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Parameter Estimation of the Exponentiated Pareto Distribution Using Ranked Set Sampling and Simple Random Sampling

Hossein Jabbari Khamnei [1], Ieva Meidute-Kavaliauskiene [2,*], Masood Fathi [3,4], Asta Valackienė [5] and Shahryar Ghorbani [6]

1 Department of Statistics, Faculty of Mathematics-Statistics and Computer Science, University of Tabriz, Tabriz 5166616471, Iran; h_jabbari@tabrizu.ac.ir
2 Faculty of Business Management, Vilnius Gediminas Technical University, Sauletekio al. 11, 10223 Vilnius, Lithuania
3 Division of Intelligent Production Systems, School of Engineering Science, University of Skövde, 54128 Skövde, Sweden; masood.fathi@his.se
4 Division of Industrial Engineering and Management, Department of Civil and Industrial Engineering, Uppsala University, 75121 Uppsala, Sweden
5 Institute of Business and Economics, Faculty of Public Governance and Business, Mykolas Romeris University, Ateities Street 20, Room C-V-509, 08303 Vilnius, Lithuania; avala@mruin.eu
6 Production Management Department, University of Sakarya, Sakarya 54050, Turkey; mg.shahryar@gmail.com
* Correspondence: ieva.meidute-kavaliauskiene@vilniustech.lt

Abstract: In this paper, we have considered that ranked set sampling is able to estimate the parameters of exponentiated Pareto distribution. The method with which the maximum likelihood estimators for the parameters of exponentiated Pareto distribution is studied is numerical since there is no presence or possibility of a closed-form at the hands of estimators or any other intellectual. The numerical approach is a well-suited one for this study as there has been struggles in achieving it with any other technique. In order to compare the different sampling methods, simulation studies are performed as the main technique. As for the illustrative purposes, analysis of a simulated dataset is desired for the objective of the presentation. The conclusion that we can reach based on these is that the estimators based on the ranked set sample have far better efficiency than the simple random sample at the same sample size.

Keywords: efficiency; exponentiated Pareto distribution; maximum likelihood estimator; order statistics; ranked set sampling; simple random sampling

MSC: 62F10; 62F99

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1. Introduction

A two-parameter distribution, called the exponentiated Pareto distribution EP(β, λ), has been introduced by [1]. They provided the cumulative distribution function (cdf) and probability density function (pdf) of this distribution as follows:

$$F(x; \beta, \lambda) = \left[1 - (1+x)^{-\lambda}\right]^{\beta}, \quad x > 0, \tag{1}$$

and

$$f(x; \beta, \lambda) = \beta\lambda(1+x)^{-(\lambda+1)}\left[1 - (1+x)^{-\lambda}\right]^{\beta-1}, \quad x > 0, \tag{2}$$

where $\beta > 0$ and $\lambda > 0$. The parameters β and λ are two shape parameters. When $\beta = 1$, the above distribution corresponds to the standard Pareto distribution of the second kind (see [2]). It is essential to mention here that when β is a positive integer, $EP(\beta, \lambda)$ cdf is the cdf of the maximum of a random sample size β from the standard Pareto distribution of the second kind (see [3]). Therefore, the EP distribution has a survival function

$$S(x;\beta,\lambda) = 1 - [1-(1+x)^{-\lambda}]^{\beta}, \quad x > 0, \tag{3}$$

and a hazard function

$$h(x;\beta,\lambda) = \frac{\beta\lambda(1+x)^{-(\lambda+1)}[1-(1+x)^{-\lambda}]^{\beta-1}}{1-[1-(1+x)^{-\lambda}]^{\beta}}, \quad x > 0. \tag{4}$$

For $\beta > 1$, the distribution has a unique model, which is $(\frac{\lambda\beta+1}{\lambda+1})^{\frac{1}{\lambda}} - 1$. The median of the distribution is $(1-0.5^{\frac{1}{\beta}})^{\frac{-1}{\lambda}} - 1$.

