*Article*

# An Improved Evaluation Methodology for Mining Association Rules

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**Abstract:** At present, association rules have been widely used in prediction, personalized recommendation, risk analysis and other fields. However, it has been pointed out that the traditional framework to evaluate association rules, based on *Support* and *Confidence* as measures of importance and accuracy, has several drawbacks. Some papers presented several new evaluation methods; the most typical methods are *Lift*, *Improvement*, *Validity*, *Conviction*, *Chi-square* analysis, etc. Here, this paper first analyzes the advantages and disadvantages of common measurement indicators of association rules and then puts forward four new measure indicators (i.e., *Bi-support*, *Bi-lift*, *Bi-improvement*, and *Bi-confidence*) based on the analysis. At last, this paper proposes a novel Bi-directional interestingness measure framework to improve the traditional one. In conclusion, the bi-directional interestingness measure framework (*Bi-support* and *Bi-confidence* framework) is superior to the traditional ones in the aspects of the objective criterion, comprehensive definition, and practical application.

**Keywords:** association rules mining; *Bi-support*; *Bi-confidence*; bi-directional interesting measure framework

## 1. Introduction

Nowadays, the amount of data stored in databases has grown in an impressive way. Thus, one of the main motivations for data storage is to obtain access to data so as to further analyze them and to obtain valuable and useful information from the data. As data mining methodology and technology have received extensive attention, association rules mining, as a vital subject in the domain of data mining and knowledge discovery, is now widely used in the field of electronic commerce [1–5]. Apriori algorithm can mine a host of association rules; however, only a few rules could be laid down and implemented by makers on account of limited resources [6,7]. Consequently, the evaluation methodology for mining association rules is of great significance in both theory design and practical application.

In recent years, quite a few results have been achieved in the study on the interestingness measure of association rules [8–11]. Tseng et al. [10] put forward incremental maintenance of generalized association rules under taxonomy evolution. Chen [12] used data envelopment analysis (DEA) as a post-processing approach. After the rules had been discovered from the association rule mining algorithms, DEA was applied to the use of ranking those discovered rules based on the specified criteria. Toloo et al. [13] proposed a new integrated DEA model, which was able to find the most efficient association rule only by solving one mixed integer linear programming (MILP). Objective interestingness mainly considers statistical significance features of objective data, including Support, Confidence

and *Lift*, which are classic, as well as *Validity*, *Conviction*, *Improvement*, and *Chi-square* analysis, which are relatively new [14]. Some scholars came up with some methods to generate both frequent and infrequent patterns by using a multi-objective genetic algorithm [15,16]. It is the common goal of researchers to mine association rules that can reflect users' interests truly and effectively.

Firstly, the traditional methods of association rule mining generate excessive association rules that include many invalids or even erroneous association rules. Secondly, with the current surge in online transactions, data on online transactions and user evaluation are extremely sparse. Thirdly, a combination of explosion problems may occur when the *Support* and *Confidence* thresholds are low, while some new knowledge of user interest will be filtered out because of data sparseness when the Support and Confidence thresholds are high. Lastly, the traditional *Support* and *Confidence* frameworks can no longer measure association rules in an effective way.

The main contributions of this paper are summarized as follows:

(1) This paper firstly expounds several aspects of association rules and then makes a summary of relevant research results on the objective interestingness measure of association rules. At the same time, we find that all have some defects and problems by discussing and comparing the various measurement methods of association rules.

(2) Then, we proposed four more effective measurement methods of association rules (*Bi-support*, *Bi-lift*, *Bi-improvement*, and *Bi-confidence*) based on the improvement of some methods. The *Bi-confidence* method makes an adjustment to the non-occurrence possibility of an antecedent based on the *Confidence*.

(3) Through the experimental analysis, we propose a novel measure framework (*Bi-support* and *Bi-confidence* framework) to improve the traditional one.

In Section 2, a review of related work is provided. In Section 3, interestingness measure indicators for association rules are described. In Section 4, four more effective measurement methods of association rules (*Bi-support*, *Bi-lift*, *Bi-improvement*, *Bi-confidence*) are proposed. In Section 5, we propose a novel Bi-directional interestingness framework to improve the traditional one through the experimental analysis. Finally, we draw conclusions in Section 6.

## 2. A Review of Relevant Research on Association Rules

This paper describes the research status of association rules from three parts: the algorithm, the application, and the evaluation method of association rules.

### 2.1. Algorithm of Association Rules

The Apriori algorithm is the earliest proposed classical association rules mining algorithm, whose implementation process is relatively simple. However, the database needs to be scanned many times, leading to a large I/O load and low algorithm execution efficiency. In order to solve the defects of Apriori, Pal and Kumar [17] proposed a distributed frequent itemset generation and association rule mining algorithm based on the MapReduce programming model. This scheme uses integrated distributed technology to generate frequent itemsets and mine association rules. Rules are mined in a distributed way, after which weights are assigned to the data subsets and association rules, and the association rules generated in a distributed way are mixed by the weighting method. This method solves the problem of multifarious operation when the data are large. Huo et al. [18] proposed an improved FP-tree algorithm to solve the defects of the Apriori algorithm. They set up the FP-tree-based (initial-FP-tree and new-FP-tree) structures to maintain fuzzy frequent itemsets in the original database and newly inserted transactions, respectively. The incremental mining strategy of fuzzy frequent itemsets is implemented by breathing priority traversal of the initial-FP-tree and new-FP-tree. This method has advantages in execution time. When the minimum *Support* threshold is low, the memory cost of this algorithm is lower than that of existing algorithms. In order to improve the speed and accuracy of association rule mining, Liu et al. [19] proposed a parallel FP growth method called SSPFP

on the basis of Spark Streaming, which can mine frequent itemsets and association rules in real-time Streaming data in parallel.

### 2.2. Application of Association Rules

Association rules are applied in various fields, mainly focusing on prediction, recommendation, cause analysis, and so on. Rafiqul et al. [20] used association rules to identify fraudulent behavior of policyholders, predict bad behavior of policyholders, and help insurance companies improve business strategies and overcome fraudulent claims. For the application of association rules in the recommendation domain, Yang et al. [21] proposed a recommendation algorithm based on temporal association rules, which could discover potential interests of users by using historical behavior data without domain knowledge. Moreover, a personalized hybrid recommendation system, TP-HR, based on time-aware CF and temporal association rules was proposed, which was used to track dynamic changes in user preferences. For feature analysis, Sanmiquel et al. [22] used association rules to analyze occupational accidents in the mining industry and identified the 20 most important factors leading to accidents.

### 2.3. Evaluation Method of Association Rules

The most widely accepted evaluation indexes of association rules are *Support* and *Confidence.* Meanwhile, *Lift*, *Validity*, *Conviction*, *Influence*, and other indicators have been gradually applied in various studies related to association rules. However, with the explosive growth of data volume and data types, these indicators are faced with problems, such as a large amount of computation and sparse data. One solution is to innovate new algorithms to improve operational efficiency. For example, Zhang et al. [23] proposed a multi-objective optimization algorithm to mine frequent itemsets of high-dimensional data. The other is to improve the evaluation method of association rules. For example, *Lift* and *Improvement* evaluation methods are optimized, and the concepts of *new-lift* and *new-improvement* are proposed in this paper.

