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Optimizing Material Selection with Fermatean Fuzzy Hybrid Aggregation Operators

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Abstract: In the pursuance of engineering excellence and sustainable practices, the optimization of material selection processes plays a crucial role. Using Fermatean fuzzy aggregation Operators (AOs), this study introduces an innovative method for improving material selection procedures. Combining the advantages of Fermatean fuzzy set (FrFS) and AOs, the proposed method enables a comprehensive evaluation of materials based on multiple criteria. The authors propose two operators: the "Fermatean fuzzy hybrid weighted arithmetic geometric aggregation (FrFHWAGA) operator" and the "Fermatean fuzzy hybrid ordered weighted arithmetic geometric aggregation (FrFHOWAGA) operator". This method facilitates informed decision making in a number of industries by taking into account factors such as cost, durability, environmental impact, and availability. This research enables engineers, designers, and decision makers to optimize material selection, resulting in more efficient, cost-effective, and sustainable solutions across multiple domains.

Keywords: decision making; aggregation operators; material selection; optimization; Fermatean fuzzy set

MSC: 03E72; 94D05; 90B50

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1. Introduction

The process of decision making emerges as a critical and influential factor that profoundly influences the trajectory of human endeavors across all domains amidst the dynamic and ever-changing global landscape [1]. In the current era, which is characterized by technological advancements, global interconnectedness, and complex dilemmas, effective decision making is of paramount importance. The long-term consequences of present-day decisions have a significant impact on numerous industries, communities, and the global environment. It is imperative that we recognize the uttermost significance of decision making if we are to thrive in our contemporary, complex society characterized by rapid change. It is crucial that we cultivate and hone our decision-making skills and employ them effectively to confront the diverse and complex dilemmas and opportunities of our time. By engaging in this process, we enhance our ability to make significant contributions to the progression of knowledge, the improvement of society, and the responsible management of our planet [2].

Within the domain of finance and economics, the process of decision making plays a crucial role in providing direction to businesses as they navigate through a complex network of potential possibilities and associated dangers. From the perspective of business strategy, investments and market expansion, as well as managing supply chains and allocation of resources, have far-reaching ramifications that impact different industries and countries [3]. The discipline of healthcare and medical care is characterized by the significant and tangible consequences of decision making, as it directly affects the physical and mental health of individuals and communities [4]. The importance of environmental sustainability on a global scale highlights the crucial role of decision making in ensuring the preservation of our planet's future. Policymakers, environmental advocates, and industrial stakeholders are confronted with the complex task of making decisions on energy sources, techniques for conservation, and regulatory interventions. In an epoch characterized by swift technological progress and growing multidisciplinary cooperation, the procedure of material selection assumes a crucial position in the evolution of inventive items and technologies inside diverse industries [5]. The careful selection of materials has a significant impact on the performance, longevity, and ecological responsibility of engineered systems, as well as their cost effectiveness and overall usefulness. As a result, professionals in the fields of engineering, science, and research are consistently in pursuit of approaches that can optimize the efficiency and precision of material selection processes [6].

The determination of material attributes and the establishment of selection criteria may lack clear-cut or exact definitions. Frequently, they demonstrate varying levels of ambiguity and imprecision. Fuzzy logic enables the depiction of uncertainty by utilizing fuzzy sets (FSs) and membership functions. Rather than categorizing a material as either "suitable" or "unsuitable" in a binary manner, FSs are used to quantify the extent to which a material exhibits a particular attribute. One illustrative instance involves representing the hardness of a material as an FS, wherein membership functions are employed to indicate the degree of "hardness" on a continuous spectrum. The utilization of this approach facilitates a more intricate and authentic portrayal of the process of decision making, resulting in enhanced material selection decisions that are well informed and in accordance with the particular requirements of the given application. The selection criteria utilized in the field of material science frequently involve the utilization of qualitative descriptors such as "strong", "durable", or "resistant". Fuzzy logic facilitates the representation of imprecise criteria through the establishment of associations between linguistic concepts and FSs. As an example, the concept of "strong" can be modeled as an FS by utilizing a membership function that effectively represents the range of strength values. Fuzzy logic facilitates the process of evaluating and prioritizing items by considering their varying degrees of acceptability in relation to the combined fuzzy criteria. Materials that possess greater aggregated membership values are seen as being more appropriate [7]. This ranking methodology offers a systematic approach to prioritizing materials by considering their overall performance across numerous criteria, particularly in cases when these criteria include intrinsic imprecision. Fuzzy logic additionally facilitates sensitivity analysis, a crucial aspect in comprehending the impact of alterations in criteria weights or membership function shapes on the outcomes of material selection.

Real-world challenges, such as agglomeration, segmentation, decision-making, and supplier evaluation, are significantly influenced by the presence of ambiguities. In the absence of processing imprecise, ambiguous, or misleading input, it is not possible for decision-making systems to obtain dependable results. According to Zadeh [8], an FS is a broadened concept of a classical set, where the membership function is employed to quantify the degrees of membership for the elements of the FS. Zadeh's FSs have been subject to thorough examination by researchers from various angles connected to MCDM. Atanassov [9] introduced the notion of "intuitionistic fuzzy sets" (IFS) as an extension of FS, wherein the combined value of the membership degree (MSD) and non-membership degree (NMSD) is constrained to be no greater than one. This particular characteristic is highly regarded by numerous professionals in the different fields. The IFSs has limitations in effectively addressing intricate decision-making problems due to the requirement of identical values for the MSD and the NMSD. Yager's concept of the "Pythagorean fuzzy set" (PFS) is a notable extension of the IFS framework, wherein the constraint of the sum of squares of MSD and the NMSD is limited to a value of one [10]. This phenomenon exhibits various uses, one of which is selection. The authorization of PFS is not always guaranteed. To illustrate, a panel of experts was partitioned into two distinct groups with the purpose of evaluating training institutions. The initial panel of experts issued a MSD rating of 0.92, whereas the subsequent panel assigned a NMSD value of 0.84. The sum of the squares of MSD and NMSD exceeded one. The IFS and PFS were not able to effectively demonstrate this phenomenon. In order to address this issue, Senapati and Yager [11] introduced the concept of FrFS as a natural expansion of IFS and PFS. The FrFS theory holds considerable importance across various domains due to its robust conceptual framework employed for addressing contradictory and inaccurate data inside an FrF framework.

The process of data aggregation holds significant importance in several sectors, including business-related, management, social, medical, technical, mental disorders and artificial intelligence areas, since it plays a crucial role in facilitating informed decision making. Historically, the concept of alternate consciousness is seen as a distinct entity or a linguistic numeral. However, due to the level of uncertainty associated with the data, its aggregation is not a straightforward task. Indeed, it is evident that decision makers known as AOs hold a significant role within the realm of MCDM challenges. The fundamental aim of MCDM is to obtain a singular numerical value by amalgamating several distinct inputs. Numerous research studies have centered on FrFSs exclusively [12]. When multiple potential solutions to a given problem exist, the concept of AOs is crucial for determining the most advantageous option [13]. Extensive research conducted by numerous scholars demonstrates that significant progress has been made in the field of FrFSs. Senapati and Yager [14] provided the basic AOs, Einstein AOs were offered by Rani et al. [15], Jeevaraj offered the concept of interval-valued FrFSs [16], Garg et al. [17] proposed the idea of Yager AOs and Shahzadi et al. [18] initiated the concept of Hamacher Interactive AOs for FrFSs. In their study, Chen et al. [19] introduced a framework for MCDM in the context of selecting sustainable construction materials. Chen et al. [20] introduced a novel approach for assessing passenger wants and measuring passenger satisfaction by utilizing online-review analysis. The concept of "Pythagorean fuzzy power AOs" was introduced by Wei and Lu [21]. Wu and Wei [22] offered the notion of "Pythagorean fuzzy Hamacher AOs", while Garg [23] proposed "confidence level-based Pythagorean fuzzy AOs" in the context of their application to MCDM. The concept of "Yager operators with the picture fuzzy set environment and its application to emergency program selection" was introduced by Qiyas et al. [24].