Ref. [1] showed that the EP distribution could be used quite effectively in analyzing many lifetime data. The EP distribution can have decreasing and upside-down bathtub-shaped failure rates depending on the shape parameter (MLE) β. The authors of [4] consider the maximum likelihood estimation of the different parameters of an EP distribution, and they studied how the different estimators of the unknown parameters of an EP distribution can behave for different sample sizes and for different parameter values. The authors of [5] studied the mixture of exponentiated Pareto and exponential distributions. The authors of [6] derived Bayes and classical estimators of the parameters of exponentiated Pareto distributions under different sample schemes. The authors of [7] introduced the generalized exponential distribution, which has been studied quite extensively. The authors of [8] also discussed a different method of estimations of a generalized exponential distribution of parameters. The authors of [9] derived the best linear unbiased estimates $(BLEUs)$ and the maximum likelihood estimates $(MLEs)$ of the location and scale parameters for the EP distribution based on progressively Type II right censored order statistics. Some of the gamma-Pareto and the exponentiated Pareto properties, including distribution shapes, limit behavior, hazard function, Renyi and Shannon entropies, moments, and deviations from the mean and median, are discussed in [10]. Ref. [11] applied four real datasets for the issues of parameter estimation and provided a visual inspection of the goodness-of-fit of the complementary beta model.

In this article, ranked set sampling (RSS) is considered. The MLE of the two parameters based on RSS will be investigated, and simulation will illustrate the mathematical finding.

2. Ranked Set Sampling

A method of sampling based on ranked sets is an efficient alternative to simple random sampling (SRS) that has been shown to outperform simple random sampling in many situations by reducing the variance of an estimator, thereby providing the same accuracy with a smaller sample size than is needed in SRS. The authors of [12] introduced RSS to estimate mean pasture yields. RSS can be applied in many studies where the exact measurement of an element is complicated (in terms of money, time, labor, and organization). Still, although not easily measurable, the variable of interest can be relatively easily ranked (order) at no cost or minimal additional cost. The ranking can be performed based on visual inspection, preliminary information, earlier sampling episodes, or other rough methods not requiring actual measurement. The procedure of using RSS is as follows

1. Select m units at random from a specified population.
2. Rank these m units with some expert judgment without measuring them.
3. Retain the smallest judged unit and return the others.
4. Continue the process until m ordered units are measured.
5. These m ordered observations $X_{[1]i}, \ldots, X_{[m]i}$ are called a cycle.
6. Process repeated $i = 1, \ldots, k$ cycle to get km observations.

These km observations are called a standard ranked set sample.

The sample mean based on RSS is $\overline{X}_{RSS} = \frac{1}{km}\sum_{j=1}^{m}\sum_{i=1}^{k} x_{[j]i}$ where $x_{[j]i}$ is the unit of order j in cycle i. $\overline{X}_{RSS}$ is an unbiased estimator for the population mean, with variance

$$Var\left(\overline{X}_{RSS}\right) = \frac{\sigma_x^2}{km} - \frac{1}{km^2}\sum_{j=1}^{m}\left(\mu_{x[j]} - \mu_x\right)^2$$

RSS yields a sample of observations that tends to be more representative of the underlying population than a SRS of equivalent size. The authors of [13] established a foundation for the theory of RSS. The method has become widely applicable, and many modifications have been made. The authors of [14] showed that RSS is more efficient than SRS even with an error in ranking. The authors of [15] suggested using extreme RSS for estimating a population mean. The authors of [16] introduced median RSS to estimate the population mean. The authors of [17] considered double RSS, as a procedure that increases the efficiency of the RSS estimator without increasing the set size m. It was shown that the double RSS estimator of the mean is more efficient than that using RSS. For details on RSS, see [18,19]. The derivation of the parameter estimation based on Simple Random Sampling and Ranked Set Sampling for Gumbel distribution and logistic distribution are discussed by [20,21]. The derivation of the likelihood function for parameter estimation based on double-ranked set sampling $(DRSS)$ designs were used by [22] for the estimation of the parameters of the power-generalized Weibull distribution. The authors of [23] derived the estimate of the finite population total under Ranked Set Sampling Without Replacement $(RSSWOR)$, employing the model relationship, especially Gamma Population Model (GPM), between the study and auxiliary variables. The authors of [24] considered the estimation of the scale parameter of Levy distribution using a ranked set sample; they derived the best linear unbiased estimator and its variance based on a ranked set sample.