### 2.4. Evaluation Method and Framework of Association Rules

Most association rule mining algorithms will generate invalid or even wrong association rules. How to identify and remove these association rules is one of the research hotspots. Song et al. [24] proposed predictability-based collective class association rule mining based on cross-validation with a new rule evaluation step. This method could remove redundant association rules and retain most of the high-value association rules. Among all kinds of entropy-based measurements, the important measures to evaluate association rules are joint entropy, conditional entropy, mutual information, cross-entropy, and equilibrium cross-entropy. Park et al. [25] proposed symmetrically balanced cross-entropy by considering the advantages of a symmetric J measure and balanced cross-entropy in association rule evaluation. Shaharani et al. [26] presented a systematic evaluation of the rules of association identified on the basis of frequent, closed, and maximum detailed exploration algorithms by combining general measures of data mining and statistical interest and described the appropriate sequence of usage. This method can also remove redundant association rules.

## 3. Review on Interestingness Measure Indicators for Association Rules

Let $A = \{A_1, A_2, \ldots, A_j\} \subset I$, $B = \{B_1, B_2, \ldots, B_k\} \subset I$. $I$ indicates itemsets, and $A \cap B = \phi$. Firstly, we suppose formal description of association rules is as follows:

$$A \rightarrow B \tag{1}$$

Rules should meet a certain *Support* threshold, $s$ (min *sup*), and *Confidence* threshold, $c$ (min *conf*).

Data in Table 1 are produced by data extraction and transformation from shopping mall receipts. Each line (a tuple) is shopping list data (shopping receipts); "1" refers to the list containing this item, while "0" means the opposite.

**Table 1.** A data set of transaction.

| Tid | E | F | G | H | I | J | K | L | M | N | R | Total |
|-----|----|---|---|---|---|---|---|---|---|---|---|-------|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 8 |
| 2 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 8 |
| 3 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 5 |
| 4 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 8 |
| 5 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 6 |
| 6 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 6 |
| 7 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 4 |
| 8 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 5 |
| 9 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 4 |
| 10 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 4 |
| Total | 10 | 8 | 7 | 4 | 4 | 5 | 3 | 2 | 4 | 6 | 5 | 58 |

### 3.1. Support and Confidence

*Support* [27] is the percentage of transactions where the rule holds. *Support* can be used to assess the usefulness of association rules. When the frequencies of *A* and *B* occurring simultaneously are equal to or higher than the set *Support* threshold, itemsets (*A*, *B*) as a frequent pattern. The formula is as follows:

$$\begin{aligned} Supp(A \to B) &= Supp(A \cup B) \\ &= P(AB) = N(AB)/|D| \end{aligned} \tag{2}$$

where $N(AB)$ is the number of records that *A* and *B* appear together, and $|D|$ is the total record number of transactions in data sets.

Firstly, *Support* has the defects of an artificially controlled threshold and rare itemsets. Secondly, at present, the number of subjects (users) and the number of projects increase exponentially in large electronic commerce systems, where online transaction data and user evaluation data are very sparse. Therefore, many infrequent patterns in data sets may have a potential value.

If both rule $A \to B$ and rule $\neg B \to \neg A$ are strong, then the rule $A \to B$ would be very strong. The rationale is that rule $A \to B$ and rule $\neg B \to \neg A$ are logically equivalent. We should look for strong evidence of these two rules to believe that they are interesting. Therefore, we proposed *Bi-support* instead of *Support*. *Bi-support* should meet the conditions where $Supp(A \to B) \geq \min supp$ and $Supp(\neg A \to \neg B) \geq \min supp$.

*Confidence* [27] is the statistic of probability $P(B|A)$ that subsequent events occur under the condition of occurrence of the precursor events in trading data sets. It is used to measure the reliability of the rules. The formula is as follows:

$$\begin{aligned} Conf(A \to B) &= P(B|A) \\ &= P(AB)/P(A) \\ &= Sup(A \cup B)/Sup(A) \end{aligned} \tag{3}$$

Association rules mining can be divided into two steps: (1) find out all of the maximal frequent patterns for meeting the condition; (2) generate the association rules from the frequent patterns. It is important to combine *Confidence* with *Support* from the *Support-Confidence* framework for mining association rules. If *Support* is larger than the designated minimum *Support* threshold and *Confidence* is larger than the designated minimum *Confidence* threshold, the rules will be called strong association rules. However, strong association rules are not always effective. Some are of no interest to users, and some are even misleading.

*3.2. Lift*

Owing to the defects of the *Support-confidence* framework, some scholars performed correlation analysis on association rules mined in the framework, namely *Lift* [24]. *Lift* means the ratio of the rule's *Confidence* to the occurrence probability of the consequent occurring, which reflects a positive or negative correlation of the antecedent and consequent of rules. It relates to the ratio of the occurrence probability of *B* under the condition *A* to that without considering condition *A*, which reflects the relationship between "*A*" and "*B*".

$$
\begin{aligned}
Lift(A \rightarrow B) &= Conf(A \rightarrow B)/P(B) \\
&= P(AB)/P(A)P(B)
\end{aligned}
\tag{4}
$$

The domain of *Lift* values is $[0, +\infty)$. As the value of *Lift* is equal to 1, it shows that the simultaneous appearance of event *A* and event *B* belongs to independent random events. That is to say, *A* and *B* do not affect each other. Additionally, we call these rules uncorrelated rules. If the value of *Lift* is less than 1, it shows that the occurrence of "*A*" reduces the occurrence of "*B*", and then we call them negative correlation rules. If the value of the *Lift* value is larger than 1, it shows that the occurrence of "*A*" promotes the occurrence of "*B*", and then we call them positive correlation rules. The problem is as follows: *Lift* takes events *A* and *B* in the equivalence position. According to the *Lift*, $A \rightarrow B$ and $B \rightarrow A$ are the same, that is to say, if we accept rule $A \rightarrow B$, $B \rightarrow A$ should also be accepted, but this is not fact.

*3.3. Validity*

Article [28] introduced a new measurement method of association rules, known as *Validity*. *Validity* is defined as the difference between the probability of "*A*" and "*B*" occurring simultaneously and the occurrence probability of "*B*" without "*A*" in database *D*. Given that the value domain of $P(AB)$ and $P(\overline{A}B)$ are [0, 1], the value domain of *Validity* is obviously $[-1, 1]$.

$$
Validity(A \rightarrow B) = P(AB) - P(\overline{A}B)
\tag{5}
$$

In fact, our research revealed that the *Validity* is not effective. Taking Table 1 for example, there is the rule $F \rightarrow R$, whose value of *Support* is 0.4, and the value of *Validity* is $0.4 - 0.1 = 0.3$. According to the literature [26], we can determine that it is a very valid association rule. However, the calculation of $Lift(F \rightarrow R) = P(FR)/P(F)P(R) = 1$ shows that *F* and *R* have a certain negative correlation. We call these rules uncorrelated rules. Then taking Table 2 for example, this case has already met the basic requirements of *Support* and *Confidence*. At the same time, $Validity(A \rightarrow B) = 0.5 - 0.2 = 0.3$. According to the measure standard of *Validity*, the rule $A \rightarrow B$ should be considered "very effective". The occurrence of "*A*" can promote the occurrence of "*B*". However, in fact, the overall occurrence frequency of event *B* is 0.70, and the occurrence probability of "*B*" under the occurrence of "*A*" is $50/80 = 0.625$, which is 7.5 percent lower.

