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An Integrated Multicriteria Sorting Methodology with *q*-Rung Orthopair Fuzzy Sets for Evaluating the Impacts of Delays on Residential Construction Projects

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Abstract: This study offers an integrated evaluation methodology for construction project delay causes viewed as a multicriteria sorting (MCS) problem. Time, cost, and quality were the three project management factors considered as criteria to evaluate 38 identified delay causes. The priority weights were extracted using the integration of Weighted Influence Non-linear Gauge Systems (WINGS) and Level-Based Weight Assessment (LBWA) to capture the inherent interdependencies of the criteria. The sorting of 38 delay causes was performed using FlowSort. To handle the uncertainty and vagueness of the judgments of the decision makers in the evaluation process, *q*-rung fuzzy orthopair fuzzy sets (*q*-ROFS) were integrated within the proposed computational framework. The proposed novel *q*-ROF–WINGS–LBWA–FlowSort method was applied in an actual case study in residential construction projects. The delay causes were categorized under three categories of construction firm vulnerability into four levels of impact. In highly vulnerable construction firms, thirty-five delay causes have a high impact, two have a moderate impact, and one has the least impact. In moderately vulnerable and least-vulnerable construction firms, 32 and 28 delay causes have a medium impact, respectively. The results may provide insights for decision makers in highly vulnerable construction firms, i.e., small companies with limited resources and networks. Layers of sensitivity and comparative analyses were put forward to test the robustness of the approach.

Keywords: construction delays; multicriteria sorting; residential projects; WINGS; LBWA; FlowSort; *q*-rung orthopair fuzzy sets

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1. Introduction

The construction industry plays a vital role in society's social and economic dimensions by providing jobs, infrastructure, and other services essential to development [1]. According to Fei et al. [2], the industry plays a critical role in addressing and achieving the United Nations Sustainable Development Goals. Some studies highlighted an important association between the construction industry and economic growth, particularly in developing countries [3]. In Ghana, a third-world country like the Philippines, the construction industry is considered the main contributor to economic progress [4]. In the Philippines, estimates by the Philippines Statistics Authority (PSA) suggest that, from 2016–2030, 5.7 million houses will be in demand [5]. In other reports, although a decline in the number of approved building permits for residential buildings has been observed at 126,429 [6] and 124,275 [7] in the pre-pandemic period, to 87,419 [8] and 106,999 [9–12] in the post-pandemic period, a significant number of residential building constructions still exist. Even globally, countries' gross domestic product and employment generation are directly impacted by the movements of their construction industry [13].

As of June 2018, 10,112 registered, licensed contractors were recorded in the Philippines. Sixty-one percent (61%) are considered small-sized contractors with appropriate licenses, comprising a significant proportion of all contractors in the country. These firms do not involve heavy investments [14]. They have few personnel, lower inputs, lesser predictability, and more standardized processes; are defined by short duration, low cost, less complexity, and limited formal documentation; and occur in active environments, with repetition of work, maintenance, renovation, remodeling, and upgrade being key processes attributed to them [15–17]. In a global context, small-sized construction projects are worth 0.1 to 5 million USD [18], in which project costs form the basis of determining the project size [19]. While important leverage is evident in small-sized contractors, critical aspects of the industry limit their potential. For instance, the capacity to cope with financial challenges varies depending on the company's size, with exacerbated impacts on small-sized contractors [20]. Delays in payments and limited financial access have noticeable effects on small-to-medium-sized enterprises [21]. In terms of the workforce, the organization's productivity and output are affected by a shortage of skilled workers [22,23]. These conditions are amplified in small-sized firms with only a few employees. In the Philippines, the level of awareness and readiness of small-sized contractors to implement the ISO 9001 series hinders them from gaining benefits from the new project-based management system [24]. This certification is widely considered an effective tool for guiding the management of quality systems of an adopting enterprise [25].

Delays commonly occur in construction projects, causing deviations from the initially estimated project duration and cost [26,27]. In general, the impacts of these construction delays are considered significant and may cause far-reaching ripple effects. In Malaysia, construction project delays decelerate national plans [28]. These delays adversely affect stakeholders' interests by increasing the cost associated with the projects [29], predominantly concerning cost overruns, time overruns, and disputes [30,31]. Tariq and Gardezi [32] pointed out that project delays and conflicts are highly intertwined. For instance, delays may ultimately result in pursuing legal actions such as arbitration, termination of contracts, and litigation [30], and these actions may eventually induce more delays. Due to these impacts, identifying the causes of construction delays has become a popular agenda in the recent literature. Islam and Trigunarsyah [33] argued that construction delays depend on the varying economic situation of countries, where delays in the majority of the developed countries are related to project conditions, while those from developing countries are highly linked to micro conditions, such as cashflow issues, scheduling, site management, and change orders. Various studies enumerating the causes of delays have been presented in the literature [26,28,34–36]. Some recent overarching reviews of these causes were reported [37,38], including a focus on developing countries [33,39] and their association with risks [40,41], among others. Durdyev and Hosseini [35] offered a comprehensive list of the causes of these delays.

Some attempts were made to address the causes of construction delays and minimize their impact on the project. Mbala and Aliu [42] emphasized that delays in construction projects can be minimized through the joint efforts of players in the construction industry. Durdyev and Hosseini [35] introduced tools such as integrated project delivery, lean construction, and building information modeling for project management. Quality management systems, such as the ISO 9001 series, can also help address problems with supervision and standardization of construction activities [24]. Meanwhile, management espouses a crucial role in mitigating delays, and the contribution of the firm's key players gives rise to establishing project management infrastructures [42]. Kineber et al. [43] found that the absence of an efficient management system negatively affects the project and the company. Thus, Aghimien et al. [44] recommended the adoption of value management by companies to remove unnecessary costs, eliminate redundancy in processes, and save workers' time as well as materials. However, the application of value management may vary depending on the size of companies and projects, as smaller projects receive more impacts due to limited resources, schedules, and manpower, as Abd El-Karim et al. [45] suggested. Despite these efforts, a rigorous evaluation of the causes of construction delays in view of their impacts on projects may offer a new direction in managing and mitigating project delays. Such an evaluation promotes the identification of specific causes highly critical to the projects, which would help design initiatives to minimize, if not eliminate, construction delays. In addition, this agenda would provide a better understanding of the delays and their impacts on the project success parameters, such as cost, time, and quality [35], necessary in improving project performance, as Tariq and Gardezi [32] pointed out. In effect, identifying those causes yields better management of resources directed at addressing project delays, which is highly relevant to small-sized contractors having resource limitations and minimal economies of scale. While such an evaluation is an important agenda of inquiry, a comprehensive approach is missing in the current literature.

Thus, the main departure of this work is to bridge the gap in the literature by offering a rigorous approach to evaluating construction delay causes on their impact on the implementation of projects. Specifically, it addresses the following question: How do we determine the impact of construction delays on the operation of projects? Due to the various factors associated with managing construction projects, such as those identified by Durdyev and Hosseini [35] (i.e., cost, time, and quality), a comprehensive approach compels this research question to be viewed as a multicriteria sorting (MCS) problem. In general, MCS problems comprise two specific computational actions: (1) determining the priority weights of different criteria (e.g., cost, time, and quality) and (2) sorting the alternatives (e.g., causes of project delays) into predefined impact categories. The first action prompts relevant stakeholders to assign priorities to factors linked with managing projects, while the second action associates the causes of delays with project impacts. In most MCS problems (e.g., [46]), the priority weights of the criteria become inputs to the sorting of alternatives. In this work, following real-life conditions that govern the interrelationships of factors such as project cost, time, and quality, the integration of the Weighted Influence Non-linear Gauge Systems (WINGS) [47] and the Level-Based Weight Assessment (LBWA) [48] methods is proposed. The WINGS method handles the inherent relationships of the factors, which are treated in this work as the set of criteria, and the LBWA method assigns the weights of these factors. While other methods, such as the analytic hierarchy process, best-worst method, and full consistency method, among other related criteria-weighting tools popular in the literature, the integrated WINGS–LBWA method augments the limitation of criteria independence necessary for evaluating construction delays. Despite the popularity of the WINGS (e.g., [49–51]) and LBWA (e.g., [52–54]) methods in recent applications, their integration is still unexplored in the literature.

In our proposed approach, the second action required in MCS problems is handled using FlowSort [55]—a widely known MCS tool based on the highly regarded Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE) method. The strength of the FlowSort tool in relation to other emerging MCS tools (see the review of Alvarez et al. [56]) lies in the capability of the PROMETHEE to allow decision makers to assign a preference function to a specific criterion. The popularity of FlowSort and its extensions as MCS tools is evident in the literature, with applications ranging from disaster management [57], manufacturing operations [58], and online reviews [59] to mobile finance [60]. Due to the inherent vagueness and imprecision in FlowSort, some advances adopted the integration of fuzzy sets and their extensions to augment its efficacy. For instance, some works introduce the notion of fuzzy sets [61], interval type-2 fuzzy sets [57], probabilistic linguistic environment [59], and intuitionistic fuzzy sets (IFS) [62,63]. Despite these extensions, especially the IFS, decision makers have limited space to elicit ambiguity and imprecision, which are highly relevant in most applications. Thus, in this work, the integration of q-rung orthopair fuzzy sets (q-ROFS) within the computational framework of FlowSort is explored to handle the MCS of the problem domain. The notion of q-ROFS proposed by Yager [64] augments the limitation of previously developed tools (e.g., fuzzy set theory, IFS, Pythagorean fuzzy sets, Fermatean fuzzy sets) in handling judgment uncertainties brought about by incomplete information, lack of understanding of the domain problem, and the idiosyncrasies at which decisions are made. Such integration of q-ROFS and FlowSort, hereby termed q-ROF–FlowSort, remains unexplored in the literature.

Thus, this work espouses two-fold contributions: (1) it rigorously evaluates the impact of causes of delays on construction projects, which may offer important insights for practice, and (2) methodologically, it offers a novel integration of WINGS–LBWA and FlowSort under a q-ROFS environment to address an MCS problem. An actual case study evaluating the impact of the causes of delays in residential construction projects viewed by small-sized contractors in the central Philippines is implemented to demonstrate the efficacy of the proposed approach. The insights of the case, although they may be confined to some idiosyncrasies, help key decision makers design initiatives that would efficiently address project delays. The remainder of the article is arranged as follows. Section 2 presents some relevant preliminary concepts of q-ROFS, the WINGS, the LBWA, and FlowSort. Section 3 outlines the case environment and demonstrates the integration of q-ROF–WINGS–LBWA–FlowSort in an actual case study. Sensitivity and comparative analyses are offered in Section 4 to evaluate the variations in the findings in view of some parameter changes and compare how the proposed approach augments similar tools. A discussion of the findings is presented in Section 5. It ends with some concluding remarks and future works in Section 6.

2. Preliminaries

This section discusses the preliminaries of the q-ROFS, WINGS, LBWA, and FlowSort methodologies. The preceding methodologies were used to assign all the alternatives into predefined categories—demonstrating an MCS process. The q-ROFS were used for representing and handling uncertainty and imprecision in the judgment elicitation of decision makers. The q-ROFS framework has been applied to a wide range of environments and applications characterized by the uncertainty of judgments within a decision-making framework [65]. LBWA and WINGS were carried out to calculate the priority weights of the decision criteria necessary in the q-ROF-FlowSort. The application of these methodologies is presented in Section 3.2.

2.1. q-Rung Orthopair Fuzzy Sets

The concept of fuzzy sets was introduced by Zadeh [66] as a computational approach for handling information uncertainties. Due to its wide range of applications, various extensions of the theory were proposed. For instance, the intuitionistic fuzzy set theory proposed by Atanassov [67] extends the notion of membership functions in Zadeh's fuzzy sets to include non-membership functions. Furthermore, Yager [64] proposed the q-ROFS, an extension of Atanassov's intuitionistic fuzzy sets that is more flexible in a range of uncertainties inherent in the decision-making process, and then applied it in the integrals of Archimedean t-Norms and t-Conorms [68]. The following provides the basic notions of q-ROFS, with a background starting from intuitionistic fuzzy sets.

