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Using the Ordered Weighted Average Operator to Gauge Variation in Agriculture Commodities in India

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Abstract: Agricultural product prices are subject to various uncertainties, including unpredictable weather conditions, pest infestations, and market fluctuations, which can significantly impact agricultural yields and productivity. Accurately assessing and understanding price is crucial for farmers, policymakers, and stakeholders in the agricultural sector to make informed decisions and implement appropriate risk management strategies. This study used the ordered weighted average (OWA) operator and its extensions as mathematical aggregation techniques incorporating ordered weights to capture and evaluate the factors influencing price variation. By generating different vectors related to different inputs to the traditional formulation, it is possible to aggregate information to calculate and provide a new view of the outcomes. The results of this research can help enhance risk management practices in agriculture and support decision-making processes to mitigate the adverse effects of price.

Keywords: coefficient of variation; agricultural yields; OWA operator; market condition; uncertainties; productivity; decision making

MSC: 03B52; 68T37

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1. Introduction

Agriculture serves as a primary source of food, feed, fiber, and fuel worldwide, and agriculture prices are a vital component of the agricultural sector. These prices have a significant impact on the livelihoods of farmers and the overall economy of a region or a country. Since agriculture is the primary source of food production, it is important in providing sustenance and nutrition to the population. These statements emphasize the interconnectedness of agricultural prices and the availability of food for consumption. However, the agricultural industry faces numerous uncertainties that significantly impact its prices, such as unpredictable weather conditions, pest infestations, and market fluctuations [1,2]. Accurately assessing and understanding price is essential for farmers, policymakers, and stakeholders in the agricultural sector to make informed decisions and implement effective risk management strategies.

This study delves into employing the ordered weighted averaging (OWA) operator to measure the coefficient of variation of rice prices in India [3]. This is a mathematical approach that incorporates the notion of ordered weights. This technique enables a thorough evaluation by accounting for the significance of distinct factors that impact agricultural results. By assigning appropriate weights to input variables related to weather, pests, market conditions, and other relevant factors, the OWA operator provides a robust framework for capturing and evaluating the diverse influences on price variation [4,5].

The results of this research have the potential to significantly contribute to enhancing risk management practices in agriculture [6]. Farmers can benefit from a better understanding of the factors contributing to price variation, enabling them to implement appropriate strategies to mitigate risks and optimize their yields. Policymakers can make informed decisions based on comprehensive assessments, leading to more effective policies and interventions to support agricultural stability and sustainability. Moreover, stakeholders in the agricultural sector, such as suppliers, distributors, and financial institutions, can use the insights gained from this research to develop tailored risk management solutions and support farmers in managing and minimizing the adverse effects of price.

In 2021–2022, agricultural products worth USD 37.3 billion were exported from India, exhibiting a notable growth of 17% compared to the year 2021–2022. India's prowess in the agriculture sector and its ability to maintain a steady upward trajectory in exports further solidify its prominent position in the global market. In 2021–2022, rice emerged as India's foremost agricultural export, constituting more than 19% of the total agricultural exports. Other significant exported products included sugar, spices, and buffalo meat, which accounted for 9%, 8%, and 7%, respectively, of the agriculture exports during the same period. Notably, wheat exports witnessed substantial growth and wheat worth USD 2.1 billion was exported in 2021–2022, a significant increase from USD 568 million in 2021–2022 [7].

Another noteworthy milestone was achieved in the coffee sector, as coffee worth more than USD 1 billion was exported for the first time. This achievement has positively impacted coffee growers in Karnataka, Kerala, and Tamil Nadu, improving realizations. Furthermore, the coastal states of West Bengal, Andhra Pradesh, Odisha, Tamil Nadu, Kerala, Maharashtra, and Gujarat have experienced the benefits of increased export of marine products, worth USD 7.7 billion. This has proven advantageous for farmers in the coastal regions, providing them with increased opportunities and economic gains [8].

The use of aggregation operators, such as the OWA operator, in the traditional coefficient-of-variation formula offers several benefits that contribute to a deeper understanding of the market [9–11]. These operators generate diverse results not previously observed, expanding the insights gained. Additionally, the study delves into specific scenarios that encompass extensions of OWA with the coefficient of variation (OWA_{cv}) and its extensions [3,12].