**Table 2.** The occurrence of "*A*" and "*B*".

|  | *B* Occur | *B* Not Occur | Total |
|---|---|---|---|
| *A* occur | 50 | 30 | 80 |
| *A* not occur | 20 | 0 | 20 |
| Total | 70 | 30 | 100 |

*3.4. Conviction*

Brin introduced the concept of *Conviction* (*Conv.*) as early as 1997 [29].

$$
Conv(A \rightarrow B) = P(A)P(\overline{B})/P(A\overline{B})
\tag{6}
$$

Its value domain is $[0, +\infty)$. When the value of *Conviction* is "1", it means "A" has no relation with "B". In addition, the greater the value of *Conviction* is, the higher interest of the rule will have. Values in [0, 1) mean negative dependence. However, the constraint requirements of *Conviction* are too high to retain all valuable association rules. Taking Table 1 for example, $Conv(J \rightarrow G) = 0.75 < 1$, the value of its *Conviction* is low, but in fact, a high value of interest may exist in $J$ and $G$. Thus, it should also meet the condition that $P(A\overline{B}) \neq 0$.

### 3.5. Improvement

Articles [30,31] proposed a new interestingness measurement method of association rules based on the description of the defects of the traditional interestingness measurement method. We shall call it "*Improvement*". It means that the difference between the conditional probability $P(B|A)$ and the occurrence probability of "B".

$$Imp(A \rightarrow B) = [P(B|A) - P(B)] \tag{7}$$

However, shortcomings of *Improvement* (*Imp.*) are obvious. Firstly, it is hard to figure out how much improvement in probability can be considered an improvement. Secondly, the occurrence probability of the antecedent will seriously affect the evaluation of *Improvement*. That is to say, when the occurrence probability of the antecedent is very high, the measurement standard of *Improvement* will go wrong, as the value of *Improvement* will be very small all the time. Take Tables 3 and 4, for example, and calculate their *Improvement* values.

**Table 3.** The occurrence of "*A*" and "*B*".

|  | $B'$ **Occur** | $B'$ **Not Occur** | **Total** |
|---|---|---|---|
| $A'$ occur | 825 | 75 | 900 |
| $A'$ not occur | 50 | 50 | 100 |
| Total | 875 | 125 | 1000 |

**Table 4.** The occurrence of "*C*" and "*D*".

|  | *D* **Occur** | *D* **Not Occur** | **Total** |
|---|---|---|---|
| C occur | 345 | 55 | 400 |
| C not occur | 385 | 215 | 600 |
| Total | 730 | 270 | 1000 |

In terms of the *Improvement*, rule $(C \rightarrow D)$ is more valuable than rule $(A' \rightarrow B')$. However, the fact is also very clear that the occurrence probability of "B" with "A" occurring increases by up to 41.7% compared with the condition that "A" does not occur, while the occurrence probability of "D" with "C" occurring increases by 22.2% compared with the condition that "C" does not occur. Therefore, rule $(A' \rightarrow B')$ should be more meaningful than rule $(C \rightarrow D)$.

$$Imp(A' \rightarrow B') = [P(B'|A') - P(B')] = 0.032 \tag{8}$$

$$Imp(C \rightarrow D) = [P(D|C) - P(D)] = 0.133 \tag{9}$$

### 3.6. Chi-Square Analysis

Article [32] put forward an interestingness measurement standard based on a *T*-test, which used a *T*-test to analyze the difference between associated *Confidence* $P(B|A)$ and expected *Confidence* $P(B)$. If the difference is relatively bigger, it indicates that the occurrence of "*A*" has a larger influence on "*B*", and the rule $(A \rightarrow B)$ is interesting. The formula is as follows:

$$ChiSquare(A \rightarrow B) = [P(B|A) - P(B)]/\sigma \tag{10}$$

$$\sigma = \sqrt{\frac{P(B)(1 - P(B))}{n}} \tag{11}$$

If $ChiSquare(A \rightarrow B) > t_\alpha(n)$, it shows that it has a bigger difference between associated *Confidence* $P(B \,|\, A)$ and expected *Confidence* $P(B)$, and the rule $(A \rightarrow B)$ is interesting. To some extent, it enhances the traditional framework of the interestingness measure. However, there is also a defect that is similar to that of *Improvement*.

## 4. The Improvement of Objective Interestingness Measures

### 4.1. Bi-Support

If both rule $A \rightarrow B$ and rule $\neg A \rightarrow \neg B$ are strong, then the rule $A \rightarrow B$ would be very strong. Thus, we should look for strong evidence to prove these rules are interesting. We proposed the *Support* conditions (*Bi-support*) of the *Bi-directional* measure framework are:

(1)   $Supp(A \rightarrow B) \geq \min supp.$;

(2)   $Supp(\neg A \rightarrow \neg B) \geq \min supp.$

### 4.2. Bi-Lift

Related research shows that the *Lift* method helps produce good evaluation results. However, it is obvious that *Lift* puts $A$ and $B$ in equivalent positions, which shows rule $A \rightarrow B$ is equivalent to $B \rightarrow A$. If we accept rule $A \rightarrow B$, we should also accept rule $B \rightarrow A$. However, sometimes it is not true. For this problem, the paper proposes a *Bi-lift* measurement method. Since there is a need to study the relationship of $\overline{A} \rightarrow B$ when you want to evaluate the relationship of $(A \rightarrow B)$ by $Lift(A \rightarrow B)$, we introduce $Lift(\overline{A} \rightarrow B)$ to adjust $Lift(A \rightarrow B)$. The higher $Lift(A \rightarrow B)$ is, the better the rule $A \rightarrow B$ is; conversely, the higher $Lift(\overline{A} \rightarrow B)$ is, the worse the rule $A \rightarrow B$ is. Therefore, we propose a *Bi-lift* measurement method, taking $Lift(\overline{A} \rightarrow B)$ as the denominator and $Lift(A \rightarrow B)$ as the numerator to form the ratio of $Lift(A \rightarrow B)$ to $Lift(\overline{A} \rightarrow B)$. The *Bi-lift* formula is as follows:

$$\begin{aligned} Bi - lift(A \rightarrow B) &= \frac{Lift(A \rightarrow B)}{Lift(\overline{A} \rightarrow B)} \\ &= \frac{P(AB)/P(A)P(B)}{P(\overline{A}B)/P(\overline{A})P(B)} = \frac{P(AB)P(\overline{A})}{P(\overline{A}B)P(A)} \end{aligned} \tag{12}$$

Two conditions need to be satisfied. One is $P(\overline{A}B) \neq 0$, and the other is that "$A$" and "$B$" are not a certain event or an impossible event. Its value domain is $[0, \infty)$. The *Bi-lift* method takes the deduction of a negative premise as a constraint to form a bi-deduction comparing algorithm so as to improve the reliability of the mutual influence between the premise and follow-up.