Definition 1 ([67]). *Let* X *be a non-empty universe of discourse. Then, IFS* $\mathcal{I}$ *in* X *is an object having a form given by*

$$\mathcal{I} = \{\langle x, \mu_{\mathcal{I}}(x), \nu_{\mathcal{I}}(x)\rangle : x \in \mathrm{X}\} \tag{1}$$

where the functions $\mu_{\mathcal{I}}(x) : \mathrm{X} \longrightarrow [0,1]$ *and* $\nu_{\mathcal{I}}(x) : \mathrm{X} \longrightarrow [0,1]$ *refer to the degree of membership and degree of non-membership of* $x \in \mathrm{X}$ *in* F, *respectively, such that* $0 \leq \mu_{\mathcal{I}}(x) + \mu_{\mathcal{I}}(x) \leq 1$, $\forall x \in \mathrm{X}$. *The degree of hesitancy* $\pi_{\mathcal{I}}$ *is defined as follows:*

$$\pi_{\mathcal{I}}(x) = 1 - \mu_{\mathcal{I}}(x) - \nu_{\mathcal{I}}(x) \tag{2}$$

However, the restrictive condition of the IFS may fail to reflect decision-makers' judgments in practical applications. For instance, when $\mu_{\mathcal{I}} = 0.6$ and $\nu_{\mathcal{I}} = 0.5$ for some x that decision makers possibly elicit, the condition in Definition 1 is violated. To address this possible scenario, the concept of the second-type IFS was developed by Atanassov [69], which was later reintroduced by Yager [70] as Pythagorean fuzzy sets (PFS). Some applications and extensions of PFS were presented in a bibliometric analysis during 2013–2020 [71], including useful metrics such as the divergence measure of PFS and its application in medical diagnosis [72] and some distance and similarity measures [73]. The definition of a PFS is as follows:

Definition 2 [70]. *Let* X *be a non-empty universe of discourse, where the PFS* $\mathcal{P}$ *is presented as*

$$\mathcal{P} = \{\langle x, \mu_{\mathcal{P}}(x), \nu_{\mathcal{P}}(x)\rangle : x \in \mathrm{X}\} \tag{3}$$

where the functions $\mu_{\mathcal{P}}(x) : \mathrm{X} \longrightarrow [0,1]$ *and* $\nu_{\mathcal{P}}(x) : \mathrm{X} \longrightarrow [0,1]$ *refer to the degree of membership and degree of non-membership of* $x \in \mathrm{X}$ *in the set* $\mathcal{P}$, *respectively, such that* $0 \leq (\mu_{\mathcal{P}}(x))^2 + (\nu_{\mathcal{P}}(x))^2 \leq 1, \forall x \in \mathrm{X}$. *The degree of indeterminacy* $\pi_{\mathcal{P}}$ *is defined as follows:*

$$\pi_{\mathcal{P}}(x) = \sqrt{1 - (\mu_{\mathcal{P}}(x))^2 - (\nu_{\mathcal{P}}(x))^2} \tag{4}$$

However, certain conditions may exist that the PFS may fail to handle. For instance, when $\mu_{\mathcal{P}} = 0.5$ and $\nu_{\mathcal{P}} = 0.9$, then $0.5^2 + 0.9^2 > 1$, violating the condition of the PFS. Hence, Yager [64] introduced the q-ROFS. For q-ROFS $\mathcal{Q}$, the degree of membership $\mu_{\mathcal{Q}}$ and non-membership $\nu_{\mathcal{Q}}$ satisfy the condition $(\mu_{\mathcal{Q}}(x))^q + (\nu_{\mathcal{Q}}(x))^q \leq 1$ for $q \geq 1$. Obviously, for $q = 1$, then $\mathcal{Q}$ is an IFS, and $q = 2$ implies that $\mathcal{Q}$ is a PFS. Below are the formal definitions and operations governing q-ROFS.

Definition 3 ([64]). *Let* X *be a non-empty universe of discourse, where the* q *-ROFS* $\mathcal{Q}$ *is presented as*

$$\mathcal{Q} = \{\langle x, \mu_{\mathcal{Q}}(x), \nu_{\mathcal{Q}}(x)\rangle : x \in \mathrm{X}\} \tag{5}$$

where the functions $\mu_{\mathcal{Q}}(x) : \mathrm{X} \longrightarrow [0,1]$ *and* $\nu_{\mathcal{Q}}(x) : \mathrm{X} \longrightarrow [0,1]$ *refer to the degree of membership and degree of non-membership of* $x \in \mathrm{X}$ *in* $\mathcal{Q}$, *respectively, such that* $0 \leq (\mu_{\mathcal{Q}}(x))^q + (\nu_{\mathcal{Q}}(x))^q \leq 1$ *for some finite* $q \geq 1, \forall x \in \mathrm{X}$. *The degree of indeterminacy* $\pi_{\mathcal{Q}}$ *is defined as follows:*

$$\pi_{\mathcal{Q}}(x) = \left(1 - (\mu_{\mathcal{Q}}(x))^q - (\nu_{\mathcal{Q}}(x))^q\right)^{\frac{1}{q}} \tag{6}$$

For convenience, $\langle \mu_{\mathcal{Q}}(x), \nu_{\mathcal{Q}}(x)\rangle$ is referred to as a q-rung orthopair fuzzy number (q-ROFN) on $\mathbb{R}$, and is written as $\mathcal{Q} = (\mu_{\mathcal{Q}}, \nu_{\mathcal{Q}})$.

Some interesting results were put forward by Yager [64], including the result in Theorem 1.

Theorem 1 ([64]). *If* $(\mu_{\mathcal{Q}}, \nu_{\mathcal{Q}})$ *is a valid* q_1*-rung orthopair membership grade, then it is a valid* q_2*-rung orthopair membership grade for* $q_2 > q_1$.

Proof. Since $(\mu_{\mathcal{Q}})^{q_1} + (\nu_{\mathcal{Q}})^{q_1} \leq 1$, then $(\mu_{\mathcal{Q}})^{q_2} + (\nu_{\mathcal{Q}})^{q_2} \leq 1$ for $q_2 > q_1$. Thus, $(\mu_{\mathcal{Q}}, \nu_{\mathcal{Q}})$ is a q_2-rung orthopair membership grade. □

Theorem 1 implies an important Corollary, as shown below.

Corollary 1. *For $q_2 > q_1$, all q_1-rung orthopair fuzzy sets are q_2-rung orthopair fuzzy sets.*

To illustrate, suppose $\mu_Q = 0.9$ and $\nu_Q = 0.3$. For $q = 2$, the condition $0.9^2 + 0.3^2 \leq 1$ suffices; therefore, $(0.9, 0.3)$ is a valid orthopair membership grade. The same is valid for $q = 3$, since $0.9^3 + 0.3^3 \leq 1$. Thus, $(0.9, 0.3)$ is also a 3-rung orthopair membership grade.

The following presents certain operations of q-ROFS.

Definition 4 ([74,75]). *Let $\ddot{q}_1 = (\mu_1, v_1)$ and $\ddot{q}_2 = (\mu_2, v_2)$ be two q-ROFNs and $\lambda > 0$, then corresponding operations are defined as follows:*

$$\ddot{q}_1^c = (v_1, \mu_1), \text{ where } \ddot{q}_1^c \text{ is the complement of } \ddot{q}_1 \tag{7}$$

$$\ddot{q}_1 \cup \ddot{q}_2 = (\mu_1 \vee \mu_2, v_1 \wedge v_2) \tag{8}$$

$$\ddot{q}_1 \cap \ddot{q}_2 = (\mu_1 \wedge \mu_2, v_1 \wedge v_2) \tag{9}$$

$$\ddot{q}_1 \oplus \ddot{q}_2 = \left(\sqrt[q]{\mu_1^q + \mu_2^q - \mu_1^q \mu_2^q}, v_1 v_2 \right) \tag{10}$$

$$\ddot{q}_1 \otimes \ddot{q}_2 = \left(\mu_1 \mu_2, \sqrt[q]{v_1^q + v_2^q - v_1^q v_2^q} \right) \tag{11}$$

$$\lambda \ddot{q} = \left(\sqrt[q]{1 - (1 - \mu^q)^\lambda}, v^\lambda \right) \tag{12}$$

$$\ddot{q}^\lambda = \left(\mu^\lambda, \sqrt[q]{1 - (1 - \mu^q)^\lambda} \right) \tag{13}$$

$$\ddot{q}_1 \ominus \ddot{q}_2 = \left(\mu_1 v_2, \sqrt[q]{v_1^q + \mu_2^q - v_1^q \mu_2^q} \right) \tag{14}$$

$$\ddot{q}_1 \oslash \ddot{q}_2 = \left(\sqrt[q]{\mu_1^q + v_2^q - \mu_1^q v_2^q}, v_1 \mu_2 \right) \tag{15}$$

Definition 5 ([74]). *Suppose that $\ddot{q} = (\mu, \nu)$ is a q-ROFN, then a score function $\mathbb{S}(\ddot{q})$ is defined as*

$$\mathbb{S}(\ddot{q}) = \mu^q - \nu^q \tag{16}$$

Definition 6 ([74]). *Suppose that $\ddot{q} = (\mu, \nu)$ is a q-ROFN, then an accuracy function $\mathbb{H}(\ddot{q})$ is defined as*

$$\mathbb{H}(\ddot{q}) = \mu^q + \nu^q \tag{17}$$

Theorem 2 ([74]). *For any two q-ROFNs $\ddot{q}_1 = (\mu_1, v_1)$, and $\ddot{q}_2 = (\mu_2, v_2)$, with the score function, a comparison method using the score function $\mathbb{S}$ and $\mathbb{H}$ is defined as follows:*

(1) If $\mathbb{S}(\ddot{q}_1) > \mathbb{S}(\ddot{q}_2)$, then $\ddot{q}_1 > \ddot{q}_2$;
(2) If $\mathbb{S}(\ddot{q}_1) < \mathbb{S}(\ddot{q}_2)$, then $\ddot{q}_1 < \ddot{q}_2$
(3) If $\mathbb{S}(\ddot{q}_1) = \mathbb{S}(\ddot{q}_2)$, then
    If $\mathbb{H}(\ddot{q}_1) > \mathbb{H}(\ddot{q}_2)$, then $\ddot{q}_1 > \ddot{q}_2$;
    If $\mathbb{H}(\ddot{q}_1) = \mathbb{H}(\ddot{q}_2)$, then $\ddot{q}_1 = \ddot{q}_2$.

Theorem 2 allows for the ordering of q-ROFNs, which has an important role in various areas of applications, especially in dealing with multi-attribute decision-making (MADM) problems. However, some limitations exist for the score and accuracy functions of Liu and

Wang [74], prompting others in the literature to offer other formulations. Listed in Table 1 are the existing formulations of score functions. Note that the list is not comprehensive.

Table 1. Selected existing score functions.

Authors	**Score Functions**
Peng et al. [76]	$\mathbb{S}(\ddot{q}) = \frac{1}{2}\left(\mu^2 + \left(\sqrt[q]{1 - v^q}\right)^2\right)$
Jana et al. [77]; Wei et al. [78]	$\mathbb{S}_{JW}(\ddot{q}) = \frac{\mu^q - v^q + 1}{2}$
Banerjee et al. [79]	$\mathbb{S}_b(\ddot{q}) = \frac{1 - v^q}{2 - \mu^q - v^q}$
Farhadinia and Liao [80]	$\mathbb{S}_{fl}(\ddot{q}) = \mu^q + \lambda\pi^q$
Rani and Mishra [81]	$\mathbb{S}_{rm}(\ddot{q}) = \mu^q(1 + \pi)$

In addition to the basic operations of the q-ROFNs introduced by Liu and Wang [74], they also proposed the aggregation operator, namely the q-rung orthopair fuzzy weighted averaging operator (q-ROFWA), which is defined as follows.

Theorem 3. *Suppose that* $\mathcal{Q}_k = \left(\mu_{\mathcal{Q}_k}, \nu_{\mathcal{Q}_k}\right)(k = 1, 2, \ldots, n)$ *is a collection of q-ROFNs, then the q-ROFWA is obtained by*

$$q\text{-ROFWA}(\mathcal{Q}_1, \mathcal{Q}_2, \ldots, \mathcal{Q}_n) = \left(\left(1 - \prod_{k=1}^{n}\left(1 - \mu_{\mathcal{Q}_k}^{q}\right)^{w_k}\right)^{\frac{1}{q}}, \prod_{k=1}^{n} \nu_{\mathcal{Q}_k}^{w_k}\right) \tag{18}$$

where $w_k > 0$ *(*$\forall k$*) and* $\sum_{k=1}^{n} w_k = 1$*. Here,* w_k *denotes the weight assigned to* $\mathcal{Q}_k$*.*

2.2. Weighted Influence Non-Linear Gauge Systems

Michnik [47] introduced the WINGS method, an approach that enhances the ability of the Decision-Making Trial and Evaluation Laboratory (DEMATEL) to evaluate the intertwined relationships among the components (or factors) that affect a particular system. In contrast to the DEMATEL, WINGS integrates both the individual strength and intensity of influence of the system components in the computational model. The DEMATEL outcome varies from the WINGS due to this intrinsic strength being integrated into the methodology. Michnik [47,82] emphasized that WINGS makes it possible to evaluate a specific system when it is essential to consider the interrelationships between its components. As an alternative, when system components are independent, WINGS reduce to an additive aggregation approach, similar to classical MADM approaches. Numerous applications demonstrate the efficacy of the WINGS method, as seen in applications such as improving agricultural green supply chain management [49], evaluation of consumption barriers of refurbished mobile phones [51], and sustainable partner selection [50], among other strengths. This list is not meant to be comprehensive. The following illustrates the methodological steps of WINGS.

Step 1. Construct the direct strength–influence matrix.

The components of the system (i.e., barriers, concepts, drivers) may be obtained through a thorough assessment of the literature, a focus group discussion, or a blended strategy. Having identified the system components, decision makers elicit their evaluations of the strength and influence of each component. These evaluations are used to construct the direct strength–influence matrix $D = \left(d_{ij}\right)_{n \times n}$, for n system components. The strength of the ith component, represented by d_{ii}, is placed in the main diagonal of D. Meanwhile, values representing the intensity of the influence of ith component on jth component, such that $i \neq j$, is represented by d_{ij}.

