

Article

Application of the Multi-Criteria Optimization Method to Repair Landslides with Additional Soil Collapse

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Abstract: In current practice, the remediation of landslides has shown that the biggest problem is the increase in the number of works, and therefore the price of the works. This is due to several factors, including characteristic of the soil, such as the collapse (collapse) of the surrounding ground around the main slide during landslide remediation. Unless these soil erosion effects are taken into account, recovery costs will overrun, which can jeopardize the planned budget. This paper presents a multi-criteria optimization of landslide remediation using the PROMETHEE method and determines the optional number of walls for the additional soil erosion. In a case study on examples of real landslides in the Republic of Serbia, the application of the method is presented and appropriate conclusions are drawn.

Keywords: landslide; repair costs; multi-criteria optimization; PROMETHEE method

1. Introduction

The main goal of this paper is to determine the required number of exploratory wells for additional landslides for different types of landslides, and methods used for their remediation, using the method of multicriteria optimization (PROMETHEE), taking into account the effects of increasing workloads, which can greatly affect the total costs of landslide remediation and which may lead to exceeding the planned budget.

The article is organized as follows. Section [2](#page-1-0) points out the importance of applying multicriteria optimization methods in solving landslide remediation problems and lists the most commonly used methods. For the purposes of research in this paper, the PROMETHEE method was chosen and the theoretical foundations on which it is based are presented.

In Section [3,](#page-3-0) case studies on the examples of seven landslides located in the territory of the Republic of Serbia are presented, in order to consider a wide range of effects of multicriteria optimization of the application of the PROMETHEE method. Activities, criteria, and appropriate weighting coefficients are defined. For the given scenarios (and selected preferences) and based on the ranking of the flow function, the function of the number of wells was optimized during additional soil subsidence. The obtained results are presented and a comment is made on the obtained ranking results.

Sensitivity analysis of the obtained results was performed. For sensitivity analysis, in addition to the given criteria, data were used to increase the amount of material ∆V of

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the performed state of landslide remediation V_i according to the amount of material of the project solution of landslide remediation

In Section [4,](#page-20-0) the corresponding conclusions are derived from the multicriteria method of optimization of the given problem presented in the previous section.

Section [5](#page-20-1) describes the contribution of the work to science.

Estimating the condition of the landslide and the need for remediation of the same representation is a complex engineering-technological-economic problem that is solved by analyzing several parameters in the process of making the final decision. In the preliminary phase of the project development, the methods of remediation are considered, taking into account the engineering (rehabilitation technology, change in the geometry of the section, the system of drains, different types of support structures, application of deep foundation (piles) in combination with other geotechnical structures, application of coatings, application geosynthetics, geo and geochemicals, injection of masses of soil, application of electro osmosis, etc.) and economic (work premise and calculations) aspects of rehabilitation. The final design solution implies the selection of an optimal remediation method with the appropriate safety factor.

One of the most important aspects in landslide remediation, in addition to the analysis of landslide stability, is the problem of estimating the quantity of earthworks. When landscaping the landslide, situations arise such that, when excavated, the influence of destabilizing forces is further increased, which was not originally taken into account. The soil that forms the slopes of the excavation plays a role in secondary landslides that may or may not have to be activated. In the case of activation, an additional quantity of earth is being ruined, increasing the effect of the existing landslide. Experience in practice so far has shown that if these soil erosion effects are not taken into account at the stage of development of project documentation, the cost of rehabilitation can be up to 20% higher. In certain situations, this percentage increase may be even higher.

For a mass of soil that collapses during the rehabilitation of the main landslide, additional research and analyses are carried out (length of landslide and number of boreholes with increase of excavation and embankment, surface of landslide, number of boreholes per hectare with increase of excavation and embankment, drilling per hectare with increase of excavation and embankments, cost of drilling with projected price, and cost of drilling costs with the cost of execution). The big problem for the investor is the number of exploration wells, since the increase in the number of wells also increases the cost of landslide remediation. The cost of geomechanical exploration increases, and therefore the cost of the project. Determining the safety factor requires more detailed input parameters, which are directly correlated with the number of exploratory wells.

2. Application of Multi-Criteria Optimization to Landslide Rehabilitation

Solving such problems can be achieved through the application of multi-criteria optimization methods (MCDM—Multi-Criteria Decision Making). Multi-criteria optimization of landslide remediation is considered through the function of the amount of work in the additional soil erosion. Using multi-criteria optimization in the analysis of landslides with additional soil erosion can emphasize the good and bad aspects of the landslide removal method itself [\[1,](#page-20-2)[2\]](#page-20-3).

Today there is a considerable number of multi-criteria optimization methods in which the solution of a multi-criteria problem is obtained by choosing the best alternative from a set of defined alternatives (MADM—Multi-Attribute Decision Making) or by programming the best alternative (MODM—Multi-Objective Decision Making). The most commonly used methods are TOPSIS [\[3,](#page-20-4)[4\]](#page-21-0), VIKOR, ELECTRE [\[5](#page-21-1)[–9\]](#page-21-2), PROMETHEE, etc.

For the purposes of this research, the PROMETHEE method was selected, given its proven reliability in the application of various multi-criteria problems in different engineering and economic fields. The key elements of PROMETHEE methods are predefined scenarios, activities, criteria, and appropriate weighting coefficient ranking; its interaction

provides a spectrum of results that can be used in making final conclusions and decisions for the following facts. $\frac{1}{2}$ for the following forts a spectrum of results that can be used in making final conclusions at $p_{\rm c}$ following forts can be used in making final conclusions and decisions ρ in the application of various multi-criteria problems in different problems in diffe ral conclusions and decisions ρ in the application of various multi-criteria problems in different problems in different problems in different problems in σ $\frac{1}{2}$ for the following facts province reliability in the application of various multi-criteria problems in different pr $F_{\rm e}$ the purposes of the PROMETHEE method was selected, given its respectively. p_{ref} religion of various multi-criteria problems in different σ the purposes of this research, the PROMETHEE method was selected, given its research, given its σ σ provides a spectrum of results that can be used in making final conclusions and decisions
for the following facts.
PROMETHEE Method

PROMETHEE Method interaction provides a spectrum of results that can be used in making final conclusions that can be used in making final conclusions that can be used in making final conclusions that can be used in maki predefined scenarios, activities, criteria, and appropriate weighting coefficient ranking; its predefined scenarios, activities, criteria, and appropriate weighting coefficient ranking; its $\mathbf{p}_{\mathcal{D}}$ and $\mathbf{p}_{\mathcal{D}}$ are key elements of P predefined schemarios, and appropriate weighting contains a propriate α $DDOMT T U T L M \sim 1$ predefined schemarios, and appropriate weighting coefficient ranking; its induction α engineering and economic fields. The key elements of PROMETHEE methods are engine and exception and economic fields. The key elements of PROMETHEE methods are \mathbb{R}^n

Table 1. Matrix relates activities and criteria.

