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Asynchronous Switching Control of Discrete Time Delay Linear Switched Systems Based on MDADT

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Abstract: Ideally, switching between subsystems and controllers occurs synchronously. In other words, whenever a subsystem requires switching, its corresponding sub-controller will be promptly activated. However, in reality, due to network delays, system detection, etc., the activation of candidate controllers frequently lags, which causes issues with asynchronous switching between controllers and subsystems. This asynchronous switching problem may affect system performance and even make the system unstable because the state between the subsystem and the controller may be inconsistent, resulting in the controller not being able to control the subsystem correctly. To keep the system stable while using asynchronous switching, this work suggests an asynchronous control technique for a class of discrete linear switching systems with time delay based on the mode-dependent average dwell time (MDADT). First, we construct a state feedback controller and establish a closed-loop system. In the asynchronous and synchronous intervals of subsystems and controllers, different Lyapunov functions are selected, and sufficient conditions for exponential stability and the H_∞ performance of the closed-loop system under asynchronous switching are obtained. In addition, using the MDADT switching strategy, the relevant parameters of each subsystem are designed and the corresponding state–feedback controller gain matrix can be obtained. Finally, a switching system with three subsystems is shown. The approach is confirmed by simulating it using the average dwell time (ADT) switching strategy and the MDADT switching strategy separately.

Keywords: asynchronous control; average dwell time; exponential stability; mode dependent; time delay

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1. Introduction

A discrete system refers to all systems that are not continuous in time and space. They are ubiquitous in practical problems. It is expected that when computers assist people in simulating, controlling, and analyzing systems, they usually need to discretize time. In addition, there are some discrete mathematical models in the fields of biology, industry, and economics. For example, research on image encryption [1], human infectious diseases [2], and changes in the market economy [3] all require the application of discrete systems. A discrete switching system is an important type of discrete system. It realizes different functions and behaviors through state switching, and it can also realize complex logic operations, control, and decision-making functions.

The benefits of switching systems in terms of model composition have garnered increased attention. Switched systems are not only widely studied in theory but also have extensive applications in many engineering fields, such as electronic equipment [4], aerospace technology [5], traffic management [6], environmental governance [7], and others. The switched system is often a dynamical system made up of switching signals and a finite number of continuous or discrete dynamic subsystems. A piecewise constant function that depends on state or time is the switching signal, which is also known as the switching rule, switching strategy, or switching law. The stability of the switching system is somewhat impacted by the choice of switching signal. When an inappropriate switching signal is selected, the trajectories of the switched system that may make all subsystems stable are divergent; similarly, when a suitable switching signal is selected, it is possible to stabilize a switched system with unstable subsystems.

Research on switching signals is mainly reflected in the design of its dwell time. At present, typical research on switching signals mainly includes arbitrary switching signals [8], dwell time switching signals (DT) [9], average dwell time switching signals (ADT) [10], mode-dependent average dwell time switching signals (MDADT) [11], and persistent dwell time switching signals (PDT) [12]. DT restricts the switching time of two consecutive subsystems to be no less than a constant τ. ADT requires a constant τ_a for the average running time of the subsystem in the limited switching interval. MDADT makes each subsystem have its own ADT, which reduces the conservativeness of research, so it has a broader range of applications compared with the former two. The PDT switching mechanism consists of fast switching and slow switching alternately: the τ part can represent slow switching and T part can represent fast switching. However, DT and ADT cannot represent fast switching because they have strict constraints on the number of switches in a given period. As this implies, PDT is more general than DT and ADT. The complexity of PDT is much higher than that of DT and ADT, so there are few related studies.

There have been abundant research achievements in switched systems on system stability analysis [13], tracking control problems [14], time delay problems [15], robust control [16], gain analysis [17], etc. In recent years, scholars have also paid more attention to asynchronous switching control [18–23]. Asynchronous switching refers to a situation where the subsystem does not match the controller; that is, the switching signals of the subsystem and the controller are inconsistent. Since the identification of subsystems and the matching of corresponding controllers takes a certain amount of time, asynchronous situations cannot be avoided. In [24], based on the asynchronous switching of subsystems and filters, the design of the filter for discrete switched T-S fuzzy systems was discussed. The challenge of designing a controller for time delay nonlinear switching systems with asynchronous switching was investigated in [25]. During this operation of the system, the system cannot always operate in an ideal state, and signal interference and fault phenomena are inevitable. If the cause and location of the fault cannot be found in time, certain losses will be caused. Therefore, introducing fault diagnosis and detection mechanisms is an essential means to ensure system security. Considering the filter and subsystem asynchrony, fault detection on switched systems is carried out in [26]. The time trigger control is frequently used in sampling control to sample periodically; however, because it samples in the form of cycles, wasting system resources, the proposed event trigger mechanism aims to overcome this problem by monitoring changes in system performance in real time by designing event triggers. Ref. [27] studies the multi-asynchronous switching issue in switching systems with event triggering. In contrast to the prior asynchrony, the system's stability and controller design are investigated, along with the many asynchronous challenges of subsystems, event triggers, and controllers. Due to the reasons of the system itself or the technical limitations of the measurement means, it is impossible to measure all of the system's status information. By constructing an observer and using the input or output information of the original system to construct a new system, studying the new system allows one to discover the pertinent characteristics of the previous system. In [28], the issue of observer design for nonlinear switching systems with asynchronous situations is covered. There are few studies on asynchronous control of discrete switched systems, and most of them are based on the ADT switching strategy. Thus, combining the MDADT approach with asynchronous switching is essential.