Step 2. Construct the scaled strength–influence matrix $S = \left(S_{ij}\right)_{n\times n}$. Here, D is scaled according to the following relation:

$$S = \frac{1}{s}D \tag{19}$$

where the scaling factor s is obtained through the following

$$s = \sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij} \tag{20}$$

Step 3. Determine the total strength–influence matrix $T = \left(t_{ij}\right)_{n\times n}$, where T is generated using the expression

$$T = S(I - S)^{-1} \tag{21}$$

Step 4. Calculate the total impact I_i and total receptivity R_i as follows:

$$I_i = \sum_{j=1}^{n} t_{ij}, \forall i = 1, \ldots, n \tag{22}$$

$$R_i = \sum_{i=1}^{n} t_{ij}, \forall i = 1, \ldots, n \tag{23}$$

The four indicators that characterize the system components through WINGS are the following:

- Total impact I_i represents the influence of the component i on all other components in the system.
- Total receptivity R_i represents the influence of all the other components in the system on the component i.
- Total involvement $I_i + R_i$ represents the sum of all influences exerted on and received by the component i.
- Cause and result role of the component i indicated by a negative $I_i - R_i$ or a positive $I_i - R_i$, respectively.

Michnik [47] recommends using the total involvement vector to evaluate the components' priority (or ranking).

2.3. Level-Based Weight Assessment

The LBWA method, offered by Žižović and Pamucar [48], is a newly developed attribute (or factor, criterion) weighting method. When adopted within MADM, it is based on a pairwise comparison of criteria by creating non-decreasing strings at criteria relevance levels. After level-based grouping, the relevance of the criteria is determined in relation to the decision makers' preferences. Recently, the LBWA has been applied for the evaluation of healthcare sectors (i.e., [52]), renewable energy resources assessment (i.e., [53]), and assessment of ideal smart network strategies for logistics companies (i.e., [54]), among others. Considering a MADM problem with n criteria $S = \{c_1, \ldots, c_n\}$, assume that the priority weights of these criteria must be determined since they are not known beforehand. The following presents the process of obtaining the priority weights of criteria using the LBWA model.

Step 1: Determine the most important criterion from the set of criteria $S = \{c_1, \ldots, c_n\}$. Let the most important criterion, determined by the decision maker and denoted by the criterion $c_{(1)}$, be the criterion in S that is deemed most significant for the decision problem.

Step 2: Group the criteria by levels of significance. Let the decision maker establish subsets of criteria in the following manner:

Level S_1: At level S_1, group the criteria from S whose significance is equal to the significance of $c_{(1)}$ or up to twice as less as the significance of $c_{(1)}$;

Level S_2: At level S_2, group the criteria from S whose significance is exactly twice as less as the significance of $c_{(1)}$ or up to three times less than the significance of $c_{(1)}$;

Level S_3: At level S_3, group the criteria from S whose significance is exactly three times less than the significance of $c_{(1)}$ or up to four times less than the significance of $c_{(1)}$;

Level S_k: At level S_k, group the criteria from S whose significance is exactly k times as less as the significance of $c_{(1)}$ or up to $k+1$ as less as the significance of $c_{(1)}$.

Here, $S = \bigcup_{i=1}^{k} S_i$. By applying these rules, the decision maker establishes a rough classification of the observed criteria, i.e., groups the criteria according to the significance levels. If the significance of a criterion c_j is denoted by $s\left(c_j\right)$, where $j \in \{1, 2, \ldots, n\}$, and for every level $i \in \{1, 2, \ldots, k\}$, the following applies:

$$S_i = \left\{c_j \in S : i \le s\left(c_j\right) < i+1\right\} \tag{24}$$

Additionally, for each $p, q \in \{1, 2, \ldots, k\}$, $p \ne q$, $S_p \cap S_q = \varnothing$. Thus, the subsets S_i $(\forall i)$ form a well-defined partition of S, i.e., $\bigcap_{i=1}^{k} S_i = \varnothing$.

Step 3: Within the formed subsets (levels) of the influence of the criteria, perform the comparison of criteria by their significance. Each criterion $c_j \in S_i$ is assigned with an integer $I_j \in \{0, 1, \ldots, r\}$ so that the most important criterion $c_{(1)}$ is assigned with $I_1 = 0$. If c_j is more significant than $c_{j'}$, $j \ne j'$, then $I_j < I_{j'}$; otherwise, if c_j is equivalent to $c_{j'}$, then $I_j = I_{j'}$. The maximum value on the scale for the comparison of criteria is defined by applying

$$r = \max\{|S_1|, |S_2|, \ldots, |S_k|\} \tag{25}$$

Step 4: Based on the defined maximum value of the scale for the comparison of criteria (r), define the elasticity coefficient $r_0 \in \mathbb{R}$ (where $\mathbb{R}$ presents the set of real numbers), which should meet the requirements $r_0 > r$.

Step 5: Calculate the influence function of the criteria. The influence function $f : S \to R$ is defined in the following way. For every $c_j \in S_i$, define the influence function of the criterion

$$f\left(c_j\right) = \frac{r_0}{i \cdot r_0 + I_j} \tag{26}$$

where i represents the number of the level/subset in which c_j belongs, r_0 represents the elasticity coefficient, while $I_j \in \{0, 1, \ldots, r\}$ represents the value assigned to the criterion c_j within S_i.

Step 6: Calculate the optimum values of the weight coefficients of criteria using the following:

$$w_{(1)} = \frac{1}{\sum_{j=1}^{n} f\left(c_j\right)} \tag{27}$$

where $w_{(1)}$ represents the priority weight of $c_{(1)}$.

The priority weights the remaining criteria c_j which are not $c_{(1)}$ are obtained through Equation (28):

$$w_j = f\left(c_j\right) \cdot w_{(1)} \tag{28}$$

2.4. FlowSort

FlowSort [55] is an extension of the PROMETHEE method for assigning alternatives to predefined ordered p categories, denoted as $C_1, C_2, \ldots, C_p$, such that $C_1 \rhd C_2 \rhd \cdots \rhd C_p$. FlowSort requires input data, including criteria weights, the alternatives' performances shown in a decision matrix, reference profiles, and threshold parameters. In FlowSort, categories are definable by two limiting profiles or one central profile. When defined by limiting profiles, upper and lower profiles are as follows: $l_1 \succ l_2 \succ \cdots \succ l_{p+1}$, where C_h, $h \in \{1, \ldots, p\}$, is defined within $[l_h, l_{h+1}]$. FlowSort has been applied in modeling customer satisfaction through online reviews (i.e., [59]), sorting mutual funds (i.e., [83]), and evaluating the strategies for university technology transfer [84], among other applications. The methodological step of FlowSort is as follows:

Step 1: Compute the preference function. Define $R_i = \{l_1, \ldots, l_{p+1}\} \cup \{a_i\}, i = 1, \ldots, m$, where $a_i \in A$ is part of the set of alternatives A. The preference function $p_j(x, y)(\forall j \in \{1, \ldots, n\})$ can be computed for any pair of $(x, y) \in R_i$. The mapping p_j calculates the preference strength of x over y in criterion j by considering the deviation between x and y. The amount of deviation between x and y is expressed as follows:

$$d_j(x, y) = g_j(x) - g_j(y) \tag{29}$$

where $g_j(x)$ and $g_j(y)$ signify the performance of x and y, respectively, under criterion j.

The preference function for the benefit (or maximizing) criteria is expressed in Equation (30), and the cost (or minimizing) criteria is obtained using Equation (31), where $p_j(x, y) \in [0, 1]$. As p_j approaches value 1, the experts' preference for x over y increases. Six types of preference functions are used in the PROMETHEE method [85]. For brevity, they are not presented here.

$$p_j(x, y) = F_j\left[d_j(x, y)\right] \tag{30}$$

$$p_j(x, y) = F_j\left[-d_j(x, y)\right] \tag{31}$$

Step 2: Compute the preference degree. The global preference function of each pair of alternatives can be obtained through Equation (32),

$$\pi(x, y) = \sum_{j=1}^{n} w_j p_j(x, y) \tag{32}$$

where w_j is the priority weight of criterion j.

Step 3: Compute the leaving ($\Phi^+_{R_i}(x)$), entering ($\Phi^-_{R_i}(x)$), and net flow ($\Phi_{Ri}(x)$) using the following:

$$\Phi^+_{R_i}(x) = \frac{1}{|R_i| - 1} \sum_{y \in R_i} \pi(x, y) \tag{33}$$

$$\Phi^-_{R_i}(x) = \frac{1}{|R_i| - 1} \sum_{y \in R_i} \pi(y, x) \tag{34}$$

$$\Phi_{Ri}(x) = \Phi^+_{R_i}(x) - \Phi^-_{R_i}(x) \tag{35}$$

where $|R_i|$ is the cardinality of R_i.

Step 4: Assign the alternatives to categories. The assignment of alternative a_i to category C_h can be computed based on net flows expressed in Equation (36).

$$C_\Phi(a_i) = C_h \quad \text{if } \Phi_{Ri}(l_h) \geq \Phi_{Ri}(a_i) > \Phi_{Ri}(l_{h+1}) \tag{36}$$

3. Methodology

3.1. Case Study

To demonstrate the efficacy of the proposed approach of evaluating the impact of causes of delays on residential construction projects, a two-part survey was conducted on various firms with experience in the construction of residential buildings in the central Philippines. The Philippine Contractors Accreditation Board (PCAB) issues PCAB licenses to contractors after a rigorous application process. The license represents the eligibility and accountability of the contractors. Moreover, the PCAB license is categorized according to the size of the construction firm. All government projects require contractors with PCAB licenses, while some private projects skip this requirement. The first part of the survey is intended to assign priority weights to the three project management factors. These factors, widely considered in the literature [35], were integrated into the survey: time, cost, and quality; and they are considered criteria in the proposed MCS problem. Two academics with expert construction project management backgrounds were asked in a focus group discussion (FGD) to elucidate judgments of the three factors within the framework of the WINGS and LBWA methods. Their judgments were represented as q-ROFS to capture the ambiguity and imprecision in decision making.

The second part of the survey intends to sort out the impact of the causes of residential construction project delays. As shown in Appendix A, 38 causes of construction project delays have been identified based on the literature review. These causes were validated through an FGD with decision makers to check relevance with residential projects in the Philippines. The survey yields eighteen participants, with six small-sized PCAB contractors, six large-sized PCAB contractors, and six without PCAB licenses yet registered under government agencies (i.e., Securities and Exchange Commission, Department of Trade and Industry). The construction firms' years of operations range from 1 to 50 years, with an employee count ranging from 12 to 1200. Table 2 presents the demographics of the 18 participants who are considered the decision makers of the case study. Survey questionnaires were distributed to these decision makers, who were asked to evaluate the degree of impact of each cause of delay on the three identified project management factors in the context of residential projects. These questionnaires were designed to achieve a decision matrix representing the evaluations of decision makers. Each evaluation is represented by a scale (1 to 7) with corresponding q-ROFS.

Table 2. Details of the expert decision makers.

	PCAB License		Years of Experience	No. of Site Workers in the Firm	No. of Technical Personnel in the Firm				No. of Residential Projects		Average Contract Prices for Implemented Projects (Million Php)
	Presence/ Category				Civil Engineers	Mechanical Engineers	Architects	Others	Urban Areas	Rural Areas	
R1	Yes	AA	10	500	>5	0	2	>5	0	1	2.1–3
R2	Yes	D	25	35	5	0	0	2	>10	>10	3.1–5
R3	No	-	3	12	1	0	1	1	3	1	2.1–3
R4	Yes	D	5	30	3	0	0	0	4	0	1–1.5
R5	Yes	AA	40	150	5	1	0	2	1	0	>5
R6	Yes	AAA	48	500	>5	2	3	>5	>10	>10	>5
R7	No	-	-	-	>5	0	0	>5	4	7	3.1–5
R8	Yes	D	3	50	4	1	1	1	>10	0	>5
R9	Yes	C	2	21	2	0	1	1	9	6	1.6–2
R10	No	-	20	32	3	0	1	2	10	>10	2.1–3
R11	Yes	C	5	80	2	0	1	0	1	0	3.1–5
R12	Yes	AA	50	200	>5	3	3	>5	>10	0	>5
R13	No	-	7	30	0	0	3	1	>10	3	3.1–5
R14	Yes	AA	15	1200	>5	0	0	>5	>10	>10	>5
R15	Yes	D	6	100	>5	0	0	1	1	0	2.1–3
R16	No	-	5	30	2	0	1	1	6	0	2.1–3
R17	Yes	A	30	60	3	0	0	5	5	5	3.1–5
R18	No	-	1	20	2	0	0	0	2	0	2.1–3

3.2. Application of the Proposed Approach in Evaluating the Impact of the Causes of Delays on Residential Construction Projects

Figure 1 exhibits the procedural framework of the proposed approach. It comprises two main phases: (i) obtaining the priority weight coefficients of the three criteria through the q-ROF–WINGS–LBWA method and (ii) sorting the causes of construction delays on residential projects into predefined impact categories using q-ROF–FlowSort. The integration of the WINGS and LBWA methods offers an approach that measures the priorities of decision elements (e.g., criteria) while capturing their inherent interdependencies. While the LBWA is comparable to well-known weight allocation methods such as the analytic hierarchy process, best-work method, and full consistency method, it does not support a mechanism to allow interdependencies of relevant elements in various applications. In our proposed approach, WINGS augments this limitation, and the assignment of weights reflects a more real-life characterization of decision elements. Meanwhile, FlowSort provides a powerful approach to sorting alternatives given a predefined set of classes or categories. The strength of FlowSort, as in the case of its base PROMETHEE, lies in its ability to appropriately represent the performance of the decision alternatives in view of the preference functions of the criteria. Due to the inherent uncertainty and imprecision of decision makers in eliciting judgments within the two phases, the integration of q-ROFS into WINGS–LBWA and FlowSort espouses a more flexible and natural way of handling the uncertainties. The application of the procedure is as follows.