The subsequent sections of the paper are organized in the following manner. Section 2 presents a set of fundamental definitions pertaining to FrFSs, operational rules, a scoring function, and an accuracy function for FrFNs. This section also discusses two fundamental AOs, namely FrFWA and FrFWG operators. In Section 3, we provide two operators, namely the "Fermatean fuzzy hybrid weighted arithmetic geometric aggregation (FrFHWAGA) operator and the Fermatean fuzzy hybrid ordered weighted arithmetic geometric aggregation (FrFHOWAGA) operator". In Section 4, we provide an FrFN-based MADM technique, while in Section 5, we propose a case study on material selection accompanied by a numerical example. Section 6 presents the research work's conclusion.

2. Preliminaries

In this section, we present some rudiments of FrF sets.

Definition 1 ([11]). *An FrFS in a finite universe $\mathscr{X}$ is*

$$F = \{< \daleth, \aleph^{\mu}{}_{F}(\daleth), \zeta^{\nu}{}_{F}(\daleth) >: \daleth \in \mathscr{X}\}$$

where $\aleph^{\mu}{}_{F}(\daleth) : \mathscr{X} \to [0,1]$ shows the MSD and $\zeta^{\nu}{}_{F}(\daleth) : \mathscr{X} \to [0,1]$ shows the NMSD of the element $\daleth \in \mathscr{X}$ to the set F, respectively, with the condition that $0 \leq \aleph^{\mu}{}_{F}(\daleth)^3 + \zeta^{\nu}{}_{F}(\daleth)^3 \leq 1$. The degree of indeterminacy is given as $\pi_F(\daleth) = \sqrt[3]{(\aleph^{\mu}{}_{F}(\daleth)^3 + \zeta^{\nu}{}_{F}(\daleth)^3 - \aleph^{\mu}{}_{F}(\daleth)^3 \zeta^{\nu}{}_{F}(\daleth)^3)}$. A basic element of the form $\langle \aleph^{\mu}{}_{F}(\daleth), \zeta^{\nu}{}_{F}(\daleth)\rangle$ in an FrFS F is called a Fermatean fuzzy number (FrFN).

2.1. Operational Laws of FrFSs

Definition 2 ([11]). *We ket $F_1 = \langle \aleph^{\mu}{}_{F_1}(\daleth), \zeta^{\nu}{}_{F_1}(\daleth)\rangle$ and $F_2 = \langle \aleph^{\mu}{}_{F_2}(\daleth), \zeta^{\nu}{}_{F_2}(\daleth)\rangle$ be FrFSs on a $\mathscr{X}$. Then, we have the following operations:*
(1) $\bar{F_1} = \langle \zeta^{\nu}{}_{F_1}(\daleth), \aleph^{\mu}{}_{F_1}(\daleth)\rangle$.
(2) $F_1 \subseteq F_2$ if and only if $\aleph^{\mu}{}_{F_1}(\daleth) \leqslant \aleph^{\mu}{}_{F_2}(\daleth)$ and $\zeta^{\nu}{}_{F_2}(\daleth) \leqslant \zeta^{\nu}{}_{F_1}(\daleth)$.
(3) $F_1 = F_2$ if and only if $F_1 \subseteq F_2$ and $F_2 \subseteq F_1$.
(4) $F_1 \cup F_2 = \{\langle \daleth, max\{\aleph^{\mu}{}_{F_1}(\daleth), \aleph^{\mu}{}_{F_2}(\daleth)\}, min\{\zeta^{\nu}{}_{F_1}(\daleth), \zeta^{\nu}{}_{F_2}(\daleth)\}\rangle \mid \daleth \in \mathscr{X}\}$.
(5) $F_1 \cap F_2 = \{\langle \daleth, min\{\aleph^{\mu}{}_{F_1}(\daleth), \aleph^{\mu}{}_{F_2}(\daleth)\}, max\{\zeta^{\nu}{}_{F_1}(\daleth), \zeta^{\nu}{}_{F_2}(\daleth)\}\rangle \mid \daleth \in \mathscr{X}\}$.
(6) $F_1 + F_2 = \{\langle \daleth, (\aleph^{\mu}{}_{F_1}(\daleth)^3 + \aleph^{\mu}{}_{F_2}(\daleth)^3 - \aleph^{\mu}{}_{F_1}(\daleth)^3 \aleph^{\mu}{}_{F_2}(\daleth)^3)^{1/3}, \zeta^{\nu}{}_{F_1}(\daleth)\zeta^{\nu}{}_{F_2}(\daleth)\rangle \mid \daleth \in \mathscr{X}\}$.
(7) $F_1.F_2 = \{\langle \daleth, (\aleph^{\mu}{}_{F_1}(\daleth)\aleph^{\mu}{}_{F_2}(\daleth), \zeta^{\nu}{}_{F_1}(\daleth)^3 + \zeta^{\nu}{}_{F_2}(\daleth)^3 - \zeta^{\nu}{}_{F_1}(\daleth)^3 \zeta^{\nu}{}_{F_2}(\daleth)^3)^{1/3}\rangle \mid \daleth \in \mathscr{X}\}$.
(8) $\sigma F_1 = \{\langle \daleth, (1 - (1 - \aleph^{\mu}{}_{F_1}(\daleth)^3)^{\sigma}))^{1/3}, \zeta^{\nu}{}_{F_1}(\daleth)^{\sigma}\rangle\}$.
(9) $F_1^{\sigma} = \{\langle \daleth, \aleph^{\mu}{}_{F_1}(\daleth)^{\sigma}, (1 - (1 - \zeta^{\nu}{}_{F_1}(\daleth)^3)^{\sigma}))^{1/3}, \rangle\}$.

Theorem 1 ([11]). *Let $\mathscr{A}$, $\mathscr{B}$ and $\mathscr{C}$ be any FrFSs over the reference set $\mathscr{X}$. Let $\widetilde{U}$ be absolute FrFS and $\widetilde{\varnothing}$ be the null FrFS. Then,*

(i) $\mathscr{A} \cup \mathscr{A} = \mathscr{A}$.
(ii) $\mathscr{A} \cap \mathscr{A} = \mathscr{A}$.
(iii) $(\mathscr{A} \cup \mathscr{B}) \cup \mathscr{C} = \mathscr{A} \cup (\mathscr{B} \cup \mathscr{C})$.
(iv) $(\mathscr{A} \cap \mathscr{B}) \cap \mathscr{C} = \mathscr{A} \cap (\mathscr{B} \cap \mathscr{C})$.
(v) $\mathscr{A} \cup (\mathscr{B} \cap \mathscr{C}) = (\mathscr{A} \cup \mathscr{B}) \cap (\mathscr{A} \cup \mathscr{C})$.
(vi) $\mathscr{A} \cap (\mathscr{B} \cup \mathscr{C}) = (\mathscr{A} \cap \mathscr{B}) \cup (\mathscr{A} \cap \mathscr{C})$.
(vii) $\mathscr{A} \cup \widetilde{\varnothing} = \mathscr{A}$ *and* $\mathscr{A} \cap \widetilde{\varnothing} = \widetilde{\varnothing}$.
(viii) $\mathscr{A} \cup \widetilde{U} = \widetilde{U}$ *and* $\mathscr{A} \cap \widetilde{U} = \mathscr{A}$.
(ix) $(\mathscr{A}^c)^c = \mathscr{A}$.
(x) $\widetilde{U}^c = \widetilde{\varnothing}$ *and* $\widetilde{\varnothing}^c = \widetilde{U}$.

Theorem 2. *Let $\mathscr{A}$ and $\mathscr{B}$ be two FrFSs over the reference set $\mathscr{X}$. Then,*

(a) $(\mathscr{A} \cup \mathscr{B})^c = \mathscr{A}^c \cap \mathscr{B}^c$, *and*
(b) $(\mathscr{A} \cap \mathscr{B})^c = \mathscr{A}^c \cup \mathscr{B}^c$.

