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On Convex F-Contraction in b-Metric Spaces

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Abstract: In this paper, we introduce a notion of convex F-contraction and establish some fixed point results for such contractions in b-metric spaces. Moreover, we give a supportive example to show that our convex F-contraction is quite different from the F-contraction used in the existing literature since our convex F-contraction does not necessarily contain the continuous mapping but the F-contraction contains such mapping. In addition, via some facts, we claim that our results indeed generalize and improve some previous results in the literature.

Keywords: F-contraction; convex F-contraction; fixed point; b-metric space

MSC: 47H10; 54H25

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1. Introduction and Preliminaries

In [1], Wardowski introduced the following concept of F-contraction and proved a fixed point theorem that generalizes the classical Banach contraction mapping principle.

Definition 1 ([1]). *Let (X, d) be a metric space and $T : X \to X$ be a mapping. Then T is called an F-contraction if there exists a function $F : (0, +\infty) \to \mathbb{R}$ such that*

(F_1) *F is strictly increasing on $(0, +\infty)$;*
(F_2) *for each sequence $\{\alpha_n\}$ of positive numbers,*

$$\lim_{n\to\infty} \alpha_n = 0 \text{ if and only if } \lim_{n\to\infty} F(\alpha_n) = -\infty;$$

(F_3) *there exists $k \in (0, 1)$ such that $\lim_{\alpha\to 0^+} \alpha^k F(\alpha) = 0$;*
(F_4) *there exists $\tau > 0$ such that*

$$\tau + F(d(Tx, Ty)) \le F(d(x, y)) \tag{1}$$

for all $x, y \in X$ with $x \neq y$.

Remark 1. *Definition 1 is the modification of [1] (Definition 2.1). In fact, (2) from [1] says $d(Tx, Ty) > 0$, that is, $Tx \neq Ty$. Note that $Tx \neq Ty$ implies $x \neq y$. Hence, $x \neq y$ in (F_4) is weaker condition than $d(Tx, Ty) > 0$ from (2) of [1]. Moreover, our modification does not disturb the main results of [1]. Clearly, compared with $d(Tx, Ty) > 0$ from [1], our $x \neq y$ is more convenient in applications.*

Otherwise, by (1) and (F_1), we have

$$d(Tx, Ty) < d(x, y) \tag{2}$$

for all $x, y \in X$ with $x \neq y$. Accordingly, any F-contraction is a contraction.

Remark 2. *It follows immediately from (2) that any F-contraction implies that the mapping T is a continuous mapping.*

Wardowski [1] proved that any F-contraction has a unique fixed point.

Theorem 1 ([1])**.** *Let (X, d) be a complete metric space and $T : X \to X$ be an F-contraction. Then, T has a unique fixed point x^* in X. For every $x \in X$, the sequence $\{T^n x\}$ converges to x^*.*

Since then, several authors proved fixed point results for F-contractions (see [2–13]). However, F-contraction has a great limitation since the mapping must be a continuous mapping (see Remark 2). But the continuity is a strong condition. Hence, it restricts the applications greatly.

On the other hand, the concept of b-metric space was introduced by Bakhtin [14] or Czerwik [15] which is a great generalization of usual metric space.

Definition 2. *A b-metric space (X, d, s) $(s \geq 1)$ is a space defined on a nonempty set X with a mapping $d : X \times X \to [0, +\infty)$ satisfying the following conditions:*

(1) $d(x, y) = 0$ *if and only if* $x = y$*;*
(2) $d(x, y) = d(y, x)$ *for all* $x, y \in X$*;*
(3) $d(x, y) \leq s[d(x, z) + d(z, y)]$ *for all* $x, y, z \in X$*.*

In this case, d is called a b-metric on X.

Regarding some other concepts, such as the concepts of b-convergent sequence, b-Cauchy sequence and b-completeness, the reader may refer to [16] and the references therein.

In the sequel, unless there is a special explanation, we always denote by $\mathbb{N}$, the set of positive integers, $\mathbb{R}$, the set of real numbers.