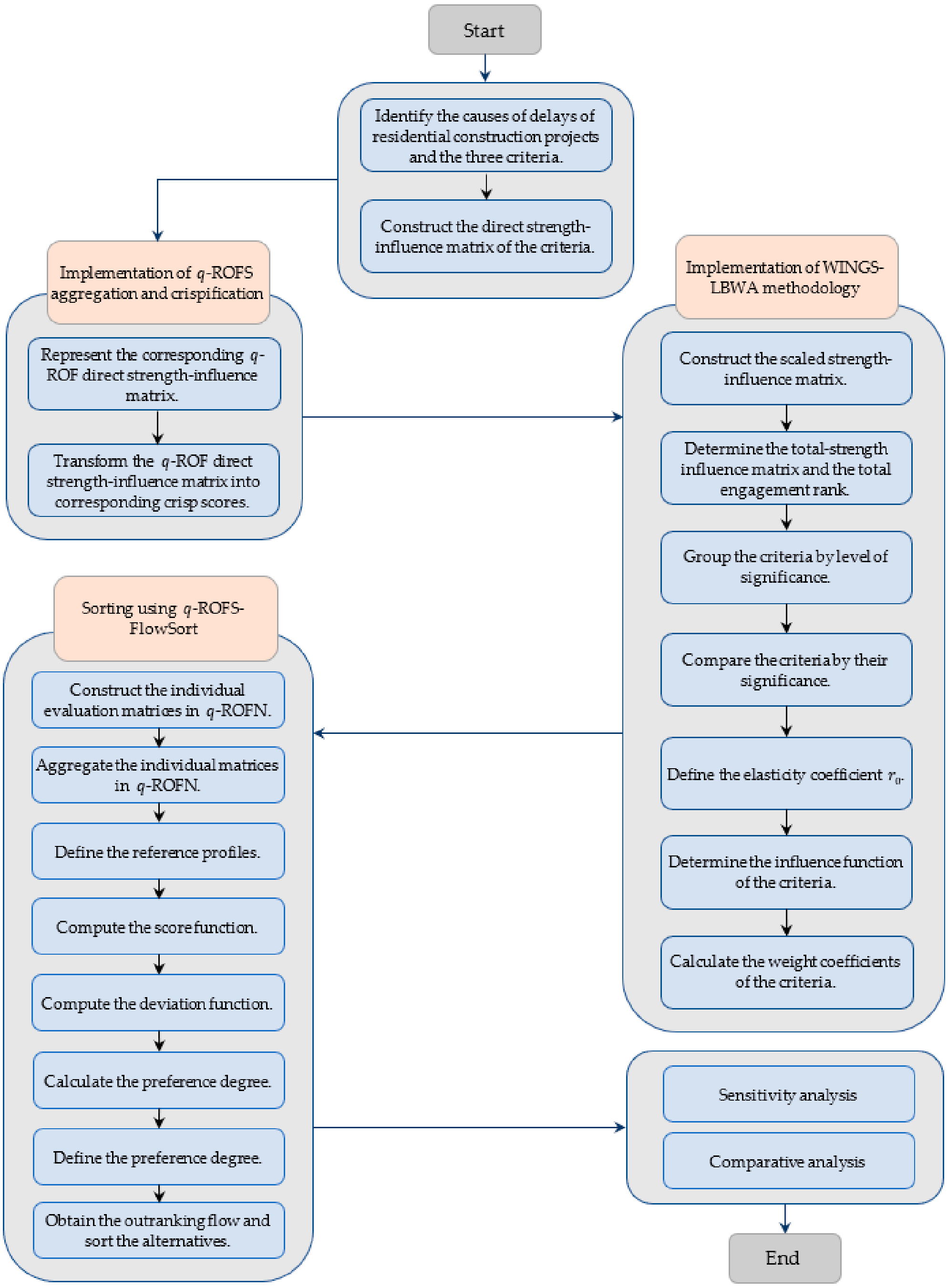

Figure 1. The proposed methodological framework.

Phase 1: q-ROF–WINGS–LBWA

Step 1. Identify the causes of delays in residential construction projects and the three criteria.

A literature survey and an FGD identified 38 specific causes of delays in residential construction projects, as shown in Appendix A. These delay causes are hereby referred to as alternatives. A literature survey also determined the criteria for the causes of construction delays. These causes of delays were evaluated under three project management factors considered as criteria in the proposed MCS problem: time, cost, and quality.

Step 2. Construct the direct strength–influence matrix of the criteria.

The first part of the survey requires decision makers to elicit judgments on the strength, intensity, and influence of the three criteria. The first set of responses requires the experts in consensus to elicit judgments on two aspects: (1) the strength of each criterion and (2) the evaluation of influence, i.e., whether or not a criterion influences another criterion. The linguistic evaluation is presented in Table 3. These evaluations constitute the direct strength–influence matrix D. The strength of the ith criterion as elicited in a consensus is represented by d_{ii} on the main diagonal, while other values of D represent the evaluation regarding the influence of the ith criterion on the jth criterion, such that $i \neq j$.

Table 3. Linguistic evaluation scale for component strength and influence.

Component Strength			**Component Influence**		
Score	**Linguistic Evaluation**	***q*-ROFN**	**Score**	**Linguistic Evaluation**	***q*-ROFN**
1	Certainly low relevance	(0.15, 0.9)	0	No influence	(0,0)
2	Very low relevance	(0.30, 0.85)	1	Certainly low influence	(0.15, 0.9)
3	Low relevance	(0.45, 0.65)	2	Very low influence	(0.30, 0.85)
4	Medium relevance	(0.50, 0.50)	3	Low influence	(0.45, 0.65)
5	High relevance	(0.75, 0.40)	4	Medium influence	(0.50, 0.50)
6	Very high relevance	(0.80, 0.25)	5	High influence	(0.75, 0.40)
7	Certainly high relevance	(0.95, 0.10)	6	Very high influence	(0.80, 0.25)
			7	Certainly high influence	(0.95, 0.1)

Step 3. Represent the corresponding q-ROF direct strength–influence matrix.

Using the predefined linguistic evaluation scale shown in Table 3, matrix D is converted into its corresponding q-ROF direct strength–influence matrix $\tilde{D} = \left(\tilde{d}_{ij}\right)_{n\times n} = \left(\left(\mu_{\tilde{d}_{ij}}, \nu_{\tilde{d}_{ij}}\right)\right)_{n\times n}$, where $\left(\mu_{\tilde{d}_{ij}}, \nu_{\tilde{d}_{ij}}\right)$ is a q-ROFS. The resultant matrix is shown in Table 4.

Table 4. q-ROF direct strength–influence matrix.

	Time	**Cost**	**Quality**
Time	(0.95, 0.10)	(0.95, 0.10)	(0.75, 0.40)
Cost	(0.80, 0.25)	(0.8, 0.25)	(0.95, 0.10)
Quality	(0.75, 0.4)	(0.8, 0.25)	(0.75, 0.40)

Step 4. Transform the q-ROF direct strength–influence matrix into a matrix with corresponding crisp scores.

The crispification of the q-ROF direct strength–influence matrix is performed using the score function by Peng et al. [76], as presented in Table 1. Throughout this work, $q = 5$. Hence, matrix $\tilde{D}$ is converted into $\overline{D} = \left(\overline{d}_{ij}\right)_{n\times n}$ which is presented in Table 5. Note that $\overline{d}_{ij} \in \mathbb{R}$. Suppose that $i = j = 1$ (i.e., Time), the score function of the element $\overline{d}_{11}$ can be computed as follows:

$$\overline{d}_{11} = \mathbb{S}\left(\tilde{d}_{11}\right) = \frac{1}{2}\left(0.95^2 + \left(\sqrt[5]{1 - 0.1^5}\right)^2\right) = 0.9513$$

The rest of the computations for the matrix $\tilde{D}$ follow the same procedure.

Table 5. Direct strength–influence matrix in crisp scores.

	Time	**Cost**	**Quality**
Time	0.9513	0.9513	0.7813
Cost	0.8200	0.8200	0.9513
Quality	0.7813	0.8200	0.7813

Step 5. Construct the scaled strength–influence matrix.

The scaled strength–influence matrix $S = \left(s_{ij}\right)_{n\times n}$ is calculated using Equations (19) and (20). Here, the scaling factor s is computed as $s = 0.9513 + 0.8200 + 0.7813 + \cdots + 0.7813 = 7.6504$. Meanwhile, the calculation for matrix S is as follows:

$$S = \begin{pmatrix} 0.9513 \times \frac{1}{7.6504} & 0.9513 \times \frac{1}{7.6504} & 0.7813 \times \frac{1}{7.6504} \\ 0.8200 \times \frac{1}{7.6504} & 0.8200 \times \frac{1}{7.6504} & 0.9513 \times \frac{1}{7.6504} \\ 0.7813 \times \frac{1}{7.6504} & 0.8200 \times \frac{1}{7.6504} & 0.7813 \times \frac{1}{7.6504} \end{pmatrix}$$

The resultant matrix is presented in Table 6.

Table 6. Scaled strength–influence matrix.

	Time	**Cost**	**Quality**
Time	0.1242	0.1242	0.1020
Cost	0.1071	0.1071	0.1242
Quality	0.1020	0.1071	0.1020

Step 6. Determine the total strength–influence matrix and the total engagement rank.

The total strength–influence matrix $T = \left(t_{ij}\right)_{n\times n}$ is computed using Equation (21) and presented in Table 7. The total engagement rank of criterion i is determined through the sum of the total impact I_i (see Equation (22)) and total receptivity R_i (see Equation (23)). Michnik [47] considered the total engagement rank with the priority ranking of the criteria. The resulting ranking will be used in the LBWA method.

Table 7. Total strength–influence matrix.

	Time	**Cost**	**Quality**	**Total Engagement Rank**
Time	0.1829	0.1837	0.1598	1
Cost	0.1633	0.1642	0.1796	2
Quality	0.1539	0.1597	0.1532	3

Step 7. Group criteria by the level of significance.

This step commences the LBWA method. The most important criterion among the set of criteria is time, based on the total engagement rank computed in Step 6. This criterion automatically belongs to the level S_1. Upon deliberation of the domain experts, it was agreed that the cost criterion also belongs to S_1, while quality belongs to the level S_2. Note that the grouping of the criteria is based on the total engagement rank computed in Step 6 and an FGD.

$$S_1 = \{Time, Cost\} \qquad S_2 = \{Quality\}$$

Step 8. Compare the criteria by their significance.

Using Equation (25), the maximum value on the scale for the comparison of the criteria is 2. Hence, $I_j \in \{0,1\}, \forall j \in \{Time, Cost, Quality\}$, where $I_{Time} = 0$, $I_{Cost} = 1$, and $I_{Quality} = 2$.

Step 9. Define the elasticity coefficient r_0.

The elasticity coefficient r_0 is set at 3, since $r_0 > r$, where $r = 1$.

Step 10. Determine the influence function of the criteria.

Through Equation (26), we obtain the following influence function $f(C_i)$:

$$f(Time) = \frac{3}{(1\times 3)+0} = 1;\ f(Cost) = \frac{3}{(1\times 3)+1} = 0.7500;\ f(Quality) = \frac{3}{(2\times 3)+2} = 0.3750$$

Step 11. Calculate the weight coefficients of the criteria.

Using Equation (27), the weight coefficient of the most significant criterion, i.e., time, is

$$w_{Time} = \frac{1}{1+0.750+0.375} = 0.4706$$

By applying this value to Equation (28), we obtain the weight coefficients of the remaining criteria.

$$w_{Cost} = 0.4706 \times 0.7500 = 0.3529$$

$$w_{Quality} = 0.4706 \times 0.3750 = 0.1765$$

Phase 2: *q*-ROF–FlowSort

Step 12. Construct the individual decision matrices in q-ROFN.

Using the set of criteria and alternatives (see Appendix A) from **Step 1**, a K number of decision makers will elicit their judgments on the relevance of the alternative i under criterion j using a predefined evaluation scale. The evaluations of the decision makers were utilized to construct the K individual decision matrices, each denoted by $X^k = \left(x_{ij}^k\right)_{m\times n}$, where $k = 1, 2, \ldots, K$. Then, the evaluations x_{ij}^k were transformed into their corresponding q-ROFN using the linguistic evaluation scale presented in Table 8. The resulting matrix is defined as $\overset{\sim k}{X} = \left(\overset{\sim k}{x}_{ij}\right)_{m\times n} = \left(\left(\mu_{\overset{\sim k}{x}_{ij}}, \nu_{\overset{\sim k}{x}_{ij}}\right)\right)_{m\times n}$, where $\left(\mu_{\overset{\sim k}{x}_{ij}}, \nu_{\overset{\sim k}{x}_{ij}}\right)$ is a q-ROFN.

Table 8. Linguistic evaluation scale for ratings of alternatives.

Score	Linguistic Evaluation	*q*-ROFN
1	Certainly low significance	(0.15, 0.9)
2	Very low significance	(0.3, 0.85)
3	Low significance	(0.45, 0.65)
4	Medium significance	(0.5, 0.5)
5	High significance	(0.75, 0.4)
6	Very high significance	(0.8, 0.25)
7	Certainly high significance	(0.95, 0.1)

Step 13. Aggregate the individual decision matrices in q-ROFN.