The fundamental idea driving these novel formulations is to obtain results that encapsulate historical data beyond standard deviation and average calculations. The volatility calculations can be enhanced by incorporating the decision maker's expectations, knowledge, and skills [2]. This is achieved by using diverse weighting vectors and distinct scenarios belonging to different categories. This approach can enhance the decision-making process for corporations, investors, governmental entities, and other stakeholders. By leveraging the power of mathematical aggregation techniques and incorporating the complexity of price dynamics, this study seeks to contribute to a more comprehensive understanding of the volatility in agriculture and facilitate evidence-based decision-making processes to ensure a stable and sustainable pricing system [13–15]. To illustrate the application of these new formulations, this study employs them to calculate the price of rice for the year 2023. A range of diverse outcomes can be generated by incorporating price data of rice from 2022 to 2023 and leveraging input from the decision maker through weights, induced vectors, and probabilistic vectors.

The remaining sections are organized as follows: Section 2 provides the preliminary formulations of the OWA operators and their extensions, the coefficient-of-variation formulation, and the OWA_{cv} operator and its extensions. Section 3 presents the results of using the aggregation operators for calculating the price of rice in India, and Section 4 details the primary insights derived from this research.

2. Preliminaries

In this section, we will introduce fundamental ideas and terminology related to the OWA operator. Additionally, we will explore various extensions of the OWA function and delve into the classical coefficient-of-variation concept.

2.1. Ordered Weighted Average (OWA) Operator and Its Extensions

The OWA function provides a means of incorporating insights from experts into mathematical operations [3]. The OWA function offers versatility by reconfiguring the average according to various criteria and providing a range of outcomes from a minimum value to a maximum value [16]. Its definition is outlined as follows:

Definition 1. *An OWA function is defined as a mapping $OWA : R^n \rightarrow R$, where R is any real number. Then, the formula for the OWA function is defined as*

$$OWA(a_1, a_2, a_3, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j, \tag{1}$$

where b_j is the j^{th}-largest element of the collection $a_1, a_2 a_3, \ldots, a_n$; each a_i is accompanied by a weight vector W of size n, where n represents the dimension such that $\sum_{j=1}^{n} w_j = 1$; and the weight w_j assigned to each element lies within the range of 0 to 1.

The heavy ordered weighted average (HOWA) operator, an extension of the OWA operator, leverages the weighting vector and its values to generate diverse scenarios [17]. Unlike the OWA operator, the HOWA function does not restrict the values of the weighting vector to 1, allowing for the creation of new scenarios that may either underestimate or overestimate the final results. The formula for the HOWA function is as follows:

Definition 2. *An HOWA function is defined as a mapping $HOWA : R^n \rightarrow R$, where **R** is any real number. Then, the formula for the HOWA function is defined as*

$$HOWA(a_1, a_2, a_3, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j, \tag{2}$$

where b_j is the j^{th}-largest element of the collection $a_1, a_2 a_3, \ldots, a_n$; each a_i is accompanied by a weight vector W of size n; and the weight w_j assigned to each element lies within the range of 0 to 1 such that $1 \leq \sum_{j=1}^{n} w_j \leq n$. Depending on the specific requirements and objectives, it is possible to expand the weighting vector from 1 to ∞ or even from $-\infty$ to ∞.

The induced heavy ordered weighted average (IOWA) function emerges as one of the extensions to the reordering step of the OWA function [4]. Unlike the OWA operator, the IOWA function offers a distinct advantage by allowing the reordering step to be guided by the decision maker's knowledge rather than relying solely on the largest element of the collection. The definition of the IOWA function is as follows:

Definition 3. *An IOWA function is defined as a mapping $IOWA : R^n \times R^n \rightarrow R$, where **R** is any real number. Then, the formula for the IOWA function is defined as*

$$IOWA(\langle u_1, a_1 \rangle, \langle u_2, a_2 \rangle, \langle u_3, a_3 \rangle, \ldots, \langle u_n, a_n \rangle) = \sum_{j=1}^{n} w_j b_j, \tag{3}$$

where b_j is the a_i value of the IOWA pair $< u_i, a_i >$ having the j^{th}-largest u_i; u_i is the order-induced variable; a_i is the argument variable, with an associated weight vector W of dimension n such that $\sum_{j=1}^{n} w_j = 1$; the weight w_j is assigned to each element that lies within the range of 0 to 1; and an induced set of ordering variables is also included (u_i).