2.2. Operational Laws of Fermatean Fuzzy Numbers (FrFNs)

Definition 3 ([14]). *Suppose $\hbar^{\delta}{}_1 = \langle \aleph^{\mu}{}_1, \zeta^{\nu}{}_1\rangle$ and $\hbar^{\delta}{}_2 = \langle \aleph^{\mu}{}_2, \zeta^{\nu}{}_2\rangle$ are the two FrFNs. Then,*
(1) $\overline{\hbar^{\delta}}{}_1 = \langle \zeta^{\nu}{}_1, \aleph^{\mu}{}_1\rangle$.
(2) $\hbar^{\delta}{}_1 \vee \hbar^{\delta}{}_2 = \langle max\{\aleph^{\mu}{}_1, \zeta^{\nu}{}_1\}, min\{\aleph^{\mu}{}_2, \zeta^{\nu}{}_2\}\rangle$.
(3) $\hbar^{\delta}{}_1 \wedge \hbar^{\delta}{}_2 = \langle min\{\aleph^{\mu}{}_1, \zeta^{\nu}{}_1\}, max\{\aleph^{\mu}{}_2, \zeta^{\nu}{}_2\}\rangle$.
(4) $\hbar^{\delta}{}_1 \oplus \hbar^{\delta}{}_2 = \langle \sqrt[3]{\aleph^{\mu 3}_1 + \aleph^{\mu 3}_2 - \aleph^{\mu 3}_1 \aleph^{\mu 3}_2}, \zeta^{\nu}{}_1\zeta^{\nu}{}_2\rangle$.
(5) $\hbar^{\delta}{}_1 \otimes \hbar^{\delta}{}_2 = \langle \aleph^{\mu}{}_1\aleph^{\mu}{}_2, \sqrt[3]{\zeta^{\nu 3}_1 + \zeta^{\nu 3}_2 - \zeta^{\nu 3}_1 \zeta^{\nu 3}_2}\rangle$.
(6) $\sigma\hbar^{\delta}{}_1 = \langle (1 - (1 - \aleph^{\mu 3}_1)^{\sigma})^{1/3}, \zeta^{\nu\sigma}_1\rangle$.
(7)$\hbar^{\delta^{\hbar^{\delta}{}_1}}_1 = \langle \aleph^{\mu\sigma}_1, (1 - (1 - \zeta^{\nu 3}_1)^{\sigma})^{1/3}\rangle$.

Theorem 3 ([14]). *Suppose $\hbar^{\delta}{}_1 = \langle \aleph^{\mu}{}_1, \zeta^{\nu}{}_1\rangle$ and $\hbar^{\delta}{}_2 = \langle \aleph^{\mu}{}_2, \zeta^{\nu}{}_2\rangle$ are any FrFNs on a $\mathscr{X}$, and $n_1, n_2 > 0$; then,*
(1) $\hbar^{\delta}{}_1 \oplus \hbar^{\delta}{}_2 = \hbar^{\delta}{}_2 \oplus \hbar^{\delta}{}_1$.
(2) $\hbar^{\delta}{}_1 \otimes \hbar^{\delta}{}_2 = \hbar^{\delta}{}_2 \otimes \hbar^{\delta}{}_1$.
(3) $n(\hbar^{\delta}{}_1 \oplus \hbar^{\delta}{}_2) = n\hbar^{\delta}{}_1 \oplus n\hbar^{\delta}{}_2$.

(4) $n_1\hbar^{\delta}{}_1 \oplus n_2\hbar^{\delta}{}_2 = (n_1 + n_2)\hbar^{\delta}{}_1$.
(5) $\hbar^{\delta n_1}_{1} \otimes \hbar^{\delta n_2}_{1} = \hbar^{\delta n_1+n_2}_{1}$.
(6) $\hbar^{\delta n}_{1} \otimes \hbar^{\delta n}_{2} = (\hbar^{\delta}{}_1 \otimes \hbar^{\delta}{}_2)^n$.

Definition 4 ([14]). *Suppose that $\mathfrak{A} = \langle \aleph^{\mu}, \zeta^{\nu} \rangle$ is a FrFN; then, score function $\mathfrak{E}$ of $\mathfrak{A}$ is defined as*

$$\mathfrak{E}(\mathfrak{A}) = \aleph^{\mu 3} - \zeta^{\nu 3}$$

$\mathfrak{E}(\mathfrak{A}) \in [-1, 1]$. *If the score is large, then the FrFN is greater. However, the score function cannot be useful in many cases of FrFN. To solve this problem, we use another function named the accuracy function.*

Definition 5 ([14]). *Consider $\mathfrak{A} = \langle \aleph^{\mu}, \zeta^{\nu} \rangle$ is the FrFN; then, the accuracy function $\mathfrak{R}$ of $\mathfrak{A}$ is defined as*

$$\mathfrak{R}(\mathfrak{A}) = \aleph^{\mu 3} + \zeta^{\nu 3}$$

2.3. Some Basic Aggregation Operators on FrFNs

Definition 6 ([14]). *Assume that $\hbar^{\delta}{}_k = \langle \aleph^{\mu}{}_k, \zeta^{\nu}{}_k \rangle$ is the conglomeration of FrFNs. Define $(FrFWA) : T^n \rightarrow T$ given by*

$$\begin{aligned}(FrFWA)(\hbar^{\delta}{}_1, \hbar^{\delta}{}_2, \ldots, \hbar^{\delta}{}_n) &= \sum_{k=1}^{n} \Re_k \hbar^{\delta}{}_k \\ &= \Re_1\hbar^{\delta}{}_1 \oplus \Re_2\hbar^{\delta}{}_2 \oplus \ldots, \Re_n\hbar^{\delta}{}_n\end{aligned}$$

where T^n is the set of all FrFNs, and $\Re = (\Re_1, \Re_2, \ldots, \Re_n)^T$ is weight vector (WV) of $(\hbar^{\delta}{}_1, \hbar^{\delta}{}_2, \ldots \hbar^{\delta}{}_n)$, such that $0 \leqslant \Re_k \leqslant 1$ and $\sum_{k=1}^{n} \Re_k = 1$. Then, the FrFWA is called the "Fermatean fuzzy weighted averaging (FRFWA) operator".

We can compute FrFWA using FrFN operating principles, as shown by the subsequent theorem.

Theorem 4 ([14]). *Let $\hbar^{\delta}{}_k = \langle \aleph^{\mu}{}_k, \zeta^{\nu}{}_k \rangle$ be the conglomeration of FrFNs. FrFWA can be obtained by*

$$(FrFWA)(\hbar^{\delta}{}_1, \hbar^{\delta}{}_2, \ldots, \hbar^{\delta}{}_n) = \left\langle \sqrt[3]{(1 - \prod_{k=1}^{n}(1 - \aleph^{\mu 3}_{k})^{\Re_k})}, \prod_{k=1}^{n} \zeta^{\nu \Re_k}_{k} \right\rangle \tag{1}$$

Definition 7 ([14]). *Assume that $\hbar^{\delta}{}_k = \langle \aleph^{\mu}{}_k, \zeta^{\nu}{}_k \rangle$ is a conglomeration of FrFN , and $(FrFWG) : T^n \rightarrow T$, if*

$$\begin{aligned}(FrFWG)(\hbar^{\delta}{}_1, \hbar^{\delta}{}_2, \ldots \hbar^{\delta}{}_n) &= \sum_{k=1}^{n} \hbar^{\delta \Re_k}_{k} \\ &= \hbar^{\delta \Re_1}_{1} \otimes \hbar^{\delta \Re_2}_{2} \otimes \ldots, \hbar^{\delta \Re_n}_{n}\end{aligned}$$

where T^n is the set of all FrFNs. Then, the FrFWG is called the "Fermatean fuzzy weighted geometric (FrFWG) operator".

We can compute the FrFWG operator using FrFN operating principles, as shown by the subsequent theorem.

Theorem 5 ([14]). *Let $\hbar^{\delta}{}_k = \langle \aleph^{\mu}{}_k, \zeta^{\nu}{}_k \rangle$ be the conglomeration of FrFNs. FrFWG can be found by*

$$(FrFWG)(\hbar^{\delta}{}_1, \hbar^{\delta}{}_2, \ldots \hbar^{\delta}{}_n) = \left\langle \prod_{k=1}^{n} \aleph^{\mu \Re_k}_{k}, \sqrt[3]{(1 - \prod_{k=1}^{n}(1 - \zeta^{\nu 3}_{k})^{\Re_k})} \right\rangle \tag{2}$$

2.4. Some Drawbacks of FrFWA and FrFWG Operators

As we know, in many MCDM problems, FrFWA and FrFWG operators are used to aggregate information. Nevertheless, the collective values may provide irrational outcomes when some values approach the upper limits of the arguments or the weights. In this analysis, we examine two distinct scenarios.