In general, the mathematical problem according to the PROMETHEE method can be *formulated as* [\[10](#page-21-3)[–17\]](#page-21-4): formulated as $[10-17]$: and decisions for the following facts. and decisions for the following facts. and decisions for the following facts. n general, the mathematical problem according to the PROMETHEE method can be

$$
\max\{f_1(a), f_2(a), \ldots, f_j(a), \ldots, f_k(a)|a \in A\}
$$
 (1)

Table 1. Matrix relates activities and criteria.

where A is the final set of *n* activity, f_k are the criteria, $f_j(a)$ is an evaluation of the activity *a* for the given criterion f_j presented in Table [1.](#page-2-0) $\frac{1}{\sqrt{2}}$ is the final set of $\frac{1}{\sqrt{2}}$ max{ } *f*1() () () () *a , f*² *a ,f ^j a ,fk a a*∈*A* (1) max{ } *f*1() () () () *a , f*² *a ,f ^j a ,fk a a*∈*A* (1) $\frac{1}{2}$ max{ } *f*1() () () () *a , f*² *a ,f ^j a ,fk a a*∈*A* (1) In general, the mathematical problem according to the mathematical problem according to the PROMETHEE method can be $\frac{1}{2}$ is the middle as $\frac{1}{2}$ In general, the mathematical problem according to the PROMETHEE method can be \mathcal{L} $f(x, y, y)$ and the crite In general, the mathematical problem according to the PROMETHEE method can be f is an evaluation ϵ *PROMETHEE Method* $\sum_{i=1}^{n}$ *PROMETHEE Method* $\sum_{i=1}^n \sum_{i=1}^n \sum_{j=1}^n \sum_{j$ *PROMETHEE Method* $\sum_{i=1}^{n}$ generally $\sum_{i=1}^{n}$ for $\sum_{i=1}^{n}$ by the Problem according to the Red May *PROMETHEE Method* $\sum_{i=1}^{n}$ generation of the Red $\sum_{i=1}^{n}$

Table 1. Matrix relates activities and criteria.

Table 1. Matrix relates activities and criteria. where *A* is the final set of *n* activity, *fk* and activity *ff* and activity *a* activity *a* activity *a* and activity *a* activity *a* and activity

Table 1. Matrix relates activities and criteria.

In the phase of setting the problem and defining the activities and criteria, it is necfunction for the given corresponding ϵ and the corresponding set of ϵ and ϵ the distribution model and the corresponding intervals of minimum and maximum tion for the given criterion and the corresponding set of activity evaluations defines the distribution model and the corresponding intervals of minimum and maximum values. The previously performed statistical data analysis and the qualitative evaluation of the obtained relations of activities and criteria determine the values of the indifference and as sufficient to generate complete preference (crucial in decision making). In general, the prevaience muerval. mum omplete preference (crucial in decision making). In be presented as: function can be presented as: f_{min} function f_{min} for the corresponding set of f_{min} and f_{min} and f_{min} evaluations defining to f_{min} f to define the rangements of preference and weight coefficients. The pref as sufficient to generate complete preference (complete preference of the main price function. Humberline is the \overline{c} *f* **f** *f f* *****f f f f f f f f f f f f f f f f <i>f f <i>f* *****f <i>f <i>f <i>f <i>f <i>f <i>f <i>f <i>f <i>f <i>f* $\frac{1}{1}$ function for the given criterion for the corresponding set of activity evaluations defines defines defines defines the distribution model and the corresponding intervals of minimum and maximum as sufficient to generate complete preference α in decision making). In general, the gen e is the largest deviation, the below σ σ () σ () () σ to generate complete preference (crucial in decision making). In general, the preference function can be presented as: function for the given corresponding set of activity evaluation ϵ to distribution model and the corresponding intervals of minimum and maximum a $\frac{1}{2}$ is generated to generate $\frac{1}{2}$ in decision making $\frac{1}{2}$ preference function can be preference function p \mathcal{L} () () () essary to define the functions of preference and weight coefficients. The preference func $f(x) = \frac{1}{2}x^2 + \frac{1}{2}$ tion for the given criterion and the corresponding set of activity t_{noncell} as a into consideration as into the language deviation the help t_{noncell} prevaience merval. Hemerence is the largest deviation, the below consideration as minor, while preference represents a slightest deviation taken as sufficient function for the given corresponding set of activity evaluation and the corresponding set of activity evaluations of activity of the given criterion and the corresponding set of activity evaluations $t_{\rm max}$ into consideration as in the present deviation, the holomorphic consequents are α as sufficient to generate the decision of the complete preference in decision makes and the as: $\overline{}$ [≤] ⁼ *^p ^f ^a ^f ^b ^f ^a ^f ^b f a f b <i>P* $\frac{1}{2}$, *p* $\frac{1}{2$ $\frac{1}{2}$ $f_{\rm H}$ function for the given coefficient set of activity evaluation $f_{\rm H}$ the corresponding set of activity evaluations defines the taken into consideration as minor, while preference represents a slightest deviation taken as the hargest deviation, the below values are not taken must [≤] ⁼ *^p ^f ^a ^f ^b ^f ^a ^f ^b f a f b ^P ^a ^b* , for $\mathcal{L}(\mathcal{L})$ for $\mathcal{L}(\mathcal{L})$ for $\mathcal{L}(\mathcal{L})$, $\mathcal{L}(\mathcal{L})$ $f(x, t) = \frac{1}{\sqrt{2\pi}} \int_{0}^{t} \frac{1}{\sqrt{2\pi}} \int$ the distribution model and the corresponding intervals of minimum and maximum and maximum and maximum and maxi taken into consideration as minor, while preference represents a slightest deviation taken as below values are not taken must ⁰ for , (2) $\frac{1}{2}$ for the given exiterion and the corresponding of politics. function for the given criterion and the corresponding set of activity prevalence interval. Indifference is the largest deviation, the below values are not taken into the consideration into consideration as minor, which preference represents a slightest deviation taken the second consideration as filliof, write preference represents a sugniest det α the given exiterion and the corresponding of positivity evaluations. function for the given criterion and the corresponding set of activity evaluations defines $t_{\rm max}$ into consideration as minor, which preference represents a slightest deviation taken the slightest deviation taken $t_{\rm max}$ aeration as minor, while preference represents a siignest deviation taken $\frac{1}{\hbar}$ to define the functions of preference and weight coefficients. The preference and weight function for the given criterion and the corresponding set of activity evaluations defines are $\frac{1}{2}$ as sufficient to generate complete preference (crucial in decision making). In general, the $\frac{1}{2}$ certificial to define define the preference and weight coefficients. The preference and $\frac{1}{2}$ function for the given corresponding set of activity evaluations defines the $t_{\rm c}$ is taken into consideration as minor, while preference represents a slightest deviation taken α ritest deviation taken as sufficient tion for the given criterion and the corresponding set of activity evaluations defines the $\frac{1}{2}$ is defined and $\frac{1}{2}$ of preference and weight coefficients. The preference and $\frac{1}{2}$ is defined as $\frac{1}{2}$ distribution flouer and the corresponding finervals of minimum and prevalence intervals. In the largest deviation, the largest deviation, the below values are not deviated are not taken into consideration as minor, while preference represents a slightest deviation taken $\frac{1}{2}$. The functions of preference and weight coefficients of preference and weight coefficients. The preference and weight coefficients. The preference and weight coefficients. The preference and $\frac{1}{2}$ punon moder and the corresponding intervals of infilmum and maxim and prevalence interval. Indifference is the largest deviation, the below values are not taken into consideration as minor, while preference represents a slightest deviation taken $\frac{1}{2}$ to define the functions of preference and weight coefficients. The preference and $\frac{1}{2}$ function for the given corresponding the corresponding set of activity evaluations defined and the corresponding ϵ and prevents are variant previous intervals. In difference is the largest deviation, the below ϵ teleference represents a sugnition deviation taken as samelent necessary to define the functions of p distribution model and the corresponding intervals of minimum and maximum values. and prevalence interval. Indifference is the largest deviation, the below values are not consideration as minor, while preference represents a slightest deviation taken as sufficient

$$
P(a,b) = \begin{cases} 0 & \text{for} \quad f(a) \le f(b) \\ p[f(a), f(b)] & \text{for} \quad f(a) > f(b) \end{cases}
$$
 (2)