The asynchronous control problem of discrete time delay switched systems is investigated in this work using MDADT. The majority of research on switched systems up to now has centered on synchronous switching. The switched system finds it challenging to sustain synchronous switching in practical circumstances. Since each subsystem and sub-controller in the system operates at a different speed and responds to commands differently, the associated controller may still be working on the current task when the subsystem has begun to execute the next task. This affects the stability of the system by causing confusion over the order in which different subsystems and sub-controllers should be executed. The main problem with current asynchronous switching is how to ensure the stability of the whole system when the subsystem does not match the corresponding sub-controller. In this paper, we present a parameter associated with exponential decay that tackles this problem by restricting the proportion of matching and mismatching durations between the controller and its corresponding subsystem. The majority of earlier investigations used the ADT technique, which meant that the dwell time and other parameters for each subsystem were constant. However, these parameters are not optimal due to the differences in each subsystem. Relatively speaking, MDADT technology is more flexible. It allows each subsystem to choose the most-suitable parameters according to its own needs to determine its dwell time, and it can set the switching delay of each sub-controller so that they do not have to be all the same. The main motivation is to use this feature of MDADT to improve system performance, and this approach to system performance verification differs from earlier ones. The following are its primary contributions:

1. An innovative asynchronous control method is provided for discrete time delay switched systems.

2. Using the MDADT switching strategy, the switched system's exponential stability and the necessary conditions for H_∞ performance are discovered, and the gain matrix of the controller is also computed.

The sections of this essay are organized as follows. The problem and definitions that apply to it are presented in Section 2. In Section 3, adequate requirements for exponential stability of discrete time delay switched systems and H_∞ performance are laid out, and controllers are also built. To test the efficacy of the asynchronous control method, Section 4 presents a discrete time delay switching system with two subsystems. Simulations are run under the ADT switching signal and the MDADT switching signal. Section 5 presents the results of the simulations of the example and the definition proof.

Notations: R^n represents n-dimensional Euclidean vector space, $L_2[0,\infty]$ is square integrable function space, $\|\cdot\|$ stands for Euclidean norm, $X > 0$ and X^T are positive definite matrices and transpose matrices, $\lambda_{min}\{\cdots\}$ and $\lambda_{max}\{\cdots\}$ represent the minimum and maximum eigenvalues of the matrix, respectively, $col\{\cdots\}$ represents a column vector, and $*$ represents the symmetric block in the block matrix.

2. Problem Statement

This section presents the discussed system and the design of the controller.

2.1. System Description

Consider the following discrete time delay linear switched system:

$$\begin{aligned} x(\varsigma+1) &= A_{\sigma(\varsigma)}x(\varsigma) + B_{\sigma(\varsigma)}u(\varsigma) + A_{1\sigma(\varsigma)}x(\varsigma-d) + B_{1\sigma(\varsigma)}\varpi(\varsigma), \varsigma \geq 0 \\ z(\varsigma) &= C_{\sigma(\varsigma)}x(\varsigma) + D_{\sigma(\varsigma)}u(\varsigma), \varsigma \geq 0 \\ x(\varsigma) &= \varphi(\varsigma), \varsigma \in \{-d, \cdots, 0\} \end{aligned} \tag{1}$$

where $x \in R^{n_x}$, $u \in R^{n_u}$, and $z \in R^{n_z}$ are the state vector, control input, and controlled output of the system, respectively; $\varpi \in R^{n_\varpi}$ represents the disturbance input and belongs to $L_2[0,\infty)$; $\varphi(\varsigma)$ represents the initial vector-valued function; and d represents a constant delay time. The symbol $\sigma(\varsigma) : N \to M = \{1, 2, \cdots, m\}$, $m \in N^+$ stands for the switching signal; $\{(\varsigma_0, \sigma(\varsigma_0)), (\varsigma_1, \sigma(\varsigma_1)), \cdots, (\varsigma_l, \sigma(\varsigma_l)), \cdots, l = 0, 1, 2, 3, \cdots\}$ represents the system's switching time sequence; ς_0 indicates the initial switching time; and ς_l indicates the

l-th switching time. If the l-th subsystem is turned on, $\varsigma \in [\varsigma_l, \varsigma_{l+1})$ results. $A_{\sigma(\varsigma)}$, $B_{\sigma(\varsigma)}$, $A_{1\sigma(\varsigma)}$, $B_{1\sigma(\varsigma)}$, $C_{\sigma(\varsigma)}$, and $D_{\sigma(\varsigma)}$ are constant matrices.

Definition 1 ([29]). *For any switching signal $\sigma(\varsigma)$, let $N_{\sigma,j}(\varsigma_0, \varsigma)$ and $T_j(\varsigma_0, \varsigma)$ denote the switching times and running times of the j-th subsystem activated on $[\varsigma_0, \varsigma)$, respectively. If $\exists N_{0j} > 0$, $\tau_{aj} > 0$, such that*

$$N_{\sigma,j}(\varsigma_0, \varsigma) \leq N_{0j}(\varsigma_0, \varsigma) + \frac{T_j(\varsigma_0, \varsigma)}{\tau_{aj}}, \forall \varsigma \geq \varsigma_0 \geq 0,$$

then τ_{aj} and N_{0j} are known as MDADT and the chatter bound, respectively.

Definition 2 ([30]). *The switched system is exponentially stable if there exist constants $c > 0$ and $0 < \zeta < 1$ such that the solution of the system satisfies:*

$$\|x(\varsigma)\|_2 < c\zeta^{(\varsigma - \varsigma_0)} \|x(\varsigma_0)\|_2, \forall \varsigma > \varsigma_0.$$

Definition 3 ([30]). *Given $\delta > 0$ and $c > 0$, if the switching system is exponentially stable and under zero initial conditions, for all nonzero ϖ, the following inequality holds:*

$$\sum_{s=k_0}^{\infty} e^{-cs} z^T(s) z(s) \leq \delta^2 \sum_{s=k_0}^{\infty} \varpi^T(s) \varpi(s).$$

Then the switched system is exponentially stable with an exponential H_∞ index δ.

Lemma 1 ([31]). *For symmetric matrices X and $W > 0$ of any appropriate dimension and for any constant ξ, the following inequalities hold:*

$$-XW^{-1}X \leq \xi^2 W - 2\xi X.$$

2.2. Controller

Establish a state–feedback controller in System (1):

$$u(\varsigma) = K_{\sigma'(\varsigma)} x(\varsigma),$$

where $\sigma'(\varsigma)$ and $K_{\sigma'(\varsigma)}$ represent the switching signal and control gain matrix of the controller, respectively. Let Δ_l be the switching delay time of the controller relative to the subsystem, and satisfy $\Delta_l < \varsigma_{l+1} - \varsigma_l$. Then $\{(\varsigma_0 + \Delta_0, \sigma'(\varsigma_0)), (\varsigma_1 + \Delta_1, \sigma'(\varsigma_1)), \cdots, (\varsigma_l + \Delta_l, \sigma'(\varsigma_l)), \cdots, l = 0, 1, 2, 3, \cdots\}$ is the controller's switching sequence.