Case 1: We take two FrFNs such that $\hbar^{\delta}{}_1 = (0.001, 0), \hbar^{\delta}{}_2 = (1, 0)$ with weights $\Re_1 = 0.9$ and $\Re_2 = 0.1$. By (1) and (2), we obtain

$$\text{FrFWA}(\hbar^{\delta}{}_1, \hbar^{\delta}{}_2) = (1, 0)$$

and

$$\text{FrFWG}(\hbar^{\delta}{}_1, \hbar^{\delta}{}_2) = (0.002, 0).$$

Case 2: We take two FrFNs such that $\hbar^{\delta}{}_1 = (0.001, 0), \hbar^{\delta}{}_2 = (1, 0)$ with weights $\Re_1 = 0.1$ and $\Re_2 = 0.9$. By (1) and (2), we obtain

$$\text{FrFWA}(\hbar^{\delta}{}_1, \hbar^{\delta}{}_2) = (1, 0)$$

and

$$\text{FrFWG}(\hbar^{\delta}{}_1, \hbar^{\delta}{}_2) = (0.501, 0)$$

Based on the obtained data, it is evident that the FrFWA and FrFWG operators did not yield satisfactory outcomes in these two instances. Hence, it is imperative to enhance the operators of aggregation in order to address these limitations or deficiencies.

3. Some Hybrid AOs of FrFNs

In this part, we provide a novel hybrid approach to address the limitations of the FrFWA and FrFGA operators.

3.1. FrFHWAGA Operator

Definition 8. *Assume that $\hbar^{\delta}{}_k = \langle \aleph^{\mu}{}_k, \zeta^{\nu}{}_k \rangle$ is a conglomeration of FrFN, and $(FrFHWAGA) : T^n \to T$, if*

$$\textit{(FrFHWAGA)}(\hbar^{\delta}{}_1, \hbar^{\delta}{}_2, \ldots, \hbar^{\delta}{}_n) = \Big(\sum_{k=1}^{n} \Re_k \hbar^{\delta}{}_k \Big)^{\beth} \Big(\sum_{k=1}^{n} \hbar^{\delta}{}_k^{\Re_k} \Big)^{1-\beth}$$

where T^n is the set of all FrFNs, $\beth$ is any real number in the interval $[0, 1]$ and $\Re = (\Re_1, \Re_2, \ldots, \Re_n)^T$ is WV of $(\hbar^{\delta}{}_1, \hbar^{\delta}{}_2, \ldots, \hbar^{\delta}{}_n)$, such that $0 \leqslant \Re_k \leqslant 1$ and $\sum_{k=1}^{n} \Re_k = 1$. Then, the FrFHWAGA is called the Fermatean fuzzy hybrid weighted arithmetic geometric aggregation (FrFHWAGA) operator.

We can compute FrFHWAGA operator using FrFN operating principles as shown by the subsequent theorem.

Theorem 6. *Let $\hbar^{\delta}{}_k = \langle \aleph^{\mu}{}_k, \zeta^{\nu}{}_k \rangle$ be a conglomeration of FrFN. FrFHWAGA can be found by*

$$\begin{aligned} \textit{(FrFHWAGA)}(\hbar^{\delta}{}_1, \hbar^{\delta}{}_2, \ldots, \hbar^{\delta}{}_n) &= \Big(\sum_{k=1}^{n} \Re_k \hbar^{\delta}{}_k \Big)^{\beth} \Big(\sum_{k=1}^{n} \hbar^{\delta}{}_k^{\Re_k} \Big)^{1-\beth} \\ &= \Big\langle (1 - \textstyle\prod_{k=1}^{n} (1 - (\aleph^{\mu}{}_k)^3)^{w_k})^{\frac{\beth}{3}} (\prod_{k=1}^{n} \aleph^{\mu}{}_k^{w_k})^{1-\beth}, \\ &\sqrt[3]{1 - (1 - (\textstyle\prod_{k=1}^{n} (\zeta^{\nu}{}_k)^{w_k})^3)^{\beth} (\prod_{k=1}^{n} (1 - (\zeta^{\nu}{}_k)^3)^{w_k})^{1-\beth}} \Big\rangle \end{aligned} \quad (3)$$

Proof. Based on the operational laws of FrFNs,

$$(\text{FrFHWAGA})(\hbar^{\delta}{}_1, \hbar^{\delta}{}_2, \ldots \hbar^{\delta}{}_n) = \Big(\sum_{k=1}^{n} \Re_k \hbar^{\delta}{}_k\Big)^{\beth} \Big(\sum_{k=1}^{n} \hbar^{\delta}{}^{\Re_k}_k\Big)^{1-\beth}$$

$$= \Big\langle (1 - \textstyle\prod_{k=1}^{n} (1 - \aleph^{\mu}{}^3_k)^{\Re_k})^{1/3}, \prod_{k=1}^{n} \zeta^{\nu}{}^{\Re_k}_k) \Big\rangle^{\beth} \Big\langle (\prod_{k=1}^{n} \aleph^{\mu}{}^{\Re_k}_k, (1 - \prod_{k=1}^{n} (1 - \zeta^{\nu}{}^3_k)^{\Re_k})^{1/3}) \Big\rangle^{1-\beth}$$

$$= \left\langle \Big((1 - \textstyle\prod_{k=1}^{n} (1 - \aleph^{\mu}{}^3_k)^{\Re_k})^{\beth/q}, (1 - (1 - (\prod_{k=1}^{n} \zeta^{\nu}{}^{\Re_k}_k)^3)^{\beth})^{\frac{1}{3}}\Big) \Big((\prod_{k=1}^{n} \aleph^{\mu}{}^{\Re_k}_k)^{1-\beth}, (1 - (\prod_{k=1}^{n} (1 - \zeta^{\nu}{}^3_k)^{\Re_k})^{1-\beth})^{\frac{1}{3}}\Big) \right\rangle$$

$$= \left\langle (1 - \textstyle\prod_{k=1}^{n} (1 - \aleph^{\mu}{}^3_k)^{\Re_k})^{\beth/q} . (\prod_{k=1}^{n} \aleph^{\mu}{}^{\Re_k}_k)^{1-\beth}, \sqrt[3]{\begin{array}{l}(1 - (1 - (\prod_{k=1}^{n} \zeta^{\nu}{}^{\Re_k}_k)^3)^{\beth})^{\frac{3}{3}} + (1 - (\prod_{k=1}^{n} (1 - \zeta^{\nu}{}^3_k)^{\Re_k})^{1-\beth})^{\frac{3}{3}} \\ -(1 - (1 - (\prod_{k=1}^{n} \zeta^{\nu}{}^{\Re_k}_k)^3)^{\beth})(1 - (\prod_{k=1}^{n} (1 - \zeta^{\nu}{}^3_k)^{\Re_k})^{1-\beth})\end{array}} \right\rangle$$

$$= \left\langle (1 - \textstyle\prod_{k=1}^{n} (1 - (\aleph^{\mu}{}_k)^3)^{\Re_k})^{\frac{\beth}{3}} (\prod_{k=1}^{n} \aleph^{\mu}{}^{\Re_k}_k)^{1-\beth}, \sqrt[3]{\begin{array}{l}(1 - (1 - (\prod_{k=1}^{n} \zeta^{\nu}{}^{\Re_k}_k)^3)^{\beth}) + (1 - (\prod_{k=1}^{n} (1 - \zeta^{\nu}{}^3_k)^{\Re_k})^{1-\beth})^{\frac{3}{3}} \\ -1 + (\prod_{k=1}^{n} (1 - \zeta^{\nu}{}^3_k)^{\Re_k})^{1-\beth}) + (1 - (\prod_{k=1}^{n} \zeta^{\nu}{}^{\Re_k}_k)^3)^{\beth}) - (1 - (\prod_{k=1}^{n} (1 - \zeta^{\nu}{}^3_k)^{\Re_k})^{1-\beth})(1 - (\prod_{k=1}^{n} \zeta^{\nu}{}^{\Re_k}_k)^3)^{\beth})\end{array}} \right\rangle$$

$$= \left\langle (1 - \textstyle\prod_{k=1}^{n} (1 - (\aleph^{\mu}{}_k)^3)^{\Re_k})^{\frac{\beth}{3}} (\prod_{k=1}^{n} \aleph^{\mu}{}^{\Re_k}_k), \sqrt[3]{1 - (1 - (\prod_{k=1}^{n} (\zeta^{\nu}{}_k)^{\Re_k})^3)^{\beth} (\prod_{k=1}^{n} (1 - (\zeta^{\nu}{}_k)^3)^{\Re_k})^{1-\beth}} \right\rangle$$

Therefore, this completes the proof of (3).