Let (X, d, s) be a b-metric space and T be a self-mapping on X. The Picard sequence of T is given by $\{x_n\}_{n \in \mathbb{N} \cup \{0\}} = \{T^n x\}_{n \in \mathbb{N} \cup \{0\}}$ for any $x \in X$, where $T^0 x = x$. In this case, for the convenience, throughout this paper, we always denote $d(x_{n+1}, x_n)$ by d_n, for all $n \in \mathbb{N} \cup \{0\}$.

In this paper, we introduce the concept of convex F-contraction and give some sufficient conditions when the Picard sequence of convex F-contraction on b-metric space satisfies the Cauchy condition. Our results improve the results of Cosentino and Vetro [17]. Our conclusions are some real generalizations of the results of Popescu and Stan [18]. Moreover, we also expand the main results of Wardowski and Dung [13]. Additionally, we pose two problems at the end of the main text. We aim to continue to work in order to solve the problems in the near future.

2. Main Results

In this section, we first define a notion called convex F-contraction in b-metric spaces. Moreover, we give two examples to illustrate our notion is well-defined. Further, we present a fixed point result for such contraction.

Definition 3. *Let (X, d, s) be a b-metric space and T be a self-mapping on X. We say that T is a convex F-contraction if there exists a function $F : (0, +\infty) \to \mathbb{R}$ such that Condition (F_1) holds and*

(F_2^α) *for each sequence $\{\alpha_n\}$ of positive numbers, if $\lim_{n\to\infty} F(\alpha_n) = -\infty$, then $\lim_{n\to\infty} \alpha_n = 0$;*
(F_3^s) *there exists $k \in (0, \frac{1}{1+\log_2 s})$ such that $\lim_{\alpha\to 0^+} \alpha^k F(\alpha) = 0$;*
(F_4^λ) *there exist $\tau > 0$ and $\lambda \in [0, 1)$ such that*

$$\tau + F(d_n) \leq F(\lambda d_n + (1 - \lambda) d_{n-1}), \tag{3}$$

for all $d_n > 0$, where $n \in \mathbb{N}$.

Remark 3. *Definition 3 improves Definition 1 greatly. Indeed, (F_2^α) is weaker than Condition (F_2). If $s = 1$, then Condition (F_3^s) is Condition (F_3). That is to say, (F_3^s) expands Condition (F_3). Moreover, if $\lambda = 0$, then Condition (F_4^λ) is a consequence of Condition (F_4).*

Example 1. *Let (X, d, s) be a b-metric space and $T : X \to X$ be a mapping. Suppose that T is an F-contraction of Kannan type, i.e., there exists $\tau > 0$ such that*

$$\tau + F(d(Tx, Ty)) \leq F\left(\frac{1}{2}[d(x, Tx) + d(y, Ty)]\right) \tag{4}$$

for all $x, y \in X$ with $x \neq y$.

Choose $F(\alpha) = \ln \alpha$, $\alpha \in (0, +\infty)$, then T is a convex F-contraction. Indeed, it is obvious that F satisfies Conditions (F_1), (F_2^α) and (F_3^s). Moreover, T satisfies Condition (F_4^λ) based on the fact that there exists $\lambda = \frac{1}{2}$ such that

$$\tau + F(d_n) \leq F\left(\frac{d_n}{2} + \frac{d_{n-1}}{2}\right)$$

for all $d_n > 0$, where $n \in \mathbb{N}$. That is, (4) becomes (F_4^λ).

Otherwise, if $F(\alpha) = \ln \alpha$, $\alpha \in (0, +\infty)$, then from (4) we have

$$d(Tx, Ty) \leq K[d(x, Tx) + d(y, Ty)],$$

where $K = \frac{e^{-\tau}}{2} < \frac{1}{2}$, i.e., the contraction of Kannan type (see [19]) holds.