2.3. Build a Closed-Loop System

Substituting the dynamic output feedback controller into the switching System (1), the following closed-loop switched system can be obtained:

$$\begin{aligned} x(\varsigma+1) &= (A_{\sigma(\varsigma)} + B_{\sigma(\varsigma)} K_{\sigma'(\varsigma)}) x(\varsigma) + A_{1\sigma(\varsigma)} x(\varsigma - d) + B_{1\sigma(\varsigma)} \varpi(\varsigma), \varsigma \geq 0 \\ z(\varsigma) &= (C_{\sigma(\varsigma)} + D_{\sigma(\varsigma)} K_{\sigma'(\varsigma)}) x(\varsigma), \varsigma \geq 0 \\ x(\varsigma) &= \varphi(\varsigma), \forall \varsigma \in \{-d, \cdots, 0\} \end{aligned} \quad (2)$$

If $\sigma(\varsigma_l) = i$, the i-th subsystem is activated at the moment of system switching ς_l; likewise, if $\sigma(\varsigma_{l-1}) = j$, the j-th subsystem is enabled at the time of system switching ς_{l-1}. $T^-(\varsigma_0, \varsigma)$ and $T^+(\varsigma_0, \varsigma)$ are utilized to indicate the matching and mismatching intervals between the subsystem and the controller while the system is running in $[\varsigma_0, \varsigma)$.$T^-(\varsigma - \varsigma_0)$, and $T^+(\varsigma - \varsigma_0)$ represent the interval lengths of $T^-(\varsigma_0, \varsigma)$ and $T^+(\varsigma_0, \varsigma)$.

In $[\varsigma_l, \varsigma_l + \Delta_l)$, the l-th subsystem has already started running, but due to factors such as controller model recognition, the controller at this time is from the $(l-1)$-th subsystem, which results in a mismatch between the subsystem and the controller. In $[\varsigma_l + \Delta_l, \varsigma_{l+1})$, the l-th controller is activated and the subsystem matches the controller. Thus, System (2) is expressed as follows:

$$\begin{cases} x(\varsigma+1) = \bar{A}_{ij}x(\varsigma) + A_{1i}x(\varsigma-d) + B_{1i}\varpi(\varsigma), \\ z(\varsigma) = \bar{C}_{ij}x(\varsigma), \qquad \varsigma \in [\varsigma_l, \varsigma_l + \Delta_l), \\ x(\varsigma) = \varphi(\varsigma), \end{cases} \tag{3a}$$

$$\begin{cases} x(\varsigma+1) = \bar{A}_{i}x(\varsigma) + A_{1i}x(\varsigma-d) + B_{1i}\varpi(\varsigma), \\ z(\varsigma) = \bar{C}_{i}x(\varsigma), \qquad \varsigma \in [\varsigma_l + \Delta_l, \varsigma_{l+1}), \\ x(\varsigma) = \varphi(\varsigma), \end{cases} \tag{3b}$$

where $\bar{A}_{ij} = A_i + B_iK_j$, $\bar{C}_{ij} = C_i + D_iK_j$, $\bar{A}_i = A_i + B_iK_i$, $\bar{C}_i = C_i + D_iK_i$.

When $\varpi = 0$, considering the stability of the system, System (3) becomes:

$$x(\varsigma+1) = \begin{cases} \bar{A}_{ij}x(\varsigma) + A_{1i}x(\varsigma-d), \varsigma \in [\varsigma_l, \varsigma_l + \Delta_l), \\ \bar{A}_{i}x(\varsigma) + A_{1i}x(\varsigma-d), \varsigma \in [\varsigma_l + \Delta_l, \varsigma_{l+1}), \end{cases} \tag{4}$$

where $\bar{A}_{ij} = A_i + B_iK_j$,$\bar{A}_i = A_i + B_iK_i$.

3. Main Results

For the above switched System (2), this section mainly discusses two issues:

(1) Solve the sufficient conditions for the time delay closed-loop switched System (2) to be exponentially stable and have the H_∞ performance index under asynchronous switching;

(2) Solve for the H_∞ controller gains based on the stability condition.

The following theorem gives sufficient conditions to ensure the exponential stability of System (4) by using multiple Lyapunov functions and the MDADT technique.

Theorem 1. *For System (4), given the parameters $\alpha_i > 0$, $\beta_i > 0$, $0 < \varepsilon_i^* < \beta_i$, and $\mu_i > 1$, if there exist matrices $P_i > 0$, $Q_i > 0$, and $R_i > 0$ satisfying*

$$\begin{bmatrix} -e^{\alpha_i}P_i + Q_i & 0 & (\bar{A}_{ij} - I)^T R_i & \bar{A}_{ij}^T P_i \\ * & -e^{d\alpha_i}Q_i & A_{1i}^T R_i & A_{1i}^T P_i \\ * & * & -d^{-1}R_i & 0 \\ * & * & * & -P_i \end{bmatrix} < 0, \tag{5}$$

$$\begin{bmatrix} -e^{-\beta_i}P_i + Q_i & 0 & (\bar{A}_{i} - I)^T R_i & \bar{A}_{i}^T P_i \\ * & -e^{-d\beta_i}Q_i & A_{1i}^T R_i & A_{1i}^T P_i \\ * & * & -d^{-1}R_i & 0 \\ * & * & * & -P_i \end{bmatrix} < 0, \tag{6}$$

$$P_i \le \mu_i P_j, Q_i \le \mu_i Q_j, R_i \le \mu_i R_j, \tag{7}$$

$$\underset{\varsigma > \varsigma_0}{inf} \frac{T^-_{\sigma(\varsigma_f)}(\varsigma_{f+1} - \varsigma_f)}{T^+_{\sigma(\varsigma_f)}(\varsigma_{f+1} - \varsigma_f)} \ge \frac{\alpha_i + \varepsilon_i^*}{\beta_i - \varepsilon_i^*}, \tag{8}$$

where $f \in \psi(i) = \sigma(k_f) = i$, $i \in M$, then System (4) is exponentially stable, and any MDADT switching signal satisfies

$$\tau_{ai} > \frac{ln(\mu_i\theta_i)}{\varepsilon_i^*}. \tag{9}$$

Proof. When $\varsigma \in [\varsigma_l, \varsigma_l + \Delta_l)$, the subsystem does not match the controller at this time. Consider the following Lyapunov function:

$$V_\alpha(x_\varsigma,\varsigma) = x^T(\varsigma)P_i x(\varsigma) + \sum_{s=\varsigma-d}^{\varsigma-1} e^{\alpha_i(\varsigma-1-s)}x^T(s)Q_i x(s) + \sum_{r=1-d}^{0}\sum_{s=\varsigma+r-1}^{\varsigma-1} e^{\alpha_i(\varsigma-1-s)}h^T(s)R_i h(s), \quad (10)$$

where $h(s) = x(s+1) - x(s)$.