### 4.3. Bi-Improvement

Owing to the defects of *Improvement*, the *Bi-improvement* [33] is put forward. When the occurrence probability of the antecedent is high, the value of *Improvement* will be very small all the time. In order to eliminate the influence, we make a correction by multiplying by the ratio of the occurrence possibility of the antecedent to the nonoccurrence probability of it. The *Bi-improvement* formula is as follows:

$$\begin{aligned} Bi - imp(A \rightarrow B) &= [P(B|A) - P(B)] * P(A)/P(\overline{A}) \\ &= \frac{P(AB) - P(A)P(B)}{P(\overline{A})} \end{aligned} \tag{13}$$

Taking Tables 3 and 4 as examples, $Imp(A' \rightarrow B') = 0.032$ and $Imp(C \rightarrow D) = 0.133$ in terms of *Improvement*; rule $(C \rightarrow D)$ is more valuable than rule $(A' \rightarrow B')$. However, the fact is that it can increase "the occurrence probability of $B$" with "$A$" occurring by 41.7% compared to the condition that "$A$" does not occur. Conversely, the occurrence possibility of "$D$" with "$C$" occurring increases by 22.2% compared to the condition that "$C$" does not

occur. Thus, rule $(A' \rightarrow B')$ should be more meaningful than rule $(C \rightarrow D)$. Calculate Bi-Improvement value through Formulas (14) and (15).

$$\begin{aligned} &Bi - imp(A \rightarrow B) \\ &= [P(B|A) - P(B)] * P(A)/P(\overline{A}) = 0.288 \end{aligned} \tag{14}$$

$$\begin{aligned} &Bi - imp(C \rightarrow D) \\ &= [P(D|C) - P(D)] * P(C)/P(\overline{C}) = 0.089 \end{aligned} \tag{15}$$

$Bi - imp(A \rightarrow B)$ is higher than $Bi - imp(C \rightarrow D)$, which is in agreement with the real condition.

### 4.4. Bi-Confidence

*Confidence* indicates that the occurrence of some itemsets will lead to the occurrence of other itemsets. However, we see that the *Confidence* of association rules only considers the occurrence possibility of "*B*" when "*A*" occurs but gives less consideration to the relation between "*A*" and "*B*" when "*A*" does not occur. Thus, it makes a lot of association rules mined invalid. For the above problems of association rules, we found that the description of *Confidence* is not perfect and cannot fully show the degree of correlation between itemsets. Thus, we put forward the concept of *Bi-confidence*, and its definition is as follows:

$$\begin{aligned} &Bi - conf(A \rightarrow B) = conf(A \rightarrow B) - conf(\overline{A} \rightarrow B) \\ &= \frac{P(AB) - P(A)P(B)}{P(A) \times [1 - P(A)]} \end{aligned} \tag{16}$$

The value domain of *Bi-confidence* is $[-1, 1]$. If the *Bi-confidence* value is greater than 0, then $P(AB) > P(A)P(B)$, which shows "*A*" and "*B*" have a positive correlation. If the *Bi-confidence* is equal to 1, then $P(AB) = P(A) = P(B)$, which shows that both "*A*" and "*B*" in record set appear together. If the *Bi-confidence* is equal to 0, then $P(AB) = P(A)P(B)$, which shows "*A*" has no relation to "*B*". If the *Bi-confidence* is less than 0, then $P(AB) < P(A)P(B)$, which shows that "*A*" and "*B*" have a negative correlation, and negative rules also have research value. The definition of *Bi-confidence* not only contains the correlation factors but also the $P(B)$ factor. Therefore, *Bi-confidence* can fully embody the effectiveness of the rules. By using the *Bi-support-Bi-confidence* framework to replace the *Support-confidence* framework, it can not only mine association rules effectively but also reduce the occurrence of rules with weak correlations.

## 5. The Bi-Directional Measure Framework of Association Rules and Experimental Analysis

### 5.1. Numerical Analysis of Simulated Data Sets

Based on a set of business data in Table 1, we test all kinds of measurement methods and design the measurement framework. Since item *E* appears in all affairs, we take it as a kind of certain event without taking into account those association rules about *E*. Set minimum *Support* to 20% and minimum *Confidence* to 50%. There are 25 rules of frequent 2-itemsets calculated through the Apriori algorithm, which are given in Table 5.

From Table 5, we can find that the *Support-Confidence* framework (min *supp.* = 0.2, min *conf.* = 0.5) mainly possesses the basic function of classic association rules measurement methods, but this framework cannot distinguish positive and negative correlation nor figure out the value of various rules. There are five rules that have negative correlation and four rules that have no correlation. Therefore, these nine rules are useless.

**Table 5.** Rules list with the Support-confidence framework (min *supp.* = 0.2, min *conf.* = 0.5).

| Rules | Supp. | Conf. | Lift | Val. | Conv. | Imp. | Csa. | Bi-lift | Bi-imp. | Bi-conf. |
|---|---|---|---|---|---|---|---|---|---|---|
| $M{\to}J$ | 0.3 | 0.75 | 1.5 | 0.1 | 2 | 0.25 | 1.58 | 2.25 | 0.16 | 0.42 |
| $M{\to}G$ | 0.3 | 0.75 | 1.08 | 0.1 | 1.2 | 0.05 | 0.35 | 1.13 | 0.03 | 0.08 |
| $J{\to}G$ | 0.4 | 0.8 | 1.14 | 0.1 | 0.75 | 0.1 | 0.69 | 1.33 | 0.1 | 0.2 |
| $I{\to}H$ | 0.3 | 0.75 | 1.88 | 0.2 | 2.4 | 0.35 | 2.26 | 4.5 | 0.23 | 0.58 |
| $I{\to}F$ | 0.4 | 1 | 1.25 | 0 | / | 0.2 | 1.58 | 1.5 | 0.13 | 0.33 |
| $H{\to}F$ | 0.3 | 0.75 | 0.95 | −0.2 | 0.8 | −0.05 | −0.4 | 0.9 | −0.03 | −0.08 |
| $R{\to}G$ | 0.5 | 1 | 1.42 | 0.3 | / | 0.3 | 2.1 | 2.5 | 0.3 | 0.6 |
| $N{\to}G$ | 0.4 | 0.67 | 0.95 | 0.1 | 0.9 | −0.03 | −0.21 | 0.89 | −0.05 | −0.08 |
| $R{\to}F$ | 0.4 | 0.8 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| $N{\to}F$ | 0.5 | 0.83 | 1.04 | 0.2 | 1.2 | 0.03 | 0.24 | 1.11 | 0.05 | 0.08 |
| $M{\to}F$ | 0.4 | 1 | 1.25 | 0 | / | 0.2 | 1.58 | 1.5 | 0.13 | 0.33 |
| $J{\to}F$ | 0.4 | 0.8 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| $G{\to}F$ | 0.5 | 0.71 | 0.89 | 0.2 | 0.7 | −0.09 | −0.71 | 0.71 | −0.21 | −0.28 |
| $J{\to}M$ | 0.3 | 0.6 | 1.5 | 0.2 | 1.5 | 0.2 | 1.29 | 3 | 0.2 | 0.4 |
| $G{\to}J$ | 0.4 | 0.57 | 1.14 | 0.3 | 1.17 | 0.07 | 0.44 | 1.71 | 0.18 | 0.24 |
| $H{\to}I$ | 0.3 | 0.75 | 1.88 | 0.2 | 2.4 | 0.35 | 2.26 | 4.5 | 0.23 | 0.58 |
| $F{\to}I$ | 0.4 | 0.5 | 1.25 | 0.4 | 1.2 | 0.1 | 0.65 | / | 0.4 | 0.5 |
| $G{\to}R$ | 0.5 | 0.71 | 1.42 | 0.5 | 1.75 | 0.21 | 1.33 | / | 0.49 | 0.71 |
| $G{\to}N$ | 0.4 | 0.57 | 0.95 | 0.2 | 0.93 | −0.03 | −0.19 | 0.86 | −0.07 | −0.1 |
| $F{\to}R$ | 0.4 | 0.5 | 1 | 0.3 | 1 | 0 | 0 | 1 | 0 | 0 |
| $F{\to}N$ | 0.5 | 0.63 | 1.04 | 0.4 | 1.07 | 0.03 | 0.19 | 1.25 | 0.12 | 0.13 |
| $F{\to}M$ | 0.4 | 0.5 | 1.25 | 0.4 | 1.2 | 0.1 | 0.65 | / | 0.4 | 0.5 |
| $F{\to}J$ | 0.4 | 0.5 | 1 | 0.3 | 1 | 0 | 0 | 1 | 0 | 0 |
| $F{\to}G$ | 0.5 | 0.63 | 0.89 | 0.3 | 0.8 | −0.07 | −0.48 | 0.63 | −0.28 | −0.38 |
| $M{\to}L$ | 0.2 | 0.5 | 2.5 | 0.2 | 0.64 | 0.3 | 2.37 | / | 0.2 | 0.5 |