3. Estimation Using Ranked Set Sampling

This section aims to find the MLE for the parameters β and λ of EP distribution using SRS and RSS.

The MLE of the parameters, when SRS is used, is the solution of the following equations given by [4].

If $X_1, X_2, \ldots, X_n$ is a random sample from $EP(\beta, \lambda)$, then the log-likelihood function, $L(\beta, \lambda)$, is

$$L(\beta,\lambda) = nln(\beta) + nln(\beta\lambda) + (\beta-1)\sum_{i=1}^{n} ln\left[1-(1+x_i)^{-\lambda}\right] - (\lambda+1)\sum_{i=1}^{n} ln(1+x_i).$$

The normal equations become:

$$\frac{\partial L(\beta,\lambda)}{\partial\beta} = \frac{n}{\beta} + \sum\nolimits_{i=1}^{n} ln\left[1-(1+x_i)^{-\lambda}\right] = 0 \quad (5)$$

$$\frac{\partial L(\beta,\lambda)}{\partial\lambda} = \frac{n}{\lambda} - \sum\nolimits_{i=1}^{n} ln(1+x_i) + (\theta-1)\sum\nolimits_{i=1}^{n}\frac{(1+x_i)^{-\lambda} ln(1+x_i)}{1-(1+x_i)^{-\lambda}} = 0 \quad (6)$$

From (5), we obtain the MLE of θ as a function of λ, say $\hat{\theta}(\lambda)$, where

$$\hat{\beta}_{MLE} = \hat{\beta}(\lambda) = -\frac{n}{\sum_{i=1}^{n} ln\left[1-(1+x_i)^{-\lambda}\right]}$$

Putting $\hat{\beta}(\lambda)$ in the log-likelihood function, we obtain

$$g(\lambda) = L\left(\hat{\beta}(\lambda), \lambda\right)$$
$$= nln(n) - nln\left(\sum_{i=1}^{n} ln\left[1-(1+x_i)^{-\lambda}\right]\right) + nln(\lambda) - \text{n}$$
$$-\sum_{i=1}^{n} ln\left[1-(1+x_i)^{-\lambda}\right] - (\lambda+1)\sum_{i=1}^{n} ln(1+x_i)$$

Therefore, the MLE of λ, say $\hat{\lambda}_{MLE}$, can be obtained by maximizing $g(\lambda)$ with respect to λ. It is observed that $g(\lambda)$ is a unimodal function, and the $\hat{\lambda}_{MLE}$ that maximizes $g(\lambda)$ can be obtained from the fixed-point solution of $h(\lambda) = \lambda$, where

$$h(\lambda) = \left\{ \frac{\sum_{i=1}^{n} \frac{(1+x_i)^{-\lambda} ln(1+x_i)}{1-(1+x_i)^{-\lambda}}}{\sum_{i=1}^{n} ln\left[1-(1+x_i)^{-\lambda}\right]} + \frac{1}{n}\sum_{i=1}^{n} \frac{ln(1+x_i)}{1-(1+x_i)^{-\lambda}} \right\}^{-1}.$$

Since $\hat{\lambda}$ is a fixed-point solution of the nonlinear equation $h(\lambda) = \lambda$, therefore, it can be obtained by using a simple iterative scheme as follows:

$$h\left(\lambda_{(j)}\right) = \lambda_{(j+1)},$$

where $\lambda_{(j)}$ is the j^{th} iterate of $\hat{\lambda}$. The iteration procedure should be stopped when $\left|\lambda_{(j)} - \lambda_{(j+1)}\right|$ is sufficiently small.

