

Editorial

# Introduction to the Special Issue in Axioms Titled Current Research on Mathematical Inequalities

Christophe Chesneau

Department of Mathematics, LMNO, University of Caen, 14032 Caen, France; christophe.chesneau@unicaen.fr

The importance of inequalities in Mathematics is beautifully summarized in a citation attributed to Respected Professor Andrey Nikolaevich Kolmogorov:

*Behind every theorem lies an inequality.*

There are many types of inequalities: Bernoulli's inequality, the Cauchy–Schwarz inequality, Hölder's inequality, the inequality of arithmetic and geometric means, the Minkowski inequality, etc. Mathematical inequalities have always played a major role in all branches of mathematics. They are often used to bind important but unmanageable mathematical terms of interest with other mathematical terms simple enough to draw some conclusions.

The Special Issue titled “Current Research on Mathematical Inequalities” was created with the intention of publishing significant research on a variety of new mathematical inequalities. We included studies based on their novelty, excellence, and significance.

Seventeen articles were chosen for the final Special Issue after a thorough peer-review process involving top assistants and reviewers. They are written by respected academic researchers from various nations and describe the state of current research projects and advancements in mathematical inequalities.

The first paper, “A Note on Hermite–Hadamard–Fejér Type Inequalities for Functions Whose  $n$ -th Derivatives Are  $m$ -Convex or  $(\alpha, m)$ -Convex Functions”, by Kovač, S. [1], introduces some Hermite–Hadamard–Fejér-type inequalities for  $n$ -times differentiable functions whose absolute values of  $n$ -th derivatives are  $(\alpha, m)$ -convex functions. The findings in this paper are expansions and generalizations of earlier findings.

The second paper, “Post-Quantum Midpoint-Type Inequalities Associated with Twice-Differentiable Functions”, by Sitthiwiratham et al. [2], establishes a  $(p, q)$ -integral identity involving the second  $(p, q)$ -derivative. This finding is used to demonstrate some new midpoint-type inequalities for twice- $(p, q)$ -differentiable convex functions.

In the third paper, entitled “Perturbation of One-Dimensional Time-Independent Schrödinger Equation with a Near-Hyperbolic Potential” by Kim and Jung [3], the authors look into a particular type of near-hyperbolic potential Hyers–Ulam stability of the Schrödinger equation, which extends, in some sense, one of their previous work.

In their paper, “C. Pólya–Szegő Integral Inequalities Using the Caputo–Fabrizio Approach”, Nale et al. [4] use the Caputo–Fabrizio fractional integral to establish some Pólya–Szegő and Minkowsky-type fractional integral inequalities and provide some examples of Pólya–Szegő inequalities in particular situations.

In their paper, “K-Nearest Neighbor Estimation of Functional Nonparametric Regression Model under NA Samples”, Hu et al. [5] offer  $k$ -nearest neighbor ( $k$ NN) estimators of the nonparametric functional regression model when the observed variables take values from negatively associated (NA) sequences. The proofs rely on well-established inequalities to arrive at some novel inequalities and results in this statistical context.

The sixth paper, “On Some New Ostrowski–Mercer-Type Inequalities for Differentiable Functions”, by Sial et al. [6], establishes a new integral identity involving differentiable functions and uses it for some new Ostrowski–Mercer-type inequalities for differentiable convex functions. Some new generalized inequalities are proved.



**Citation:** Chesneau, C. Introduction to the Special Issue in Axioms Titled Current Research on Mathematical Inequalities. *Axioms* **2023**, *12*, 109. <https://doi.org/10.3390/axioms12020109>

Received: 13 January 2023  
Accepted: 17 January 2023  
Published: 19 January 2023



**Copyright:** © 2023 by the author. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

In the seventh paper, “Coefficient Estimates and Fekete-Szegő Functional Inequalities for a Certain Subclass of Analytic and Bi-Univalent Functions” written by Illafe et al. [7], a brand-new class of symmetric domain, Gegenbauer polynomial-defined bi-univalent functions is introduced. The authors derive some significant estimates. When the parameters used in our primary results are specialized, several new results are produced.