If both rule $A \to B$ and rule $\neg A \to \neg B$ are strong, then rule $A \to B$ would be very strong. Thus, we should look for strong evidence to prove these rules are interesting. We proposed a novel *Bi-directional* measure framework of association rules, and the pseudocode of the proposed measure framework is shown in Algorithm 1. There are 30 rules of frequent 2-itemsets calculated through the Apriori algorithm, which are shown in Table 6.

**Table 6.** Rules list with the new framework (min *supp.*($AB$) = 0.2, min *supp.*($\overline{AB}$) = 0.2, min *conf.* = 0.2, and *Bi-conf.* = 0.2).

| Rules | Supp. AB | Supp. $\overline{AB}$ | Conf. | Lift | Imp. | Csa. | Bi-lift | Bi-Imp. | Bi-conf. |
|---|---|---|---|---|---|---|---|---|---|
| $F{\to}I$ | 0.4 | 0.2 | 0.50 | 1.25 | 0.10 | 0.65 | / | 0.40 | 0.50 |
| $F{\to}K$ | 0.3 | 0.2 | 0.38 | 1.25 | 0.08 | 0.52 | / | 0.30 | 0.38 |
| $F{\to}L$ | 0.2 | 0.2 | 0.25 | 1.25 | 0.05 | 0.40 | / | 0.20 | 0.25 |
| $F{\to}M$ | 0.4 | 0.2 | 0.50 | 1.25 | 0.10 | 0.65 | / | 0.40 | 0.50 |
| $G{\to}J$ | 0.4 | 0.2 | 0.57 | 1.14 | 0.07 | 0.45 | 1.71 | 0.17 | 0.24 |
| $G{\to}R$ | 0.5 | 0.3 | 0.71 | 1.43 | 0.21 | 1.36 | / | 0.50 | 0.71 |
| $H{\to}I$ | 0.3 | 0.5 | 0.75 | 1.88 | 0.35 | 2.26 | 4.50 | 0.23 | 0.58 |
| $H{\to}K$ | 0.2 | 0.5 | 0.50 | 1.67 | 0.20 | 1.38 | 3.00 | 0.13 | 0.33 |
| $I{\to}K$ | 0.2 | 0.5 | 0.50 | 1.67 | 0.20 | 1.38 | 3.00 | 0.13 | 0.33 |
| $J{\to}K$ | 0.2 | 0.4 | 0.40 | 1.33 | 0.10 | 0.69 | 2.00 | 0.10 | 0.20 |
| $J{\to}L$ | 0.2 | 0.5 | 0.40 | 2.00 | 0.20 | 1.58 | / | 0.20 | 0.40 |
| $J{\to}M$ | 0.3 | 0.4 | 0.60 | 1.50 | 0.20 | 1.29 | 3.00 | 0.20 | 0.40 |
| $J{\to}R$ | 0.3 | 0.3 | 0.60 | 1.20 | 0.10 | 0.63 | 1.50 | 0.10 | 0.20 |
| $K{\to}M$ | 0.2 | 0.5 | 0.67 | 1.67 | 0.27 | 1.72 | 2.33 | 0.11 | 0.38 |
| $L{\to}M$ | 0.2 | 0.6 | 1.00 | 2.50 | 0.60 | 3.87 | 4.00 | 0.15 | 0.75 |
| $I{\to}F$ | 0.4 | 0.2 | 1.00 | 1.25 | 0.20 | 1.58 | 1.50 | 0.13 | 0.33 |
| $K{\to}F$ | 0.3 | 0.2 | 1.00 | 1.25 | 0.20 | 1.58 | 1.40 | 0.09 | 0.29 |

**Table 6.** *Cont.*

| Rules | Supp. AB | Supp. $\overline{AB}$ | Conf. | Lift | Imp. | Csa. | Bi-lift | Bi-Imp. | Bi-conf. |
|---|---|---|---|---|---|---|---|---|---|
| L→F | 0.2 | 0.2 | 1.00 | 1.25 | 0.20 | 1.58 | 1.33 | 0.05 | 0.25 |
| M→F | 0.4 | 0.2 | 1.00 | 1.25 | 0.20 | 1.58 | 1.50 | 0.13 | 0.33 |
| J→G | 0.4 | 0.2 | 0.80 | 1.14 | 0.10 | 0.69 | 1.33 | 0.10 | 0.20 |
| R→G | 0.5 | 0.3 | 1.00 | 1.43 | 0.30 | 2.07 | 2.50 | 0.30 | 0.60 |
| I→H | 0.3 | 0.5 | 0.75 | 1.88 | 0.35 | 2.26 | 4.50 | 0.23 | 0.58 |
| K→H | 0.2 | 0.5 | 0.67 | 1.67 | 0.27 | 1.72 | 2.33 | 0.11 | 0.38 |
| K→I | 0.2 | 0.5 | 0.67 | 1.67 | 0.27 | 1.72 | 2.33 | 0.11 | 0.38 |
| K→J | 0.2 | 0.4 | 0.67 | 1.33 | 0.17 | 1.05 | 1.56 | 0.07 | 0.24 |
| L→J | 0.2 | 0.5 | 1.00 | 2.00 | 0.50 | 3.16 | 2.67 | 0.13 | 0.63 |
| M→J | 0.3 | 0.4 | 0.75 | 1.50 | 0.25 | 1.58 | 2.25 | 0.17 | 0.42 |
| R→J | 0.3 | 0.3 | 0.60 | 1.20 | 0.10 | 0.63 | 1.50 | 0.10 | 0.20 |
| M→K | 0.2 | 0.5 | 0.50 | 1.67 | 0.20 | 1.38 | 3.00 | 0.13 | 0.33 |
| M→L | 0.2 | 0.6 | 0.50 | 2.50 | 0.30 | 2.37 | / | 0.20 | 0.50 |

(a)    *Support* conditions (*Bi-support*):

(1) $Supp(A \rightarrow B) \geq \min Supp.$;
(2) $Supp(\neg A \rightarrow \neg B) \geq \min Supp.$

(b)    *Confidence* conditions:

(1) $Conf(A \rightarrow B) \geq \min conf.$

(c)    *Bi-confidence* conditions:

(1) $Bi\text{-}conf(A \rightarrow B) \geq \min Bi\text{-}conf.$

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**Algorithm 1:** Pseudocode of the proposed measure framework.