□

Remark 1. *For different values of $\beth \in [0,1]$, it is possible to investigate the various families of the FrFHWAGA operator individually. If we consider some special case like $\beth = 1$, the FrFHWAGA operator is reduced to the FrFWA operator. If $\beth = 0$, the FrFHWAGA operator is reduced to the FrFWG operator. If $\beth = 0.5$, the FrFHWAGA operator is the mean of the FrFWA and FrFWG operators.*

Example 1. *Let $\hbar^{\delta}{}_1 = (0.710, 0.520), \hbar^{\delta}{}_2 = (0.341, 0.562)$, and $\hbar^{\delta}{}_3 = (0.572, 0.681)$ be the three FrFNs, $\Re = (0.50, 0.30, 0.20)$ be the WV of $(\hbar^{\delta}{}_1, \hbar^{\delta}{}_2, \hbar^{\delta}{}_3)$, $\beth = 0.5$. Use the FrFHWAGA operator to aggregate the three FrFNs by using (3).*

$$\begin{aligned}(\mathit{FrFHWAGA})(\hbar^{\delta}{}_1, \hbar^{\delta}{}_2, \hbar^{\delta}{}_3) &= \Bigg\langle (1 - \prod_{k=1}^{3} (1 - (\aleph^{\mu}{}_k)^3)^{w_k})^{\frac{0.5}{3}} (\prod_{k=1}^{3} \aleph^{\mu}{}^{w_k}_k)^{1-0.5}, \\ &\quad \sqrt[3]{1 - (1 - (\prod_{k=1}^{3} (\zeta^{\nu}{}_k)^{w_k})^3)^{0.5} (\prod_{k=1}^{3} (1 - (\zeta^{\nu}{}_k)^3)^{w_k})^{1-0.5}} \Bigg\rangle \\ &= (0.586, 0.561)\end{aligned}$$

Theorem 7. Let $\hbar^{\delta}{}_k = \langle \aleph^{\mu}{}_k, \zeta^{\nu}{}_k \rangle$ be a conglomeration of FrFNs. Then,
1. (Idempotency) if $\hbar^{\delta}{}_k = \hbar^{\delta} = \langle \aleph^{\mu}, \zeta^{\nu} \rangle$ for all k, then

$$\text{FrFHWAGA}(\hbar^{\delta}{}_1, \hbar^{\delta}{}_2, \ldots, \hbar^{\delta}{}_n) = \hbar^{\delta} \tag{4}$$

2. (Boundedness) if $\hbar^{\delta^-} = (min(\aleph^{\mu}{}_k), max(\zeta^{\nu}{}_k))$ and $\hbar^{\delta^+} = (max(\aleph^{\mu}{}_k), min(\zeta^{\nu}{}_k))$, then

$$\hbar^{\delta^-} \leq \text{FrFHWAGA}(\hbar^{\delta}{}_1, \hbar^{\delta}{}_2, \ldots, \hbar^{\delta}{}_n) \leq \hbar^{\delta^+} \tag{5}$$

3. (Monotonicity) if $\hbar^{\delta}{}_k = \langle \aleph^{\mu}{}_k, \zeta^{\nu}{}_k \rangle$ and $\hbar^{\delta^*}_k = \langle \aleph^{\mu^*}_k, \zeta^{\nu^*}_k \rangle$ are two sets of FrFNs and if $\aleph^{\mu}{}_k \geq \aleph^{\mu^*}_k, \quad \zeta^{\nu}{}_k \leq \zeta^{\nu^*}_k$ for all k, then

$$\text{FrFHWAGA}(\hbar^{\delta}{}_1, \hbar^{\delta}{}_2, \ldots, \hbar^{\delta}{}_n) \geq \text{FrFHWAGA}(\hbar^{\delta^*}_1, \hbar^{\delta^*}_2, \ldots, \hbar^{\delta^*}_n) \tag{6}$$

3.2. FrFHOWAGA Operator

Definition 9. *Assume that $\hbar^{\delta}{}_k = \langle \aleph^{\mu}{}_k, \zeta^{\nu}{}_k \rangle$ is a conglomeration of FrFNs, and (FrFHOWAGA) : $T^n \rightarrow T$, if*

$$\textit{(FrFHWAGA)}(\hbar^{\delta}{}_1, \hbar^{\delta}{}_2, \ldots \hbar^{\delta}{}_n) = \Big(\sum_{k=1}^{n} \Re_k \hbar^{\delta}{}_{\daleth(k)} \Big)^{\daleth} \Big(\sum_{k=1}^{n} \hbar^{\delta \Re_k}_{\daleth(k)} \Big)^{1-\daleth}$$

where T^n is the set of all FrFNs,$(\daleth(1), \daleth(2), \ldots, \daleth(k))$ is a permutation of $(1, 2, \ldots, n)$ such that $\hbar^{\delta}{}_{\daleth(j-1)} \geq \hbar^{\delta}{}_{\daleth(j)}$ for any k, $\daleth$ is any real number in the interval $[0, 1]$. Then, the FrFHOWAGA is called the Fermatean fuzzy hybrid ordered weighted arithmetic geometric aggregation (FrFHOWAGA) operator.

Theorem 8. *Let $\hbar^{\delta}{}_k = \langle \aleph^{\mu}{}_k, \zeta^{\nu}{}_k \rangle$ be a conglomeration of FrFNs. FrFHOWAGA can be found by*

$$\begin{aligned}
\textit{(FrFHWAGA)}(\hbar^{\delta}{}_1, \hbar^{\delta}{}_2, \ldots, \hbar^{\delta}{}_n) &= \Big(\sum_{k=1}^{n} \Re_k \hbar^{\delta}{}_{\daleth(k)} \Big)^{\daleth} \Big(\sum_{k=1}^{n} \hbar^{\delta \Re_k}_{\daleth(k)} \Big)^{1-\daleth} \\
&= \Bigg\langle (1 - \prod_{k=1}^{n} (1 - (\aleph^{\mu}{}_{\daleth(k)})^3)^{\Re_k})^{\frac{\daleth}{3}} (\prod_{k=1}^{n} \aleph^{\mu \Re_k}_{\daleth(k)})^{1-\daleth}, \\
&\sqrt[3]{1 - (1 - (\prod_{k=1}^{n} (\zeta^{\nu}{}_{\daleth(k)})^{\Re_k})^3)^{\daleth} (\prod_{k=1}^{n} (1 - (\zeta^{\nu}{}_{\daleth(k)})^3)^{\Re_k})^{1-\daleth}} \Bigg\rangle
\end{aligned}$$

where $\daleth$ is any real number in the interval $[0, 1]$. $(\daleth(1), \daleth(2), \ldots, \daleth(k))$ is a permutation of $(1, 2, \ldots, n)$ such that $\hbar^{\delta}{}_{\daleth(k-1)} \geq \hbar^{\delta}{}_{\daleth(k)}$ for any k.

Proof. The evidence may be constructed using a similar approach to that of Theorem 6; hence, we omit the proof. □

Example 2. *We consider $\ddot{a}_1 = (0.710, 0.520)$, $\ddot{a}_2 = (0.340, 0.560)$, and $\ddot{a}_3 = (0.570, 0.680)$ to be the three FrFNs, $\Re = (0.50, 0.30, 0.20)$ be the WV of $(\ddot{a}_1, \ddot{a}_2, \ddot{a}_3)$, $\daleth = 0.5$.*
By score function, we rank these FrFNs

$$\begin{aligned}
\mathfrak{E}(\ddot{a}_1) &= 0.181, \\
\mathfrak{E}(\ddot{a}_2) &= -0.085, \\
\mathfrak{E}(\ddot{a}_3) &= -0.108;
\end{aligned}$$

now, $\hbar^{\delta}{}_1 = \ddot{a}_1, \quad \hbar^{\delta}{}_2 = \ddot{a}_2, \quad \hbar^{\delta}{}_3 = \ddot{a}_3$. We use the FrFHOWAGA operator to aggregate by using (4).