The aggregation operator in Equation (18) is utilized to aggregate the individual decision matrices $\overset{\sim k}{X}$. The aggregate evaluation matrix is denoted as $\widetilde{X} = \left(\widetilde{x}_{ij}\right)_{m\times n} = \left(\left(\mu_{\widetilde{x}_{ij}}, \nu_{\widetilde{x}_{ij}}\right)\right)_{m\times n}$. To demonstrate the computation of matrix $\widetilde{X}$, the computation of the element $\widetilde{x}_{11}$, where $k = 1, 2, \ldots, 18$, can be represented as such:

$$\widetilde{x}_{11} = \left(\sqrt[5]{1-\left[(1-0.3)^{0.0556}\times(1-0.15)^{0.0556}\ldots\times(1-0.15)^{0.0556}\right]}, 0.85^{0.0556}\times 0.90^{0.0556}\ldots\times 0.90^{0.0556}\times 0.90^{0.0556}\right)$$

The calculations for the remaining elements of the matrix $\tilde{X}$ also follow the same computation. The resultant matrix is presented in Table 9.

Table 9. Aggregate decision matrix.

	Time	**Cost**	**Quality**
A1	(0.0000, 0.8295)	(0.7290, 0.8024)	(0.9382, 0.3349)
A2	(0.9235, 0.6744)	(0.9382, 0.3158)	(0.9413, 0.2385)
A3	(0.9235, 0.6177)	(0.9335, 0.3973)	(0.9382, 0.3146)
A4	(0.0000, 0.7061)	(0.9361, 0.4938)	(0.9335, 0.3386)
A5	(0.7290, 0.7897)	(0.9361, 0.4544)	(0.9413, 0.2622)
A6	(0.7290, 0.7665)	(0.9235, 0.5829)	(0.9425, 0.2313)
A7	(0.7290, 0.6843)	(0.9298, 0.4350)	(0.9382, 0.2974)
A8	(0.7290, 0.7133)	(0.9298, 0.4480)	(0.9436, 0.2037)
A9	(0.0000, 0.7875)	(0.9361, 0.4181)	(0.9382, 0.2891)
A10	(0.0000, 0.7387)	(0.9335, 0.4069)	(0.9413, 0.2723)
A11	(0.9235, 0.5303)	(0.9235, 0.6797)	(0.9462, 0.1704)
A12	(0.0000, 0.8546)	(0.7290, 0.8219)	(0.9382, 0.3660)
A13	(0.0000, 0.8727)	(0.9298, 0.5351)	(0.9361, 0.4376)
A14	(0.0000, 0.8340)	(0.9235, 0.4737)	(0.9425, 0.2652)
A15	(0.7290, 0.6970)	(0.9335, 0.4980)	(0.9454, 0.1819)
A16	(0.0000, 0.7622)	(0.9235, 0.5227)	(0.9361, 0.3007)
A17	(0.7290, 0.7895)	(0.9361, 0.3732)	(0.9382, 0.2814)
A18	(0.7830, 0.5781)	(0.9235, 0.5391)	(0.9462, 0.1555)
A19	(0.7340, 0.6990)	(0.9298, 0.5427)	(0.9398, 0.2569)
A20	(0.7777, 0.7033)	(0.9235, 0.5628)	(0.9413, 0.2542)
A21	(0.7340, 0.7010)	(0.7861, 0.5646)	(0.9398, 0.2567)
A22	(0.7340, 0.6886)	(0.7777, 0.6381)	(0.9398, 0.2593)
A23	(0.0000, 0.7622)	(0.9335, 0.4334)	(0.9335, 0.3464)
A24	(0.9235, 0.6306)	(0.9335, 0.4396)	(0.9454, 0.1706)
A25	(0.9235, 0.5647)	(0.9235, 0.5374)	(0.9436, 0.2065)
A26	(0.7290, 0.6552)	(0.9235, 0.5587)	(0.9436, 0.1882)
A27	(0.0000, 0.7165)	(0.9361, 0.3779)	(0.9398, 0.2637)
A28	(0.9235, 0.5937)	(0.9398, 0.3093)	(0.9436, 0.2010)
A29	(0.9235, 0.5866)	(0.9361, 0.3649)	(0.9436, 0.2015)
A30	(0.7777, 0.5938)	(0.9298, 0.4845)	(0.9446, 0.1836)
A31	(0.9235, 0.5817)	(0.9235, 0.5507)	(0.9436, 0.2063)
A32	(0.9235, 0.6353)	(0.9298, 0.5053)	(0.9413, 0.2285)
A33	(0.9235, 0.6358)	(0.9335, 0.5013)	(0.9382, 0.2842)
A34	(0.9298, 0.5793)	(0.9235, 0.6010)	(0.9398, 0.3153)
A35	(0.9235, 0.6341)	(0.9335, 0.4767)	(0.9382, 0.3046)
A36	(0.9335, 0.4680)	(0.9382, 0.3299)	(0.9398, 0.3021)
A37	(0.9298, 0.5319)	(0.9398, 0.2953)	(0.9398, 0.2406)
A38	(0.9298, 0.5010)	(0.9413, 0.2893)	(0.9425, 0.2208)

Step 14. Define the reference profiles.

This study introduced three sets of reference profiles to represent the vulnerability degree of firms implementing residential construction projects. The first set represents the small-sized construction firms that may still have less experience and limited resources. These firms are highly vulnerable to the effects of construction project delays. It may take significant resources (i.e., time and finances) for them to recuperate when experiencing time and cost overruns in projects. These firms are hereby classified as High-Vulnerability Construction Firms (HVCFs). Another set of limiting profiles was generated to represent more established firms less vulnerable to the effects of construction delays. Unlike the HVCFs, these firms have a higher capacity to adapt to the impacts of these project delays. These firms are classified as Low-Vulnerability Construction Firms (LVCFs). The last set of limiting profiles represents the average construction firms with moderate vulnerability to the impacts of construction delays and are subsequently classified as Medium-Vulnerability Construction Firms (MVCFs).

Four categories of impacts were defined from the three sets of limiting profiles shown in Table 10. The set of categories $C = \{C_1, C_2, C_3, C_4\}$ was identified, where C_1 is referred to as the "insignificant impact" category, C_2 as the "low impact" category, C_3 as the "medium impact" category, and C_4 as the "high impact" category.

Table 10. Limiting profiles.

HVCF	**Time**	**Cost**	**Quality**
l_1^{HVCF}	(0, 1)	(0, 1)	(0, 1)
l_2^{HVCF}	(0.04, 0.96)	(0.067, 0.933)	(0.28, 0.72)
l_3^{HVCF}	(0.23, 0.77)	(0.4, 0.6)	(0.56, 0.44)
l_4^{HVCF}	(0.33, 0.67)	(0.6, 0.4)	(0.7, 0.3)
l_5^{HVCF}	(1, 0)	(1, 0)	(1, 0)
LVCF	Time	Cost	Quality
l_1^{LVCF}	(0, 1)	(0, 1)	(0, 1)
l_2^{LVCF}	(0.17, 0.83)	(0.33, 0.67)	(0.42, 0.58)
l_3^{LVCF}	(0.5, 0.5)	(0.67, 0.33)	(0.7, 0.3)
l_4^{LVCF}	(0.83, 0.17)	(0.84, 0.16)	(0.84, 0.16)
l_5^{LVCF}	(1, 0)	(1, 0)	(1, 0)
MVCF	Time	Cost	Quality
l_1^{MVCF}	(0, 1)	(0, 1)	(0, 1)
l_2^{MVCF}	(0.28, 0.72)	(0.28, 0.72)	(0.28, 0.72)
l_3^{MVCF}	(0.56, 0.44)	(0.56, 0.44)	(0.56, 0.44)
l_4^{MVCF}	(0.84, 0.16)	(0.84, 0.16)	(0.84, 0.16)
l_5^{MVCF}	(1, 0)	(1, 0)	(1, 0)

Step 15. Compute the score function.

Each set of limiting profiles is integrated into the aggregate matrix $\widetilde{X}$ to form an extended decision matrix. The score function of Peng et al. [76] was used to transform the aggregate evaluation scores $\widetilde{x}_{ij}$ in q-ROFN to their corresponding crisp scores. For brevity, the crisp-extended decision matrices (i.e., corresponding to each set of limiting profiles) are stored in the Supplementary Material. The demonstration of the crispification for $i = j = 1$ element of the matrix $\widetilde{X}$ is presented as follows:

$$\mathbb{S}\left(\widetilde{x}_{11}\right) = \frac{1}{2}\left(0.3380^2 + \left(\sqrt[5]{1 - 0.8295^5}\right)^2\right) = 0.4667$$

Step 16. Compute the deviation function.

The deviation values using Equation (29) were computed for each extended decision matrix. These values are presented in the Supplementary Material.

Step 17. Calculate the preference function.

All criteria were allowed to assume the type III criterion function, which is defined as follows:

$$F(d_{ii'}) = \begin{cases} 1 & d_{ii'} \leq -p_* \\ \frac{-d_{ii'}}{p_*} & -p_* \leq d_{ii'} < 0 \\ 0 & d_{ii'} \geq 0 \end{cases}$$

where $p_* \in \left\{p_{Time}, p_{Cost}, p_{Quality}\right\}$ is determined a priori by the decision makers. Here, $p_{Time} = 0.2$, $p_{Cost} = 0.2$, and $p_{Cost} = 0.3$.

Step 18. Define the preference degrees.

The preference degrees, in this case, were obtained using Equation (34) and are presented in the Supplementary Material.

Step 19. Obtain the outranking flow and sort the alternatives.

Sorting the causes of delays was based on the net flow defined in Equation (36). The net flows for the three sets of limiting profiles corresponding to the vulnerability degrees of construction firms are highlighted in Tables 11–13. Visualized in Figures 2–4 are the results of the MCS problems. Furthermore, a summary of the three sorting results from HVCF, LVCF, and MVCF is illustrated in Figure 5.

Table 11. Net outranking flows of the causes of delays with respect to the HVCF-limiting profiles.

	l_1^{HVCF}	l_2^{HVCF}	l_3^{HVCF}	l_4^{HVCF}	l_1^{HVCF}	a_i
ϕ_{a_1}	1	0.5767	−0.0958	−0.3479	−0.9783	−0.1547
ϕ_{a_2}	1	0.5767	0.0838	−0.1390	−0.9566	−0.5649
ϕ_{a_3}	1	0.5767	0.0833	−0.1503	−0.9746	−0.5351
ϕ_{a_4}	1	0.5767	0.0255	−0.2137	−0.9799	−0.4086
ϕ_{a_5}	1	0.5767	0.0241	−0.2133	−0.9721	−0.4155
ϕ_{a_6}	1	0.5767	0.0102	−0.2419	−0.9728	−0.3722
ϕ_{a_7}	1	0.5767	0.0458	−0.1980	−0.9771	−0.4474
ϕ_{a_8}	1	0.5767	0.0459	−0.1984	−0.9713	−0.4529
ϕ_{a_9}	1	0.5767	0.0065	−0.2306	−0.9741	−0.3785
$\phi_{a_{10}}$	1	0.5767	0.0238	−0.2151	−0.9737	−0.4117
$\phi_{a_{11}}$	1	0.5767	0.0620	−0.1623	−0.9690	−0.5075
$\phi_{a_{12}}$	1	0.5688	−0.1146	−0.3667	−0.9790	−0.1085
$\phi_{a_{13}}$	1	0.5580	−0.0438	−0.2959	−0.9824	−0.2360
$\phi_{a_{14}}$	1	0.5767	−0.0176	−0.2698	−0.9731	−0.3162
$\phi_{a_{15}}$	1	0.5767	0.0508	−0.1957	−0.9697	−0.4621
$\phi_{a_{16}}$	1	0.5767	0.0022	−0.2499	−0.9777	−0.3512
$\phi_{a_{17}}$	1	0.5767	0.0224	−0.2079	−0.9670	−0.4242
$\phi_{a_{18}}$	1	0.5767	0.0844	−0.1613	−0.9685	−0.5313
$\phi_{a_{19}}$	1	0.5767	0.0494	−0.2028	−0.9748	−0.4485
$\phi_{a_{20}}$	1	0.5767	0.0400	−0.2121	−0.9737	−0.4309
$\phi_{a_{21}}$	1	0.5767	0.0237	−0.2284	−0.9747	−0.3972
$\phi_{a_{22}}$	1	0.5767	0.0120	−0.2401	−0.9752	−0.3734
$\phi_{a_{23}}$	1	0.5767	0.0099	−0.2310	−0.9812	−0.3745
$\phi_{a_{24}}$	1	0.5767	0.0913	−0.1479	−0.9694	−0.5508
$\phi_{a_{25}}$	1	0.5767	0.0765	−0.1633	−0.9712	−0.5187
$\phi_{a_{26}}$	1	0.5767	0.0401	−0.2120	−0.9709	−0.4338
$\phi_{a_{27}}$	1	0.5767	0.0283	−0.2027	−0.9666	−0.4357
$\phi_{a_{28}}$	1	0.5767	0.0896	−0.1157	−0.9507	−0.5999
$\phi_{a_{29}}$	1	0.5767	0.0894	−0.1258	−0.9615	−0.5789
$\phi_{a_{30}}$	1	0.5767	0.0829	−0.1672	−0.9702	−0.5222
$\phi_{a_{31}}$	1	0.5767	0.0731	−0.1664	−0.9715	−0.5119
$\phi_{a_{32}}$	1	0.5767	0.0871	−0.1594	−0.9730	−0.5315
$\phi_{a_{33}}$	1	0.5767	0.0840	−0.1607	−0.9767	−0.5233
$\phi_{a_{34}}$	1	0.5767	0.0637	−0.1606	−0.9765	−0.5032
$\phi_{a_{35}}$	1	0.5767	0.0839	−0.1542	−0.9768	−0.5296
$\phi_{a_{36}}$	1	0.5767	0.0848	−0.1101	−0.9591	−0.5923
$\phi_{a_{37}}$	1	0.5767	0.0865	−0.1031	−0.9521	−0.6080
$\phi_{a_{38}}$	1	0.5767	0.0884	−0.0983	−0.9474	−0.6194