Other extensions enrich the final results, including probability-based approaches [10]. A notable case is the probabilistic ordered weighted average (POWA) operator, which employs two vectors in the calculation: the conventional OWA function weighting vector and a probabilistic vector reflecting event probabilities. Both vectors have values less than 1. The formal definition of the POWA operator is as follows [14]:

Definition 4. *The POWA function is defined as a mapping* $POWA : R^n \rightarrow R$, *where* $\boldsymbol{R}$ *is any real number. Then, the formula for the POWA function is defined as*

$$POWA(a_1, a_2, a_3, \ldots, a_n) = \sum_{j=1}^{n} \hat{p}_j b_j, \tag{4}$$

where b_j *is the* j^{th}*-biggest argument of* $a_1, a_2, a_3, \ldots, a_n$; *each argument* a_i *is associated with a probability* p_i *between 0 and 1; the sum of the probability is 1;* $\hat{p}_j = \beta\omega_j + (1-\beta)p_j$ *with* $\beta \in [0,1]$; *and* p_j *is the probability of* p_i *ordered according to* b_j, *that is, according to the* j^{th}*-largest element of* a_i.

The induced probabilistic ordered weighted average (IPOWA) function can encompass the features of induced and probabilistic operators within a single formulation. Merigó [10] defines the IPOWA operator. The definition of the IPOWA function is as follows:

Definition 5. *An IPOWA function is defined as a mapping* $IPOWA : R^n \times R^n \rightarrow R$, *where* $\boldsymbol{R}$, *any real number, incorporates an associated weight vector W of dimension n. Then, the formula for the IPOWA function is defined as*

$$IPOWA(\langle u_1, a_1 \rangle, \langle u_2, a_2 \rangle, \langle u_3, a_3 \rangle, \ldots, \langle u_n, a_n \rangle) = \sum_{j=1}^{n} \hat{v}_j b_j, \tag{5}$$

where b_j *is the* j^{th}*-largest* u_i, u_i *is the order-induced variable, and* a_i *is the argument variable where each element* a_i *has an associated probability* p_i *where* $\sum_{i=1}^{n} p_i = 1$ *and* $p_i \in [0,1]$, $\hat{v}_j = \beta\omega_j + (1-\beta)p_j$ *with* $\beta \in [0,1]$ *and* p_j *is the probability of* p_i *ordered according to* b_j, *that is according to the* j^{th}*-largest* u_i.

Ultimately, the reordering attribute of the IOWA function converges with the unrestricted weighted vector of the HOWA function within the induced heavy ordered weighted average (IHOWA) function [18], encapsulating both features in a unified formulation. Its definition is as follows:

Definition 6. *An IHOWA function is defined as a mapping* $IHOWA : R^n \times R^n \rightarrow R$, *where* $\boldsymbol{R}$ *is any real number and incorporates an associated weight vector W of dimension n. Then, the formula for the IHOWA function is defined as*

$$IHOWA(\langle u_1, a_1 \rangle, \langle u_2, a_2 \rangle, \langle u_3, a_3 \rangle, \ldots, \langle u_n, a_n \rangle) = \sum_{j=1}^{n} w_j b_j, \tag{6}$$

where b_j *is the* a_i *value of the IHOWA pair* $< u_i, a_i >$ *having the* j^{th}*-largest* u_i, u_i *is the order-induced variable, and* a_i *is the argument variable. Depending on the specific requirements and objectives, it is possible to expand the weighting vector from 1 to* ∞ *or even from* $-\infty$ *to* ∞.

2.2. Coefficient of Variation

The rapid development of agricultural products in the futures market has led to a significant increase in price volatility [19,20]. Particularly since the second half of 2008, there has been a decrease in prices for various commodities, particularly agricultural ones, accompanied by an escalation in price variation [21]. This trend can be attributed to multiple factors, for example, low inventory levels, the depreciation of the US dollar, and the growing production of biofuels [22].

Given the prevailing circumstances, it becomes crucial to generate new scenarios that account for the potential volatility in the prices of various products, including agricultural ones. One of the traditional approaches for calculating volatility uses the coefficient of variation, which can be defined as follows:

Definition 7. *The coefficient of variation of a given set of arguments* $a_1, a_2, a_3, \ldots, a_n$ *can be defined as*

$$cv(a_1, a_2, a_3, \ldots, a_n) = \frac{\sigma(a_1, a_2, a_3, \ldots, a_n)}{\mu(a_1, a_2, a_3, \ldots, a_n)}, \tag{7}$$

where cv represents the coefficient of variation, σ *denotes the standard deviation, and* μ *represents the mean or average of the set of arguments* $(a_1, a_2, a_3, \ldots, a_n)$*. This formulation quantifies the relationship between the standard deviation and the mean, providing insight into the volatility of the set [23].*

The classical formulation for the coefficient of variation has a significant drawback: it solely relies on historical data to obtain final results. However, it becomes crucial to incorporate information from past and future scenarios to obtain more realistic outcomes that align with the subject under study [24,25]. In this regard, using the OWA function and its extensions offers a solution by allowing the aggregation of knowledge, expectations, and the decision maker's attitude in the final results. The OWA_{cv} formulation can be defined as follows [6]:

Definition 8. *An* OWA_{cv} *function is defined as a mapping* $OWA_{cv} : R^n \rightarrow R$*, where* $\mathbf{R}$ *is any real number. Then, the formula for the* OWA_{cv} *function is defined as*

$$OWA_{cv}(a_1, a_2, a_3, \ldots, a_n) = \frac{\sigma_{OWA}}{\mu_{OWA}}, \tag{8}$$

where σ_{OWA} *is the OWA standard deviation and* μ_{OWA} *is the OWA average, with an associated weight vector W of dimension n such that* $\sum_{j=1}^{n} w_j = 1$ *and the weight* w_j *assigned to each element lies within the range of 0 to 1.*

As seen in the formulation, it is possible to visualize different scenarios using the OWA operator in the standard deviation and the average. Depending on the specific problem under analysis, the suitability of employing a reordering process in both formulation components might be questionable. This becomes apparent when the information lacks subjectivity or the outcomes demonstrate inconsistencies arising from the applied reordering procedure. This integration is possible by including a weighting vector within the standard deviation and average calculations. Traditionally, the average assigns equal importance to all data (using a weighting vector of 1/n). However, the decision maker's expectations and knowledge may warrant assigning greater or lesser importance to the newest or oldest data points.

The formulations for the coefficient of variation using the IOWA, HOWA, POWA, IHOWA, and IPOWA operators follow the same structure as that of the OWA_{cv}, for example, the $IOWA_{cv} = \frac{\sigma_{IOWA}}{\mu_{IOWA}}$, derived from the redundancy of the formulations. These are not written in detail.

As has been defined by [26], the general formulation will be the case where both the μ and the σ use the aggregation operator. Case 1 will be using the operator just in the μ, and case 2 will be using the operator just in the σ. In this context, the OWA function and its extensions are vital in generating scenarios that align with the information provided by the decision maker. By modifying the weighting vector and implementing diverse reordering steps, these operators help generate alternative scenarios that reflect the decision maker's preferences and insights.

3. Employing OWA Operators and Their Extensions for Analyzing Rice Price Variation

3.1. Definition of the Process for Analyzing the Information

One of the main drawbacks of the coefficient-of-variation formulation is that it only includes historical information, leaving out any knowledge or expectation of the immediate future that the experts and decision makers have about the topic being analyzed [27]. Therefore, in addition to the traditional analysis, using different operators that provide new insight by aggregating different elements of information will provide a better understanding of the problem and generate new, unforeseen scenarios [28,29]. In agricultural production, price volatility becomes an important element to analyze because it will help the farmers decide which product to sow [30,31].

The following steps should be followed to obtain the variation based on the OWA operators and their extensions:

Step 1. Determine the period for calculation (e.g., quarter-year, half-year, full-year) based on decision criteria.

Step 2. Collect price data.

Step 3. Define a weighting vector reflecting the importance of the information and the decision maker's knowledge. In this step, the values of the vector used in the OWA operator are obtained.

Step 4. Obtain the weight vector for heavy values, adjustable from $-\infty$ to ∞, enabling under- or over-estimation of the result as needed. The main idea is to obtain weights that will correspond to the expectations of the following year's prices. If the price is expected to rise, then the value of the vector must be higher than 1, and if the price is expected to fall, then the weights must add up to less than 1.

Step 5. Establish an ordered vector to arrange weights according to the expectations of the decision maker. The idea is to obtain induced values in which the ordered weights align to the values. The induced vector incorporates the visualization of specific situations that are not included in the original weight's values.

Step 6. Develop a probability vector. Also, ensure that the sum of the percentages allocated to weighting and probability vectors is 100%. Probabilities will help incorporate the information considering different elements, this time using the concept of the percent of occurrence of that value. An important thing to consider in this step is that the relative importance of the weighting and probability vector is defined.

Step 7. Using information from Steps 1 to 6, construct diverse formulations using OWA_{cv} and its extensions. Consider the general formulation case 1 and case 2.

Step 8. Conduct an analysis using outcomes from various operator formulations.

3.2. Case of Rice Prices in India in 2022–2023

Step 1. We identify how often the latest data are available. The Ministry of Commerce and Industry of the Government of India provides the rice price data monthly.

Step 2. With the information provided, the average monthly spot price of the rice is obtained from April 2022 to March 2023 (see Table 1).

Table 1. The average monthly spot price of rice in USD.