$$\begin{aligned}
\textit{(FrFHOWAGA)}(\hbar^{\delta}{}_1, \hbar^{\delta}{}_2, \hbar^{\delta}{}_3) &= \Bigg\langle (1 - \prod_{k=1}^{3} (1 - (\aleph^{\mu}{}_{\daleth(k)})^4)^{w_k})^{\frac{0.5}{4}} (\prod_{k=1}^{3} \aleph^{\mu w_k}_{\daleth(k)})^{1-0.5}, \\
&\sqrt[4]{1 - (1 - (\prod_{k=1}^{3} (\zeta^{\nu}{}_{\daleth(k)})^{w_k})^4)^{0.5} (\prod_{k=1}^{3} (1 - (\zeta^{\nu}{}_{\daleth(k)})^4)^{w_k})^{1-0.5}} \Bigg\rangle \\
&= (0.582, 0.567)
\end{aligned}$$

Theorem 9. Let $\hbar^{\delta}{}_k = \langle \aleph^{\mu}{}_k, \zeta^{\nu}{}_k \rangle$ be a conglomeration of FrFNs . Then,
1. (Idempotency) If $\hbar^{\delta}{}_k = \hbar^{\delta} = \langle \aleph^{\mu}, \zeta^{\nu} \rangle$ for all k, then

$$\text{FrFHOWAGA}(\hbar^{\delta}{}_1, \hbar^{\delta}{}_2, \ldots, \hbar^{\delta}{}_n) = \hbar^{\delta} \tag{7}$$

2. (Boundedness) If $\hbar^{\delta^-} = (min(\aleph^{\mu}{}_k), max(v_k))$ and $\hbar^{\delta^+} = (max(\aleph^{\mu}{}_k), min(\zeta^{\nu}{}_k))$, then

$$\hbar^{\delta^-} \leq \text{FrFHOWAGA}(\hbar^{\delta}{}_1, \hbar^{\delta}{}_2, \ldots, \hbar^{\delta}{}_n) \leq \hbar^{\delta^+} \tag{8}$$

3. (Monotonicity) If $\hbar^{\delta}{}_k = \langle \aleph^{\mu}{}_k, \zeta^{\nu}{}_k \rangle$ and $\hbar^{\delta^*}_k = \langle \aleph^{\mu^*}_k, \zeta^{\nu^*}_k \rangle$ are two sets of FrFNs and if $\aleph^{\mu}{}_k \geq \aleph^{\mu^*}_k, \quad \zeta^{\nu}{}_k \leq \zeta^{\nu^*}_k$ for all k, then

$$\text{FrFHOWAGA}(\hbar^{\delta}{}_1, \hbar^{\delta}{}_2, \ldots, \hbar^{\delta}{}_n) \geq \text{FrFHOWAGA}(\hbar^{\delta^*}_1, \hbar^{\delta^*}_2, \ldots, \hbar^{\delta^*}_n) \tag{9}$$

3.3. Numerical Example

In order to demonstrate the suitability of the aggregated values obtained from the FrFHWAGA and FrFHOWAGA operators, we consider the first scenario outlined in Section 2.4. In this study, we consider the value of $\sqsupset$ to be 0.5. For the given scenario, we employ the FrFHWAGA and FrFHOWAGA operators. For Case 1, by (3), there is FrFHWAGA$(\hbar^{\delta}{}_1, \hbar^{\delta}{}_2) = (0.045, 0)$ which is between FrFWA$(\hbar^{\delta}{}_1, \hbar^{\delta}{}_2) = (1, 0)$ and $FrFWG(\hbar^{\delta}{}_1, \hbar^{\delta}{}_2) = (0.002, 0)$. In the aforementioned scenario, recently devised operators demonstrate the presence of moderate values. It is evident that these operators possess the capability to address the limitations of the FrFWA and FrFWG operators. Hence, the FrFHWAGA and FrFHOWAGA operators exhibit more efficacy and rationality in the context of information aggregation.

4. Decision-Making Method Based on Proposed AOs

We suppose that $A = \{(A^{\Upsilon}{}_1, A^{\Upsilon}{}_2, \ldots, A^{\Upsilon}{}_p)\}$ and $C = \{C_1, C_2, \ldots, C_q\}$ is the set of alternatives and criterions, respectively. We let $\Re$ be the WV of attributes, such that $\Re_j \in [0,1]$ and $\sum_{j=1}^{n} \Re_j = 1, (j = 1, 2, \ldots, n)$ and $\Re_j$ represent the weight of C_j. An alternative on criterions is evaluated by the decision maker, and the evaluation values must be in FrFNs. We assume that $(\digamma_{ij})_{p \times q} = \langle \aleph^{\mu}{}_{ij}, \zeta^{\nu}{}_{ij} \rangle$ is the decision matrix provided by decision maker $(\digamma_{ij})$ representing FrFNs for alternative $A^{\Upsilon}{}_i$ associated with criterions C_j. Here, we have some conditions:
1. $\aleph^{\mu}{}_{ij}$ and $\zeta^{\nu}{}_{ij} \in [0,1]$,
2. $0 \leq \aleph^{\mu}{}_A(\daleth)^3 + \zeta^{\nu}{}_A(\daleth)^3 \leq 1$.
Now, we develop Algorithm 1 to solve the given problem.

Algorithm 1: Decision-making algorithm

Step 1. The preference matrix for input is assessed.

$$(\digamma_{ij})_{p \times q} = \langle \aleph^{\mu}{}_{ij}, \zeta^{\nu}{}_{ij} \rangle$$

.

Step 2. The decision matrix should be normalized. In the context of decision making, it is common to encounter many criteria or characteristics, such as cost and benefit. In order to standardize the decision matrix, a normalization technique may be employed, wherein the complement of certain criteria, such as cost, is taken into consideration.
Step 3. Evaluate $\digamma_i = \text{FrFHWAGA}(\digamma_{i1}, \digamma_{i2}, \ldots \digamma_{in})$ or $\digamma_i = FrFHOWAGA(\digamma_{i1}, \digamma_{i2}, \ldots \digamma_{in})$ for each $i = 1, 2, \ldots, q$.
Step 4. The objective is to ascertain the scoring functions for each individual $\digamma_i$ in relation to the aggregate overall FrFNs.
Step 5. Rank all the $\digamma_i$ $(i = 1, 2, \ldots, p)$ according to the score values.

The flow chart of Algorithm 1 is given in Figure 1.

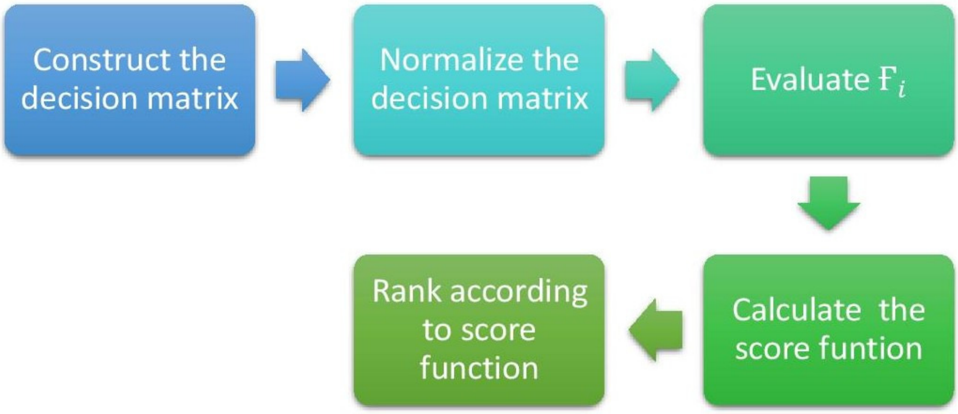

Figure 1. Flow chart of Algorithm 1.

5. Case Study

The process of choosing materials is a crucial element in the domains of design and engineering, as it significantly impacts the performance, longevity, and usefulness of a diverse array of products in numerous industries. The selection of materials has a significant influence on the ultimate result, regardless of the industry, whether it is aerospace, automotive, construction, healthcare, or consumer products.