Figure 1 shows the types of preference functions implemented in the PROMETHEE method, for $f(a) > f(b)$ are $[10-17]$: [∀] [≤] ⁼ *¹ ^з^a ^x ⁰* are $[10-17]$: [∀] [≤] ⁼ *¹ ^з^a ^x ⁰* method, for $f(a) > f(b)$ are [\[10](#page-21-3)-17]:

$$
p(x) = \begin{cases} 0 & 3a \quad \forall x \le 0 \\ 1 & 3a \quad \forall x > 0 \end{cases}
$$

$$
p(x) = \begin{cases} x/m & 3a \quad x \le m \\ 1 & 3a \quad x \ge m \end{cases}
$$

$$
p(x) = \begin{cases} 0 & 3a \quad x \le l \\ 1 & 3a \quad x > l \end{cases}
$$

$$
p(x) = \begin{cases} 0 & 3a \quad x \le q \\ 0.5 & 3a \quad q < x \le q + p \\ 1 & 3a \quad x > q + p \end{cases}
$$

$$
p(x) = \begin{cases} 0 & 3a \quad x \le s \\ (x - s)/r & 3a \quad s \le x \le s + r \\ 1 & 3a \quad x \ge s + r \end{cases}
$$

$$
p(x) = \begin{cases} 0 & 3a \quad x \le 0 \\ 1 - e^{-x^2/2\sigma^2} & 3a \quad x \ge 0 \end{cases}
$$

Based on the weight coefficients w_i assigned to each criterion, a preference index can be determined:

π() () *a,b w Pj a,b*

$$
\pi(a,b) = \sum_{j=1}^{k} w_j P_j(a,b)
$$
\n(3)

where it is If (it is) *π*(*a*,*b*) = 0, then all values are *Pj*(*a*,*b*) = 0, and if (it is) *π*(*a*,*b*) = 1, then all values

$$
\sum_{j=1}^{k} w_j = 1 \tag{4}
$$

⁼ *^j* ⁼¹ (3)

If (it is) $\pi(a,b) = 0$, then all values are $P_j(a,b) = 0$, and if (it is) $\pi(a,b) = 1$, then all values
((a,b) - 1. Now for each activity positive Φ^+ and positive Φ^- flows son be determined are $P_j(a,b) = 1$. Now, for each activity, positive Φ^+ and negative Φ^- flows can be determined: hen all values are $P_j(a,b) = 0$, and if (it is) $\pi(a,b) = 1$
ach activity, positive Φ^+ and negative Φ^- flows can ≠

$$
\Phi^{+}(a) = \frac{1}{n-1} \sum_{b \neq a} \pi(a, b), \n\Phi^{-}(a) = \frac{1}{n-1} \sum_{b \neq a} \pi(b, a)
$$
\n(5)

Rankings based on the usage of *Φ*⁺ and *Φ*[−] flows are partial rankings, while complete ranking is performed according to the term: $\overline{0}$ is performed according to the term is performed according to the term:

$$
\Phi(a) = \Phi^+(a) - \Phi^-(a) \tag{6}
$$

A positive flow of preferences shows the degree of importance (dominance) of one A positive flow of preferences shows the degree of importance (dominance) of one activity in relation to other activities, so if the value of the positive flow is greater Φ^+ \rightarrow 1, then the activity is even more significant. The negative flow of preference indicates the then the activity is even more significant. The negative flow of preference indicates the weakness of the activity, or shows how much other activities are preferred in relation to the weakness of the activity, or shows how much other activities are preferred in relation to activity for which (it is) Φ^- →0.

Figure 1. Types of preference functions. (a–c) Usual V shape U shape, (d–f) level linear Gaussian.

3. Case Study

In the case study [\[18–](#page-21-5)[22\]](#page-21-6) on real examples of landslides in the territory of the Republic of Serbia, the multicriteria optimization of landslide landscaping carried out for the budget model is presented—a determination of the required number of boreholes for additional soil erosion.

3.1. Kominje

At the site "Kominje 2" (Figure [2\)](#page-4-0) on the main road M22, the section Novi Pazar— Ribariće km $478 + 910$ (ID 0253), due to the terrain slipping, had a road embankment collapse as well as a significant carriageway collapse (on the longitudinal profile of 30 cm length) at a road stretch of 50 m [\[23\]](#page-21-7). The slopes are of elongated shape (the length of the body of the landslide is 50 m and the total height difference is 20 m) and it is made of clay and crushed material by which one erosion groove is filled, and the movement of the material itself is slow and takes place in the form of a phase of intermittent plastic

 \mathcal{L} road stretch of \mathcal{L} shape (the length of the length o $\frac{1}{\sqrt{1-\frac{1$

deformations. On steep slope sides, kinematic conditions met with the orientation of the deformations. On steep slope sides, kinematic conditions met with the orientation of the elements of the assembly; deep, blocky landslides or slopes are often formed. The sliding elements of the assembly; deep, blocky landslides or slopes are often formed. The sliding surface is formed in contact with less degraded wall mass. surface is formed in contact with less degraded wall mass.

Figure 2. "Kominja 2" location on which the landslide is formed [\[23](#page-21-7)]. **Figure 2.** "Kominja 2" location on which the landslide is formed [23].

The project includes a part of the landslide in the traffic zone. First, stability analyses The project includes a part of the landslide in the traffic zone. First, stability analyses were carried out prior to the planned remediation measures and the position of the critical were carried out prior to the planned remediation measures and the position of the critical slip surface was determined with a minimum safety factor in relation to the state of the slip surface was determined with a minimum safety factor in relation to the state of the boundary equilibrium $F_s = 1.02$. After these conducted analyses, a check of the stability the slope under the conditions of the designed rehabilitation measures of the newly of the slope under the conditions of the designed rehabilitation measures of the newly projected deep trenches D_1 and D_2 was made, including lowering of groundwater level, drainage, and regulation of surface atmospheric waters. The results of the analysis show drainage, and regulation of surface atmospheric waters. The results of the analysis show that the safety factor obtained is $F_s = 1.343$. Then, the analysis of the slope stability was carried out under the conditions of the planned remediation measures, including the carried out under the conditions of the planned remediation measures, including the impact of the earthquake with the coefficient of seismicity $K = 0.05$. The results of the analysis show that the safety factor obtained is $F_s = 1.18$.