Date	Rice Price (USD)
April 2022	806.60
May 2022	855.25
June 2022	1060.80
July 2022	930.29
August 2022	1041.70
September 2022	782.90
October 2022	703.83
November 2022	800.52
December 2022	1017.60
January 2023	826.23
February 2023	1034.90
March 2023	1127.40

Source. https://tradestat.commerce.gov.in/meidb/comq.asp?ie=e., accessed on 1 August 2023.

Step 3. From this step onward, an expert (an advisor to the agricultural sector with more than 10 years of experience, specifically with companies that grow rice) is consulted about the weights that should be used. The idea is to obtain the expectations of the decision maker about different elements. The weighting vector is defined as follows:

W = (0.05, 0.05, 0.05, 0.05, 0.08, 0.08, 0.08, 0.10, 0.10, 0.12, 0.12, 0.12)

This vector considers the relative importance of each month to the average; that is why the expert gives a higher score to the most recent months and a lower value to the further months, because as per their experience, the future price will be more related to the newer values than the older ones.

Step 4. The heavy weights vector is defined as follows:

H = (0.05, 0.05, 0.05, 0.05, 0.10, 0.10, 0.10, 0.10, 0.15, 0.15, 0.15, 0.20)

The reason for these values is that the price is expected to increase because of the macroeconomic events still having a considerable influence on the prices of agricultural products. The expert also informs that rice prices will increase according to product demands; some news agrees with this forecast [32].

Step 5. The induced vector is defined as follows:

U = (1, 3, 2, 6, 8, 9, 5, 4, 7, 10, 12, 11)

These values represent the expectation of the expert that the future prices will have values similar to the prices during the reference month, that is, they will fall within the highest and lowest prices of the reference month.

Step 6. A probability vector P consists of probabilities assigned to different events or outcomes, reflecting the likelihood or chance of each event occurring. The vector is defined as follows:

P = (0.05, 0.05, 0.05, 0.05, 0.05, 0.10, 0.10, 0.10, 0.10, 0.10, 0.10, 0.15)

The weighting vector carries a weightage of 60%, emphasizing its significance in the overall assessment. In comparison, the probability vector holds a weightage of 40%, contributing to evaluating the probabilities associated with different outcomes.

Step 7. With the information provided by the expert, the different results are calculated using the traditional formulation and the OWA operators and its extensions. To understand the process better, the results are explained in detail next.

3.2.1. Coefficient of Variation

The first result was obtained using the traditional formula of coefficient of variation (see Table 2).

Table 2. Rice price coefficient of variation.

Date	x (*Rice Price*)	$(x - \mu)^2$
April 2022	806.6	11,895.90
May 2022	855.25	3650.38
June 2022	1060.8	21,063.20
July 2022	930.29	213.79
August 2022	1041.7	15,883.98
September 2022	782.9	17,627.43
October 2022	703.83	44,875.48
November 2022	800.52	13,259.14
December 2022	1017.6	10,390.06
January 2023	826.23	7999.22
February 2023	1034.9	14,216.19
March 2023	1127.4	44,830.30
	μ = **915.67**	205,890.84

To calculate the standard deviation:

$$\sigma = \sqrt{\sum \frac{(x-\mu)^2}{n}} = \sqrt{\frac{205,890.84}{12}} = 130.987$$

The coefficient of variation is $\frac{130.987}{915.66} = 0.14305$.

Applying the traditional formula for the coefficient of variation to the provided rice price data, we obtained a mean (μ) of 915.67 and a standard deviation (σ) of 130.987. This calculation results in a coefficient of variation of approximately 0.14305, offering a quantitative measure of rice price variability essential for risk assessment and decision making.

3.2.2. Coefficient-of-Variation Calculation Using OWA Operators and Their Extensions

A specific analysis for each operator was carried out to understand how the formulations are being used. To understand the most complex situation, the general formulation is presented; this is the case where the aggregation operators are used in both σ and μ.

(a) OWA_{cv} operator (see Table 3): It assigns different weighting factors to x-prices, reflecting their varying importance in the analysis. This customization allows stakeholders to tailor the analysis to their specific objectives and data characteristics. The correspondence between weighting factors and x-prices is crucial for quantifying the impact of each data point on the calculated coefficient of variation OWA_{cv}. It offers flexibility, transparency, and sensitivity analysis to ensure robust results and informed decision making.