Rodić [8], in the paper “Some Generalizations of the Jensen-Type Inequalities with Applications”, uses a number of Green functions to provide generalizations of those findings for the actual Stieltjes measure  $d\lambda$ , which is not always positive. Some new mean value theorems of the Lagrange and Cauchy types are derived, along with some new Cauchy-type means.

In their paper, “Bounds for Quotients of Inverse Trigonometric and Inverse Hyperbolic Functions”, Thool et al. [9] look into new simple bounds for the quotients of inverse trigonometric and inverse hyperbolic functions, which can be directly used for numerous mathematical applications.

In the tenth paper, entitled “Some Generalized Euclidean Operator Radius Inequalities” written by Alomari et al. [10], the authors demonstrate a few generalized radius inequalities for Euclidean operators. Improvements to a few well-known results are offered.

In the eleventh paper, Smoljak Kalamir, [11] in “New Diamond- $\alpha$  Steffensen-Type Inequalities for Convex Functions over General Time Scale Measure Spaces”, employs the diamond-dynamic integral to extend some Steffensen-type inequalities to time scales. Additionally, using time-scale calculus’s positive  $\sigma$ -finite measures, we prove a few new Steffensen-type inequalities for convex functions. Several special cases are discussed.

In their paper, “Interval Fejér-Type Inequalities for Left and Right- $\lambda$ -Preinvex Functions in Interval-Valued Settings”, Saeed et al. [12] produce inequalities of the Hermite–Hadamard (H-H) type. Furthermore, under some mild conditions, Hermite–Hadamard–Fejér (H-H-Fejér)-type inequalities for the left and right interval-valued preinvex functions are obtained.

In their paper, “Univariate and Multivariate Ostrowski-Type Inequalities Using Atangana–Baleanu Caputo Fractional Derivative”, Desta et al. [13] obtain some single- and multivariate Ostrowski-type inequalities using the Atangana–Baleanu fractional derivative in the sense of Liouville–Caputo (ABC).

Latif [14], in their paper “Weighted Integral Inequalities for Harmonic Convex Functions in Connection with Fejér’s Result”, presents some brand-new functionals for harmonic convex functions that are connected to weighted integral inequalities. Additionally, a few brand-new inequalities of the Fejér type are found.

In the fifteenth paper, entitled “On Symmetrized Stochastic Harmonically Convexity and Hermite–Hadamard Type Inequalities”, written by Latif [15], the author goes into more depth regarding the idea of symmetrized harmonically convex stochastic processes. The use of Hermite–Hadamard-type inequalities is discussed with some specific characterizations for symmetrized harmonically convex stochastic processes.

In their paper, “Operator Jensen’s Inequality for Operator Superquadratic Functions”, Alomari et al. [16] specify a superquadratic function (or operator) for positive operators in Hilbert space. A number of examples with significant characteristics are given, along with some observations that pertain to the operator convexity. It follows that a general Bohr inequality for positive operators can be drawn. There is evidence for a Jensen-type inequality.

Finally, in the paper “Some New Estimates for the Berezin Number of Hilbert Space Operators”, Altwaijry et al. [17] discover inequalities that are advancements over earlier ones involving the Berezin norm and Berezin number of bounded linear operators defined on a reproducing kernel Hilbert space.

As a final thought, we hope that this Special Issue will encourage future generations to make significant and novel discoveries by disseminating fresh ideas about inequalities.

**Funding:** This research received no external funding.

**Acknowledgments:** We want to express our gratitude to the authors for their great work on this special edition.

**Conflicts of Interest:** The authors declare that they have no conflicts of interest to report regarding the present study.