Denote $\Delta V_\alpha(x_\varsigma,\varsigma) = V_\alpha(x_{\varsigma+1},\varsigma+1) - e^{\alpha_i}V_\alpha(x_\varsigma,\varsigma)$ and $\xi(\varsigma) = col(x(\varsigma), x(\varsigma-d))$. Then

$$\begin{aligned}\Delta V_\alpha(x_\varsigma,\varsigma) &= V_\alpha(x_{\varsigma+1},\varsigma+1) - e^{\alpha_i}V_\alpha(x_\varsigma,\varsigma)\\ &\le x^T(\varsigma+1)P_i x(\varsigma+1) - e^{\alpha_i}x^T(\varsigma)P_i x(\varsigma) + x^T(\varsigma)Q_i x(\varsigma)\\ &- e^{d\alpha_i}x^T(\varsigma-d)Q_i x(\varsigma-d) + dh^T(\varsigma)R_i h(\varsigma)\end{aligned} \quad (11)$$

It follows from (4) that

$$x^T(\varsigma+1)P_i x(\varsigma+1) = \xi^T(\varsigma)\begin{bmatrix}\bar{A}_{ij} & A_{1i}\end{bmatrix}^T P_i\begin{bmatrix}\bar{A}_{ij} & A_{1i}\end{bmatrix}\xi(\varsigma), \quad (12)$$

and

$$\begin{aligned}dg^T(\varsigma)R_i g(\varsigma) &= d(x(\varsigma+1) - x(\varsigma))^T R_i(x(\varsigma+1) - x(\varsigma))\\ &= d\xi^T(\varsigma)\begin{bmatrix}\bar{A}_{ij} - I & A_{1i}\end{bmatrix}^T R_i\begin{bmatrix}\bar{A}_{ij} - I & A_{1i}\end{bmatrix}\xi(\varsigma).\end{aligned} \quad (13)$$

From (11)–(13), we can obtain

$$\Delta V_\alpha(x_\varsigma,\varsigma) = V_\alpha(x_\varsigma,\varsigma) - e^{\alpha_i}V_\alpha(x_\varsigma,\varsigma) \le \xi^T(\varsigma)\Omega_{ij}\xi(\varsigma), \quad (14)$$

where

$$\begin{aligned}\Omega_{ij} = &\begin{bmatrix}\bar{A}_{ij} & A_{1i}\end{bmatrix}^T P_i\begin{bmatrix}\bar{A}_{ij} & A_{1i}\end{bmatrix} + diag(-e^{\alpha_i}P_i + Q_i, -e^{d\alpha_i}Q_i)\\ &+ d\begin{bmatrix}\bar{A}_{ij} - I & A_{1i}\end{bmatrix}^T R_i\begin{bmatrix}\bar{A}_{ij} - I & A_{1i}\end{bmatrix}.\end{aligned} \quad (15)$$

According to Schur's complement, Equations (5) and (15) are equivalent; we can obtain

$$\Delta V_\alpha(x_\varsigma,\varsigma) = V_\alpha(x_{\varsigma+1},\varsigma+1) - e^{\alpha_i}V_\alpha(x_\varsigma,\varsigma) \le 0. \quad (16)$$

This implies

$$V_{\alpha\sigma(\varsigma_l)}(x_\varsigma) \le e^{\alpha_{\sigma(\varsigma_l)}(\varsigma-\varsigma_l)}V_{\alpha\sigma(\varsigma_l)}(x_{\varsigma_l}). \quad (17)$$

When $\varsigma \in [\varsigma_l + \Delta_l, \varsigma_{l+1})$, the subsystem is matched with the controller at this time. Consider the following Lyapunov function:

$$V_\beta(x_\varsigma,\varsigma) = x^T(\varsigma)P_i x(\varsigma) + \sum_{s=\varsigma-d}^{\varsigma-1} e^{\beta_i(s-\varsigma+1)}x^T(s)Q_i x(s) + \sum_{r=1-d}^{0}\sum_{s=\varsigma+r-1}^{\varsigma-1} e^{\beta_i(s-\varsigma+1)}h^T(s)R_i h(s), \quad (18)$$

Denote $\Delta V_\beta(x_\varsigma,\varsigma) = V_\beta(x_{\varsigma+1},\varsigma+1) - e^{-\beta_i}V_\beta(x_\varsigma,\varsigma)$. Similarly, we have

$$\Delta V_\beta(x_\varsigma,\varsigma) = V_\beta(x_\varsigma,\varsigma) - e^{-\beta_i}V_\beta(x_\varsigma,\varsigma) \le \xi^T(\varsigma)\Omega_i\xi(\varsigma), \quad (19)$$

where

$$\begin{aligned}\Omega_i = &\begin{bmatrix}\bar{A}_i & A_{1i}\end{bmatrix}^T P_i\begin{bmatrix}\bar{A}_i & A_{1i}\end{bmatrix} + diag(-e^{-\beta_i}P_i + Q_i, -e^{-d\beta_i}Q_i)\\ &+ d\begin{bmatrix}\bar{A}_i - I & A_{1i}\end{bmatrix}^T R_i\begin{bmatrix}\bar{A}_i - I & A_{1i}\end{bmatrix}.\end{aligned} \quad (20)$$

According to Schur's complement, Equations (6) and (20) are equivalent; we can obtain

$$\Delta V_\beta(x_\varsigma,\varsigma)=V_\beta(x_{\varsigma+1},\varsigma+1)-e^{-\beta_i}V_\beta(x_\varsigma,\varsigma)\le 0. \tag{21}$$