3.2. Zavlaka 3.2. Zavlaka

On the section of the state road $111-137$ (R-221a) Sabac-Tekeris-Zaviaka at km 10 + 225, there was a landslide [\[24\]](#page-21-8) (Figure [3\)](#page-4-1). The rehabilitation road route is side out and the destruction involved the left side of the road, the embankment slope. The frontal width of destruction involved the left side of the road, the embankment slope. The frontal width of the landslide was 50 m. The activation of the landslide was, due to the sudden drowning the landslide was 50 m. The activation of the activation of the field α such the landslide was α moduling the sudden drowning the sudden of the sudden and interest α is the field α such the sum of the sum of t of the soil with water and intense swelling of water in the field, as well as the large amount
of atmospheric precipitation of atmospheric precipitation. of atmospheric precipitation. On the section of the state road IIA-137 (R-221a) Šabac-Tekeriš-Zavlaka at km 10 + 225,

Figure 3. Slope on the section: Šabac-Tekeriš-Zavlaka at km 10 + 225 [\[24](#page-21-8)]. **Figure 3.** Slope on the section: Šabac-Tekeriš-Zavlaka at km 10 + 225 [24].

The implemented remedial measures—the drainage system development and the The implemented remedial measures—the drainage system development and the surface regulation of atmospheric waters. Appropriate analyses of the stability of the surface regulation of atmospheric waters. Appropriate analyses of the stability of the slopes at the moment of slipping and after the implemented remediation measures were carried out. The analysis of the slope stability was made using the method *Morgenstern-Price* with

the conventionally assumption of the sine function of the inclination of inter-cellular forces. After the development of the longitudinal drainage trench D_1 and the cross-drain trench A fter the development of the longitudinal drainage trench D_1 and the cross-drain trench Extra are no trep part of the congruencial trench D₂, a new stability analysis was performed. The results of the analysis show that the safety F_2 , which can expect the same safety factor was $F_s = 1.40$.

3.3. Jezgrovi´ce 3.3. Jezgroviće

At the "Jezgroviće 2" (Figure [4\)](#page-5-0) locality of the main M2 road, the Ribariće-Vitkovići section, a $1185 + 100$ (ID 0066) km, road route is side cut and the slipping terrain led to the carriageway depression (collapse) at the road stretch of 120 m [\[25\]](#page-21-9). Large cracks and denivelations of 30 cm appeared on carriageway. Some sections of the existing supporting denivelations of 30 cm appeared on carriageway. Some sections of the existing supporting wall were separated and inclined. Traces of rupture separating individual structural wall were separated and inclined. Traces of rupture separating individual structural blocks of massifs, partially visible on the ground and partly assumed, formed kinematic blocks of massifs, partially visible on the ground and partly assumed, formed kinematic active fields. active fields.

Figure 4. Locality "Jezgroviće 2" which was formed in landslide [\[25](#page-21-9)].

The implemented remediation measures were also adopted—moving one track of The implemented remediation measures were also adopted—moving one track of the road to the hill, making a reinforced concrete wall and regulation of atmospheric and the road to the hill, making a reinforced concrete wall and regulation of atmospheric and underground waters. For the characteristic geotechnical profile of the terrain, stability underground waters. For the characteristic geotechnical profile of the terrain, stability analyses were first performed prior to the planned remediation measures. On the basis of analyses were first performed prior to the planned remediation measures. On the basis of the computational analysis, the position of the critical sliding surface has been determined the computational analysis, the position of the critical sliding surface has been determined with a minimum safety factor in relation to the state of the boundary equilibrium, whereby $\overline{}$ a safety factor $F_s = 1.055$. The stability of embankments has been examined for the case when the slope is side cut and intersects with a supporting structure $F_s = 1.48$.

3.4. Footwear Factory 3.4. Footwear Factory

on the part of the regional road II of the line R234, Novi Pazar-Rajetice (Figure 5), km 1 + 000, (ID 1086), at the site of the footwear factory (Rudera Bošković street in Novi Pazar), excavation in the pier from the embankment sheath caused the collapse of the shoulder and part of the pavement [\[26\]](#page-21-10). To plan the plot on the right side of the road, On the part of the regional road II of the line R234, Novi Pazar-Rajetiće (Figure [5\)](#page-5-1), an excavation of a steep cliff was made. After the precipitation and under the load of the road there was a collapse of the mentioned scarp. there was a collapse of the mentioned scarp.

Figure 5. Slipway on the part of the regional road II of line R234, Novi Pazar-Rajetiće (a) right view; **central view** $[26]$ **.**

The adopted and implemented recovery solution wasthe development of a reinforced concrete wall and regulation of rain and underground water. The first model analyzed

the case of a non-concealed slope after excavation. The method of reverse analysis was to examine to what extent the shear strength of the soil was mobilized. The obtained result is valid when groundwater filtration takes place in the field. When a rise in groundwater level in the field forms a line of scattering due to the increase in the pore pressure in the soil, at the expense of effective strength, the slope hits the state of the boundary balance and breakdown (F_s = 0.996). After these conducted analyses, a check of the stability of the slope was achieved in the conditions of the designed rehabilitation measures, i.e., the construction of a reinforced concrete wall, and the safety factor is $F_s = 2.194$.

The adopted and implemented recovery solution was the development of and implement of and α

3.5. 6 + 900 $+900$

On the part of the state road IIA-137 (R-127), the section Krupanj-Mačkov Kamen, km $6 + 900$, due to heavy precipitation, an active landslide was formed [\[27\]](#page-21-11) (Figure 6). Slipping completely destroyed the hull of the road at the length of 60 m with a leading scar of height of 3–5 m. The length of the landslide is 140 m along the local road and about 50 m below the local road with visible traces of the picking of the fine material from the formed sliding body. In addition to the above-mentioned sliding body, two more sliding bodies were registered in the direction of the growth of the station to Mačkov Kamen. The total width of all three slopes formed on the hill side is 160 m, 60 m of which are interrupted, 100 m along on the slope below the route. At a minimum distance of 5 m below the bank road, the scar was formed at a length of 15 m with a sub vertical denivelations of 8 m. $\,$

Figure 6. Formed slip on the part of the state road IIA-137 (P-127) [\[27\]](#page-21-11).

The adopted and implemented remediation measures are construction of a stone threshold as a support to the new embankment, regulation of underground water (construction of longitudinal drainage on the left side of the road), replacement of a part of the embankments on the right side of the road, regulation of atmospheric waters and protection of the slope from erosion. First, stability analyses were carried out prior to the planned remediation measures; the position of critical sliding surfaces with a minimum safety factor was determined in relation to the state of the boundary equilibrium $F_s = 1.001$. After this analysis, a check of slope stability was carried out under the conditions of the planned measures for remediation of the newly projected drain, riparian drainage, lowering the level of groundwater, replacing part of the roadway as well as regulating surface waters. The results of the analysis show that the safety factor is $F_s = 1.541$.

