axioms

Editorial

Approximation Theory and Related Applications

Yurii Kharkevych

Faculty of Information Technologies and Mathematics, Lesya Ukrainka Volyn National University, 43025 Lutsk, Ukraine; kharkevich.juriy@gmail.com

The theory of approximation of functions is one of the central branches of mathematical analysis. It arose as a result of not only the internal development of mathematical science, but also the demands of practice. In terms of the concept of a function, it reflects a fundamental idea—the approximation (or replacement) of complex objects by ones that are simpler and more convenient to use. This idea is decisive regarding the relationship between mathematics and practice, which has always stimulated the development of approximation theory and which we hope will continue to garner interest in in the future.

This book is a collection of 14 papers included in the Special Issue "Approximation Theory and Related Applications" of the journal *Axioms*; it discusses contemporary problems in approximation theory, its applications in other areas of mathematics, and its practical uses. The main purpose of this Special Issue is to disseminate ideas and methods in approximation theory, to present new and significant results in this area, and to highlight related issues.

In [1], the authors discussed the well-known BMO class of functions of bounded mean oscillation by John–Nirenberg, which, long ago, became one of the most important concepts in harmonic analysis, partial differential equations, and related areas. Specifically, they investigated its applications to modern mapping theory. The authors established a series of criteria involving BMO to determine the existence of approximate solutions to the Beltrami equations in the whole complex plane, with asymptotic homogeneity at infinity. Note that such mappings inherit the main geometric properties of conformal mappings. These results can be applied to the fluid mechanics in strongly anisotropic and inhomogeneous media because the Beltrami equation is a complex form of the main equation of hydromechanics.

In [2], the authors considered abstract rings of multisets with components in a Banach algebra. These investigations are related to symmetric and supersymmetric polynomials and function calculus in algebras of analytic functions.

In [3], the authors showed that the Fréchet algebras of all entire bounded-type symmetric functions in the complex Banach space of all integrable essentially bounded functions in the arbitrary union of Lebesgue–Rohlin-measurable spaces are isomorphic to the Fréchet algebra of all entire bounded-type symmetric functions in the complex Banach space L_∞ of all Lebesgue-measurable essentially bounded functions in $[0,1]$.

The problems considered in paper [4] relate to rational approximations of the analytical functions of several variables—one of the main directions in the modern theory of continued and branched continued fractions. The authors constructed and investigated branched continued fraction expansions for ratios of the confluent hypergeometric function $\Phi_D^{(N)}$. Several numerical experiments are presented to indicate the power and efficiency of branched continued fractions as an approximation tool compared to multiple power series.

In [5], the authors solved one extremal problem of the theory of approximation of functional classes using linear methods. Namely, asymptotic equalities were obtained for the least upper bounds of the approximations of functions from the classes $W^r_{\beta,\infty}$ via generalized Abel–Poisson integrals in a uniform metric. These formulas provide a solution to the corresponding Kolmogorov–Nikol'skii problem.

check for updates

Citation: Kharkevych, Y. Approximation Theory and Related Applications. *Axioms* **2022**, *11*, 736. https://doi.org/10.3390/axioms11120736

Received: 14 December 2022
Accepted: 14 December 2022
Published: 16 December 2022

In [6], the authors used the averaging method, which is one of the most effective tools for constructing approximate solutions, including optimal control problems for ODEs and PDEs. The Krasnoselski–Krein theorem and its various modifications play an essential role in all such considerations, since it guarantees a limit transition in perturbed problems with fast-oscillating coefficients. The authors used this approach for a nonlinear parabolic system with fast-oscillating (w.r.t. time variable) coefficients on an infinite time interval. They proved that control of the problem using averaging coefficients could be considered "approximately" optimal for the initial perturbed system.

In [7], the authors investigated the ideas of deferred weighted statistical Riemann integrability and statistical deferred weighted Riemann summability for sequences of functions. They prove the existence of an inclusion theorem connecting these two concepts and two Korovkin-type approximation theorems using algebraic test functions.

In [8], with the help of Fubini's theorem, as well as a straightforward outcome of Keller's chain rule on time scales, the authors demonstrated new dynamic Hardy-type inequalities, which are reverse inequalities on time scales. Moreover, they generalized a number of other inequalities to a general time scale and obtained the discrete and continuous inequalities as special cases of the main results.

In [9], the authors studied the pointwise estimations of an unknown function in a regression model with multiplicative and additive noise. The authors found a linear wavelet estimator using the wavelet method and studied the order of the pointwise convergence of this estimator in the local Hölder space. They also constructed a nonlinear wavelet estimator using the hard thresholding method.

In [10], the authors refined the notion of the partial modular metric defined by Hosseinzadeh and Parvaneh to eliminate the occurrence of discrepancies in the non-zero self-distance and triangular inequality. The common fixed-point theorem for four self-mappings was proven, and the authors applied their results to establishing the existence of a solution for a system of Volterra integral equations.