Thus,

$$V_{\beta\sigma(\varsigma_l)}(x_\varsigma)\le e^{-\beta_{\sigma(\varsigma_l)}(\varsigma-\varsigma_l-\Delta_l)}V_{\beta\sigma(\varsigma_l)}(x_{\varsigma_l}+\Delta_l). \tag{22}$$

In the whole interval $[0,\varsigma]$, the Lyapunov function consists of (10) and (18):

$$V_{\sigma(\varsigma)}(\varsigma)=\begin{cases}V_{\alpha\sigma(\varsigma)},\varsigma\in[\varsigma_l,\varsigma_l+\Delta_l),\\ V_{\beta\sigma(\varsigma)},\varsigma\in[\varsigma_l+\Delta_l,\varsigma_{l+1}).\end{cases} \tag{23}$$

From (7), (10) and (18), we have

$$V_\beta(x_\varsigma,\varsigma)\le\mu_iV_\alpha(x_\varsigma,\varsigma),V_\alpha(x_\varsigma,\varsigma)\le\theta_iV_\beta(x_\varsigma,\varsigma),(\theta_i=\mu_ie^{(\alpha_i+\beta_i)d}). \tag{24}$$

When $\varsigma\in[\varsigma_l,\varsigma_{l+1})$, from (17), (22) and (24), we have

$$\begin{aligned}V_{\sigma(\varsigma_l)}(x_\varsigma)&\le e^{-\beta_{\sigma(\varsigma_l)}T^-(\varsigma_{l+1}-\varsigma_l)}V_{\beta\sigma(\varsigma_l)}(x_{\varsigma_l+\Delta_l})\\&\le\mu_{\sigma(\varsigma_l)}e^{-\beta_{\sigma(\varsigma_l)}T^-(\varsigma_{l+1}-\varsigma_l)}V_{\alpha\sigma(\varsigma_l)}(x^-_{\varsigma_l+\Delta_l})\\&\le\mu_{\sigma(\varsigma_l)}e^{-\beta_{\sigma(\varsigma_l)}T^-(\varsigma_{l+1}-\varsigma_l)}e^{\alpha_{\sigma(\varsigma_l)}T^+(\varsigma_{l+1}-\varsigma_l)}V_{\alpha\sigma(\varsigma_l)}(x_{\varsigma_l})\\&\le\mu_{\sigma(\varsigma_l)}\theta_{\sigma(\varsigma_l)}e^{-\beta_{\sigma(\varsigma_l)}T^-(\varsigma_{l+1}-\varsigma_l)}e^{\alpha_{\sigma(\varsigma_l)}T^+(\varsigma_{l+1}-\varsigma_l)}V_{\beta\sigma(\varsigma_l)}(x^-_{\varsigma_l})\\&\le\mu_{\sigma(\varsigma_l)}\theta_{\sigma(\varsigma_l)}e^{-\beta_{\sigma(\varsigma_l)}T^-(\varsigma_{l+1}-\varsigma_l)}e^{\alpha_{\sigma(\varsigma_l)}T^+(\varsigma_{l+1}-\varsigma_l)}\\&\times\mu_{\sigma(\varsigma_{l-1})}\theta_{\sigma(\varsigma_{l-1})}e^{-\beta_{\sigma(\varsigma_{l-1})}T^-(\varsigma_l-\varsigma_{l-1})}e^{\alpha_{\sigma(\varsigma_{l-1})}T^+(\varsigma_l-\varsigma_{l-1})}\times V_{\beta\sigma(\varsigma_{l-1})}(x^-_{\varsigma_{l-1}})\\&\le\cdots\le\mu_{\sigma(\varsigma_l)}\theta_{\sigma(\varsigma_l)}\mu_{\sigma(\varsigma_{l-1})}\theta_{\sigma(\varsigma_{l-1})}\cdots\mu_{\sigma(\varsigma_1)}\theta_{\sigma(\varsigma_1)}\times e^{-\beta_{\sigma(\varsigma_l)}T^-(\varsigma_{l+1}-\varsigma_l)}e^{\alpha_{\sigma(\varsigma_l)}T^+(\varsigma_{l+1}-\varsigma_l)}\\&\times e^{-\beta_{\sigma(\varsigma_{l-1})}T^-(\varsigma_l-\varsigma_{l-1})}e^{\alpha_{\sigma(\varsigma_{l-1})}T^+(\varsigma_l-\varsigma_{l-1})}\times\cdots\times e^{-\beta_{\sigma(\varsigma_0)}T^-(\varsigma_1-\varsigma_0)}e^{\alpha_{\sigma(\varsigma_0)}T^+(\varsigma_1-\varsigma_0)}V_{\sigma(\varsigma_0)}\\&=\prod_{i\in M}(\mu_i\theta_i)^{N_{\sigma,i}}\times e^{\sum\limits_{i\in M,f\in\psi(i)}\alpha_iT^+(\varsigma_{f+1}-\varsigma_f)-\beta_iT^-(\varsigma_{f+1}-\varsigma_f)}\times V_{\sigma(\varsigma_0)}(x_{\varsigma_0}).\end{aligned} \tag{25}$$

It follows from (8) that

$$\alpha_iT^+(\varsigma_{l+1}-\varsigma_l)-\beta_iT^-(\varsigma_{l+1}-\varsigma_l)\le-\varepsilon_i^*(\varsigma_{l+1}-\varsigma_l). \tag{26}$$

From (25) and (26), we can obtain

$$\begin{aligned}&V_{\sigma(\varsigma_l)}(x_\varsigma)\\&\le\prod_{i\in M}(\mu_i\theta_i)^{N_{\sigma,i}}\times e^{\sum\limits_{i\in M,f\in\psi(i)}\alpha_iT^+(\varsigma_{f+1}-\varsigma_f)-\beta_iT^-(\varsigma_{f+1}-\varsigma_f)}\times V_{\sigma(\varsigma_0)}(x_{\varsigma_0})\\&\le\prod_{i\in M}(\mu_i\theta_i)^{N_{\sigma,i}}\times e^{\sum\limits_{i\in M,f\in\psi(i)}-\varepsilon_i^*(\varsigma_{f+1}-\varsigma_f)}V_{\sigma(\varsigma_0)}(x_{\varsigma_0})\\&\le e^{\sum\limits_{i\in M}N_{0,i}ln(\mu_i\theta_i)}e^{\sum\limits_{i\in M,f\in\psi(i)}(\frac{ln(\mu_i\theta_i)}{\tau_{ai}}-\varepsilon_i^*)(\varsigma_{f+1}-\varsigma_f)}V_{\sigma(\varsigma_0)}(x_{\varsigma_0}).\end{aligned} \tag{27}$$