Here we use the Newton–Raphson method to solve the nonlinear system of equations. It is shown in [25] that if the initial guess is sufficiently close to the exact solution, the Newton–Raphson method will be convergent to the exact solution.

These equations are solved numerically to compare the MLE based on SRS to that found on RSS.

Now to simplify notations, let $\{Y_{ij} : i = 1, 2, \ldots, m, j = 1, 2, \ldots, k\}$ denote the ranked set sample of size $n = km$ from an EP population, where m is the set size, and k is the number of cycles. Then, the pdf of Y_{ij} is given by:

$$g_i(y_{ij}) = \tfrac{m!}{(i-1)!(m-i)!}\left[F(y_{ij})\right]^{i-1}\left[1-F(y_{ij})\right]^{m-i} f(y_{ij})$$
$$= \tfrac{m!}{(i-1)!(m-i)!}\beta\lambda(1+y_{ij})^{-(\lambda+1)}\left[1-(1+y_{ij})^{-\lambda}\right]^{\beta i-1}\{1$$
$$-\left[1-(1+y_{ij})^{-\lambda}\right]^{\beta}\}^{m-i}; \quad y_{ij} > 0 \tag{7}$$

The likelihood function is given by

$$L(\beta, \lambda) = \Pi_{j=1}^{k}\Pi_{i=1}^{m} g_i(y_{ij})$$
$$= c\beta^{mk}\lambda^{mk}\Pi_{j=1}^{k}\Pi_{i=1}^{m}(1+y_{ij})^{-(\lambda+1)}[1-(1$$
$$+y_{ij})^{-\lambda}]^{\beta i-1}\Pi_{j=1}^{k}\Pi_{i=1}^{m}\{1-\left[1-(1+y_{ij})^{-\lambda}\right]^{\beta}\}^{m-i}$$

where c is a constant. Then, the log-likelihood function is:

$$
\begin{aligned}
logL(\beta,\lambda) = lnl = lnc + mkln\beta + mkln\lambda - (\lambda+1)\sum_{j=1}^{k}\sum_{i=1}^{m} ln\left(1+y_{ij}\right) \\
+\sum_{j=1}^{k}\sum_{i=1}^{m}(\beta i - 1)ln\left[1-\left(1+y_{ij}\right)^{-\lambda}\right] + \sum_{j=1}^{k}\sum_{i=1}^{m}(m-i)ln\{1 \\
-\left[1-\left(1+y_{ij}\right)^{-\lambda}\right]^{\beta}\}
\end{aligned} \tag{8}
$$

By finding the first derivation of $logL$ with respect to β and λ and setting them to zero, the following equations are found

$$
\begin{aligned}
\frac{\partial}{\partial\beta} logL = \frac{mk}{\beta}\sum_{j=1}^{k}\sum_{i=1}^{m} + iln\left[1-\left(1+y_{ij}\right)^{-\lambda}\right] - \sum_{j=1}^{k}\sum_{i=1}^{m}(m \\
-i)\frac{\left[1-\left(1+y_{ij}\right)^{-\lambda}\right]^{\beta} ln\left[1-\left(1+y_{ij}\right)^{-\lambda}\right]}{1-\left[1-\left(1+y_{ij}\right)^{-\lambda}\right]^{\beta}} = 0
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
\frac{\partial}{\partial\lambda} logL = \frac{mk}{\lambda} - \sum_{j=1}^{k}\sum_{i=1}^{m} ln\left(1+y_{ij}\right) + \sum_{j=1}^{k}\sum_{i=1}^{m}(\beta i \\
-1)\frac{\left(1+y_{ij}\right)^{-\lambda} ln\left(1+y_{ij}\right)}{1-\left(1+y_{ij}\right)^{-\lambda}} - \beta\sum_{j=1}^{k}\sum_{i=1}^{m}(m \\
-i)\frac{\left(1+y_{ij}\right)^{-\lambda}\left[1-\left(1+y_{ij}\right)^{-\lambda}\right]^{\beta-1} ln\left(1+y_{ij}\right)}{1-\left[1-\left(1+y_{ij}\right)^{-\lambda}\right]^{\beta}} = 0
\end{aligned} \tag{10}
$$

