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Editorial

Henri Poincaré's Comment on Calculus and Albert Einstein's Comment on Entropy: Mathematical Physics on the Tenth Anniversary of *Axioms*

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This Special Issue of the journal *Axioms* collates submissions in which the authors report their perceptions and results in the field of mathematical physics and/or physical mathematics without any preconditions of the specific research topic. The papers are intended to provide the reader with a broad window into the status of the research field showing our understanding of how a known concept changes our thinking in that area of science.

The history of interactions between physics and mathematics is old and complex. Physics cannot flourish without mathematics and mathematics frequently takes its inspiration from physics. The inward-bound trajectory of 20th century physics towards the discovery of the most fundamental laws of physics resulted in the creation of quantum field theory (QFT) and string theory (ST). QFT/ST was a revelation of 20th century scientists well into the 21st century and is widely recognized as being far from fully understood. Research into QFT/ST has made use of ever more sophisticated mathematics, including cutting edge mathematics at the focus of present-day research. Conversely, many developments in QFT/ST have also led to profound new insights, constructions, and even entire subfields of mathematics (examples include vertex operator algebra theory and homological mirror symmetry). A community of scientists, involving both mathematicians and physicists, are vigorously engaged in the pursuit of investigating QFT/ST and its relationship with mathematics. There is dual and equal emphasis on both the discovery of the fundamental laws of nature as well as on mathematical discovery. This field of intellectual research has been termed physical mathematics. Physical mathematics is a subfield of the much broader field of mathematical physics [1].

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1. Solvay 1911: Poincaré (Leibniz Newton Calculus) and Einstein (Boltzmann Gibbs Entropy)

A similar situation occurred at the beginning of the 20th century in physics and mathematics. The situation was reflected in the proceedings of the first Solvay Council more than 100 years ago in 1911. The central topic at this time was not QFT but quantum mechanics and the Solvay Council proceeded in elaborating on questions concerning Planck's quantum of action in terms of mathematics and physics [2].

For this Special Issue of *Axioms*, the question of what the eminent physicists and mathematicians contributed to the deliberations of the first Solvay Council had in mind for their research to develop "the theory of [photon] radiation and quanta" into quantum physics on a similar standing as classical physics was considered. When planning this Special Issue of *Axioms* the question was asked if we were in a similar situation today asking for the discovery of "the theory of neutrino radiation and quanta" generalizing the mathematics and physics of the standard model of elementary particle interactions.

The Solvay Councils have been devoted to understanding preeminent open problems in physics by applying modern mathematics. Hendrik A. Lorentz was chairman of the first Solvay Conference on Physics, held in Brussels from 30 October to 3 November 1911. The subject was The Theory of Radiation and Quanta. This council looked at the problems of having two approaches, namely, classical physics and quantum physics.

On one side, at the first Solvay Council, a time between the discovery of Planck's quantum of action and the birth of Heisenberg's and Schrödinger's quantum mechanics, Poincaré asked if it was still possible to represent basic physical laws (mechanics) in terms of differential equations [3]?. Today a follow-up question to Poincare's question could be, in principle: What physics is behind the so-called fractional calculus and fractional differential equations [4]?

On the other side, although the equation on Boltzmann's grave at the Vienna Central Cemetery captures his insight into entropy, he never wrote it down himself. It was Planck who, in 1900, first wrote into the form that became Boltzmann's epitaph and supported the birth of quantum theory. In 1905, in one of his papers, Einstein termed it Boltzmann's principle. This equation reflects the fundamental insight that the second law of thermodynamics can only be understood in terms of a connection between entropy and probability and thus the second law is statistical in nature. Einstein's perspective on classical statistical mechanics and particularly on Boltzmann's principle is reflected by his written words:

> "What I find strange about the way Mr. Planck applies Boltzmann's equation is that he introduces a state probability W without giving this quantity a physical definition. If one proceeds in such a way, then, to begin with, Boltzmann's equation does not have a physical meaning. The circumstance that W is equated to the number of complexions belonging to a state does not change anything here; for there is no indication of what is supposed to be meant by the statement that two complexions are equally probable. Even if it were possible to define the complexions in such a manner that the S obtained from Boltzmann's equation agrees with experience, it seems to me that with this conception of Boltzmann's principle it is not possible to draw any conclusions about the admissibility of any fundamental theory whatsoever on the basis of the empirically known thermodynamic properties of a system." [5].

Even earlier, Einstein emphasized with respect to the equation $\mathrm{S} = (\mathrm{R}/N)\,\mathrm{l\,gW} + \mathrm{const.}$, that:

> "Neither Herr Boltzmann nor Herr Planck gave a definition of W. They put formally W = number of complexions of the state under consideration".

2. Mathematical Physics

What is the meaning and purpose of the field of mathematical physics or physical mathematics? The common understanding is that mathematical physics applies rigorous mathematical ideas to problems inspired by physics and to investigate the mathematical structure of physical theories, and vice versa. As such, it is a remarkably broad subject. Mathematics and physics are traditionally tightly linked subjects and many historical figures, such as Isaac Newton and Carl Friedrich Gauss, were both physicists and mathematicians. Traditionally mathematical physics has been closely associated with ideas in calculus, particularly those of differential equations.

In recent years, in part due to the rise of QFT, quantum gravity, and cosmology, many more branches of mathematics have become major contributors to physics. The section Mathematical Physics of the Journal *Axioms* covers a wide field for research in the mathematical and physical sciences and their applications, including applications in chemistry, biology and the social sciences. Depending on the inclination of the authors of research papers in *Axioms*, one may prefer mathematics from the point of view of physics or vice versa.