## References

1. Kovač, S. A Note on Hermite-Hadamard-Fejer Type Inequalities for Functions Whose  $n$ -th Derivatives Are  $m$ -Convex or  $(\alpha, m)$ -Convex Functions. *Axioms* **2022**, *11*, 16. [[CrossRef](#)]
2. Sitthiwiratham, T.; Murtaza, G.; Ali, M.A.; Promsakon, C.; Sial, I.B.; Agarwal, P. Post-Quantum Midpoint-Type Inequalities Associated with Twice-Differentiable Functions. *Axioms* **2022**, *11*, 46. [[CrossRef](#)]
3. Kim, B.; Jung, S.-M. Perturbation of One-Dimensional Time-Independent Schrödinger Equation with a Near-Hyperbolic Potential. *Axioms* **2022**, *11*, 63. [[CrossRef](#)]
4. Nale, A.B.; Chinchane, V.L.; Panchal, S.K.; Chesneau, C. Pólya-Szegő Integral Inequalities Using the Caputo-Fabrizio Approach. *Axioms* **2022**, *11*, 79. [[CrossRef](#)]
5. Hu, X.; Wang, J.; Wang, L.; Yu, K. K-Nearest Neighbor Estimation of Functional Nonparametric Regression Model under NA Samples. *Axioms* **2022**, *11*, 102. [[CrossRef](#)]
6. Sial, I.B.; Patanarapeelert, N.; Ali, M.A.; Budak, H.; Sitthiwiratham, T. On Some New Ostrowski-Mercer-Type Inequalities for Differentiable Functions. *Axioms* **2022**, *11*, 132. [[CrossRef](#)]
7. Illafe, M.; Amourah, A.; Haji Mohd, M. Coefficient Estimates and Fekete-Szegő Functional Inequalities for a Certain Subclass of Analytic and Bi-Univalent Functions. *Axioms* **2022**, *11*, 147. [[CrossRef](#)]
8. Rodić, M. Some Generalizations of the Jensen-Type Inequalities with Applications. *Axioms* **2022**, *11*, 227. [[CrossRef](#)]
9. Thool, S.B.; Bagul, Y.J.; Dhaigude, R.M.; Chesneau, C. Bounds for Quotients of Inverse Trigonometric and Inverse Hyperbolic Functions. *Axioms* **2022**, *11*, 262. [[CrossRef](#)]
10. Alomari, M.W.; Shebrawi, K.; Chesneau, C. Some Generalized Euclidean Operator Radius Inequalities. *Axioms* **2022**, *11*, 285. [[CrossRef](#)]
11. Smoljak Kalamir, K. New Diamond- $\alpha$  Steffensen-Type Inequalities for Convex Functions over General Time Scale Measure Spaces. *Axioms* **2022**, *11*, 323. [[CrossRef](#)]
12. Saeed, T.; Khan, M.B.; Treantă, S.; Alsulami, H.H.; Alhodaly, M.S. Interval Fejér-Type Inequalities for Left and Right- $\lambda$ -Preinvex Functions in Interval-Valued Settings. *Axioms* **2022**, *11*, 368. [[CrossRef](#)]
13. Desta, H.D.; Pachpatte, D.B.; Mijena, J.B.; Abdi, T. Univariate and Multivariate Ostrowski-Type Inequalities Using Atangana-Baleanu Caputo Fractional Derivative. *Axioms* **2022**, *11*, 482. [[CrossRef](#)]
14. Latif, M.A. Weighted Integral Inequalities for Harmonic Convex Functions in Connection with Fejér's Result. *Axioms* **2022**, *11*, 564. [[CrossRef](#)]
15. Latif, M.A. On Symmetrized Stochastic Harmonically Convexity and Hermite-Hadamard Type Inequalities. *Axioms* **2022**, *11*, 570. [[CrossRef](#)]
16. Alomari, M.W.; Chesneau, C.; Al-Khasawneh, A. Operator Jensen's Inequality for Operator Superquadratic Functions. *Axioms* **2022**, *11*, 617. [[CrossRef](#)]
17. Altwaijry, N.; Feki, K.; Minculete, N. Some New Estimates for the Berezin Number of Hilbert Space Operators. *Axioms* **2022**, *11*, 683. [[CrossRef](#)]

**Disclaimer/Publisher's Note:** The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.