Meanwhile, considering the Lyapunov function, there are positive numbers $\tilde{a}$ and $\tilde{b}$ satisfying

$$\tilde{a}\|x(\varsigma)\|^2\le V(x_\varsigma)\le\tilde{b}\|x(\varsigma)\|^2, \tag{28}$$

where

$$\tilde{a} = \min_{i \in M}\{\lambda_{min}(P_i)\},$$
$$\tilde{b} = \max_{i \in M}\{\lambda_{max}(P_i)\} + d \max_{i \in M}\{e^{d\alpha_i}\lambda_{max}(Q_i)\} + \frac{d(d+1)}{2} \max_{i \in M}\{e^{d\alpha_i}\lambda_{max}(R_i)\}.$$

By Definition 2, $c = \sqrt{\frac{\tilde{b}}{\tilde{a}}} e^{\frac{1}{2}\max_{i \in M}\{N_{0,i} ln(\mu_i \theta_i)\}}$ is a constant, and $0 < \zeta = e^{\frac{1}{2}\max_{i \in M}\{\frac{ln(\mu_i \theta_i)}{\tau_{ai}} - \varepsilon_i^*\}} < 1$; we have

$$\|x(\varsigma)\| \leq c\zeta^{(\varsigma - \varsigma_0)}\|x(\varsigma_0)\|. \tag{29}$$

Therefore, System (4) is exponentially stable . □

The following theorem provides sufficient conditions for resolving the controller gain of System (4) based on Theorem 1.

Theorem 2. *For System (4), given the parameters $\alpha_i > 0$, $\beta_i > 0$, $0 < \varepsilon_i^* < \beta_i$, and $\mu_i > 1$, if there exist matrices $X_i > 0$, $Q_i > 0$, $R_i > 0$, and $Y_i > 0$ satisfying*

$$\begin{bmatrix} \Theta_1 & 0 & \Theta_2 & \Theta_3 & X_j \\ * & \Theta_4 & X_i A_{1i}^T & X_i A_{1i}^T & 0 \\ * & * & -d^{-1}R_i' & 0 & 0 \\ * & * & * & -X_i & 0 \\ * & * & * & * & -Q_i' \end{bmatrix} < 0, \tag{30}$$

$$\begin{bmatrix} \Theta_1^1 & 0 & \Theta_2^1 & \Theta_3^1 & X_i \\ * & \Theta_4^1 & X_i A_{1i}^T & X_i A_{1i}^T & 0 \\ * & * & -d^{-1}R_i' & 0 & 0 \\ * & * & * & -X_i & 0 \\ * & * & * & * & -Q_i' \end{bmatrix} < 0, \tag{31}$$

$$\begin{bmatrix} -\mu_i X_j & X_j \\ * & -X_i \end{bmatrix} \leq 0, \begin{bmatrix} -\mu_i Q_j' & Q_j' \\ * & -Q_i' \end{bmatrix} \leq 0, \begin{bmatrix} -\mu_i R_j' & R_j' \\ * & -R_i' \end{bmatrix} \leq 0, \tag{32}$$

where

$$\Theta_1 = e^{\alpha_i}(X_i - 2X_j), \Theta_2 = X_j A_i^T + Y_j^T B_i^T - X_j, \Theta_3 = X_j A_i^T + Y_j^T B_i^T, \Theta_4 = e^{d\alpha_i}(Q_i' - 2X_i),$$
$$\Theta_1^1 = -e^{-\beta_i}X_i, \Theta_2^1 = X_i A_i^T + Y_i^T B_i^T - X_i, \Theta_3^1 = X_i A_i^T + Y_i^T B_i^T, \Theta_4^1 = e^{-d\beta_i}(Q_i' - 2X_i).$$

Therefore, the corresponding state–feedback controller gain matrix $K_i = Y_i X_i^{-1}$ can be obtained.

Proof. Suppose the controller gain $K_i = Y_i P_i$; let $X_i = P_i^{-1}$, $R_i^{-1} = R_i'$ and $Q_i^{-1} = Q_i'$. Multiply both sides of (5) by $diag\{X_j, X_i, R_i', X_i\}$ at the same time, and multiply both sides of (6) by $diag\{X_i, X_i, R_i', X_i\}$ at the same time. Through Lemma 1, we can obtain

$$-X_j P_i X_j \leq X_i - 2X_j, -X_i Q_i X_i \leq Q_i' - 2X_i. \tag{33}$$

By Schur's complement and (33), Conditions (30) and (31) are obtained. In addition, Condition (32) is obtained by multiplying both sides of the inequality (7) by X_i, Q_i', and R_i', respectively.

According to Theorem 1, we establish the exponential stability of System (4) in the absence of perturbations and subsequently demonstrate that System (3) with perturbations satisfies the sufficient conditions for H_∞ performance. □