Example 2. *Let T be an F-contraction of Reich type (see [20]), i.e., there exist $\tau > 0$ and $\alpha, \beta, \gamma \in [0, 1], \alpha + \beta + \gamma = 1$ such that*

$$\tau + F(d(Tx, Ty)) \leq F(\alpha d(x, y) + \beta d(x, Tx) + \gamma d(y, Ty)), \tag{5}$$

for all $x, y \in X$ with $x \neq y$.

Choose $F(\alpha) = -\frac{1}{\sqrt{\alpha}}$, $\alpha \in (0, +\infty)$, then T is a convex F-contraction. Indeed, it is clear that F satisfies Conditions (F_1), (F_2^α) and (F_3^s). Moreover, T satisfies Condition (F_4^λ) because there exists $\lambda = \beta$ such that (3) holds. That is, T satisfies Condition (F_4^λ).

Otherwise, if $F(\alpha) = -\frac{1}{\sqrt{\alpha}}$, $\alpha \in (0, +\infty)$, then (5) implies

$$d(Tx, Ty) < \alpha d(x, y) + \beta d(x, Tx) + \gamma d(y, Ty),$$

which is the contraction of Reich type.

Lemma 1. *Let (X, d, s) be a b-metric space and T be a convex F-contraction on X. Then, for every $x \in X$, the sequences $\{T^n x\}_{n \in \mathbb{N} \cup \{0\}}$ is a b-Cauchy sequence.*

Proof. Choose $x \in X$ and construct a sequence $\{x_n\}$ by $x_n = T^n x$ for all $n \in \mathbb{N} \cup \{0\}$. If there exists $n_0 \in \mathbb{N} \cup \{0\}$ such that $x_{n_0+1} = x_{n_0}$, then

$$\{x_n\} = \{x, Tx, T^2 x, \ldots, T^{n_0-1} x, x_{n_0}, x_{n_0}, \ldots\}.$$

It is valid that $\{T^n x\}_{n \in \mathbb{N} \cup \{0\}}$ is a b-Cauchy sequence. The proof is completed.

Without loss of generality, assume that $x_{n+1} \neq x_n$ for all $n \in \mathbb{N} \cup \{0\}$. That is to say, assume that $d_n > 0$ for all $n \in \mathbb{N} \cup \{0\}$. From Condition (F_4^λ), we have

$$F(d_n) < \tau + F(d_n) \leq F(\lambda d_n + (1 - \lambda) d_{n-1}).$$

Using Condition (F_1), we obtain

$$d_n < \lambda d_n + (1-\lambda)d_{n-1},$$

then $0 < d_n < d_{n-1}$ for all $n \in \mathbb{N}$. Hence, $\{d_n\}$ is a convergent sequence.

In the following, we show $\lim\limits_{n\to\infty} d_n = 0$. To this end, we show

$$\tau + F(d_n) \le F(d_{n-1}), \tag{6}$$

for all $n \in \mathbb{N}$.

Indeed, if (6) is not true, then

$$\tau + F(d_n) > F(d_{n-1}),$$

for some $n \in \mathbb{N}$. Thus, it establishes that

$$F(d_{n-1}) < \tau + F(d_n) \le F(\lambda d_n + (1-\lambda)d_{n-1}).$$

Using Condition (F_1), we get

$$d_{n-1} < \lambda d_n + (1-\lambda)d_{n-1},$$

which means $d_{n-1} < d_n$. This is a contradiction.

It follows immediately from (6) that

$$F(d_n) \le F(d_0) - n\tau, \tag{7}$$

for all $n \in \mathbb{N}$. (7) implies $\lim\limits_{n\to\infty} F(d_n) = -\infty$. Then by Condition (F_2^α), it leads to $\lim\limits_{n\to\infty} d_n = 0$.