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**INPUT:** Shopping lists
**OUTPUT:** High value association rules
**Step1:** Calculate frequent 1-itemsets $L_1$.
**Step2:** Find frequent 2-itemsets $L_2$ with $L_1$: $l_1 \bowtie l_2$, namely rule $(A \rightarrow B)$;
**Step3:** Calculate the *Bi-support* of the association rules:
        (1) $Supp(A \rightarrow B) \geq$ *Support* threshold (min *Supp.*)
        (2) $Supp(\neg A \rightarrow \neg B) \geq$ *Support* threshold (min *Supp.*)
**Step4:** Calculate the *Confidence* of the association rules, namely *Conf* $(A \rightarrow B)$:
        *Conf* $(A \rightarrow B) \geq$ *Confidence* threshold (min *conf.*)
**Step5:** Calculate the *Bi-confidence* of the association rules, namely *Bi-conf* $(A \rightarrow B)$:
        *Bi-conf* $(A \rightarrow B) \geq$ *Bi-confidence* threshold (min *Bi-conf.*)
**Step6:** Output the high value association rules.

---

The time efficiency of this algorithm is $O(n)$. Compared with traditional methods, the complexity of time efficiency has not been increased.

The comparative results of the two different frameworks are shown in Table 7. There are nine rules that are useless (five rules have a negative correlation, and four rules have no correlation) in the *Support-confidence* framework. Due to the introduction of *Bi-support* and *Bi-confidence* to filter association rules, it is necessary to reduce the *Confidence* from 0.5 to 0.2 to generate more association rules. The new framework retains 12 high-value association rules of the old framework and reduces 32 non-value association rules. The high-value association rules are shown in Tables 8 and 9. Table 7 shows that when mining association rules, setting the *Confidence* threshold and the *Support* threshold too high will lose some high-value association rules. There are only 9 high-value association rules when the *Confidence* threshold is 0.5, while there are 12 high-value association rules when the *Confidence* threshold is 0.2. In order to mine more high-value association rules, we will set lower thresholds. However, low thresholds generate more low-value and valueless association rules at the same time. When the *Confidence* threshold is 0.2, it will generate

15 low-value association rules and 23 valueless association rules, which is more than when the *Confidence* threshold is 0.5. This will undoubtedly cost us more energy to focus on the actual value of association rules. However, the *Bi-support* and *Bi-confidence* framework can retain high-value association rules and remove most of the low-value and valueless association rules at low thresholds.

**Table 7.** Comparisons of the different frameworks.

| Framework | Conditions | High Value | Low Value | No Value | Total |
|---|---|---|---|---|---|
| *support-confidence* | min *supp.* ($AB$) = 0.2, and min *conf.* ($AB$) = 0.5 | 9 | 7 | 9 | 25 |
| *support-confidence* | min *supp.* ($AB$) = 0.2, and min *conf.* ($AB$) = 0.2 | 12 | 22 | 32 | 66 |
| *Bi-support* and *Bi-confidence* | min *supp.* ($AB$) = 0.2, min *supp.* ($\overline{AB}$) = 0.2, min *conf.* ($AB$) = 0.2, and *Bi-conf.* ($AB$) = 0.2 | 12 | 18 | 0 | 30 |

**Table 8.** Top 12 rules with the new framework (min *supp.*($AB$) = 0.2, min *supp.*($\overline{AB}$) = 0.2, min *conf.* = 0.2, and *Bi-conf.* = 0.2).

| Rules | Supp. AB | Supp. $\overline{AB}$ | Conf. | Lift | Imp. | Csa. | Bi-lift | Bi-Imp. | Bi-conf. |
|---|---|---|---|---|---|---|---|---|---|
| L→M | 0.2 | 0.6 | 1.00 | 2.50 | 0.60 | 3.87 | 4.00 | 0.15 | 0.75 |
| G→R | 0.5 | 0.3 | 0.71 | 1.43 | 0.21 | 1.36 | | 0.50 | 0.71 |
| L→J | 0.2 | 0.5 | 1.00 | 2.00 | 0.50 | 3.16 | 2.67 | 0.13 | 0.63 |
| R→G | 0.5 | 0.3 | 1.00 | 1.43 | 0.30 | 2.07 | 2.50 | 0.30 | 0.60 |
| H→I | 0.3 | 0.5 | 0.75 | 1.88 | 0.35 | 2.26 | 4.50 | 0.23 | 0.58 |
| I→H | 0.3 | 0.5 | 0.75 | 1.88 | 0.35 | 2.26 | 4.50 | 0.23 | 0.58 |
| M→L | 0.2 | 0.6 | 0.50 | 2.50 | 0.30 | 2.37 | | 0.20 | 0.50 |
| F→I | 0.4 | 0.2 | 0.50 | 1.25 | 0.10 | 0.65 | | 0.40 | 0.50 |
| F→M | 0.4 | 0.2 | 0.50 | 1.25 | 0.10 | 0.65 | | 0.40 | 0.50 |
| M→J | 0.3 | 0.4 | 0.75 | 1.50 | 0.25 | 1.58 | 2.25 | 0.17 | 0.42 |
| J→L | 0.2 | 0.5 | 0.40 | 2.00 | 0.20 | 1.58 | | 0.20 | 0.40 |
| J→M | 0.3 | 0.4 | 0.60 | 1.50 | 0.20 | 1.29 | 3.00 | | 0.40 |

**Table 9.** Top 9 Rules with the Support-Confidence framework (min *supp.* = 0.2, min *conf.* = 0.5).

| Rules | Supp. | Conf. | Lift | Imp. | Csa. | Bi-lift | Bi-imp. | Bi-conf. |
|---|---|---|---|---|---|---|---|---|
| G→R | 0.5 | 0.71 | 1.42 | 0.21 | 1.33 | / | 0.49 | 0.71 |
| R→G | 0.5 | 1 | 1.42 | 0.3 | 2.1 | 2.5 | 0.3 | 0.6 |
| I→H | 0.3 | 0.75 | 1.88 | 0.35 | 2.26 | 4.5 | 0.23 | 0.58 |
| H→I | 0.3 | 0.75 | 1.88 | 0.35 | 2.26 | 4.5 | 0.23 | 0.58 |
| F→I | 0.4 | 0.5 | 1.25 | 0.1 | 0.65 | / | 0.4 | 0.5 |
| F→M | 0.4 | 0.5 | 1.25 | 0.1 | 0.65 | / | 0.4 | 0.5 |
| M→L | 0.2 | 0.5 | 2.5 | 0.3 | 2.37 | / | 0.2 | 0.5 |
| M→J | 0.3 | 0.75 | 1.5 | 0.25 | 1.58 | 2.25 | 0.16 | 0.42 |
| J→M | 0.3 | 0.6 | 1.5 | 0.2 | 1.29 | 3 | 0.2 | 0.4 |

From Tables 5 and 6, we can draw the following conclusions.