Table 3. Rice price coefficient of variation with *OWA*.

x (*Rice Price*)	*W*	*x* ∗*W*	$(x-\mu)^2$
1127.4	0.12	135.29	29,216.35
1060.8	0.12	127.30	10,884.31
1041.7	0.12	125.00	7263.79
1034.9	0.1	103.49	6150.94
1017.6	0.1	101.76	3736.62
930.29	0.08	74.42	685.50
855.25	0.08	68.42	10,245.91
826.23	0.08	66.10	16,963.00
806.6	0.05	40.33	22,461.65
800.52	0.05	40.03	24,321.06
782.9	0.05	39.15	30,127.27
703.83	0.05	35.19	63,828.03
		μ_{OWA} = **956.47**	225,884.44

To calculate the standard deviation:

$$\sigma = \sqrt{\sum \frac{(x-\mu_{OWA})^2}{n}} = \sqrt{\frac{225,884.44}{12}} = 137.19$$

The OWA_{cv} is $\frac{137.19}{956.47}$ = 0.1434.

The calculated standard deviation (σ) and resulting OWA_{cv}, of 0.1434, indicate the level of rice price variability, providing a key quantitative measure for risk assessment.

(b) $IOWA_{cv}$ operator (see Table 4): It is effective in quantifying rice price variability, offering an alternative perspective for assessing risk. The correspondence between the weighting factors and the x-prices allows for customization, reflecting the varying importance of data points in the analysis. This flexibility makes it a valuable tool for stakeholders to tailor risk assessments and decision making to their specific objectives and data characteristics.

Table 4. Rice price coefficient of variation with *IOWA*.

x (*Rice Price*)	*W*	$x * W$	$(x-\mu)^2$
1127.4	0.12	135.29	31,933.55
1060.8	0.12	127.30	12,566.32
1041.7	0.12	125.00	8648.93
1034.9	0.08	82.79	7430.37
1017.6	0.08	81.41	4747.15
930.29	0.1	93.03	338.94
855.25	0.05	42.76	8732.98
826.23	0.08	66.10	14,999.00
806.6	0.1	80.66	20,192.52
800.52	0.05	40.03	21,957.43
782.9	0.05	39.15	27,489.77
703.83	0.05	35.19	59,961.51
		$\mu_{IOWA} = \mathbf{948.70}$	218,998.48

To calculate the standard deviation:

$$\sigma = \sqrt{\sum \frac{(x - \mu_{IOWA})^2}{n}} = \sqrt{\frac{218{,}998.48}{12}} = 135.09$$

The $IOWA_{cv}$ is $\frac{135.09}{948.70}$ = 0.1424.

Using the IOWA formula for the coefficient of variation with the provided rice price data, we found a mean $IOWA_{cv}$ of 948.70 and a calculated standard deviation (σ) of 135.09. Therefore, the coefficient of variation $IOWA_{cv}$ is approximately 0.1424, offering a valuable quantitative measure of rice price variability, aiding in risk assessment and decision making.

(c) $HOWA_{cv}$ operator (see Table 5): The correspondence between the weighting factors and the x-prices allows for a customized assessment of risk, reflecting the varying importance of data points.

Table 5. Rice price coefficient of variation with *HOWA*.

x (*Rice Price*)	*H*	$x * H$	$(x-\mu)^2$
1127.4	0.2	225.48	7451.06
1060.8	0.15	159.12	23,384.37
1041.7	0.15	156.26	29,590.71
1034.9	0.15	155.24	31,976.41
1017.6	0.1	101.76	38,462.86
930.29	0.1	93.03	80,332.28
855.25	0.1	85.53	128,500.38
826.23	0.1	82.62	150,148.11
806.6	0.05	40.33	165,746.29
800.52	0.05	40.03	170,733.83
782.9	0.05	39.15	185,605.44
703.83	0.05	35.19	259,987.30
		$\mu_{HOWA} = \mathbf{1213.72}$	1,271,919.04

To calculate the standard deviation:

$$\sigma = \sqrt{\sum \frac{(x - \mu_{HOWA})^2}{n}} = \sqrt{\frac{1{,}271{,}919.04}{12}} = 325.57$$

The $HOWA_{cv}$ is $\frac{325.57}{1213.72}$ = 0.2682.

The resulting high coefficient of variation $(HOWA_{cv})$, of 0.2682, signifies substantial price variability, making this methodology valuable for stakeholders in risk assessment and decision making, particularly in scenarios with significant data heterogeneity.

(d) $POWA_{cv}$ operator (see Table 6): It quantifies rice price variability with a unique combination of weighting factors. The correspondence between the weighting factors and the x-prices allows for tailored risk assessment, accommodating the varying importance of data points.

Table 6. Rice price coefficient of variation with $POWA$.