In view of $\lim\limits_{n\to\infty} d_n = 0$, then via Condition (F_3^s), there exists $k \in (0, \frac{1}{1+\log_2 s})$ such that

$$\lim_{n\to\infty} d_n^k F(d_n) = 0. \tag{8}$$

From (7) we obtain

$$d_n^k n\tau \le d_n^k F(d_0) - d_n^k F(d_n). \tag{9}$$

Combine (8) and (9), it is easy to see that

$$\lim_{n\to\infty} d_n^k n = 0.$$

Therefore, there exists $n_0 \in \mathbb{N}$ such that

$$d_n \le \frac{1}{n^{\frac{1}{k}}},$$

for all $n \ge n_0$. Finally, using [21] (Lemma 11), we claim that $\{x_n\}$ is a b-Cauchy sequence. □

Theorem 2. *Let (X, d, s) be a b-complete b-metric space and T be a continuous convex F-contraction on X. Then, T has a fixed point in X.*

Proof. For any $x \in X$, by Lemma 1 we deduce that the sequence $\{T^n x\}$ is b-convergent. Write $x^* = \lim\limits_{n\to\infty} T^n x$. Due to the continuity of the mapping T, we conclude that x^* is a fixed point of T. □

Remark 4. *The continuous condition of Theorem 2 is necessary because there exists discontinuous convex F-contraction. See Example 3 in the sequel.*

3. Some Results Related to Convex F-Contractions

In this section, we obtain some results regarding convex F-contractions. We give a supportive example to verify that the mapping T with regard to convex F-contraction is not necessarily continuous. This fact shows that our convex F-contraction is more meaningful than the F-contraction introduced by Wardowski [1] since any F-contraction must contain the continuous mapping T (see Remark 2).

First of all, we present a fixed point theorem for F-contraction of Banach type as follows:

Theorem 3. *Let (X, d, s) be a b-complete b-metric space and T be a self-mapping on X. Suppose that there exists a function $F : (0, +\infty) \to \mathbb{R}$ satisfying Conditions (F_1), (F_2^α), (F_3^s) and (F_4). Then, T has a unique fixed point x^* in X. Moreover, for any $x \in X$, the sequence $\{T^n x\}$ b-converges to x^*.*

Proof. From Condition (F_4) we obtain Condition (F_4^λ) if we choose $\lambda = 0$. So, T is a convex F-contraction. Since (F_4) is satisfied, then by Remark 2, T is continuous. Now, from Theorem 2 and Lemma 1, we conclude that for any $x \in X$, there exists $x^* \in X$ such that $Tx^* = x^*$ and $x^* = \lim_{n\to\infty} T^n x$.

In the following, we prove that the fixed point of T is unique. Indeed, assume that T has another fixed point $y^* \in X$, then by (F_4), ones have

$$F(d(x^*, y^*)) = F(d(Tx^*, Ty^*)) < \tau + F(d(Tx^*, Ty^*)) \le F(d(x^*, y^*)),$$

which is a contraction. □

Remark 5. *Note that, from Theorem 3 we get Theorem 1 because in metric spaces $1 + \log_2 s = 1$ holds, where $s = 1$. Therefore, Theorem 3 generalizes Theorem 1.*

Secondly, we give a fixed point theorem for the F-contraction of Kannan type as follows:

Theorem 4. *Let (X, d, s) be a b-complete b-metric space with $s \in [1, 2)$. Let T be an F-contraction of Kannan type, i.e., T satisfies (4). Suppose that there exists a function $F : (0, +\infty) \to \mathbb{R}$ satisfying Conditions (F_1), (F_2^α) and (F_3^s). Then, T has a unique fixed point x^* in X. Moreover, for any $x \in X$, the sequence $\{T^n x\}$ b-converges to x^*.*

Proof. From Example 1 we obtain that T is a convex F-contraction. So, by Lemma 1 we conclude that there exists $x^* \in X$ such that $x^* = \lim_{n\to\infty} x_n$, where $x_n = T^n x$ for any $x \in X$. Next, from Condition (4) and (F_1) we obtain

$$d(Tx, Ty) < \frac{1}{2}[d(x, Tx) + d(y, Ty)] \tag{10}$$

for all $x, y \in X$ with $x \neq y$.