(1) The traditional *Support-Confidence* framework can exclude most of the irrelevant association rules. When the *Support* threshold and *Confidence* threshold are low, it can bring about the combination explosion, and with low constraints, it can also give rise to a number of frequent patterns which have little correlation with it, and can even produce some rules which are either negatively correlated or totally wrong. On the contrary, when the *Support* threshold and *Confidence* threshold are high, some interesting rules and fresh knowledge that users show great interest in will be filtered out because of data sparsity.

(2)  The *Validity (Val.)* is not effective and differs greatly from other measurement methods. There are eight rules whose values are incorrectly estimated by the *Validity* method in Table 5. Rules $F{\rightarrow}J$ (*val.* = 0.3) and $F{\rightarrow}R$(*val.* = 0.3) are in fact irrelevant, while rules, such as $F{\rightarrow}G$ (*val.* = 0.3), $G{\rightarrow}N$ (*val.* = 0.2), $G{\rightarrow}F$ (*val.* = 0.2), and $N{\rightarrow}G$ (*val.* = 0.1), may have some restraining influences. The *Validity* of rule $I{\rightarrow}F$ and rule $M{\rightarrow}F$ are both 0, but they are actually positively related.

(3)  As the value domain of *Conviction* (conv.) is the value of *Conviction* being "1" implies that "*A*" have no relation with "*B*", and, here, "1" means independence. Additionally, the greater the value of *Conviction* is, the higher Interest the rule will have. Values in [0, 1) represent a negative correlation; thus, lots of valuable association rules will be excluded. For example, in Table 1, $Conv(J \rightarrow G) = 0.75 < 1$ and $Conv(M \rightarrow L) = 0.64 < 1$ both have a low value of *Conviction*, but, in fact, it is still possible to find high interest in the rule $J \rightarrow G$ and the rule $M \rightarrow L$. Moreover, it is supposed to meet the requirement of $P(A\overline{B}) \neq 0$ at the same time, and thus, the values of many rules cannot be calculated.

(4)  The evaluation result of *Lift* is satisfactory. However, obviously, *Lift* puts event A and event B in equivalent positions, that is to say, $Lift(A \rightarrow B)$ is equivalent to $Lift(B \rightarrow A)$. Once we accept rule $A \rightarrow B$, rule $B \rightarrow A$ should also be accepted, which is sometimes against the fact. Thus, under such circumstances, we propose *Bi-lift* to solve this problem. However, *Bi-lift* also has its own problem since it has to satisfy two conditions. One is $P(\overline{A}B) \neq 0$, and the other is that both *A* and *B* are not a certain event or an impossible event. Its value domain is $[0, \infty)$.

(5)  Shortcomings of Improvement (*Imp.*) are obvious. Firstly, it is hard to figure out what level of improvement is needed to make a difference. Secondly, the probability of the antecedent's occurrence will greatly affect the *Improvement* evaluation in such a way that when it is high, the value of Improvement will be very small all the time. Thus, it is difficult to distinguish valuable rules, and it is likely to make an inaccurate value evaluation. Chi-square analysis (*Csa.*) is proposed based on the Improvement (*Imp.*). However, the evaluation results indicate that it eventually fails to solve the problem of improvement, and its evaluation performance is not as good as that of *Bi-improvement*. Therefore, this paper puts forward *Bi-Improvement* aimed at evaluating the value of rules in a more accurate way.

(6)  The evaluation results of the author's "*Bi-confidence*", which is adjusted by the nonoccurrence probability of the antecedent, will increase the differentiation and accuracy. The novel *Bi-support and Bi-confidence* framework is more efficient than the traditional *Support-confidence* framework. The comparative results of the two different frameworks are shown in Table 7. There are nine rules that are useless (five rules have a negative correlation, and four rules have no correlation) in the *Support-confidence* framework. The new framework retains 12 high-value association rules of the old framework and reduces 32 non-value association rules. The high-value association rules are shown in Tables 8 and 9. Evaluation results and comparisons of *Improvement (Imp.), Bi-improvement (Bi-Imp.), Bi-confidence (Bi-conf.), Lift, Chi-square* analysis (*Csa.*), and *Bi-lift* are shown in Figure 1.
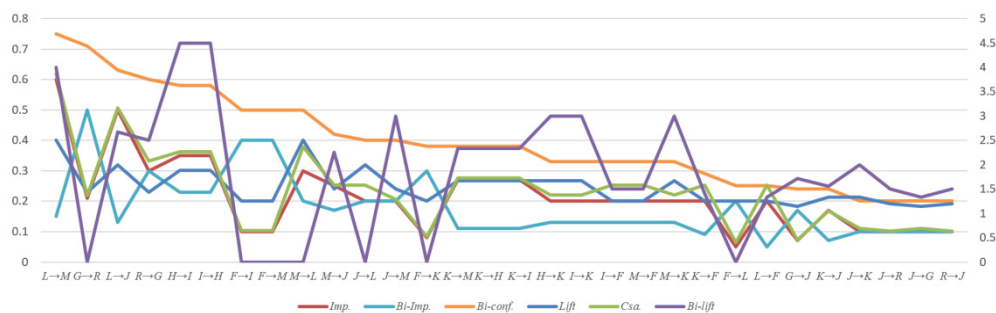


**Figure 1.** Evaluation results and comparisons of Improvement (Imp.), Bi-improvement (Bi-Imp.), Bi-confidence (Bi-conf.), Lift, Chi-square analysis (Csa.), and Bi-lift.

According to the evaluation results and the performance analysis for the measurement method, the *Validity* (*Val.*) is not effective, and it can even lead to some essential mistakes under some circumstances. For instance, sometimes, when the evaluation shows negative, but the actual result shows to be positive.

Though essential mistakes would not occur in the *Improvement* (*Imp.*) and *Chi-square* analysis (*Csa.*), the stability of their evaluation still needs further improvement since they are likely to produce errors in calculation. *Bi-lift* has a small problem, as its premise is $P(\overline{AB}) \neq 0$, and $A$ and $B$ are not certain events or impossible events. The evaluation results of the "*Bi-confidence*" could help improve the differentiation and accuracy, and the results show its high stability of evaluations. In conclusion, we can leave out indicators, such as *Validity* (*Val.*), *Improvement* (*Imp.*), *Chi-square* analysis (*Csa.*), *Lift*, and *Bi-lift* while we try to build up a reasonable measurement framework based on the indicators of *Support*, *Confidence*, and *Bi-confidence*. Procedures are as follows: firstly, use the *Support* threshold and *Confidence* threshold to filter out the frequent set, $Supp(A \rightarrow B) \geq \min supp$, $Supp(\neg A \rightarrow \neg B) \geq \min supp$, and $conf(A \rightarrow B) \geq \min conf$; secondly, calculate the *Bi-confidence* value, and select rules by *Bi-conf* $(A \rightarrow B) \geq \min$ *Bi-conf*.