3.6. Pejˇcina Krivina

At the site "Pejčina krivina", on the part of the state road IB-23 Požega-Čačak, a slipway was formed [\[28\]](#page-21-12) (Figure [7\)](#page-7-0). The rehabilitation road route is side cut, and the right slope of the embankment road was hit by demolition, in the direction of the growth of the station and the shoulder and about half of the right-hand carriageway. The frontal width of the landslide is 25 m. According to the mechanical properties, the "Pejčina krivine"

landslide belongs to the consecutive type of landslide with the tendency of regressive landslide belongs to the consecutive type of landslide with the tendency of regressive process development. process development.

station and the shoulder and about half of the right-hand carriageway. The frontal width

Figure 7. Details of the supporting wall in the basis of the road embankment at the site "Pejčina krivina" (**a**) right view; (**b**) left view [28]. krivina" (**a**) right view; (**b**) left view [\[28\]](#page-21-12).

Remedial measures were adopted and implemented, making a supporting wall to Remedial measures were adopted and implemented, making a supporting wall to the right edge of the road, a drainage fill behind the wall, regulating atmospheric underground water. Stability analyses were conducted for geotechnical and hydrogeological conditions that led to the occurrence of slip as well as conditions that correspond to the condition after the planned remediation measures. The analyzed cases show that in the period of hydrological extremes, when it is possible to increase the level of groundwater in the field, or when a line of scattering is formed on the slope to a certain height, due to the increase in the pore pressure in the soil, at the expense of effective strength, the slope can reach the state of the boundary balance. After these conducted analyses, the stability of the slope was achieved under the conditions of the planned remediation measures by constructing a supporting structure (supporting wall), lowering the groundwater level as well as collecting surface atmospheric waters. The results of the analysis show that the safety factor is higher than necessary, $F_s = 1.368$. than necessary, $F_s = 1.368$.

3.7. Lubnica

On the R-261 road from Zaječar to Boljevac at km 6 + 412.70 to km 6 + 540.30, at the to-ponyms "Lubnica", the already existing landslide was reactivated [\[29\]](#page-21-13) (Figure [8\)](#page-7-1). The landslide is 80 m long and 130 m wide and cover about 1.0 hectare. It is estimated that a wall mass of about 50,000 m^{3} is being launched.

Figure 8. Parts of the road affected by the landslide at toponyms "Lubnica" (a) left view; (b) right view [\[29\]](#page-21-13).

To save the slope and put the road into a stable condition, several rehabilitation measures were implemented: posting a new one, quality, sandy-pebbly floor with "mattress" or stone crumbs, drainage system development (longitudinal and transverse drainage trenches), surface water collection by a concrete open channel, and the humidation of the surface of the slope of the road and surface of the landslide whose soil has been replaced.

For shorter active sliding layers, (their length is about 55 m), in high groundwater, a safety factor is obtained $F_S = 1.15$. After making a drainage layer, or by lowering the groundwater for 4–5 m, a safety factor $F_s = 1.51$.

3.8. Application of PROMETHEE Method—Optimization of the Number of Wells in Additional Soil Erosion

The optimization solutions for the selected number of scenarios are presented in the function of the flow rate *Φ*(*a*) of the complete ranking. Based on the PROMETHEE ranking, the activities and criteria are presented integrally at the GAIA level [\[30\]](#page-21-14). Here, an agreement or conflict can be established between the criteria, degree of superiority or inferiority of one activity, in relation to other activities. The final solution, according to the GAIA level, is presented using a decision vector that has a defined direction, direction, and intensity and which clearly indicates the position of optimal activity and key criterion.

Optimization Consideration Activities—models of applied landslide remediation with additional soil erosion (7 cases from practice) are:

- \bullet *a*₁—"Kominje",
- a_2 —"Zavlaka",
- a_3 —"Jezgroviće",
- *a*₄—"Footwear factory",
- a_5 —"6 + 900",
- *a*₆—"Pejčina krivina",
- *a*7—"Lubnica".

Optimization Consideration Criteria—The economic effects resulting from quantitative borehole analysis in the additional soil erosion (3 criteria) are [\[31\]](#page-22-0):

- C_1 —the ratio of the landslide length (along the road) and the number of wells $L/n(B)$,
- C_2 —the ratio of the number of wells and surface of the landslide $n(B)/A$,
- *C*3—the ratio of the amount of drilling over the surface of the landslide *H*/*A*.

Scenarios in optimization considerations—basic and combined models of weight coefficient scenarios resulting from quantitative wells analysis in additional soil erosion (seven scenarios) are:

- *S*1—minimization (for min. preference) of the ratio of the length of the landslide (by road) and the number of wells *L*/*n*(*B*),
- *S*2—maximization (for min. preference) of the length of the landslide (by road) and the number of wells *L*/*n*(*B*),
- *S*3—Maximization of (for preference max) ratio of the number of wells and the surface of the landslide *n*(*B*)/*A*,
- *S*4—minimization of (for preference max) ratio of the number of wells and the surface of the landslide *n*(*B*)/*A*,
- *S*5—Maximization of (for preference max) ratio of the quantity of drilling the surface of the landslide *H*/*A*,
- S_6 —minimization (for preference max) ratio of the quantity of drilling the surface of the landslide *H*/*A*,
- *S*7—combined scenario with equivalent weight coefficients.

The selection of activities was carried out on the basis of the defined landslide relief problem, while the selection of the criteria was based on the conducted economic analysis and the developed economic effects arising from the quantitative analysis of the wells in the additional soil erosion. The matrix relation of activity *a* and criteria *c* for optimization is shown in Table [2.](#page-9-0)

Table 2. Matrix relations of activity *a* and criteria *c*.

The selection of scenarios, as basic and combined models of weight coefficient scenarios, was carried out in order to examine a wide spectrum of the effects of multi-criteria optimization, in the function of applied models, landscaping landslides and number of wells in additional soil erosion. The matrix of the scenario *S* and weight coefficients *w* for optimization is shown in Table [3.](#page-9-1)

Table 3. Matrix of the relation between the scenario *S* and the weight coefficients *w*.

	w_1 (L/n(B))	w_2 (n(B)/A)	w_3 (H/A)
		0.5	0.5
	0.5		0.5
h	0.5	0.5	
	0.333	0.333	0.333

Optimization solutions (wells number function for additional soil erosion), according Optimization solutions (wells number function for additional soil erosion), according to GAIA and PROMETHEE methods, are shown in Figures [9–](#page-9-2)[15.](#page-12-0) to GAIA and PROMETHEE methods, are shown in Figures 9–15.

In the case of scenario *S*1—the smallest number of wells (minimization (for min. In the case of scenario *S*1—the smallest number of wells (minimization (for min. Preference) of the relationship between the length of the landslide (by road) and the Preference) of the relationship between the length of the landslide (by road) and the number of wells $L/n(B)$) was realized for the slopes "Kominja" with $\Phi = 0.279$ (Figure [9\)](#page-9-2).

Figure 9. Optimization solutions for the S_1 scenario "Kominja" with $\Phi = 0.279$.

In the case of scenario S_2 —maximization (for min. preference) of the length of the landslide (by road) and the number of wells $L/n(B)$ was realized for the slopes "Pejčina krivina" with *Φ* = 0.545 (Figure [10](#page-10-0)). krivina" with *Φ* = 0.545 (Figure 10).