If $x^* \neq Tx^*$, using (10), we have

$$\begin{aligned} d(x^*, Tx^*) &\le s[d(x^*, x_{n+1}) + d(x_{n+1}, Tx^*)] \\ &\le s\left(d(x^*, x_{n+1}) + \frac{1}{2}[d(x_n, x_{n+1}) + d(x^*, Tx^*)]\right). \end{aligned}$$

Take the limit as $n \to \infty$ from the above inequality, it follows that

$$d(x^*, Tx^*) \le \frac{s}{2} d(x^*, Tx^*) < d(x^*, Tx^*),$$

which is a contradiction. Hence, $x^* = Tx^*$.

Finally, we prove the fixed point of T is unique. As a matter of fact, if T has two distinct fixed points x^* and y^*, i.e., $x^* \neq y^*$, then by (10), it is easy to see that

$$d(x^*, y^*) = d(Tx^*, Ty^*) < \frac{1}{2}[d(x^*, Tx^*) + d(y^*, Ty^*)] = 0.$$

which is a contradiction. □

Remark 6. *Similar to Theorem 4, the mapping T has a unique fixed point if T from Theorem 4 is replaced by the F-contraction of Chatterjea type (see [22]), i.e., there exists $\tau > 0$ such that*

$$\tau + F(d(Tx, Ty)) \leq F\left(\frac{1}{2s}[d(x, Ty) + d(y, Tx)]\right)$$

for all $x, y \in X$ with $x \neq y$.

Example 3. *Let $X = \mathbb{R}$ and define a mapping $d : X \times X \to [0, +\infty)$ by*

$$d(x, y) = |x - y|^p$$

for all $x, y \in X$, where $p \in [1, 2)$. Then (X, d, s) is a b-metric space with $s = 2^{p-1} \in [1, 2)$. Let $T : X \to X$ be a mapping defined by

$$Tx = \begin{cases} 0, & \text{if } x \in (-\infty, 2], \\ \frac{1}{2}, & \text{if } x \in (2, +\infty). \end{cases}$$

Let $F(\alpha) = \ln \alpha$, $\alpha \in (0, +\infty)$, then F satisfies (F_1), (F_2^α) and (F_3^s). Moreover, there exists $\tau = -\ln(2K) > 0$ such that T is an F-contraction of Kannan type, where $K \in [\frac{1}{3^p}, \frac{1}{2})$ is a constant. Hence, T satisfies (4). Clearly, T is not continuous but by Theorem 4, it has a unique fixed point $x^* = 0$ in X.

Otherwise, it is easy to see that

$$d(Tx, Ty) \leq K[d(x, Tx) + d(y, Ty)]$$

for all $x, y \in X$. Therefore, T is a contraction for Kannan type. However, T is not a contraction for Banach type. Actually, there is not a constant $k \in (0, 1)$ such that

$$d(Tx, Ty) \leq kd(x, y)$$

for all $x, y \in X$.

Remark 7. *By Example 3, we claim that Theorem 4 has a superiority since the mapping T does not necessarily be continuous. Hence, our convex F-contraction can derive more applications than the counterpart of all the results regarding F-contraction. This is because any F-contraction must contain a continuous mapping (see Remark 2).*

Finally, we give a result on F-contraction of Hardy–Rogers type in b-metric spaces. Our result improves the results of [17,18] in b-metric spaces.

Theorem 5. *Let T be a self-mapping on a b-complete b-metric space (X, d, s). Suppose that there exists a function $F : (0, +\infty) \to \mathbb{R}$ satisfying Conditions (F_1), (F_2^α) and (F_3^s). If there exists $\tau > 0$ such that*

$$\begin{aligned} \tau + F(d(Tx, Ty)) \leq F(&ad(x, y) + b[d(x, Tx) + d(y, Ty)] \\ &+ c[d(x, Ty) + d(y, Tx)]), \end{aligned} \tag{11}$$

for all $x, y \in X$ with $x \neq y$, where $a, b, c \geq 0, a + 2b + 2cs = 1$ and $bs + cs^2 < 1$, then T has a unique fixed point x^* in X. For any $x \in X$, the sequence $\{T^n x\}$ b-converges to x^*.