3. From Solvay 1911 to *Axioms* 2022: Fractional Calculus and Non-Additive Entropy

Mathematical structures entered the development of physics, and problems emanating from physics influenced developments in mathematics. Examples include the role of Riemann's differential geometry in Einstein's general relativity, the dynamical theory of space and time, and the influence of Heisenberg's quantum mechanics in the development of functional analysis built on the understanding of Hilbert spaces. A prospective similar development occurred a couple of decades ago when non-Abelian gauge theories emerged as QFTs for describing fundamental particle interactions. Recently, attention has turned to the application of Riemann–Liouville fractional calculus [6–8] to physics, including Tsallis non-additive entropy [9,10]. Statistical mechanics concerns mechanics (classical, quantum, special or general relativistic) and the theory of probabilities through the adoption of a specific entropic functional [11]. Connection with thermodynamics and its macroscopic laws is established through this function.

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List of Papers in Special Issue:

1. Sternheimer, D. Some Multifaceted Aspects of Mathematical Physics, Our Common Denominator with Elliott Lieb. [12]
The field of mathematical physics is driven by ingenious individuals and deep international cooperation.
2. Fedorov, V.E.; Turov, M.M.; Kien, B.T. A Class of Quasilinear Equations with Riemann–Liouville Derivatives and Bounded Operators. [6]
Fractional integro-differential calculus is an important tool in modelling various phenomena that arise in natural sciences and engineering. The studies the local unique solvability of initial value problems for multi-term equations in Banach spaces with fractional Riemann–Liouville derivatives, fractional Riemann–Liouville integrals, and with nonlinearity, which depends on fractional derivatives of lower orders.
3. Tudorache, A.; Luca, R. Positive Solutions for a System of Fractional Boundary Value Problems with r-Laplacian Operators, Uncoupled Nonlocal Conditions and Positive Parameters. [7]
An investigation is pursued for existence and nonexistence of positive solutions for a system of Riemann–Liouville fractional differential equations with r-Laplacian operators, subject to nonlocal uncoupled boundary conditions that contain Riemann–Stieltjes integrals, various fractional derivatives, and positive parameters.
4. Chinchane, V.L.; Nale, A.B.; Panchal, S.K.; Chesneau, C.; Khandagale, A.D. On Fractional Inequalities Using Generalized Proportional Hadamard Fractional Integral Operator. [8]
A contribution is made to fractional calculus by using the generalized proportional Hadamard fractional integral operator to establish some new fractional integral inequalities for extended Chebyshev functionals.
5. Borges, E.P.; da Costa, B.G. Deformed Mathematical Objects Stemming from the q-Logarithm Function. [11]
The long-standing question on the emergence of nonadditive entropies based on mathematical formalism has been solved.
6. Kichenassamy, S. Hot Spots in the Weak Detonation Problem and Special Relativity. [13]
Special relativistic physics and mathematics is developed for the initiation of a detonation in an explosive gaseous mixture in the high activation energy regime, in three space dimensions, that typically leads to the formation of a singularity at one point, the "hot spot".
7. Redkina, T.V.; Zakinyan, R.G.; Zakinyan, A.R.; Novikova, O.V. Bäcklund Transformations for Liouville Equations with Exponential Nonlinearity. [14]
New results for Bäcklund transformations, that are an example of differential geometric structures generated by differential equations, to study solutions for nonlinear partial differential equations, are reported.
8. Ernst, T. A New q-Hypergeometric Symbolic Calculus in the Spirit of Horn, Borngässer, Debiard and Gaveau. [15]
The paper introduces a new complete multiple q-hypergeometric symbolic calculus, which leads to q-Euler integrals and a very similar canonical system of q-difference equations for multiple q-hypergeometric functions.
9. Barseghyan, V.; Solodusha, S. Control of String Vibrations by Displacement of One End with the Other End Fixed, Given the Deflection Form at an Intermediate Moment of Time. [16]
Mathematical modelling is studied of boundary-controlled processes with the equation of string vibration with given initial and final conditions and given the deflection form at an intermediate moment of time leads to wave equations.
10. Morgulis, A. Waves in a Hyperbolic Predator–Prey System. [17]
Mathematical results are derived for a hyperbolic predator–prey model, which is formulated with the use of the Cattaneo model for chemo sensitive movement.

11. Fu, J.; Zhang, L.; Cao, S.; Xiang, C.; Zao, W. A Symplectic Algorithm for Constrained Hamiltonian Systems. [18]
Using the symplectic method of constrained Hamiltonian systems, singular dynamic problems of nonconservative constrained mechanical systems, nonholonomic constrained mechanical systems as well as physical problems in quantum dynamics can be solved.
12. Mercorelli, P. A Theoretical Dynamical Noninteracting Model for General Manipulation Systems Using Axiomatic Geometric Structures. [19]
A new theoretical approach is developed to the study of robotics manipulators dynamics, based on the geometric approach to system dynamics, according to which some axiomatic definitions of geometric structures concerning invariant subspaces are used.
13. Steinle, R.; Kleiner, T.; Kumar, P.; Hilfer, R. Existence and Uniqueness of Nonmonotone Solutions in Porous Media Flow. [20]
Mathematical models exhibiting nonlinearity and hysteresis are longstanding "hot topics" that continue to generate fundamental insights and progress in mathematics, physics, and engineering.
14. Apostol, B.F. Near-Field Seismic Motion: Waves, Deformations and Seismic Moment. [21]
A tensorial force acting in a localized seismic focus is introduced and the corresponding seismic waves as solutions of the elastic wave equation in a homogeneous and isotropic body are derived.

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