***x* (*Rice Price*)**	***W***	***P***	**$(x * W * 0.60)+$ $(x * P * 0.40)$**	**$(x-\mu)^2$**
1127.4	0.12	0.15	148.82	28,848.87
1060.8	0.12	0.1	118.81	10,660.47
1041.7	0.12	0.1	116.67	7081.15
1034.9	0.1	0.1	103.49	5982.95
1017.6	0.1	0.1	101.76	3605.95
930.29	0.08	0.1	81.87	743.13
855.25	0.08	0.1	75.26	10,465.38
826.23	0.08	0.05	56.18	17,245.06
806.6	0.05	0.05	40.33	22,786.04
800.52	0.05	0.05	40.03	24,658.57
782.9	0.05	0.05	39.15	30,502.78
703.83	0.05	0.05	35.19	64,374.07
			$\mu_{POWA} = \mathbf{957.55}$	226,954.42

To calculate the standard deviation:

$$\sigma = \sqrt{\sum \frac{(x-\mu_{POWA})^2}{n}} = \sqrt{\frac{226,954.42}{12}} = 137.52$$

The $POWA_{cv}$ is $\frac{137.52}{957.55} = 0.1436$.

The resulting coefficient of variation $(POWA_{cv})$, of 0.1436, provides a valuable measure of price variability, offering insights for stakeholders in risk assessment and decision making in the context of this particular formulation.

(e) $IHOWA_{cv}$ operator (see Table 7): It is a specialized mathematical aggregation operator that allows decision makers to assign higher weights or importance to specific criteria or attributes within the decision process.

Table 7. Rice price coefficient of variation with $IHOWA$.

***x* (*Rice Price*)**	***H***	**$x * H$**	**$(x-\mu)^2$**
1127.4	0.15	169.11	5674.08
1060.8	0.2	212.16	20,143.13
1041.7	0.15	156.26	25,929.53
1034.9	0.1	103.49	28,165.73
1017.6	0.1	101.76	34,271.82
930.29	0.15	139.54	74,221.65
855.25	0.05	42.76	120,739.92
826.23	0.1	82.62	141,749.61
806.6	0.1	80.66	156,916.20
800.52	0.05	40.03	161,770.07
782.9	0.05	39.15	176,254.29
703.83	0.05	35.19	248,897.72
		$\mu_{IHOWA} = \mathbf{1202.73}$	1,194,733.76

To calculate the standard deviation:

$$\sigma = \sqrt{\sum \frac{(x - \mu_{IHOWA})^2}{n}} = \sqrt{\frac{1,194,733.76}{12}} = 315.53$$

The $POWA_{cv}$ is $\frac{315.53}{1202.73}$ = 0.2623.

A higher $IHOWA_{cv}$ indicates that the data have a higher level of variability relative to their mean, while a lower $IHOWA_{cv}$ suggests that the data are more stable and have less variability relative to their mean.

(f) $IPOWA_{cv}$ operator (see Table 8): This operator allows for the customization of weighting factors based on the importance of specific criteria, combining both weighted and probabilistic considerations.

Table 8. Rice price coefficient of variation with $IPOWA$.

x **(*Rice Price*)**	***W***	***P***	**(*x* ∗ *W* ∗ 0.60)+(*x* ∗ *P* ∗ 0.40)**	**(*x*−*μ*)²**
1127.4	0.12	0.1	126.27	32,632.60
1060.8	0.12	0.15	140.03	13,006.25
1041.7	0.12	0.1	116.67	9014.55
1034.9	0.08	0.1	91.07	7769.53
1017.6	0.08	0.05	69.20	5019.01
930.29	0.1	0.1	93.03	271.10
855.25	0.05	0.05	42.76	8373.17
826.23	0.08	0.1	72.71	14,526.29
806.6	0.1	0.1	80.66	19,643.44
800.52	0.05	0.05	40.03	21,384.69
782.9	0.05	0.05	39.15	26,848.47
703.83	0.05	0.05	35.19	59,012.58
			μ_{IPOWA} = **946.76**	217,501.67

To calculate the standard deviation:

$$\sigma = \sqrt{\sum \frac{(x - \mu_{IHOWA})^2}{n}} = \sqrt{\frac{217,501.67}{12}} = 134.63$$

The $IPOWA_{cv}$ is $\frac{134.63}{946.76}$ = 0.1422.

The result is in an $IPOWA_{cv}$ value of 0.1422, signifying rice price variability. This information, complemented by the standard deviation (σ) and mean (μ), supports risk assessment and decision making. Lower $IPOWA_{cv}$ values indicate price stability, while higher values imply greater variability, enabling stakeholders to align strategies with their risk preferences effectively.

All the information and the results of the coefficient of variation using the different aggregation operators and the generalized formulation (case 1 and case 2) are presented in Table 9.