Table 12. Net outranking flows of the causes of delays with respect to the LVCF-limiting profiles.

	l_1^{LVCF}	l_2^{LVCF}	l_3^{LVCF}	l_4^{LVCF}	l_1^{LVCF}	a_i
ϕ_{a_1}	1	0.4303	−0.1641	−0.5736	−0.9208	0.2281
ϕ_{a_2}	1	0.5788	0.0449	−0.4928	−0.8991	−0.2317
ϕ_{a_3}	1	0.5757	0.0335	−0.5041	−0.9171	−0.1880
ϕ_{a_4}	1	0.5383	−0.0299	−0.5232	−0.9224	−0.0628
ϕ_{a_5}	1	0.5368	−0.0294	−0.5154	−0.9146	−0.0774
ϕ_{a_6}	1	0.5349	−0.0581	−0.5507	−0.9153	−0.0109
ϕ_{a_7}	1	0.5586	−0.0142	−0.5248	−0.9196	−0.1000
ϕ_{a_8}	1	0.5558	−0.0145	−0.5194	−0.9138	−0.1080
ϕ_{a_9}	1	0.5193	−0.0468	−0.5174	−0.9166	−0.0386
$\phi_{a_{10}}$	1	0.5366	−0.0313	−0.5170	−0.9162	−0.0721
$\phi_{a_{11}}$	1	0.5625	0.0222	−0.5155	−0.9115	−0.1577
$\phi_{a_{12}}$	1	0.4116	−0.1792	−0.5743	−0.9215	0.2633
$\phi_{a_{13}}$	1	0.4726	−0.1105	−0.5419	−0.9249	0.1047
$\phi_{a_{14}}$	1	0.5023	−0.0859	−0.5373	−0.9156	0.0366
$\phi_{a_{15}}$	1	0.5590	−0.0119	−0.5200	−0.9122	−0.1150
$\phi_{a_{16}}$	1	0.5262	−0.0661	−0.5450	−0.9202	0.0051
$\phi_{a_{17}}$	1	0.5351	−0.0241	−0.5103	−0.9095	−0.0913
$\phi_{a_{18}}$	1	0.5788	0.0225	−0.5151	−0.9110	−0.1752
$\phi_{a_{19}}$	1	0.5670	−0.0189	−0.5356	−0.9173	−0.0951
$\phi_{a_{20}}$	1	0.5657	−0.0283	−0.5478	−0.9162	−0.0734
$\phi_{a_{21}}$	1	0.5498	−0.0446	−0.5611	−0.9172	−0.0269
$\phi_{a_{22}}$	1	0.5381	−0.0563	−0.5705	−0.9177	0.0063
$\phi_{a_{23}}$	1	0.5227	−0.0471	−0.5260	−0.9237	−0.0260
$\phi_{a_{24}}$	1	0.5788	0.0359	−0.5017	−0.9118	−0.2012
$\phi_{a_{25}}$	1	0.5788	0.0205	−0.5171	−0.9137	−0.1685
$\phi_{a_{26}}$	1	0.5629	−0.0282	−0.5434	−0.9134	−0.0779
$\phi_{a_{27}}$	1	0.5411	−0.0189	−0.5099	−0.9091	−0.1033
$\phi_{a_{28}}$	1	0.5788	0.0681	−0.4696	−0.8932	−0.2842
$\phi_{a_{29}}$	1	0.5788	0.0581	−0.4796	−0.9040	−0.2533
$\phi_{a_{30}}$	1	0.5788	0.0166	−0.5211	−0.9127	−0.1616
$\phi_{a_{31}}$	1	0.5761	0.0175	−0.5202	−0.9140	−0.1593
$\phi_{a_{32}}$	1	0.5788	0.0245	−0.5132	−0.9155	−0.1746
$\phi_{a_{33}}$	1	0.5763	0.0232	−0.5145	−0.9192	−0.1658
$\phi_{a_{34}}$	1	0.5694	0.0267	−0.5110	−0.9190	−0.1661
$\phi_{a_{35}}$	1	0.5762	0.0296	−0.5080	−0.9193	−0.1786
$\phi_{a_{36}}$	1	0.5771	0.1013	−0.4363	−0.9016	−0.3405
$\phi_{a_{37}}$	1	0.5788	0.0939	−0.4437	−0.8946	−0.3344
$\phi_{a_{38}}$	1	0.5788	0.1028	−0.4348	−0.8899	−0.3569

Table 13. Net outranking flows of the causes of delays with respect to the MVCF-limiting profiles.

	l_1^{MVCF}	l_2^{MVCF}	l_3^{MVCF}	l_4^{MVCF}	l_1^{MVCF}	a_i
ϕ_{a_1}	1	0.3770	−0.0695	−0.6153	−0.9221	0.2299
ϕ_{a_2}	1	0.5283	0.1394	−0.5384	−0.9004	−0.2289
ϕ_{a_3}	1	0.5350	0.1281	−0.5497	−0.9184	−0.1949
ϕ_{a_4}	1	0.4768	0.0647	−0.5649	−0.9237	−0.0529
ϕ_{a_5}	1	0.4694	0.0651	−0.5571	−0.9159	−0.0615
ϕ_{a_6}	1	0.4761	0.0365	−0.5923	−0.9166	−0.0036
ϕ_{a_7}	1	0.4941	0.0803	−0.5665	−0.9209	−0.0871
ϕ_{a_8}	1	0.4883	0.0800	−0.5611	−0.9151	−0.0922
ϕ_{a_9}	1	0.4541	0.0478	−0.5590	−0.9179	−0.0250
$\phi_{a_{10}}$	1	0.4692	0.0633	−0.5587	−0.9175	−0.0563
$\phi_{a_{11}}$	1	0.5400	0.1167	−0.5611	−0.9128	−0.1828
$\phi_{a_{12}}$	1	0.3589	−0.0812	−0.6159	−0.9228	0.2610
$\phi_{a_{13}}$	1	0.4134	0.0004	−0.5836	−0.9262	0.0960
$\phi_{a_{14}}$	1	0.4348	0.0086	−0.5790	−0.9169	0.0524

Table 13. *Cont.*

	l_1^{MVCF}	l_2^{MVCF}	l_3^{MVCF}	l_4^{MVCF}	l_1^{MVCF}	a_i
$\phi_{a_{15}}$	1	0.4916	0.0827	−0.5617	−0.9135	−0.0992
$\phi_{a_{16}}$	1	0.4623	0.0284	−0.5866	−0.9215	0.0174
$\phi_{a_{17}}$	1	0.4697	0.0704	−0.5520	−0.9108	−0.0773
$\phi_{a_{18}}$	1	0.5382	0.1171	−0.5607	−0.9123	−0.1822
$\phi_{a_{19}}$	1	0.5002	0.0756	−0.5773	−0.9186	−0.0798
$\phi_{a_{20}}$	1	0.5030	0.0663	−0.5894	−0.9175	−0.0623
$\phi_{a_{21}}$	1	0.4929	0.0499	−0.6028	−0.9185	−0.0215
$\phi_{a_{22}}$	1	0.4817	0.0383	−0.6121	−0.9190	0.0112
$\phi_{a_{23}}$	1	0.4623	0.0474	−0.5676	−0.9250	−0.0171
$\phi_{a_{24}}$	1	0.5337	0.1305	−0.5473	−0.9131	−0.2037
$\phi_{a_{25}}$	1	0.5442	0.1151	−0.5627	−0.9150	−0.1815
$\phi_{a_{26}}$	1	0.4987	0.0663	−0.5851	−0.9147	−0.0652
$\phi_{a_{27}}$	1	0.4744	0.0757	−0.5515	−0.9104	−0.0882
$\phi_{a_{28}}$	1	0.5456	0.1612	−0.5152	−0.8945	−0.2971
$\phi_{a_{29}}$	1	0.5463	0.1526	−0.5252	−0.9053	−0.2684
$\phi_{a_{30}}$	1	0.5243	0.1111	−0.5659	−0.9140	−0.1555
$\phi_{a_{31}}$	1	0.5445	0.1120	−0.5658	−0.9153	−0.1754
$\phi_{a_{32}}$	1	0.5375	0.1190	−0.5588	−0.9168	−0.1809
$\phi_{a_{33}}$	1	0.5331	0.1177	−0.5601	−0.9205	−0.1702
$\phi_{a_{34}}$	1	0.5463	0.1212	−0.5566	−0.9203	−0.1907
$\phi_{a_{35}}$	1	0.5395	0.1242	−0.5536	−0.9206	−0.1895
$\phi_{a_{36}}$	1	0.5463	0.1959	−0.4819	−0.9029	−0.3574
$\phi_{a_{37}}$	1	0.5463	0.1853	−0.4893	−0.8959	−0.3464
$\phi_{a_{38}}$	1	0.5463	0.1913	−0.4805	−0.8912	−0.3661

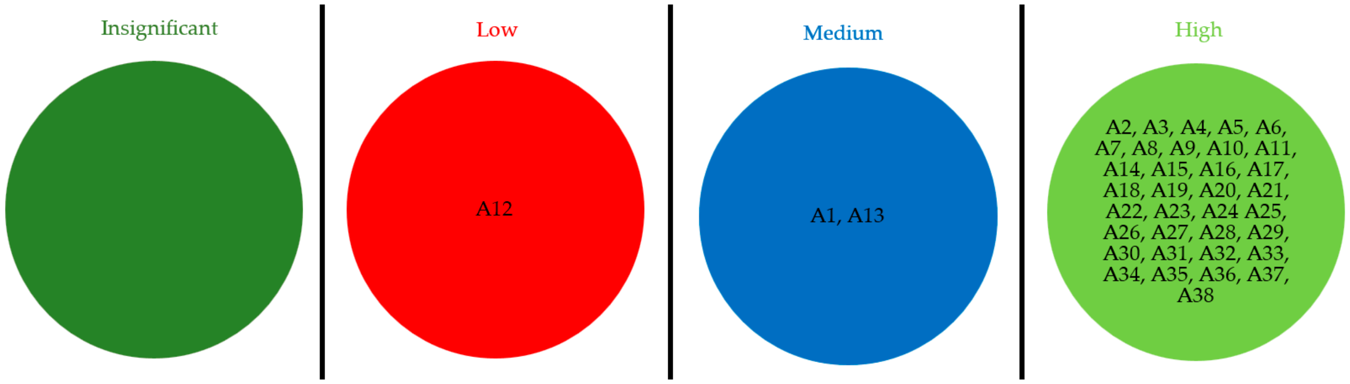

Figure 2. Sorting of construction project delay causes for HVCFs.

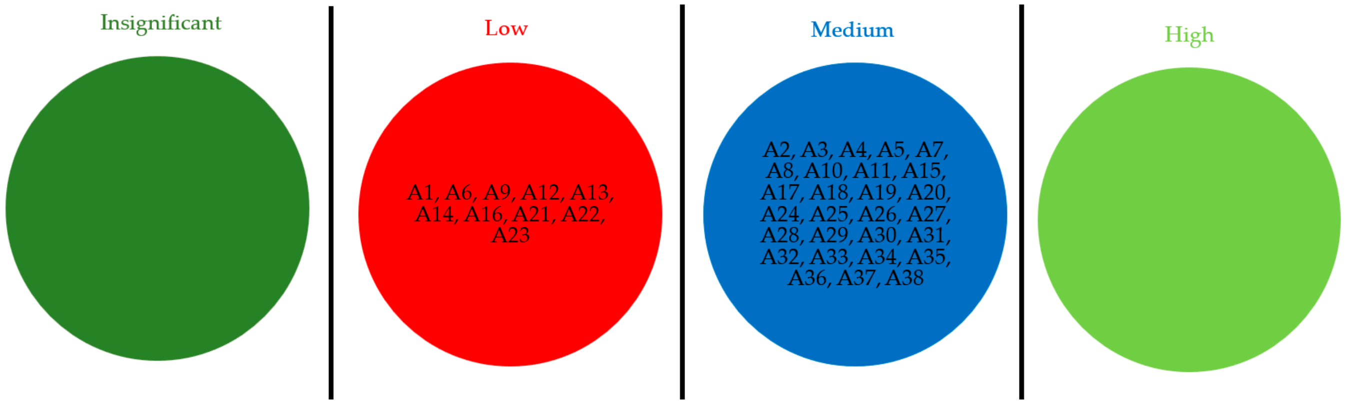

Figure 3. Sorting of construction project delay causes for LVCFs.