Proof. Let $x \in X$ and $x_n = T^n x$, for all $n \in \mathbb{N} \cup \{0\}$. If there exists $n_0 \in \mathbb{N} \cup \{0\}$ such that $x_{n_0+1} = x_{n_0}$, that is, $Tx_{n_0} = x_{n_0}$, then x_{n_0} is a fixed point of T.

Without loss of generality, we always assume that $x_{n+1} \neq x_n$ for any $n \in \mathbb{N} \cup \{0\}$. Making full use of (11), we speculate

$$\begin{aligned}\tau + F(d_n) &\leq F(ad_{n-1} + b(d_n + d_{n-1}) + cd(x_{n-1}, x_{n+1})) \\ &\leq F(ad_{n-1} + b(d_n + d_{n-1}) + cs(d_{n-1} + d_n)) \\ &= F((b + cs)d_n + (a + b + cs)d_{n-1}).\end{aligned}$$

That is, (F_4^λ) holds. Consequently, T is a convex F-contraction. Via Lemma 1, there exists $x^* \in X$ such that $x^* = \lim\limits_{n\to\infty} x_n$.

In the following, we prove that x^* is a fixed point of T. To this end, we suppose that $x^* \neq Tx^*$ is absurd. Then

$$d(x^*, Tx^*) \leq s[d(x^*, x_{n+1}) + d(x_{n+1}, Tx^*)]$$

and

$$d(x_{n+1}, Tx^*) \leq s[d(x_{n+1}, x^*) + d(x^*, Tx^*)]$$

imply that

$$\frac{1}{s} d(x^*, Tx^*) \leq \liminf_{n\to\infty} d(x_{n+1}, Tx^*) \leq \limsup_{n\to\infty} d(x_{n+1}, Tx^*) \leq sd(x^*, Tx^*). \tag{12}$$

Put $l = \liminf\limits_{n\to\infty} d(x_{n+1}, Tx^*)$ and $L = \limsup\limits_{n\to\infty} d(x_{n+1}, Tx^*)$. Using Condition (11) and (F_1), we have

$$\begin{aligned}d(Tx_n, Tx^*) < ad(x_n, x^*) + b[d(x_n, x_{n+1}) + d(x^*, Tx^*)] \\ + c[d(x_n, Tx^*) + d(x^*, x_{n+1})].\end{aligned} \tag{13}$$

Hence, taking the limit as $n \to \infty$ from both sides of (13) and considering (12), we get

$$l \leq bd(x^*, Tx^*) + cL. \tag{14}$$

Hence, using (12) and (14), we obtain

$$\frac{1}{s} d(x^*, Tx^*) \leq l \leq bd(x^*, Tx^*) + cL \leq bd(x^*, Tx^*) + csd(x^*, Tx^*),$$

which means that $bs + cs^2 \geq 1$. This is a contradiction. Therefore, $x^* = Tx^*$.

Finally, we need to prove the uniqueness of the fixed point. To this end, assume that T has another fixed point y^*. Taking advantage of (11), we arrive at

$$\begin{aligned}F(d(x^*, y^*)) = F(d(Tx^*, Ty^*)) &< \tau + F(d(Tx^*, Ty^*)) \\ &\leq F(ad(x^*, y^*) + b[d(x^*, Tx^*) + d(y^*, Ty^*)] \\ &\quad + c[d(x^*, Ty^*) + d(y^*, Tx^*)]) \\ &= F((a + 2c)d(x^*, y^*)),\end{aligned}$$

which follows immediately from Condition (F_1) that

$$d(x^*, y^*) < (a + 2c)d(x^*, y^*) \leq d(x^*, y^*).$$

This is a contradiction. □

Remark 8. *Theorem 5 generalizes [13] (Corollary 2.5). By virtue of convex F-contractions and Lemma 1, we can get [13] (Theorem 2.4) and [23] (Theorem 3).*

We finally pose the following problems:

Problem 1. Can Condition (F_3^s) be replaced with Condition (F_3) in our all results?
Problem 2. Does Theorem 4 hold if $s \geq 1$ is arbitrary?

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