Figure 10. Optimization solutions for scenario S_2 —the best solution is "Pejčina krivina" with $\Phi = 0.545.$

In the case of scenario S_3 —Maximization of (for preference max) ratio of the number of wells and the surface of the landslide *n*(*B*)/*A* was realized for the slopes "Footwear of wells and the surface of the landslide *n*(*B*)/*A* was realized for the slopes "Footwear Factory" *Φ* = 0.859 (Figure 11). Factory" *Φ* = 0.859 (Figure [11\)](#page-10-1).

Figure 11. S_3 Optimization Solutions—The optimal solution is "Footwear Factory" $\Phi = 0.859$.

In the case of scenario S_4 —minimization of (for preference max) ratio of the number of wells and the surface of the landslide $n(B)/A$ was realized for the slopes "Pejčina krivina" krivina" with *Φ* = 0.545 (Figure 12). with *Φ* = 0.545 (Figure [12\)](#page-10-2). krivina" with *Φ* = 0.545 (Figure 12).

Figure 12. Solutions obtained by optimizing the scenario *S*4—the best solution is "Pejčina krivina" **12.** Solutions by the scenario *S*4—the best solution is "Pejina krivina" with $Φ = 0.545$. with $\Phi = 0.545$.
With $\Phi = 0.545$. **Figure 12.** Solutions obtained by optimizing the scenario *S*₄—the best solution is "Pejčina krivina" with Φ = 0.545.

In the case of scenario *S*₅—Maximization of (for preference max) ratio of the quantity of drilling the surface of the landslide H/A was realized for the slopes "Pejčina krivina" with *Φ* = 0.839 (Figure [13\)](#page-11-0). with *Φ* = 0.839 (Figure 13).

Figure 13. Solutions obtained by optimizing the scenario S_5 —optimal solution is "Pejčina krivina" with *Φ* = 0.839. with *Φ* = 0.839.

In the case of scenario S_6 —minimization (for preference max) ratio of the quantity of drilling the surface of the landslide *H*/*A* was realized for the slopes "Footwear factory" drilling the surface of the landslide *H*/*A* was realized for the slopes "Footwear factory" with *Φ* = 0.535 (Figure [14\)](#page-11-1). with *Φ* = 0.535 (Figure 14).

Figure 14. Solutions obtained by optimizing the scenario S_6 —optimal solution is "Footwear factory" with $\Phi = 0.535$.

In the case of scenario *S*7—combined scenario with equivalent weight coefficients In the case of scenario *S*7—combined scenario with equivalent weight coefficients was realized for the slopes "Pejčina krivina" with $Φ = 0.446$ (Figure [15\)](#page-12-0).

For the scenario: *S*² (maximization (for min. Preference) of the relationship between the length of the landslide (by road) and the number of wells $L/n(B)$) with $\Phi = 0.544$, *S*⁴ (maximization of (for preference max) ratio of the number of wells and the surface of the landslide $n(B)/A$) with $\Phi = 0.545$, S_5 (maximization (for preference max) ratio of the quantity of drilling the surface of the landslide H/A) with $\Phi = 0.839$ and $S₇$ with an equivalent weight coefficients with $\Phi = 0.446$ optimal solution is "Pajčina krivina".

For scenarios: *S*³ (maximization (preference max) the ratio of the number of wells and surface of the landslide $n(B)/A$) with $\Phi = 0.859$ and S_6 (minimization (for preference max) the ratio of the amount of drilling over the surface of the landslide *H*/*A*) with *Φ* = 0.535 the optimal solution is "Footwear Factory".

Based on the implemented optimization, it can be concluded that it forms a relationship:

- the length of the landslide (by the way) and the number of wells *L*/*n*(*B*)—the minimum required number of wells is 3 (scenario S_2),
- number of wells and surface of landslide *n*(*B*)/*A*—the minimum required number of wells is 3 (scenario S_3),
- the amount of drilling on the surface of the landslide H/A —that the minimum required number of wells is 3 (scenario S_5).

An identical solution was obtained for the combined scenario *S*₇ with equivalent weight coefficients. *was obtained for the combined secriting σ*

Figure 15. Solution obtained by optimizing for the combined scenario S_7 —the optimal solution is "Pejčina krivina" with $\Phi = 0.446$.

For the scenario: *S*2 (maximization (for min. Preference) of the relationship between *3.9. Sensitivity Analysis*

Section 3.2 presents the results of optimization based on the given criteria C_1 ($L/n(B)$), C_2 ($n(B)/A$) and C_3 (H/A). For the analysis of the sensitivity of the obtained results, in addition to these criteria, data on the increase in the amount of material ΔV of the performed state of landslide remediation Vi according to the amount of material of the project solution for landslide remediation were used.

Figures 16-21 shows the ratios of the increase in the amount of materials and works of the parameters of the third group (individual parameters) as a function of the considered remedied landslide. The units of measure are as follows: for the number of wells is *pcs* (pieces), for the length of the landslide (along the road) m', for the surface of the landslide ha (10,000 square meters = 10^4 m²) and the amount of drilling m'. we help to the landslide (along the road) m', for the barrace of the landslide

Figure 16. The ratio of the length of the landslide and the number of wells $L/n(B)$ and the ratio of the increase in the amount of excavation (performed/projected excavation) ΔV. increase in the amount of excavation (performed/projected excavation) ∆V.

Figure 16. The ratio of the length of the landslide and the number of wells *L/n*(*B*) and the ratio of the

Figure 17. Ratio of landslide length and number of wells $L/n(B)$ and ratio of embankment volume increase (constructed/designed embankment) ΔV.

> Criterion *C*¹ should have the lowest possible value, because, in that case, for the Criterion *C*1 should have the lowest possible value, because, in that case, for the appropriate number of wells $n(B)$, the engineering-geological profile of the terrain is better described. For criterion C_1 , and considering the parameter ΔV , a conditionally satisfactory solution for scenario S_1 was obtained in the rehabilitation of the landslide "Zavlaka" and "Jezgrovići", while a conditionally satisfactory solution for scenario S_2 was obtained in the rehabilitation of the landslide "Footwear Factory".

Figure 18. Ratio of number of wells and landslide area $n(B)/Ai$ Ratio of excavation increase (performed/projected excavation) $ΔV$.

Figure 19. Ratio of number of wells and landslide area $n(B)/Ai$ ratio of embankment volume increase (constructed/designed embankment) ΔV. (constructed/designed embankment) ∆V.

Criterion *C*² should have the highest possible value, because, in that case, for the appropriate number of wells *n*(*B*), the engineering-geological profile of the terrain is better described. For criterion *C*2, and considering the parameter ∆V, no conditionally satisfactory solution was achieved for scenario S_3 in landslide remediation, while conditionally satisfactory solution for scenario S_4 was obtained in landslide remediation "Footwear Factory".

Figure 20. The ratio of the amount of drilling on the surface of the landslide H/A and the ratio of the increase in the amount of excavation (performed/projected excavation) ΔV. increase in the amount of excavation (performed/projected excavation) ∆V. increase in the amount of excavation (performed/projected excavation) ΔV.