- In the aerospace sector, where the highest priority is placed on safety and dependability, the process of material selection has critical significance. Aircrafts and spacecrafts necessitate the ability to endure exceedingly challenging circumstances, encompassing elevated temperatures, heightened pressures, and the inhospitable surroundings of the extraterrestrial realm. The selection of materials for various components, including airframes, engines, and avionics, has a direct influence on the performance and safety of these vehicles.
- The selection of materials in the automotive sector exerts a substantial influence on the safety of vehicles, their fuel efficiency, and their overall performance. In the realm of car manufacturing, engineers and designers are confronted with the task of effectively managing several considerations, including weight, strength, and cost, in order to develop automobiles that align with both legal requirements and market preferences. The utilization of high-strength steel, aluminum, and composite materials is a common practice in the automotive industry with the aim of diminishing vehicle mass and enhancing fuel economy [25]. Furthermore, it is vital to have materials that possess exceptional crashworthiness characteristics in order to augment passenger safety in the occurrence of a collision. The advancement of electric automobiles has moreover prompted the utilization of lightweight materials in order to optimize the capacity of the battery.
- The process of choosing materials in the field of construction has a crucial role in determining the structural stability, energy efficiency, and durability of a structure. The selection of appropriate materials for the construction of foundations, walls, roofs, and insulation can have a substantial influence on the long-term performance of a structure. Concrete and steel are frequently employed in the construction of tall structures because to their robustness and long-lasting properties [26]. The utilization of timber and wood-based items is very prevalent in the domain of residential construction due to their commendable sustainability and aesthetic attributes. Energy-efficient materials, such as insulated glass and improved insulation materials, play a crucial role in mitigating heating and cooling expenses within buildings, therefore promoting sustainability and enhancing occupant comfort.
- The careful selection of materials is of the utmost importance in the healthcare industry, as it directly impacts the design and production of medical devices, implants, and

medications. When selecting materials for medical purposes, it is crucial to prioritize factors such as biocompatibility, sterilizability, and durability. Surgical equipment and implants frequently employ medical-grade polymers, stainless steel, and titanium alloys due to their biocompatibility and corrosion resistance properties. Furthermore, it is vital to exercise meticulousness in the selection of pharmaceutical packing materials in order to guarantee the durability and security of medications during their storage and transit processes [27].

- The selection of materials in the domain of energy production is of the utmost importance in ensuring the optimal efficiency and sustainability of power generating and storage systems. In the context of solar panels and wind turbines, the selection of materials for photovoltaic cells and turbine blades plays a crucial role in determining both the efficiency of energy conversion and the longevity of these devices. Advanced materials, such as lithium-ion batteries, play a crucial role in the storage of energy for electric cars and renewable energy systems. The aforementioned materials have a significant influence on the energy density, charge/discharge rates, and overall longevity of batteries, hence exerting an impact on the feasibility of clean energy solutions [28].
- The consideration of costs has the utmost importance when it comes to the choosing of materials in many domains. The entire cost of production is influenced by several factors, including the availability and cost of raw materials, manufacturing processes, and labor. In order to maintain competitiveness in the market, engineers and designers are required to achieve a harmonious equilibrium between performance, quality, and cost effectiveness. Efficiency constitutes an additional key element. Materials that possess the ability to undergo efficient processing and fabrication, resulting in the attainment of specified forms and sizes, while minimizing energy consumption, are crucial in enhancing industrial efficiency and generating cost savings [29].

Figure 2 shows the connection of material selection in different fields.

The automobile business is characterized by intense competition and a rapidly evolving landscape, as it continually strives to discover novel approaches for improving vehicle performance, safety, and sustainability. The process of choosing materials for vehicles is pivotal; it has a direct impact on the overall quality, financial implications, and environmental consequences. This case study aims to examine a scenario whereby AutoTech, a prominent automotive company, is confronted with the decision of choosing the optimal material for the suspension arm, a critical component of their next electric vehicle (EV) model. The individual responsible for making decisions in this particular scenario is the chief engineer.

AutoTech has conducted a comprehensive analysis and has successfully reduced the range of material possibilities to a total of four alternatives.

${A^{\gamma}}_1$ High-Strength Steel (HSS) refers to a type of steel that possesses superior strength properties compared to conventional steels. Advantages: The material exhibits exceptional strength and durability, making it very suitable for many applications. Additionally, it offers a cost-effective solution due to its affordability and widespread availability. There are several drawbacks associated with this phenomenon. The object has a substantial weight and possesses a diminished level of resistance to corrosion.

${A^{\gamma}}_2$ Titanium alloy, also referred to as Ti, is a metallic material that has a combination of titanium and other elements. There are several advantages associated with this material, including its lightweight nature, exceptional strength, and resistance to corrosion. One of the drawbacks of this option is that it has the highest material cost compared to other solutions.

${A^{\gamma}}_3$ Carbon Fiber Reinforced Polymer (CFRP) is a composite material that consists of carbon fibers embedded in a polymer matrix. There are several advantages associated with the use of this material. Firstly, it possesses an exceptional strength-to-weight ratio, which means that it can withstand high levels of stress while remaining relatively lightweight. Additionally, it exhibits a high level of resistance to corrosion, making it particularly appropriate for use in environments where exposure to moisture or other corrosive substances is a concern. Lastly, there are several drawbacks associated with this technology, including the high cost of manufacturing and the restricted availability of the product.

A^{γ}_{4} Aluminum Alloy (AA) is a type of material that is often used in many industries. There are several advantages associated with the use of this material. Firstly, it is characterized by its lightweight nature, which contributes to its overall appeal. Additionally, it possesses a commendable strength-to-weight ratio, further enhancing its desirability. Furthermore, this material exhibits corrosion-resistant properties. There are several drawbacks associated with this approach, namely increased material expenses and the necessity for specialized welding procedures.

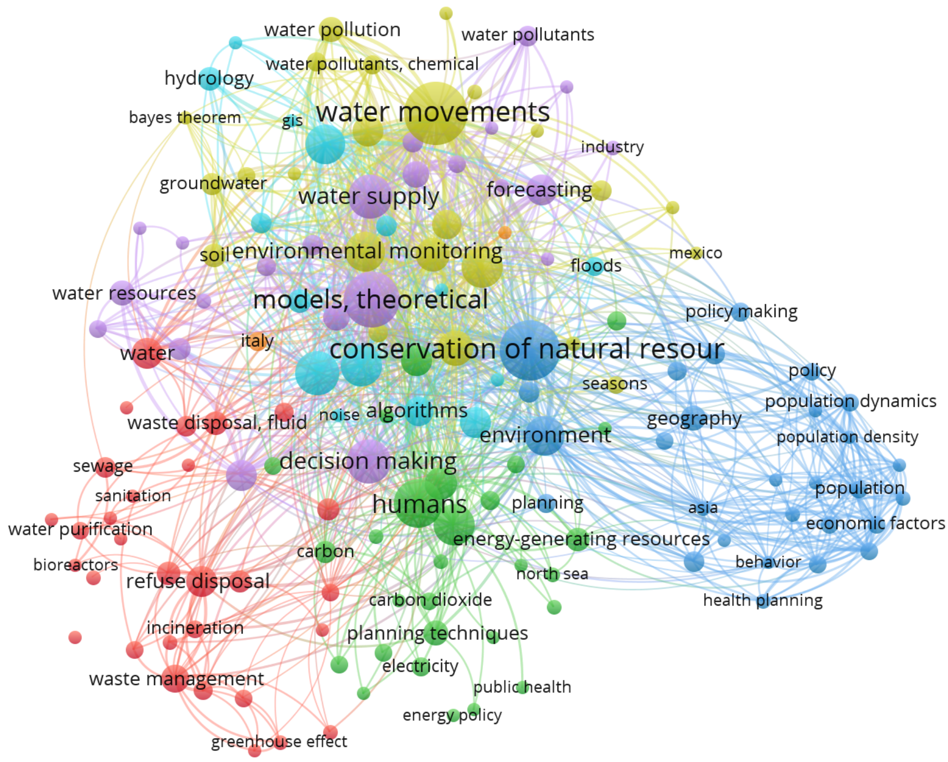

Figure 2. Relation of material selection in different fields.

The individual holding the position of chief engineer at AutoTech assumes the responsibility of making the ultimate choice. The individual possesses a significant background in the field of automotive engineering and demonstrates a strong commitment to developing an electric vehicle that surpasses expectations in terms of performance, safety, and sustainability, all while effectively managing manufacturing costs [30,31].

To make an informed decision, the deciion maker considers the following four criteria:

C_1 Weight Reduction: The optimization of performance and battery life in electric vehicles is contingent upon the criticality of lowering weight, owing to their emphasis on energy economy and range.

C_2 Cost Efficiency: The effective management of manufacturing costs is crucial for ensuring the profitability and affordability of the EV model.