The ML estimators of β and λ, say $\overline{\beta}_{RSS}$ and $\overline{\lambda}_{RSS}$, are the solution of the two nonlinear equations. Since it is difficult to find a closed-form solution for the parameters, a numerical technique is needed to solve them.

4. Simulation Study

In this section, a numerical study is considered to compare the ML estimators of the unknown parameters β and λ for EP distribution based on RSS and SRS. Comparison studies between these estimators will be carried out through $MSEs$, Bias, and relative efficiency. Monte Carlo simulation is applied for different set sizes, different numbers of cycles, and different parameter values. The simulation procedures are described through the following algorithm.

Step 1: A random sample of sizes $n = 9$, 12, 15, 20, 24, 30, 32 and 40 with set size $m = (3, 4)$, number of cycles $k = (3, 5, 8, 10)$, where $n = m \times k$ are generated from EP distribution.

Step 2: The parameter values are selected as $\beta = 0.5$, 1, 1.5 for $\lambda = 1$ in the estimation procedure.

Step 3: For the chosen set of parameters and each sample of size n, four estimators $\left(\hat{\beta}_{SRS}, \hat{\lambda}_{SRS}, \overline{\beta}_{RSS}, \overline{\lambda}_{RSS}\right)$ are computed under SRS and RSS.

Step 4: Repeat the previous steps from 1 to 3, N times representing different samples, where $N = 1000$. Then, the MSE and Bias of the estimates are computed.

Step 5: Compute the efficiency of estimators, that defined as, $Efficiency = \frac{MSE(SRS)}{MSE(RSS)}$.

All simulated studies presented here are obtained via Maple. The results are reported in Tables 1–3. Each table contains the estimates of parameters for EP distribution under SRS and RSS, along with the efficiency of the estimators for SRS relative to RSS for the different sample sizes and parameters values.

Table 1. The bias (MSE) and efficiency of the MLE parameters β and λ when $\beta = 0.5$ and $\lambda = 1$.

(*k*, *s*)	*RSS*		*n*	*SRS*		**Efficiency**	
	β	λ		β	λ	β	λ
(3, 3)	0.115 (0.091)	0.328 (0.658)	9	0.197 (0.291)	0.493 (1.262)	3.207	1.918
(4, 3)	0.072 (0.035)	0.209 (0.298)	12	0.127 (0.099)	0.367 (0.819)	2.751	2.851
(3, 5)	0.059 (0.031)	0.171 (0.240)	15	0.092 (0.057)	0.282 (0.474)	1.975	1.841
(4, 5)	0.043 (0.019)	0.130 (0.149)	20	0.063 (0.033)	0.180 (0.260)	1.744	1.773
(3, 8)	0.032 (0.013)	0.087 (0.105)	24	0.055 (0.025)	0.180 (0.225)	2.142	1.984
(3, 10)	0.026 (0.010)	0.074 (0.082)	30	0.036 (0.016)	0.098 (0.116)	1.419	1.630
(4, 8)	0.026 (0.009)	0.077 (0.076)	32	0.034 (0.015)	0.108 (0.128)	1.694	1.755
(4, 10)	0.021 (0.007)	0.066 (0.057)	40	0.031 (0.012)	0.088 (0.094)	1.645	1.828

Table 2. The bias (MSE) and efficiency of the MLE parameters β and λ when $\beta = 1$ and $\lambda = 1$.