Table 9. Coefficient of variation for rice in India using aggregation operators.

Formulation	σ	μ	**General**	**Case 1**	**Case 2**
Traditional	130.99	915.66	0.1431		
OWA_{cv}	137.19	956.47	0.1434	0.1369	0.1498
$IOWA_{cv}$	135.09	948.70	0.1424	0.1381	0.1475
$HOWA_{cv}$	325.57	1213.72	0.2682	0.1079	0.3556
$POWA_{cv}$	137.52	957.55	0.1436	0.1368	0.1502
$IHOWA_{cv}$	315.53	1202.73	0.2623	0.1089	0.3446
$IPOWA_{cv}$	134.63	946.76	0.1422	0.1384	0.1470

Step 8. With results obtained from Table 9, it is possible to visualize that the coefficient of variation of the price of rice in India goes from 0.1079 to 0.3556. In a more detailed analysis, it is possible to assume that the price can vary from 10% to 36% for the following months. This interpretation is critical because a heavy-weighting vector may not provide the most accurate estimate of rice prices in India, but stakeholders still want to incorporate it into their calculations because they believe it aligns with expectations of high demand for rice in the future, which is in line with what they have seen in news reports. This decision reflects a balance between potentially biased data and market insights.

Furthermore, this examination has the potential to enable the prediction of various factors, including the projected quantity of rice cultivation. A farmer's choice of agricultural product to cultivate hinges on their anticipation of future product prices. Presently, a product might have a certain price, but the actual harvest occurs later. Consequently, having diverse methodologies that facilitate the visualization of potential price ranges for the product becomes crucial. This insight helps ascertain whether engaging in its cultivation would be a lucrative decision.

Simultaneously, policymakers can leverage these formulations to gain enhanced foresight into the future. Through these emerging scenarios, they can develop more strategic policies and regulations to oversee the engagement of distinct economic sectors. For instance, when the visualization of price fluctuations exceeds governmental projections, a measure such as acquiring derivatives can be employed. This serves to mitigate the risk of losses within the agricultural sector.

Employing distinct formulations within the traditional coefficient-of-variation framework enhances decision-making perspectives, enabling the exploration of new scenarios inaccessible via the traditional approach. Integrating these novel operators empowers subject matter experts to incorporate decision makers' knowledge, expectations, and attitudes, enriching the result.

The analysis results, derived from various formulations assessing rice price volatility, hold significant implications for diverse stakeholders. These findings, characterized by differing levels of price variability, play a pivotal role in shaping economic decisions and policy considerations. Stakeholders, including farmers, traders, and policymakers, can employ these insights to gauge and mitigate risks associated with price fluctuations. The comparative analysis underscores the importance of selecting the appropriate volatility measure, enabling more informed decision making. Furthermore, the results may spur further research and the identification of actionable strategies to enhance price stability in the rice market, benefiting both the agricultural sector and consumers.

4. Conclusions

The main objective of this paper is to integrate different aggregation operators, such as OWA operators, and their extensions to calculate the coefficient of variation in the price of agricultural products, specifically in rice prices in India. The purpose of using these methods is to integrate into the formulation not just the historical data but the decision maker's knowledge, expectations, and experience in the calculation process. The idea is to include that information about the future in a formulation that usually just considers the past, providing a better insight into the problem being analyzed and generating new scenarios that the decision maker must consider. The first thing that was accomplished was to define the coefficient of variation using the OWA operator, which was called OWA_{cv}. Also, the induced, heavy, and probability OWA operator and their combinations were presented.

This paper extends its contributions by applying the formulated methodologies to assess rice prices within the 2022–2023 timeframe. Given rice's pivotal role as a staple commodity in India, accurately identifying variations in its price is critical to ensuring that local farmers make a profit in rice farming. Notably, our findings reveal that introducing more intricate aggregation scenarios and incorporating new vectors in the process provide a fresh perspective. Such insights, unattainable through conventional formulas, empower decision makers to comprehend market intricacies better and make informed choices that mitigate the company's risks. This innovative approach broadens the understanding of market dynamics and offers alternative avenues for risk mitigation.

In our upcoming research, we aim to enrich the volatility formulation by integrating diverse mathematical techniques, such as Bonferroni means [33,34], moving averages [35,36], linguistic variables [29,37], and prioritized operators [38,39]. We will focus on applying these enhanced formulations within group decision-making scenarios and expansive large-scale contexts. Furthermore, our investigation will extend to the synergistic integration of these formulations with the theory of expertons [40,41], forgotten effects [42,43], and a range of aggregation function methodologies.

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