Figure 21. Ratio of the amount of drilling on the landslide surface H/A and the ratio of the increase in the amount of embankment (constructed/designed embankment) ΔV. in the amount of embankment (constructed/designed embankment) ΔV. in the amount of embankment (constructed/designed embankment) ∆V.

Criterion C_3 should have the highest possible value, because, in that case, for the appropriate amount of drilling H, the engineering-geological profile of the terrain is better described.

With regards to this criterion, and considering the parameter ∆V, a conditionally satisfactory solution for scenario S_5 was obtained in the rehabilitation of landslides "Zavlaka" and "Jezgrovići", while a conditionally satisfactory solution for scenario S_6 was obtained in the rehabilitation of landslide "Zavlaka".

Additionally, the relations of previously presented parameters were considered and regression analysis was performed by searching for the optimal degree of polynomials for the selected function. Regression analysis (Reg), for the general case of the considered parameters, is represented by a polynomial function [\[32\]](#page-22-1):

$$
y_{Reg} = a_1 + a_2 x_{Reg} + \ldots + a_n x_{Reg}^n
$$
 (7)

x therein y_{Reg} considered the ordinate parameter according to regression analysis, x_{Reg} considered the abscissa parameter according to regression analysis, a_1, \ldots, a_n unknown considered the abscissa parameter according to regression analysis, *a*1,…*an* unknown

coefficients determined using the least squares method and matrix algebra. The general formula for least squares regression is:

$$
\sum_{i=1}^{n} \left(y_{Reg,i} - f(x_i, a_1, \dots, a_n) \right) \frac{\partial}{\partial a_i} f(x_i, a_1, \dots, a_n) = 0 \tag{8}
$$

where the other half of Expression (8) can be written as:

$$
\frac{\partial}{\partial a_i} f(x_i, a_1, \dots, a_n) = \frac{\partial}{\partial a_i} [a_1 g_1 + a_2 g_2 + \dots + a_n g_n] = g_i(x_i)
$$
\n(9)

therefore, we obtain the following:

$$
\sum_{i=1}^{n} [y_{Reg,i} - a_i g_i(x_i)] g_i(x_i) = 0
$$
\n(10)

while in matrix form the previous expression reads:

$$
\begin{bmatrix}\n\sum_{i=1}^{n} g_1(x_i)g_1(x_i) & \cdots & \sum_{i=1}^{n} g_1(x_i)g_n(x_i) \\
\cdots & \cdots & \cdots \\
\sum_{i=1}^{n} g_n(x_i)g_1(x_i) & \cdots & \sum_{i=1}^{n} g_n(x_i)g_n(x_i)\n\end{bmatrix}\n\begin{bmatrix}\na_1 \\
\cdots \\
a_n\n\end{bmatrix} = \begin{bmatrix}\n\sum_{i=1}^{n} y_{\text{Reg},i}g_1(x_i) \\
\cdots \\
\sum_{i=1}^{n} y_{\text{Reg},i}g_n(x_i)\n\end{bmatrix}
$$
\n(11)

By solving the matrix form (11) they are obtained:

$$
f(x_i, a_1, \dots, a_n) = a_1 g_1(x) + a_2 g_2(x) + \dots + a_n g_n(x).
$$
 (12)

where in:

$$
g_1(t) = 1
$$
, $g_2(x) = x$, $g_3(x) = x^2$,..., $g_n(x) = x^{n-1}$ (13)

Determination of unknown regression coefficients is carried out according to:

$$
\begin{bmatrix}\n\sum_{i=1}^{n} 1 & \sum_{i=1}^{n} x_i & \dots & \dots & \sum_{i=1}^{n} x_i^{n-5} & \sum_{i=1}^{n} x_i^{n-4} \\
\sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 & \dots & \dots & \sum_{i=1}^{n} x_i^{n-4} & \sum_{i=1}^{n} x_i^{n-3} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
\sum_{i=1}^{n} x_i^{n-5} & \sum_{i=1}^{n} x_i^{n-4} & \dots & \dots & \dots & \dots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\sum_{i=1}^{n} x_i^{n-5} & \sum_{i=1}^{n} x_i^{n-4} & \dots & \dots & \sum_{i=1}^{n} x_i^{n-2} & \sum_{i=1}^{n} x_i^{n-1} \\
\sum_{i=1}^{n} x_i^{n-4} & \sum_{i=1}^{n} x_i^{n-3} & \dots & \dots & \sum_{i=1}^{n} x_i^{n-1} & \sum_{i=1}^{n} x_i^{n}\n\end{bmatrix}\n\begin{bmatrix}\na_1 \\
\vdots \\
a_n\n\end{bmatrix}\n=\n\begin{bmatrix}\n\sum_{i=1}^{n} y_i \\
\vdots \\
\sum_{i=1}^{n} y_i x_i^{n-1} \\
\vdots \\
\sum_{i=1}^{n} y_i x_i^{n-1}\n\end{bmatrix}
$$
\n(14)

Since polynomials of higher degree are considered, the change in the value of the correlation coefficient is monitored:

$$
r = \frac{\sum_{i=1}^{n} (x_i - x_m)(y_i - y_m)}{\sqrt{\sum_{i=1}^{n} (x_i - x_m)^2 (y_i - y_m)^2}}
$$
(15)

wherein x_i and y_i are discrete values of abscissa and ordinate, respectively, x_m and y_m mean values.

Both absolute and relative parameter values were considered in this group:

• discrete values and regression analysis by polynomial function of landslide length (along the road) Li number of wells *n*(*B*) (Figure [22\)](#page-17-0):

$$
n(B)_{Reg1} = 3.97 - 0.1118L + 0.003L^2 - 0.00002L^3
$$
\n(16)

• discrete values and regression analysis by polynomial function of landslide surface (along the road) *A* and number of wells $n(B)$ (Figure [23\)](#page-17-1):

$$
n(B)_{Reg2} = 2.96 - 0.48A + 0.56A^2 - 0.053A^3
$$
\n(17)

• discrete values and regression analysis by polynomial function of the amount of drilling *H* and the number of wells $n(B)$ (Figure [24\)](#page-17-2):

$$
n(B)_{Reg3} = 3.81 - 0.15H + 0.0036H^2 - 0.00002H^3
$$
\n(18)

• discrete values and regression analysis by polynomial function of the ratio of the length of the landslide (along the road) Li of the number of wells *n*(*B*) and the ratio of the increase in the amount of excavation (performed/projected excavation)) ∆V (Figure [25\)](#page-18-0):

$$
\Delta V_{Reg1} = 41.9 + 18.4(L/n(B)) - 1.39(L/n(B))^{2} + 0.029(L/n(B))^{3}
$$
 (19)

• discrete values and regression analysis by polynomial function of the ratio of the number of wells $n(B)$ and the landslide area *A* and the ratio of the increase in the amount of excavation (performed/projected excavation) ∆V (Figure [26\)](#page-18-1):

$$
\Delta V_{Reg2} = 9.12 + 10.6(n(B)/A) + 0.75(n(B)/A)^{2} - 0.36(n(B)/A)^{3}
$$
 (20)

