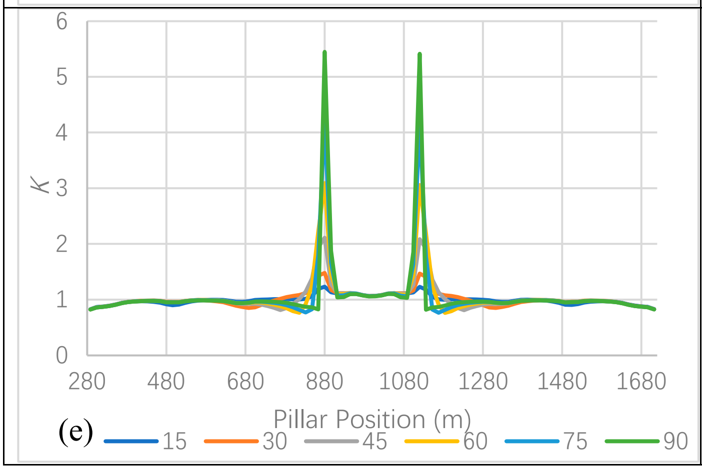


Figure 7.13 Effect of L_v on K when L_v=256 m: (a) H_D=500 m; (b) H_D=400 m; (c) H_D=300 m; (d) H_D=200 m; (e) H_D=100 m.

Due to the superimposed effects of two slopes, the K_{vm} is larger than the K_{m} (compare Figure 11a and Figure 4a in the paper), and there will be two K_{vm} of equal value beneath the two slope toes of a valley. The K_{vm} will be larger if the slope toe lies between two adjacent pillars (such as ground surface represented by red lines in Figure 7.14) instead of lying directly above a pillar (such as ground surface represented by red lines in Figure 7.14). e.g., the slope toe lies directly above a pillar in Figure 11(a) (in the paper) and the maximum K_{vm} is about 18, while the slope toe lies between two pillars in Figure 11(c) (in the paper) and the maximum K_{vm} exceeds 35. Therefore, it is better to ensure the slope toes lie directly above a pillar when conducting a pillar design to reduce the stress concentration.

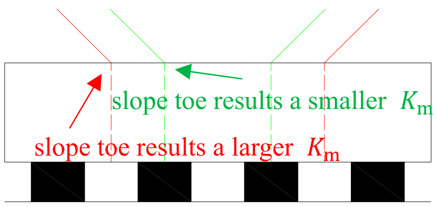


Figure 7.14 The relative position of slope toes and pillars

The K_{vm} decreases with the increase of L_v and finally reaches a relative stable value (Figure 11a,d-f). To determine the differences between single slope models and valley models, the K_{vm} of valley models with different L_v is divided by the K_m of the single slope model that is described in Section 3.1, and the results are listed in Table 4. Table 4 shows that K_{vm} is larger than K_m for most cases, which means the valley terrain enhances the stress concentration. The superimposed effects of two slopes are non-linear and are affected by H_D and α. Considering the most unfavorable conditions, the maximum ratio in a column in Table 4 (such as the column marked with a gray background) is used for regression analysis. In addition, K_{vm} can be calculated by:

$$K_{mv} = [(7.368 \times 10^{-8}\alpha^3 - 1.409 \times 10^{-5}\alpha^2 + 7.677 \times 10^{-4}\alpha - 0.007896)H_B + 1.043]K_m \quad (10)$$

The RMSE of Eq. (10) is 0.1223 and the R^2 is 0.9687. The K_{vm} of a valley terrain can be estimated based on the K_m of a single slope terrain according to Eq. (10). As the concentrated stress will decrease if H_B or L_v increases, Eq. (10) should overestimate the pillar stress to ensure it is safe enough to be used.

Table 4 Ratio of K_{vm} to K_m

L_v (m)	H_D (m)	α (°) 15	30	45	60	75	90	H_D (m)	α (°) 15	30	45	60	75	90
16		1.3	3.0	3.7	2.6	1.8	0.2		1.3	1.9	2.0	1.6	1.4	0.3
48		1.2	2.2	2.7	2.0	1.5	1.4		1.2	1.6	1.6	1.4	1.2	1.2
80		1.2	2.1	2.6	1.9	1.4	1.3		1.2	1.6	1.6	1.4	1.2	1.1
112	500	1.2	2.0	2.4	1.8	1.4	1.4	200	1.2	1.5	1.5	1.3	1.2	1.1
144		1.2	1.9	2.3	1.8	1.3	1.3		1.2	1.5	1.5	1.3	1.2	1.1
176		1.2	1.8	2.2	1.7	1.3	1.3		1.2	1.5	1.5	1.3	1.2	1.1
208		1.2	1.7	2.1	1.7	1.3	1.3		1.2	1.4	1.5	1.3	1.2	1.1
16		1.3	2.8	3.2	2.3	1.6	0.2		1.2	1.4	1.4	1.3	1.3	0.5
48		1.2	2.1	2.4	1.8	1.4	1.3		1.1	1.2	1.2	1.2	1.1	1.1
80		1.2	2.0	2.3	1.7	1.4	1.3		1.1	1.2	1.2	1.2	1.1	1.1
112	400	1.2	1.9	2.2	1.7	1.3	1.3	100	1.1	1.2	1.2	1.2	1.1	1.1
144		1.2	1.8	2.1	1.6	1.3	1.3		1.1	1.2	1.2	1.2	1.1	1.1
176		1.2	1.8	2.0	1.6	1.3	1.3		1.1	1.2	1.2	1.2	1.1	1.1
208		1.2	1.7	1.9	1.5	1.3	1.3		1.1	1.2	1.2	1.2	1.1	1.1
16		1.3	2.4	2.6	2.0	1.5	0.2							
48		1.2	1.9	2.0	1.6	1.3	1.3							
80		1.2	1.8	1.9	1.6	1.3	1.2							
112	300	1.2	1.7	1.9	1.5	1.3	1.2							
144		1.2	1.7	1.8	1.5	1.3	1.2							
176		1.2	1.6	1.7	1.5	1.3	1.2							
208		1.2	1.6	1.7	1.4	1.2	1.2