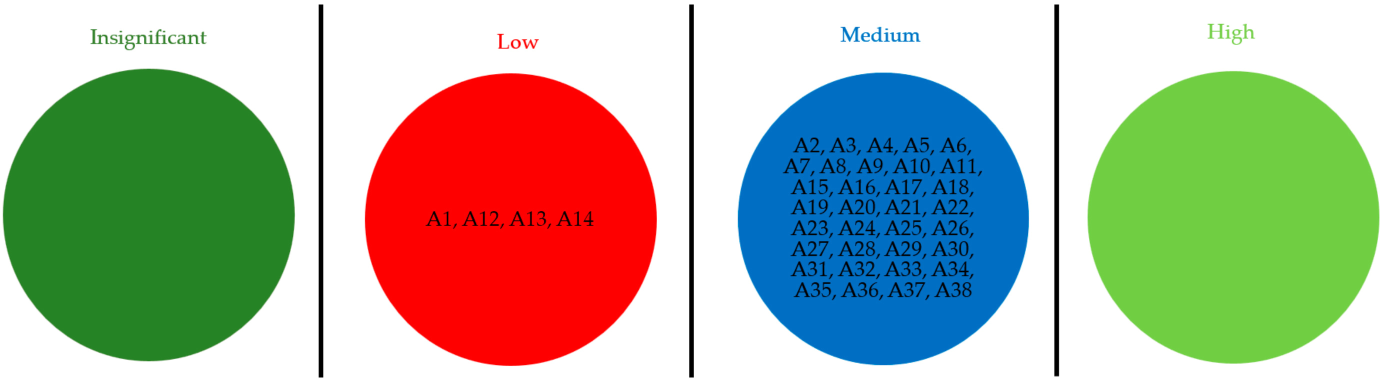

Figure 4. Sorting of construction project delay causes for MVCFs.

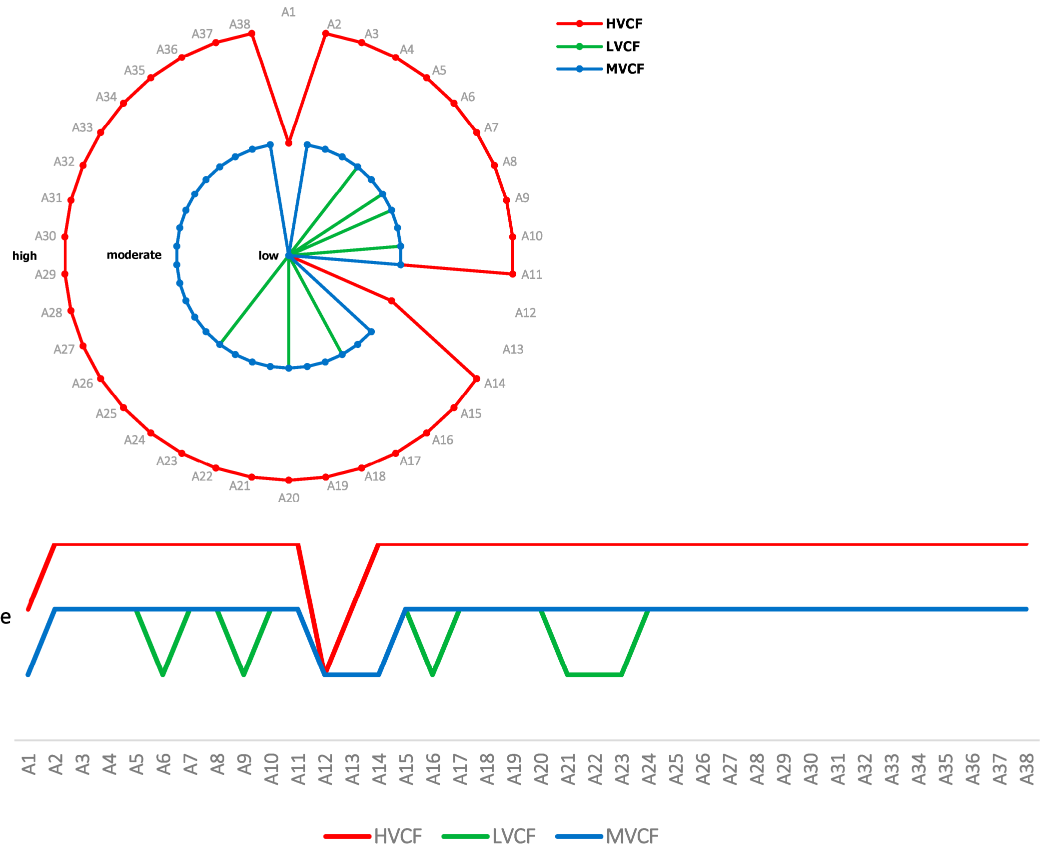

Figure 5. Summary of sorting results of construction project delay causes for HVCFs, LVCFs, and MVCFs.

4. Sensitivity and Comparative Analyses

4.1. Sensitivity Analysis

A sensitivity analysis was conducted to assess the robustness of the proposed integrated q-ROF–WINGS–LBWA–FlowSort method. Changes in the assignment of the causes of construction delays were observed with respect to the modifications of the q parameter. In dealing with q-ROFS, the q parameter is determined using the domain knowledge of the decision makers. To demonstrate the stability of the proposed method regardless of the value of the q parameter, T iterations of the q-ROF–WINGS–LBWA–FlowSort method were performed. For demonstrative purposes, the limiting profiles of MVCFs are presented. To evaluate the changes in the assignment of the proposed sorting method, the percentage of the assignment of each cause of delay to a specific category is calculated through

$\mathcal{P}_{ih} = \frac{\sum_{q=2}^{T} p_{qih}}{T}$, $p_{qih} \in \{0, 1\}$, where $p_{qih} = 1$ represents that the ith delay cause is assigned to hth category at a given q, $q = 2, \ldots, T$; otherwise, $p_{qih} = 0$. T is the total number of iterations. For illustration, $T = 100$. Table 14 summarizes the $\mathcal{P}_{ih}$ values of all delay causes.

Table 14. The percentage of frequency of the assignment of delay causes.

Delay Causes	High	Medium	Low	Insignificant
A_1	0	0.8990	0.1010	0
A_2	0.8687	0.1313	0	0
A_3	0.8485	0.1515	0	0
A_4	0	0.9798	0.0202	0
A_5	0	0.9798	0.0202	0
A_6	0	0.9697	0.0303	0
A_7	0	0.9899	0.0101	0
A_8	0	0.9899	0.0101	0
A_9	0	0.9697	0.0303	0
A_{10}	0	0.9798	0.0202	0
A_{11}	0.8283	0.1616	0.0101	0
A_{12}	0	0.8990	0.0909	0.0101
A_{13}	0	0.9495	0.0505	0
A_{14}	0	0.9596	0.0404	0
A_{15}	0	0.9899	0.0101	0
A_{16}	0	0.9697	0.0303	0
A_{17}	0	0.9798	0.0202	0
A_{18}	0.4343	0.5657	0	0
A_{19}	0	0.9798	0.0202	0
A_{20}	0.3232	0.6566	0.0202	0
A_{21}	0	0.9697	0.0303	0
A_{22}	0	0.9697	0.0303	0
A_{23}	0	0.9697	0.0303	0
A_{24}	0.8586	0.1414	0	0
A_{25}	0.8182	0.1818	0	0
A_{26}	0	0.9798	0.0202	0
A_{27}	0	0.9899	0.0101	0
A_{28}	0.8788	0.1212	0	0
A_{29}	0.8687	0.1313	0	0
A_{30}	0.4242	0.5758	0	0
A_{31}	0.8182	0.1818	0	0
A_{32}	0.8384	0.1515	0.0101	0
A_{33}	0.8485	0.1414	0.0101	0
A_{34}	0.8485	0.1414	0.0101	0
A_{35}	0.8485	0.1414	0.0101	0
A_{36}	0.9293	0.0707	0	0
A_{37}	0.9091	0.0909	0	0
A_{38}	0.9192	0.0808	0	0

It can be observed that 15 delay causes are predominantly assigned in the high category. Delay causes A_2, A_3, A_{11}, A_{24}, A_{25}, A_{28}, A_{29}, A_{31}, A_{32}, A_{33}, A_{34}, A_{35}, A_{36}, A_{37}, and A_{38} are assigned to the high category for more than 80% of the total number of iterations. Meanwhile, the remaining 23 delay causes are mainly in the medium category. As illustrated in Figure 6, the higher the value of q, the more likely the delay will be assigned to the medium and high categories. Consequently, according to Yager [64], a higher value of q translates to a higher hesitancy degree. Hence, assigning delay causes to the medium and high categories may be attributed to the increasing value of hesitancy degrees in evaluating such delay causes.

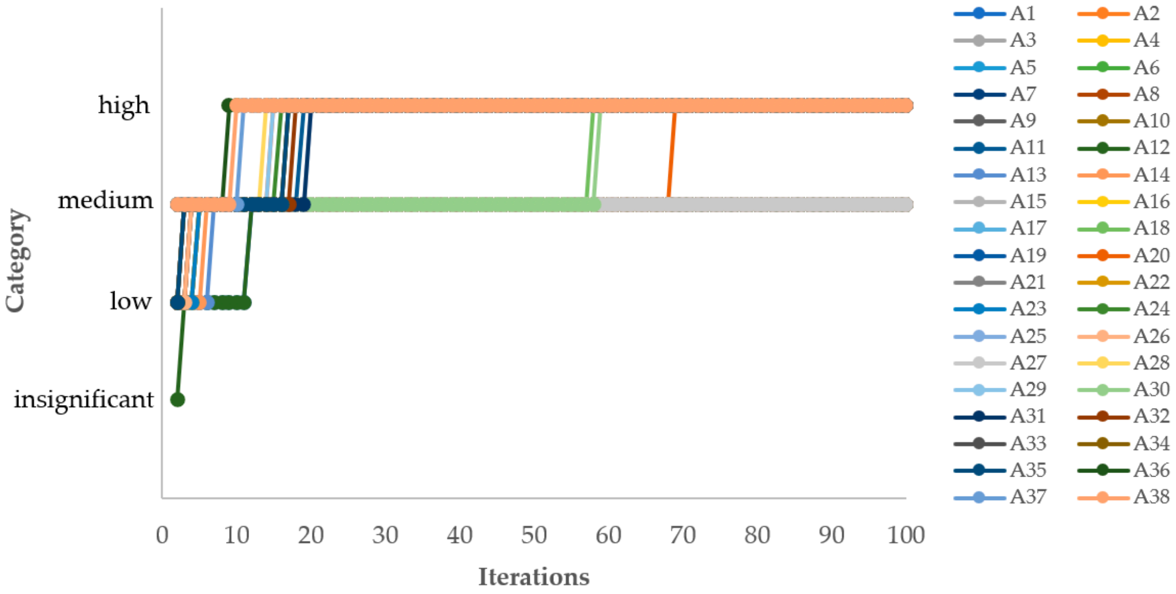

Figure 6. Scatter graph of the assignment of delay causes at t iterations.

4.2. Comparative Analysis

Several aggregation operators have been employed in decision-making problems to determine the consensual agreement of multiple evaluations of a given alternative. In the proposed q-ROF–WINGS–LBWA–FlowSort method, the q-ROF weighted average operator (q-ROFWA) by Liu and Wang [74] was utilized to aggregate the individual evaluation responses. Aside from the utilized q-ROFWA, several q-ROF aggregation operators anchored in the Hamacher operator, point operator, Dombi operator, and exponential operator were proposed by various scholars. Darko and Liang [86] introduced the weighted q-ROF Hamacher average (Wq-ROHA) operator. The q-ROF point weighted average (q-ROFPoWA) was presented by Xing et al. [87]. Meanwhile, the q-ROF Dombi weighted average (q-ROFDWA) was explored by Jana et al. [77]. Lastly, the q-ROF weighted exponential aggregation (q-ROFWEA) operator was defined by Peng et al. [88]. A comparative analysis of the assignment of alternatives to the categories was conducted using the five aggregation operators. The analysis is performed utilizing the similarity ratio metric S_r proposed by Keshavarz Ghorabee et al. [89], illustrated as follows:

$$S_r = \frac{\sum_{i=1}^{m} w_i(x_i, y_i)}{m}, x_i, y_i \in \{High, Medium, Low, Insignificant\} \tag{37}$$

where $w_i(x_i, y_i) = \begin{cases} 1 \; if \; x_i = y_i \\ 0 \; if \; x_i \neq y \end{cases}$ and m is the number of alternatives, x_i is the category of the ith alternative using one aggregation operator, while y_i is the category of the ith alternative using another aggregation operator. When $S_r = 1$, then the two methods have full agreement on all assignments of alternatives. The resulting similarity ratio metric is presented in Table 15. It can be seen in Table 15 that the assignments using QRFOWA have a high similarity with the assignments using Wq-ROFHA and q-ROFDWA, wherein the similarity is more than 80%. On the other hand, the results using q-ROFPoWA yield a low similarity ratio compared to other aggregation operators. These results may be attributed to the concept of point operators, wherein the hesitancy function of the q-ROFS was distributed to the membership and non-membership functions. Furthermore, they generate no similar assignment results with other aggregation operators. The exponential operator considers the weight coefficient as the bases of the operators, resulting in low values of aggregate membership and non-membership functions. Hence, all alternatives are sorted into the low category.

Table 15. Comparing the assignments using QRFOWA, Wq-ROFHA, q-ROFDWA, q-ROFPoWA, and q-ROFWEA.