Theorem 3. *For System (3), given the parameters $\alpha_i > 0$, $\beta_i > 0$, $0 < \varepsilon_i^* < \beta_i$, $\mu_i > 1$, and $\gamma > 0$, if there exist matrices $P_i > 0$, $Q_i > 0$, and $R_i > 0$ satisfying*

$$\begin{bmatrix} -e^{\alpha_i}P_i + Q_i & 0 & 0 & (\bar{A}_{ij} - I)^T R_i & \bar{A}_{ij}^T P_i & \bar{C}_{ij}^T \\ * & -e^{d\alpha_i}Q_i & 0 & A_{1i}^T R_i & A_{1i}^T P_i & 0 \\ * & * & -\gamma I & B_{1i}^T R_i & B_{1i}^T P_i & 0 \\ * & * & * & -d^{-1}R_i & 0 & 0 \\ * & * & * & * & -P_i & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{34}$$

$$\begin{bmatrix} -e^{-\beta_i}P_i + Q_i & 0 & 0 & (\bar{A}_i - I)^T R_i & \bar{A}_i^T P_i & \bar{C}_i^T \\ * & -e^{-d\beta_i}Q_i & 0 & A_{1i}^T R_i & A_{1i}^T P_i & 0 \\ * & * & -\gamma I & B_{1i}^T R_i & B_{1i}^T P_i & 0 \\ * & * & * & -d^{-1}R_i & 0 & 0 \\ * & * & * & * & -P_i & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{35}$$

$$P_i \leq \mu_i P_j, Q_i \leq \mu_i Q_j, R_i \leq \mu_i R_j, \tag{36}$$

$$\underset{\varsigma > \varsigma_0}{inf} \frac{T^{-}_{\sigma(\varsigma_f)}(\varsigma_{f+1} - \varsigma_f)}{T^{+}_{\sigma(\varsigma_f)}(\varsigma_{f+1} - \varsigma_f)} \geq \frac{\alpha_i + \varepsilon_i^*}{\beta_i - \varepsilon_i^*}, \tag{37}$$

where $f \in \psi(i) = \sigma(k_f) = i$, $i \in M$, then the switched System (3) is exponentially stable and has H_∞ performance index $\bar{\gamma} = \sqrt{\gamma}$. Meanwhile, any MDADT switching signal satisfies

$$\tau_{ai} > \frac{ln(\mu_i \theta_i)}{\varepsilon_i^*}. \tag{38}$$

Proof. Consider System (3): denote $F(\varsigma) = z^T(\varsigma)z(\varsigma) - \gamma\varpi^T(\varsigma)\varpi(\varsigma)$.

When $\varsigma \in [\varsigma_l, \varsigma_l + \Delta_l)$, the subsystem does not match the controller at this time; we can obtain

$$V_{\alpha\sigma(\varsigma_l)}(x_\varsigma) \leq e^{\alpha_{\sigma(\varsigma_l)}} V_{\alpha\sigma(\varsigma_l)}(x_{\varsigma-1}) - F(\varsigma - 1). \tag{39}$$

By iterating through the formula, we have

$$\begin{aligned} & V_{\alpha\sigma(\varsigma_l)}(x_\varsigma) \\ & \leq e^{\alpha_{\sigma(\varsigma_l)}} V_{\alpha\sigma(\varsigma_l)}(x_{\varsigma-1}) - F(\varsigma - 1) \\ & \leq e^{\alpha_{\sigma(\varsigma_l)}} (e^{\alpha_{\sigma(\varsigma_l)}} V_{\alpha\sigma(\varsigma_l)}(x_{\varsigma-2}) - F(\varsigma - 2)) - F(\varsigma - 1) \\ & \ldots \\ & \leq e^{\alpha_{\sigma(\varsigma_l)}(\varsigma - \varsigma_l)} V_{\alpha\sigma(\varsigma_l)}(x_{\varsigma_l}) - \sum_{s=\varsigma_l}^{\varsigma_l + \Delta_l - 1} e^{\alpha_{\sigma(\varsigma_l)}(\varsigma_l + \Delta_l - s - 1)} F(s). \end{aligned} \tag{40}$$

When $\varsigma \in [\varsigma_l + \Delta_l, \varsigma_{l+1})$, the subsystem is matched with the controller at this time; we can obtain

$$V_{\beta\sigma(\varsigma_l)}(x_\varsigma) \leq e^{-\beta_i} V_{\beta\sigma(\varsigma_l)}(x_{\varsigma-1}) - F(\varsigma - 1). \tag{41}$$

Similarly, we can obtain

$$\begin{aligned} & V_{\beta\sigma(\varsigma_l)}(x_\varsigma) \\ & \leq e^{-\beta_{\sigma(\varsigma_l)}} V_{\beta\sigma(\varsigma_l)}(x_{\varsigma-1}) - F(\varsigma - 1) \\ & \leq e^{-\beta_{\sigma(\varsigma_l)}} (e^{-\beta_{\sigma(\varsigma_l)}} V_{\sigma(\varsigma_l)}(x_{\varsigma-2}) - F(\varsigma - 2)) - F(\varsigma - 1) \\ & \ldots \\ & \leq e^{-\beta_{\sigma(\varsigma_l)}(\varsigma - \varsigma_l - \Delta_l)} V_{\beta\sigma(\varsigma_l)}(x_{\varsigma_l + \Delta_l}) - \sum_{s=\varsigma_l + \Delta_l}^{\varsigma - 1} e^{-\beta_{\sigma(\varsigma_l)}(\varsigma - s - 1)} F(s). \end{aligned} \tag{42}$$

From (24), combined with equations (40) and (42), when $\varsigma \in [\varsigma_l, \varsigma_{l+1})$, it can be known that