(*k*, *s*)	*RSS*		*n*	*SRS*		**Efficiency**	
	β	λ		β	λ	β	λ
(3, 3)	0.318 (0.698)	0.328 (0.320)	9	0.552 (2.917)	0.328 (0.563)	1.759	4.180
(4, 3)	0.191 (0.233)	0.147 (0.162)	12	0.339 (0.721)	0.243 (0.366)	2.263	3.096
(3, 5)	0.155 (0.200)	0.118 (0.131)	15	0.239 (0.369)	0.190 (0.231)	1.765	1.847
(4, 5)	0.114 (0.121)	0.092 (0.086)	20	0.157 (0.194)	0.122 (0.137)	1.596	1.609
(3, 8)	0.081 (0.074)	0.061 (0.061)	24	0.140 (0.147)	0.126 (0.120)	1.964	2.989
(3, 10)	0.066 (0.058)	0.052 (0.048)	30	0.089 (0.087)	0.069 (0.068)	1.416	1.501
(4, 8)	0.068 (0.056)	0.055 (0.045)	32	0.085 (0.090)	0.074 (0.073)	1.542	1.616
(4, 10)	0.054 (0.039)	0.047 (0.034)	40	0.077 (0.068)	0.062 (0.055)	1.641	1.751

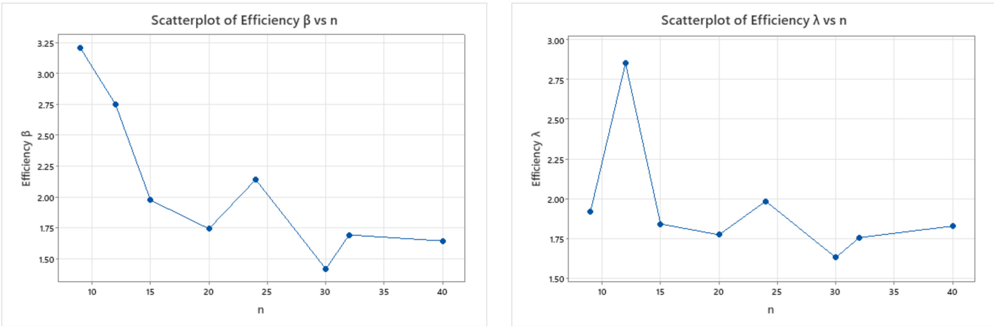
Table 3. The bias (MSE) and efficiency of the MLE parameters β and λ when $\beta = 1.5$ and $\lambda = 1$.

(*k*, *s*)	*RSS*		*n*	*SRS*		**Efficiency**	
	β	λ		β	λ	β	λ
(3, 3)	0.593 (2.496)	0.193 (0.243)	9	1.056 (3.094)	0.280 (0.417)	5.244	1.716
(4, 3)	0.345 (0.737)	0.128 (0.127)	12	0.620 (2.529)	0.206 (0.269)	2.121	3.432
(3, 5)	0.278 (0.624)	0.103 (0.102)	15	0.428 (0.426)	0.108 (0.092)	1.927	2.030
(4, 5)	0.204 (0.368)	0.079 (0.068)	20	0.277 (0.571)	0.105 (0.106)	1.558	1.551
(3, 8)	0.143 (0.210)	0.053 (0.048)	24	0.248 (0.426)	0.108 (0.092)	1.927	2.030
(3, 10)	0.117 (0.166)	0.046 (0.039)	30	0.155 (0.243)	0.060 (0.054)	1.384	1.463
(4, 8)	0.119 (0.161)	0.047 (0.035)	32	0.150 (0.262)	0.063 (0.058)	1.660	1.631
(4, 10)	0.095 (0.112)	0.041 (0.027)	40	0.133 (0.188)	0.054 (0.044)	1.637	1.686

From Tables 1–3, many conclusions can be made on the performance of both methods of estimation based on RSS and SRS. Further, the sensitivity analysis of results is very important [25] and Figures 1–3 show the effectiveness of the results. These conclusions are summarized as follows:

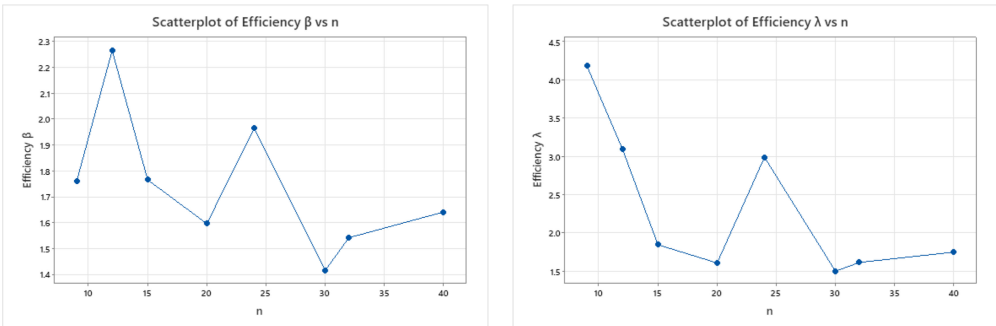
1. Based on SRS, the bias and MSE for estimates of β and λ are more significant than that based on RSS.
2. For both methods of estimations, it is clear that the bias and MSE decrease as set sizes increase for fixed values of β.

3. As the value of β increases, the bias and MSE increase in almost all cases.



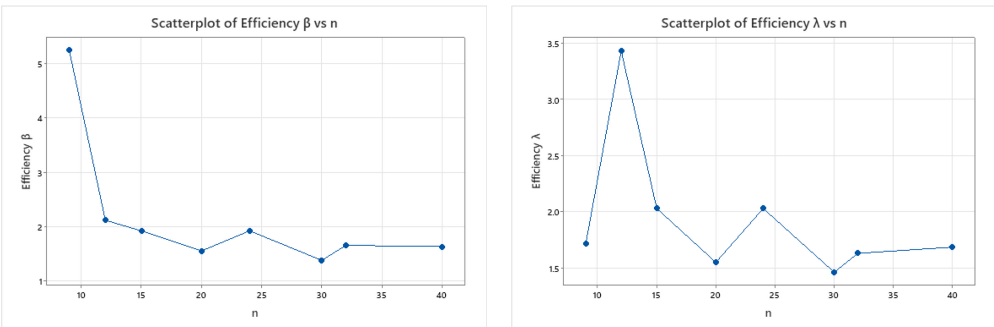
(**a**) (**b**)

Figure 1. The efficiency of the MLE parameters β (**a**) and λ (**b**) when $\beta = 1$ and $\lambda = 1$.



(**a**) (**b**)

Figure 2. The efficiency of the MLE parameters β (**a**) and λ (**b**) when $\beta = 1$ and $\lambda = 1$.



(**a**) (**b**)

Figure 3. The efficiency of the MLE parameters β (**a**) and λ (**b**) when $\beta = 1.5$ and $\lambda = 1$.

It is clear from Tables 1–3 and Figures 1–3 that the efficiency of estimators increases as the sample sizes increase. The estimators based on RSS have a smaller MSE than the corresponding ones based on SRS. The efficiency of RSS estimators with respect to SRS estimators is greater than one and increases when the sample size increases.

5. Conclusions

This paper deals with the estimation problem of unknown parameters of an EP distribution based on RSS. Maximum likelihood estimators (MLE) for the parameters of EP distribution were investigated mathematically and numerically. It was found that the $MLEs$ were not in closed form, so simulation was conducted to study the behaviors of the proposed estimators. The numerical results show that the bias and MSE of the estimates for both shape parameters relative to RSS are smaller than the corresponding SRS. This study revealed that the estimators based on *RSS* are more efficient than those from *SRS* at the same sample size.

It would be interesting to have a numerical application with real data to illustrate its usefulness in future work. The addition of this section was the kind suggestion of an anonymous referee of the journal.

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