To sum up, in this paper, we propose an improved evaluation methodology for mining association rules. However, given that only a small number of data are used in verification, the new method and the new framework need to be further tested through the analysis of practical cases in the following studies.

### 5.2. Verification Analysis of Public Data Sets

In order to test the rationality and stability of the Bi-directional measure framework of association rules, this paper uses the public built-in data set of IBM SPSS Modeler for verification. The data set consists of more than 1000 shopping records with 11 items, including fruits, fresh meat, dairy products, and so on. Each shopping record contains 1 to 11 items. This paper uses traditional methods and *Bi-support* and *Bi-confidence* framework to generate association rules and verifies the stability of the *Bi-support* and *Bi-confidence* framework in removing low-value and valueless association rules. When the amount of data becomes larger, it is necessary to reduce the *Confidence* threshold or *Support* threshold to generate sufficient association rules, or the valuable association rules will be lost. If the *Support* threshold is maintained at 0.2, as mentioned above, less than 10 association rules are mined, failing to validate the ability of the *Bi-support* and *Bi-confidence* framework to remove low-value and valueless association rules. Therefore, this paper adjusts the *Support* threshold to 0.05, generating 72 association rules.

The comparative results of two different frameworks are shown in Table 10. Of the 72 association rules mined in the *Support-Confidence* framework, only 10 are of high value, but 48 are invalid. In the *Bi-support* and *Bi-confidence* framework, 48 invalid association rules are successfully filtered out, and 10 high-value association rules are retained. High-value association rules are shown in Table 11.

**Table 10.** Comparisons of the different frameworks with data from the IBM SPSS Modeler.

| Framework | Conditions | High Value | Low Value | No Value | Total |
|---|---|---|---|---|---|
| *support-confidence* | min *supp.* $(AB) = 0.05$, and min *conf.* $(AB) = 0.2$ | 10 | 14 | 48 | 72 |
| *Bi-support* and *Bi-confidence* | min *supp.* $(AB) = 0.05$ min *supp.* $(\overline{AB}) = 0.05$, min *conf.* $(AB) = 0.2$, and $Bi - conf.(AB) = 0.2$ | 10 | 5 | 0 | 15 |

**Table 11.** Top 10 rules with the new framework (min *supp.*(AB) = 0.05, min *supp.*($\overline{AB}$) = 0.05, min *conf.* = 0.2, and *Bi-conf.* = 0.2).

| Rules | Supp. AB | Supp. $\overline{AB}$ | Conf. | Bi-conf. |
|---|---|---|---|---|
| Beer→Frozen meat | 0.29 | 0.57 | 0.58 | 0.39 |
| Frozen meat→beer | 0.30 | 0.57 | 0.56 | 0.39 |
| Frozen meat→Canned vegetables | 0.30 | 0.56 | 0.57 | 0.39 |
| Canned vegetables→Frozen meat | 0.30 | 0.56 | 0.57 | 0.39 |
| Beer→Canned vegetables | 0.29 | 0.57 | 0.57 | 0.38 |
| Canned vegetables→beer | 0.30 | 0.57 | 0.55 | 0.37 |
| Candy→wine | 0.27 | 0.58 | 0.52 | 0.32 |
| Wine→candy | 0.28 | 0.58 | 0.50 | 0.32 |
| Fish→Fruits and vegetables | 0.29 | 0.55 | 0.50 | 0.28 |
| Fruits and vegetables→fish | 0.29 | 0.55 | 0.48 | 0.28 |
| Beer→Frozen meat | 0.29 | 0.57 | 0.58 | 0.39 |
| Frozen meat→beer | 0.30 | 0.57 | 0.56 | 0.39 |

The bi-directional measure framework of association rules can stably remove low-value association rules and retain high-value rules in the verification of a small volume of data (11 instances, 10 pieces of data) and medium volume of data (11 instances, 1000 pieces of data). In order to test whether the bi-directional measure framework of association rules can perform well in the case of a large number of data, we use a groceries data set (https://github.com/stedy/Machine-Learning-with-R-datasets/blob/master/groceries.csv accessed in February 2021), which is a real transaction record of a grocery store for one month, with a total of 9835 consumption records and 169 items. The comparative results of the two different frameworks are shown in Table 12. There are 55 association rules with no value or low value among the 114 association rules mined in the *Support-Confidence* framework, while in the *Bi-support* and *Bi-confidence* framework, there are only 7 association rules with no value or low value, and 50 high-value association rules are retained.

**Table 12.** Comparisons of different Framework with Groceries data set.

| Framework | Conditions | High Value | Low Value | No Value | Total |
|---|---|---|---|---|---|
| *support-confidence* | min *supp.* (AB) = 0.02, and min *conf.* (AB) = 0.1 | 59 | 37 | 18 | 114 |
| *Bi-support* and *Bi-confidence* | min *supp.* (AB) = 0.02 min *supp.* ($\overline{AB}$) = 0.02, min *conf.* (AB) = 0.1 and *Bi-conf.* (AB) = 0.1 | 50 | 7 | 0 | 57 |

## 6. Conclusions

This paper presents several key indicators for analyzing and comparing the various measures developed in the statistics and data mining literature. Given differences in some of their indicators, a significant number of these measures may provide conflicting information about the interestingness of rules. Therefore, how to estimate the reliability of the obtained association rules has become a research hotspot. We demonstrate the benefits of using Support in eliminating irrelevant and poorly correlated patterns. Studying the traditional association rules mining relies on the *Support-confidence* framework, and only rules that meet both the *Support-Confidence* threshold can be called strong association rules. However, sometimes strong association rules are of no interest to users, and even worse, these association rules can be misleading to some extent. Thus, in order to find the most valuable association rules, it is necessary to further analyze and evaluate the mined rules. Finally, we build up a reasonable measure framework (*Bi-support* and *Bi-confidence* framework) for mining appropriate association rules.

After mining association rules, how to judge the value of the mined rules is another hot topic. Generally, since most rules with high *Support* are either obvious or known to users, rules with low *Support*, bringing users something new, may outperform the former in terms of novelty. However, a relatively low *Support* threshold can also give rise to the combination explosion. Thus, the best way to resolve this dilemma is to set a low *Support* threshold first or use the dynamic *Support* threshold to complete a series of mining and then employ the new association rules measure framework to screen the mining results and extract the most valuable and interesting association rules at the same time. The new association rule measurement framework has high accuracy and stability for the screening and selecting of association rules. Moreover, it can be widely used for mixed or combined recommendations in commercial areas.

Although our proposed method improves the accuracy of the traditional interestingness measure methods and framework, it suffers from two limitations. The first limitation is that the experiment did not test for massive data. The second limitation is that the limited experimental results in various related fields. In the future, we will design parallel methods to deal with massive data and apply our findings to various related fields to observe their robustness [34–41].

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