	QRFOWA	Wq-ROFHA	q-ROFDWA	q-ROFPoWA	q-ROFWEA
QRFOWA	1	0.8684	0.8947	0.5263	0.0000
Wq-ROFHA	-	1	0.7632	0.3947	0.0000
q-ROFDWA	-	-	1	0.5789	0.0000
q-ROFPoWA	-	-	-	1	0.0000
q-ROFWEA	-	-	-	-	1

Aside from the aggregation operator, the score function is one of the significant operators in the proposed method. In most cases, the mapping of fuzzy numbers to their corresponding crisp values is conducted using score functions. Several studies have introduced score functions with distinct strengths in the formulation. Table 1 in Section 2 presents a selected number of score functions. To evaluate the consistency of the assignment results, a comparative analysis is conducted using different score functions. The score functions $\mathbb{S}(\ddot{q})$, $\mathbb{S}_{JW}(\ddot{q})$, $\mathbb{S}_b(\ddot{q})$, $\mathbb{S}_{fl}(\ddot{q})$, and $\mathbb{S}_{rm}(\ddot{q})$ were integrated into Step 4 and Step 15 in the proposed method (see Section 3.2). The similarity ratio metric S_r in Equation (37) was utilized for the analysis. Table 16 presents the S_r results using the five score functions. Based on Table 16, the similarity ratio of the assignment results using the five score functions is more than 70%. Hence, the assignment of the alternative remains consistent regardless of the choice of a score function.

Table 16. Comparing the assignments using $\mathbb{S}(\ddot{q})$, $\mathbb{S}_{JW}(\ddot{q})$, $\mathbb{S}_b(\ddot{q})$, $\mathbb{S}_{fl}(\ddot{q})$, and $\mathbb{S}_{rm}(\ddot{q})$.

	$\mathbb{S}(\ddot{q})$	$\mathbb{S}_{JW}(\ddot{q})$	$\mathbb{S}_b(\ddot{q})$	$\mathbb{S}_{fl}(\ddot{q})$	$\mathbb{S}_{rm}(\ddot{q})$
$\mathbb{S}(\ddot{q})$	1	0.8421	0.7895	0.8684	0.9474
$\mathbb{S}_{JW}(\ddot{q})$	-	1	0.9474	0.9737	0.7895
$\mathbb{S}_b(\ddot{q})$	-	-	1	0.9211	0.7368
$\mathbb{S}_{fl}(\ddot{q})$	-	-	-	1	0.8158
$\mathbb{S}_{rm}(\ddot{q})$	-	-	-	-	1

5. Discussion and Insights

Considering the inherent interrelationships of the three project management factors and the ambiguity concerning these factors, the integration of the WINGS and LBWA methods under a q-ROFS environment becomes relevant for analyzing their priority weights. Results indicate that the assigned weight for the time criterion yields the highest, followed by cost, with the least weight assigned to quality. In construction projects, time and cost overruns are considered highly interrelated and more significant than the impact of the delays on quality. Understanding the impact of the causes of delays to these three factors can help manage projects, especially for small-sized construction firms. These firms may not easily adapt to the impact of delays and may be unable to recover overruns faster than middle- and large-sized construction firms. Understanding the impacts of various construction delays on these factors can aid decision makers in these firms.

The impact of the causes of delays to highly vulnerable, moderately vulnerable, and least-vulnerable groups vary considerably. Four categories of impacts are defined in this study, namely the High Category (HC), Medium Category (MC), Low Category (LC), and Insignificant Category (IC), with HC imposing the greatest impact on the delay of the construction project and IC causing the least impact. For the highly vulnerable construction firms, thirty-five causes fall under HC, two under MC and one under LC. For the moderately vulnerable construction firms, thirty-four causes fall under MC and four under LC. For the least vulnerable category, twenty-eight MC and ten LC were recorded. Of the thirty-eight causes of delays, a number of them have impacts that greatly vary across the different vulnerability groups of construction firms.

To gain insights into the findings of the MCS problems, we divided the implications of the results into clusters of delay causes: (1) design stage, (2) material unavailability, (3) shortage of manpower, (4) force majeure, (5) subcontractors and suppliers, and (6) equipment. First, the design stage happens before the construction stage. Delays in design information vary directly as the construction firm's vulnerability group changes. LVCFs, which are more established construction firms, are less affected as a network of consultants and in-house designers may already be available. The case is the opposite for HVCFs. LVCFs can have more control over in-house designers by strictly imposing deadlines for design outputs. Meanwhile, the approval of design drawings and information is highly dependent on the owner and beyond the contractor's control. Thus, it still has moderate impacts on LVCFs and MVCFs, while it has a high impact on HVCFs. While commencements of project implementation are targeted to suit favorable weather conditions (e.g., excavation works during summer or dry season), these projected timelines may cause an offset of schedule because of uncontrollable client demands and approval.

Secondly, in relation to construction materials, the impact of the low quality of materials varies greatly in the three vulnerability categories—HC for HVCFs, MC for MVCFs, and LC for LVCFs. This insight may be due to LVCFs having direct access and relations to local and global suppliers. LVCFs have opportunities for private product presentations and plant tours as they purchase large quantities. Moreover, they have the leverage to hire capable technical personnel who can perform quality checks and controls. However, for issues related to material unavailability, HVCFs are highly impacted, while the other two categories are moderately impacted. The criticality of material unavailability lies in the notion that a global shortage of supplies or materials impacts all construction firms almost equally. This insight can be extended to price escalation. An example is the effect of the 2001 Olympic Games in China. After China successfully hosted the Olympic games, China's steel demand increased significantly, resulting in more steel importation demand from foreign countries. Lin and Wu [90] observed that this has largely affected Taiwan's steel market. This also resulted in China becoming the main driving force behind the increase seen in global steel prices since 2002 as a result of China's growth in urbanization and construction of enormous sports facilities due to their hosting of the Olympics. Moreover, it was suggested that with the increase in demand for steel raises, construction firms, being consumers, preorder or postpone the purchase of materials to avoid profit loss as a result of price fluctuations, and the government, as policymakers, make use of the demand information to implement decisions such as raising tariffs to prevent excessive price fluctuations in the steel market [90]. For HVCFs with limited financial resources, high volume purchase is less feasible.

Third, due to the construction boom in the country, the Philippines is experiencing a shortage of manpower, both skilled and unskilled. Such a dilemma explains why a shortage of site workers and technical personnel moderately impacts MVCFs and LVCFs and highly impacts HVCFs. Employees are expected to prefer to work in more established firms over smaller and new ones. This has impacted manpower shortage in both MVCFs and LVCFs, with this shortage being slightly higher for HVCFs. Still, concerning manpower problems, labor absenteeism is another cause of delay and is uncontrollably making the impact high for HVCFs, and moderate for both MVCFs and LVCFs, as these firms have established policies regarding employee attendance and retention.

Fourth, force majeure is also experienced onsite, although some examples are avoidable. Flood incidences moderately impact HVCFs but have low impacts on MVCFs and LVCFs. With additional resources, MVCFs and LVCFs can have stand-by pumps and tools to alleviate and prevent further flood-induced damage onsite. Similarly, fires onsite highly impact HVCFs, since they may need additional resources to recover damages from the fire. Additionally, the impact of fires on-site is still moderate for LVCFs and MVCFs, since safety policies and systems are generally in place. For PCAB-licensed companies, a minimum of one safety officer is required on-site, which increases as the number of workers increases. On the other hand, fires in nearby areas in the construction site usually have a low impact on all construction firms.

Fifth, although residential projects are typically small in scale, subcontractors and suppliers are still involved in their construction, especially in supplying highly specialized products or items. Subcontractors, although bound by contracts with the main contractor or owner with specified work duration, are not directly managed by the main contractor. Thus, challenges in integrating work schedules resulting in possible delays in work accomplishment are unavoidable. Delays due to subcontractor work moderately impact LVCFs and MVCFs. For HVCFs, in addition to these delays, changes in subcontractors have high impacts due to the limited network of competent suppliers and contractors. The availability of a network of competent contractors, consultants, and subcontractors contributes to why communication problems and site inspection inadequacy do not have high impacts on LVCFs and MVCFs compared to HVCFs.

Finally, problems concerning equipment may be rare in LVCFs due to the availability of more equipment compared to smaller construction firms. However, this is the opposite in HVCFs, as the impact of equipment allocation problems is high. Similarly, unavoidable equipment breakdown has a moderate impact on both LVCFs and MVCFs and a high impact on HVCFs. Also, since LVCFs have more financial resources, equipment repairs and maintenance are regularly conducted, making the impact of an equipment-related problem low for these firms.

Overall, the causes of delays highly impact small construction firms due to the unavailability of resources and the limited network of suppliers and contractors, among others. As a construction firm becomes more established, recovery from the impacts of the causes of delays becomes faster.

6. Conclusions and Future Work

Despite the popularity of identifying causes of delays in construction projects in the current literature, an overarching evaluation of their impact on projects remains unexplored. Such an agenda would provide insights into the design of targeted efforts that would efficiently guide construction firms, especially small-sized firms with limited financial capabilities, in mitigating these delays. This work overcomes this gap by viewing the evaluation process as an MCS problem, which intends to draw insights into the impacts of delay causes on projects. In the proposed method, an integrated WINGS–LBWA method under a q-ROFS environment assigns the priority weights of project management factors relevant to construction projects, and these are time, cost, and quality. Such adoption resembles the interrelationships and ambiguity inherent in those factors. Then, sorting the impact of delay causes is carried out using the proposed q-ROF–FlowSort method, which can handle a large-scale sorting problem with judgments capturing imprecision and vagueness. The integrated q-ROF–WINGS–LBWA–FlowSort method is deemed novel in this work. An actual case study in small-sized residential projects in the central Philippines demonstrates the efficacy of the proposed approach.

The findings reveal that construction delays have different levels of impact on construction firms. The impact of the causes of delays to highly vulnerable, moderately vulnerable, and least-vulnerable construction firms varies greatly. Four categories of impact are defined in this study. For highly vulnerable construction firms, thirty-five causes of delays fall under the high category, two under the medium category, and one under the low category for highly vulnerable construction firms. For moderately vulnerable construction firms, thirty-four causes of delay fall under the medium category and four under the low category. For the least-vulnerable construction firm, twenty-eight under the medium category and ten under the low category were recorded. Overall, the causes of delays highly impact small-sized construction firms due to the unavailability of resources and a limited network of suppliers and contractors. These findings can guide decision makers from construction firms in minimizing, if not eliminating, construction delays. Such an agenda intends to improve project performance due to the better management of resources, a method that is more relevant to small-sized contractors with significant financial limitations of minimal economies of scale. Layers of sensitivity and comparative analyses were presented to test the robustness of the proposed approach to some model parameters. They show that the proposed q-ROF–WINGS–LBWA–FlowSort approach is robust to changes in aggregation operators and score functions. Meanwhile, the findings brought about by the choice of the q parameter are consistent with prior studies.

This work has some limitations. First, the findings may be confined to the idiosyncrasies of the case study. Thus, direct application of the insights to other conditions outside the case must be made with caution. Longitudinal and spatial works may be necessary to obtain generalized insights into the impact of delay causes on construction projects. Second, the choice of residential projects may be extended in future work to more complex projects, such as high-rise buildings, bridge construction, and ports and harbors. Third, a future study that examines the changes in the findings in view of an expanded set of decision makers may be warranted to gain more robust insights. Finally, other MCS tools and fuzzy environments may be used for the MCS problem posed in this study.

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Appendix A. List of Causes of Construction Delays

Code	Causes of Delays	References						
		Ref. [91]	**Ref. [34]**	**Ref. [35]**	**Ref. [28]**	**Ref. [26]**	**Ref. [36]**	**Ref. [27]**
A1	Accidents on site because of poor site safety		X				X	
A2	Changes in orders and variations	X	X	X	X	X	X	X
A3	Confined site							X
A4	Conflicts and disputes	X	X		X	X	X	X
A5	Delay in approval of drawings				X	X	X	
A6	Delay in the availability of design information				X	X	X	X
A7	Delay in material delivery	X	X		X	X	X	X
A8	Delays in suppliers' work		X		X	X	X	
A9	Equipment allocation problems			X			X	X
A10	Financial difficulties of the owner		X	X	X		X	X
A11	Fire (onsite)				X			
A12	Fire (nearby area)							
A13	Flood				X		X	
A14	Frequent change of subcontractors because of their inefficient work				X		X	
A15	Frequent equipment breakdown		X				X	X
A16	Inadequacy of site inspection							X

Code	**Causes of Delays**	**References**						
		Ref. [91]	**Ref. [34]**	**Ref. [35]**	**Ref. [28]**	**Ref. [26]**	**Ref. [36]**	**Ref. [27]**
A17	Inadequate contractor experience		X	X	X	X	X	
A18	Inappropriate overall organizational structure linking all project teams					X		X
A19	Insufficient amount of equipment			X			X	X
A20	Labor absenteeism		X				X	
A21	Lack of communication between client and contractor		X	X	X	X	X	X
A22	Long waiting time for approval of test samples of materials					X	X	
A23	Low-quality materials							X
A24	Material unavailability		X	X	X	X	X	X
A25	Mistakes and discrepancies in design documents					X	X	
A26	Planning and scheduling problems	X	X	X	X		X	X
A27	Poor site management and supervision			X		X	X	
A28	Price escalation in materials and labor		X					X
A29	Rework due to errors during construction				X			
A30	Shortage of site workers		X	X	X		X	X
A31	Shortage of technical personnel		X	X	X		X	X
A32	Slow decision making				X	X	X	X
A33	Slow permits by government agencies						X	X
A34	Slow response					X	X	X
A35	Suspensions				X		X	
A36	Unforeseen ground conditions					X	X	
A37	Unrealistic contract durations imposed by the client					X	X	
A38	Wind damage			X	X			

Note: Gray-colored cells with an "X" signify the reference source of a given cause of delay.

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