C_3 Environmental Impact: AutoTech demonstrates a strong dedication to mitigating their environmental impact by actively striving to decrease its carbon emissions. The selection of materials characterized by a minimal environmental footprint is consistent with their overarching objectives of promoting sustainability.

C_4 Safety and Durability: The suspension arm plays a critical role in ensuring the safety of passengers and withstanding the various pressures that are inherent in the operation of EVs.

5.1. Decision-Making Process

In this particular scenario, FrFNs are employed to assess the four potential alternatives, denoted as $A^{\curlyvee}{}_i$ $(i = 1, 2, 3, 4)$, based on the aforementioned criteria C_i where i takes values from one to four. The WV $\Re$ is given by $(0.20, 0.30, 0.10, 0.40)^T$ for C_i. Additionally, the controlling index is denoted as $\daleth$ and has a value of 0.5. Algorithm 1 is employed for the purpose of addressing the Multiple Criteria Decision Making (MCDM) problem. The process stages are outlined in detail as follows:

- **Step 1**: Evaluate the decision matrix given by the DM consisting of FrF information given in Table 1.

Table 1. Assessment matrix acquired from DM.

	C_1	C_2	C_3	C_4
$A^{\curlyvee}{}_1$	(0.610, 0.320)	(0.450, 0.410)	(0.330, 0.560)	(0.660, 0.530)
$A^{\curlyvee}{}_2$	(0.520, 0.610)	(0.560, 0.340)	(0.570, 0.680)	(0.610, 0.490)
$A^{\curlyvee}{}_3$	(0.320, 0.720)	(0.490, 0.360)	(0.600, 0.420)	(0.700, 0.210)
$A^{\curlyvee}{}_4$	(0.120, 0.760)	(0.320, 0.820)	(0.910, 0.120)	(0.600, 0.130)

- **Step 2**: The decision matrix is not in a normalized form, because C_1 and C_2 are the cost type criteria. Therefore, the normalized decision matrix is given in Table 2.

Table 2. Normalized matrix.

	C_1	C_2	C_3	C_4
$A^{\curlyvee}{}_1$	(0.320, 0.610)	(0.410, 0.450)	(0.330, 0.560)	(0.660, 0.530)
$A^{\curlyvee}{}_2$	(0.610, 0.520)	(0.340, 0.560)	(0.570, 0.680)	(0.610, 0.490)
$A^{\curlyvee}{}_3$	(0.720, 0.320)	(0.360, 0.490)	(0.600, 0.420)	(0.700, 0.210)
$A^{\curlyvee}{}_4$	(0.760, 0.120)	(0.820, 0.320)	(0.910, 0.120)	(0.600, 0.130)

- **Step 3**: Compute $\digamma_i = \text{FrFHWAGA}(\digamma_{i1}, \digamma_{i2}, \ldots, \digamma_{in})$ for each i. Thus, find aggregated FrFNs by using Equation (3).

$$\digamma_1 = (0.498, 0.527),$$
$$\digamma_2 = (0.531, 0.539),$$
$$\digamma_3 = (0.602, 0.351),$$
$$\digamma_4 = (0.742, 0.199).$$

- **Step 4**: Evaluate the score functions for all $\digamma_i$ for the collective overall FrFNs.

$$\mathfrak{E}(\digamma_1) = -0.023,$$
$$\mathfrak{E}(\digamma_2) = -0.007,$$
$$\mathfrak{E}(\digamma_3) = 0.175,$$
$$\mathfrak{E}(\digamma_4) = 0.401.$$

- **Step 5**: Rank all the $\digamma_i$ $(i = 1, 2, 3, 4)$ according to the score values.

$$\digamma_4 \succ \digamma_3 \succ \digamma_2 \succ \digamma_1,$$

$$A^{\gamma}{}_4 \succ A^{\gamma}{}_3 \succ A^{\gamma}{}_2 \succ A^{\gamma}{}_1,$$

and thus $A^{\gamma}{}_4$ is the most desirable alternative.

The process of selecting materials in the automobile industry is a multifaceted decision-making process that involves the consideration of several aspects and trade-offs. The selection of the aluminum alloy for the suspension arm by the DM demonstrates AutoTech's dedication to manufacturing an electric car that is both efficient and cost effective, while also prioritizing passenger safety and durability, all within the context of environmental sustainability.

5.2. Comparative Analysis

This section presents a comparison between the proposed AOs and the existing implemented AOs. By utilizing extant algorithms and optimization techniques, we were able to derive an ideal solution that aligns with our research findings. This exemplifies the robustness and effectiveness of the AO. Our proposed strategy demonstrates more applicability and superiority when compared to many previously documented alternative options. Various contemporary operators are employed to authenticate our ideal solution. The best alternatives that we identified are the same, which serves as evidence supporting the validity of the alternative options shown in Table 3.

Table 3. Comparison analysis.

Authors	AOs	Ranking of Alternatives
Garg et al. [17]	FrFYWA	$A^{\gamma}{}_4 \succ A^{\gamma}{}_1 \succ A^{\gamma}{}_2 \succ A^{\gamma}{}_3$
	FrFYWG	$A^{\gamma}{}_4 \succ A^{\gamma}{}_3 \succ A^{\gamma}{}_2 \succ A^{\gamma}{}_1$
Shahzadi et al. [18]	FrFHIWA	$A^{\gamma}{}_4 \succ A^{\gamma}{}_3 \succ A^{\gamma}{}_2 \succ A^{\gamma}{}_1$
	FrFHIWG	$A^{\gamma}{}_4 \succ A^{\gamma}{}_2 \succ A^{\gamma}{}_3 \succ A^{\gamma}{}_1$
Senapati and Yager [14]	FRFWA	$A^{\gamma}{}_4 \succ A^{\gamma}{}_3 \succ A^{\gamma}{}_1 \succ A^{\gamma}{}_2$
	FrFWG	$A^{\gamma}{}_4 \succ A^{\gamma}{}_3 \succ A^{\gamma}{}_2 \succ A^{\gamma}{}_1$
Rani et al. [15]	FrFEWA	$A^{\gamma}{}_4 \succ A^{\gamma}{}_1 \succ A^{\gamma}{}_2 \succ A^{\gamma}{}_3$
	FrFEWG	$A^{\gamma}{}_4 \succ A^{\gamma}{}_1 \succ A^{\gamma}{}_3 \succ A^{\gamma}{}_2$
Proposed	FrFHWAGA	$A^{\gamma}{}_4 \succ A^{\gamma}{}_3 \succ A^{\gamma}{}_2 \succ A^{\gamma}{}_1$
	FrFOHWAGA	$A^{\gamma}{}_4 \succ A^{\gamma}{}_3 \succ A^{\gamma}{}_1 \succ A^{\gamma}{}_2$

Comparative analysis demonstrated that our proposed AOs are preferable to the alternatives currently available. These AOs have the potential to make significant contributions to the field of optimization algorithms based on their robustness, applicability, and consistency in outperforming other operators. Additional research and real-world applications are required to validate their performance in a variety of contexts and solidify their status as valuable optimization and algorithmic operations tools. This is the first study for material selection in automotive industry using Fermatean fuzzy AOs. That is why we compare our results with those of exsiting AOs.

6. Conclusions

AOs used for the aggregation of FrFNs, namely FrFWA operators and FrWG operators, are important mathematical tools. To address some limitations associated with the FrFWA and FrFWG operators in real-world scenarios, we introduced two new operators called the FrFHWAGA operator and the FrFHOWAGA operator. Several properties of the FrFHWAGA and FrFHOGA operators were also discovered. Compared to the currently specified operators on FrFNs, the proposed operators are more efficient. Utilizing apparent aides, the suggested operators were elucidated in greater detail. In the context of FrF, the proposed operators exhibited greater efficacy and adaptability than the currently available operators. This paper presented a comprehensive case study of the process of material selection in the EV industry.

Subsequent investigations will explore the potential applications of the suggested operators across diverse data formats and their operational dynamics within different domains. The principles elucidated in this essay possess the potential for application across a diverse array of real-world scenarios. These methods have the potential to effectively mitigate ambiguity in various domains such as business, machine intelligence, cognitive science, the electoral system, pattern recognition, learning techniques, trade analysis, predictions, agricultural estimation, microelectronics, and other related topics.

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