In [11], a fractional model of the Hopfield neural network was considered in the application of the generalized proportional Caputo fractional derivative. The authors studied the stability of the Hopfield neural network using the generalized proportional Caputo-type fractional derivative and defined the equilibrium of the studied model.

In [12], the authors studied the relationship between the intuitionistic fuzzy reasoning and interval-valued fuzzy reasoning algorithms, and proved that there is a bijection between the solutions of the intuitionistic fuzzy triple I algorithm and the interval-valued fuzzy triple I algorithm. They also showed that there is a bijection between the solutions of the intuitionistic fuzzy reverse triple I algorithm and the interval-valued fuzzy reverse triple I algorithm.

In [13], the authors developed symbolic regression models for waste gasification. When evaluating CEET models based on input data, two different statistical metrics are usually used to quantify their accuracy: the mean square error and the Pearson correlation coefficient. The authors also demonstrated a universal method based on dynamic system criteria that can detect suitable models with good properties following statistical metrics.

In [14], the authors defined the quasi-density of natural number subsets, and they determined the necessary conditions to ensure that the quasi-statistical convergence was equivalent to that of the matrix summability method for a special class of triangular matrices with real coefficients.

Funding: This research received no external funding.

Data Availability Statement: Not applicable.

Conflicts of Interest: The author declares no conflict of interest.

References

1. Gutlyanskii, V.; Ryazanov, V.; Sevost'yanov, E.; Yakubov, E. BMO and Asymptotic Homogeneity. *Axioms* **2022**, *11*, 171. [CrossRef]
2. Chernega, I.; Zagorodnyuk, A. Supersymmetric Polynomials and a Ring of Multisets of a Banach Algebra. *Axioms* **2022**, *11*, 511. [CrossRef]
3. Vasylyshyn, T.; Zhyhallo, K. Entire Symmetric Functions on the Space of Essentially Bounded Integrable Functions on the Union of Lebesgue-Rohlin Spaces. *Axioms* **2022**, *11*, 460. [CrossRef]
4. Antonova, T.; Dmytryshyn, R.; Kurka, R. Approximation for the Ratios of the Confluent Hypergeometric Function by the Branched Continued Fractions. *Axioms* **2022**, *11*, 426. [CrossRef]
5. Kal'chuk, I.; Kharkevych, Y. Approximation Properties of the Generalized Abel-Poisson Integrals on the Weyl-Nagy Classes. *Axioms* **2022**, *11*, 161. [CrossRef]
6. Kapustian, O.A.; Kapustyan, O.V.; Ryzhov, A.; Sobchuk, V. Approximate Optimal Control for a Parabolic System with Perturbations in the Coefficients on the Half-Axis. *Axioms* **2022**, *11*, 175. [CrossRef]
7. Srivastava, H.M.; Jena, B.B.; Paikray, S.K. Some Korovkin-Type Approximation Theorems Associated with a Certain Deferred Weighted Statistical Riemann-Integrable Sequence of Functions. *Axioms* **2022**, *11*, 128. [CrossRef]
8. El-Deeb, A.A.; Cesarano, C. On Some Generalizations of Reverse Dynamic Hardy Type Inequalities on Time Scales. *Axioms* **2022**, *11*, 336. [CrossRef]
9. Kou, J.; Huang, Q.; Guo, H. Pointwise Wavelet Estimations for a Regression Model in Local Hölder Space. *Axioms* **2022**, *11*, 466. [CrossRef]
10. Das, D.; Narzary, S.; Singh, Y.M.; Khan, M.S.; Sessa, S. Fixed Point Results on Partial Modular Metric Space. *Axioms* **2022**, *11*, 62. [CrossRef]
11. Almeida, R.; Agarwal, R.P.; Hristova, S.; O'Regan, D. Quadratic Lyapunov Functions for Stability of the Generalized Proportional Fractional Differential Equations with Applications to Neural Networks. *Axioms* **2021**, *10*, 322. [CrossRef]
12. Luo, M.; Li, W.; Shi, H. The Relationship between Fuzzy Reasoning Methods Based on Intuitionistic Fuzzy Sets and Interval-Valued Fuzzy Sets. *Axioms* **2022**, *11*, 419. [CrossRef]
13. Praks, P.; Lampart, M.; Praksová, R.; Brkić, D.; Kozubek, T.; Najser, J. Selection of Appropriate Symbolic Regression Models Using Statistical and Dynamic System Criteria: Example of Waste Gasification. *Axioms* **2022**, *11*, 463. [CrossRef]
14. Masarova, R.; Visnyai, T.; Vrabel, R. Quasi-Density of Sets, Quasi-Statistical Convergence and the Matrix Summability Method. *Axioms* **2022**, *11*, 88. [CrossRef]