• discrete values and regression analysis by polynomial function of the relationship between the length of the landslide (along the road) *L* and the number of wells $n(B)$ and the relationship between the increase in the amount of embankment (constructed/designed embankment)) ∆V (Figure [27\)](#page-18-2):

$$
\Delta V_{Reg3} = 80.3 - 20.36(L/n(B)) + 1.86(L/n(B))^2 - 0.044(L/n(B))^3
$$
 (21)

• discrete values and regression analysis by polynomial function of the ratio of the number of wells *n*(*B*) and the landslide surface *A* and the ratio of the increase in the amount of embankment (constructed/designed embankment)) ∆V (Figure [28\)](#page-19-0):

$$
\Delta V_{Reg4} = 90.47 - 50.2(n(B)/A) + 12.5(n(B)/A)^{2} - 0.96(n(B)/A)^{3}
$$
 (22)

• discrete values and regression analysis by polynomial function of the ratio of the amount of drilling *H* and the surface of landslide *A* and the ratio of increasing the amount of excavation (performed/projected excavation) ∆V (Figure [29\)](#page-19-1):

$$
\Delta V_{Reg5} = -32.8 + 5.96(H/A) - 0.16(H/A)^{2} + 0.0012(H/A)^{3}
$$
 (23)

• discrete values and regression analysis by polynomial function of the ratio of the amount of drilling *H* and the surface of landslide *A* and the ratio of increasing the amount of embankment (constructed/designed embankment) ∆V (Figure [30\)](#page-19-2):

$$
\Delta V_{Reg6} = 91.6 - 5.62(H/A) + 0.128(H/A)^{2} - 0.0009(H/A)^{3}
$$
 (24)

Figure [22](#page-17-0) shows the discrete values and regression analyzes by polynomial function of the third group (absolute and relative values) of parameters:

Figure 22. Discrete values and regression analysis by polynomial function of landslide length (along the road) Li number of wells $n(B)$.

Figure 23. Discrete values and regression analysis by polynomial function of landslide surface (along the road) A and number of wells $n(B)$.

Figure 24. Discrete values and regression analysis by polynomial function of drilling volume H and number of wells *n*(*B*). number of wells *n*(*B*).

Derived expressions ((19)–(24)), from regression analyses by polynomial function can be used for practical purposes for analyzes of remediation of other landslides, taking into account the extreme values of the number of wells *n*(*B*) from this research.

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Figure 25. Discrete values and regression analysis by polynomial function of the ratio of the length **Figure 25.** Discrete values and regression analysis by polynomial function of the ratio of the length of the landslide (along the road) Li of the number of wells *n*(*B*) and the ratio of the increase in the of the landslide (along the road) Li of the number of wells $n(B)$ and the ratio of the increase in the of the landslide (along the road) Li of the number of wells $n(B)$ and the ratio of the increase in the amount of excavation (performed/projected excavation)) ∆V. amount of excavation (performed/projected excavation)) ΔV. of the landslide (along the road) Li of the number of wells *n*(*B*) and the ratio of the increase in the

Figure 26. Discrete values and regression analysis by polynomial function of the ratio of the number (performed/projected excavation) ΔV. of wells *n*(*B*) and the landslide area *A* and the ratio of the increase in the amount of excavation (performed/projected excavation) ∆V. of wells *n*(*B*) and the landslide area *A* and the ratio of the increase in the amount of excavation

Figure 27. Discrete values and regression analysis by polynomial function of the ratio of the length **Figure 27.** Discrete values and regression analysis by polynomial function of the ratio of the length of the landslide (along the road) L and the number of wells $n(B)$ and the ratio of the increase in the amount of embankment (constructed/designed embankment)) ΔV. amount of embankment (constructed/designed embankment)) ∆V.

amount of embankment (constructed) α and α and α and α

Figure 28. Discrete values and regression analysis by polynomial function of the ratio of the number **Figure 28.** Discrete values and regression analysis by polynomial function of the ratio of the number of wells *n*(*B*) and the landslide area *A* and the ratio of the increase in the amount of embankment of wells $n(B)$ and the landslide area A and the ratio of the increase in the amount of embankment (constructed/designed embankment)) ∆V. (constructed/designed embankment)) ΔV. of wells *n*(*B*) and the landslide area *A* and the ratio of the increase in the amount of embankment

of the landslide (along the road) *L* and the number of wells *n*(*B*) and the ratio of the increase in the

Figure 29. Discrete values and regression analysis by polynomial function of the ratio of the amount **Figure 29.** Discrete values and regression analysis by polynomial function of the ratio of the amount of drilling H and the surface of landslide A and the ratio of increasing the amount of excavation (performed/projected excavation) ΔV. (performed/projected excavation) ∆V.

Figure 30. Discrete values and regression analysis by polynomial function of the ratio of the amount **Figure 30.** Discrete values and regression analysis by polynomial function of the ratio of the amount of drilling *H* and the surface of landslide *A* and the ratio of increasing the amount of embankment of drilling *H* and the surface of landslide *A* and the ratio of increasing the amount of embankment (constructed/designed embankment) ΔV. (constructed/designed embankment) ∆V.

4. Conclusions

The research conducted in this paper refers to the issue of multi-criteria optimization solutions for the repair of landslides. By optimizing the function of the number of wells in additional soil erosion through several scenarios, for selected preferences, solutions were obtained according to predefined scenarios and based on the ranking of the flow function. The research showed that it is from the ratio of the length of the landslide (along the road) and the number of wells $L/n(B)$, the ratio of the number of wells and surface of the landslide *n*(*B*)/*A* and the ratio of the amount of drilling over the surface of the landslide *H*/*A* that the minimum required number of boreholes is 3. An identical solution was obtained for the combined scenario with equivalent weight coefficients. However, by expanding the population of the number of landslides using statistical distribution and stochastic modeling, and by considering the theory of probability, it was shown that the required number of boreholes a *n*(*B*) is significantly higher than 3.

As one of the most important examples of what should be done in order to improve the research, it is more accurate to determine the weight of certain criteria and a potential addition to the existing list of criteria, with the recommendation to use the so-called Delphi method. The Delphi method is based on collecting, analyzing, and harmonizing the answers from a large number of experts to certain questions in the field under research. By hiring a certain number of experts who provide answers to precisely designed and formulated questions through an anonymous survey, and which can usually have from 10 to 12 questions (in some cases much higher), we would obtain more precisely determined weights of certain criteria. The Delphi method could be implemented in two or three rounds. In the first round of the survey, experts would provide answers to the questions asked in the questionnaire. The obtained answers would then be analyzed and systematized, and based on them, another questionnaire is made. In the second round, the experts were informed about the results of the first round of the survey, through certain statistical indicators, and would again answer the questions asked. Sometimes the third round is conducted according to the procedure described for the second round of the survey.

5. Contribution to Science

The contribution of this paper to science is reflected in the definition of criteria for optimizing the problem of landslide remediation in subsequent soil collapse. The paper investigates different scenarios expressed through appropriate weighting coefficients on the sequence of analyzed alternatives. In each of the defined scenarios, the mutual values of the orders of the defined alternatives were determined. For a realistic set of alternatives, the procedure is explained in detail.

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