$$
\begin{aligned}
& V_{\sigma(\varsigma_l)}(x_\varsigma) \\
& \leq e^{-\beta_{\sigma(\varsigma_l)}(\varsigma-\varsigma_l-\Delta_l)} V_{\beta\sigma(\varsigma_l)}(x_{\varsigma_l+\Delta_l}) - \sum_{s=\varsigma_l+\Delta_l}^{\varsigma-1} e^{-\beta_{\sigma(\varsigma_l)}(\varsigma-s-1)} F(s) \\
& \leq \mu_{\sigma(\varsigma_l)} e^{-\beta_{\sigma(\varsigma_l)}(\varsigma-\varsigma_l-\Delta_l)} V_{\alpha\sigma(\varsigma_l)}(x^-_{\varsigma_l+\Delta_l}) - \sum_{s=\varsigma_l+\Delta_l}^{\varsigma-1} e^{-\beta_{\sigma(\varsigma_l)}(\varsigma-s-1)} F(s) \\
& \leq \mu_{\sigma(\varsigma_l)} e^{-\beta_{\sigma(\varsigma_l)}(\varsigma-\varsigma_l-\Delta_l)} (e^{\alpha_{\sigma(\varsigma_l)}\Delta_l} V_{\alpha\sigma(\varsigma_l)}(x_{\varsigma_l}) - \sum_{s=\varsigma_l}^{\varsigma_l+\Delta_l-1} e^{\alpha_{\sigma(\varsigma_l)}(\varsigma_l+\Delta_l-s-1)} F(s)) \\
& - \sum_{s=\varsigma_l+\Delta_l}^{\varsigma-1} e^{-\beta_{\sigma(\varsigma_l)}(\varsigma-s-1)} F(s) \\
& \leq \mu_{\sigma(\varsigma_l)} e^{-\beta_{\sigma(\varsigma_l)}(\varsigma-\varsigma_l-\Delta_l)} (e^{\alpha_{\sigma(\varsigma_l)}\Delta_l} \theta_{\sigma(\varsigma_l)} V_{\beta\sigma(\varsigma_l)}(x^-_{\varsigma_l}) - \sum_{s=\varsigma_l}^{\varsigma_l+\Delta_l-1} e^{\alpha_{\sigma(\varsigma_l)}(\varsigma_l+\Delta_l-s-1)} F(s)) \\
& - \sum_{s=\varsigma_l+\Delta_l}^{\varsigma-1} e^{-\beta_{\sigma(\varsigma_l)}(\varsigma-s-1)} F(s) \\
& \leq \mu_{\sigma(\varsigma_l)} e^{-\beta_{\sigma(\varsigma_l)}(\varsigma-\varsigma_l-\Delta_l)} (e^{\alpha_{\sigma(\varsigma_l)}\Delta_l} \theta_{\sigma(\varsigma_l)} (\mu_{\sigma(\varsigma_{l-1})} \times e^{-\beta_{\sigma(\varsigma_{l-1})}(\varsigma_l-\varsigma_{l-1}-\Delta_{l-1})} (e^{\alpha_{\sigma(\varsigma_{l-1})}\Delta_{l-1}} \theta_{\sigma(\varsigma_{l-1})} \\
& \times V_{\beta\sigma(\varsigma_{l-1})}(x^-_{\varsigma_{l-1}}) - \sum_{s=\varsigma_{l-1}}^{\varsigma_{l-1}+\Delta_{l-1}-1} e^{\alpha_{\sigma(\varsigma_{l-1})}(\varsigma_{l-1}+\Delta_{l-1}-s-1)} F(s)) - \sum_{s=\varsigma_{l-1}+\Delta_{l-1}}^{\varsigma_l-1} e^{-\beta_{\sigma(\varsigma_{l-1})}(\varsigma-s-1)} F(s)) \\
& - \sum_{s=\varsigma_l}^{\varsigma_l+\Delta_l-1} e^{\alpha_{\sigma(\varsigma_l)}(\varsigma_l+\Delta_l-s-1)} F(s)) - \sum_{s=\varsigma_l+\Delta_l}^{\varsigma-1} e^{-\beta_{\sigma(\varsigma_l)}(\varsigma-s-1)} F(s) \\
& \leq \cdots \leq e^{-\beta_{\sigma(\varsigma_l)}(\varsigma-\varsigma_l-\Delta_l)} \times (e^{\alpha_{\sigma(\varsigma_l)}\Delta_l} \mu_{\sigma(\varsigma_l)} \theta_{\sigma(\varsigma_l)} \times \cdots \times \mu_{\sigma(\varsigma_1)} \theta_{\sigma(\varsigma_1)} \times (e^{-\beta_{\sigma(\varsigma_0)}(\varsigma_1-\varsigma_0-\Delta_0)} \\
& \times (e^{\alpha_{\sigma(\varsigma_0)}\Delta_0} V_{\sigma(\varsigma_0)}(x_{\varsigma_0}) - \sum_{s=\varsigma_0}^{\varsigma_0+\Delta_0-1} e^{\alpha_{\sigma(\varsigma_0)}(\varsigma_0+\Delta_0-s-1)} F(s)) - \sum_{s=\varsigma_0+\Delta_0}^{\varsigma_1-1} e^{-\beta_{\sigma(\varsigma_0)}(\varsigma_1-s-1)} F(s)) \\
& - \cdots - \sum_{s=\varsigma_l}^{\varsigma_l+\Delta_l-1} e^{\alpha_{\sigma(\varsigma_l)}(\varsigma_l+\Delta_l-s-1)} F(s)) - \sum_{s=\varsigma_l+\Delta_l}^{\varsigma-1} e^{-\beta_{\sigma(\varsigma_l)}(\varsigma-s-1)} F(s)
\end{aligned} \tag{43}
$$

Under zero initial conditions, i.e., $x(\varsigma_0) = 0$, from (37) and (43), we have

$$
\sum_{s=\varsigma_0}^{\varsigma-1} e^{\sum\limits_{i\in M, f\in\psi(i)} -\varepsilon_i^*(\varsigma_{f+1}-s-1)} z^T(s)z(s) \leq \gamma \sum_{s=\varsigma_0}^{\varsigma-1} e^{\sum\limits_{i\in M, f\in\psi(i)} -\varepsilon_i^*(\varsigma_{f+1}-s-1)} \varpi^T(s)\varpi(s). \tag{44}
$$

Multiply both sides of (44) by $e^{\sum\limits_{i\in M, f\in\psi(i)} N_{\sigma,i}(s,\varsigma_{f+1}) ln(\mu_i\theta_i)}$; we have

$$
\begin{aligned}
& \sum_{s=\varsigma_0}^{\varsigma-1} e^{\sum\limits_{i\in M, f\in\psi(i)} -\varepsilon_i^*(\varsigma_{f+1}-s-1)+N_{\sigma,i}(s,\varsigma_{f+1}) ln(\mu_i\theta_i)} z^T(s)z(s) \\
& \quad \leq \gamma \sum_{s=\varsigma_0}^{\varsigma-1} e^{\sum\limits_{i\in M, f\in\psi(i)} -\varepsilon_i^*(\varsigma_{f+1}-s-1)+N_{\sigma,i}(s,\varsigma_{f+1}) ln(\mu_i\theta_i)} \varpi^T(s)\varpi(s).
\end{aligned} \tag{45}
$$

Multiply both sides of inequality (45) by $e^{\sum\limits_{i\in M, f\in\psi(i)} -N_{\sigma,i}(\varsigma_f,\varsigma_{f+1}) ln(\mu_i\theta_i)}$; we have

$$\begin{aligned}\sum_{s=\varsigma_0}^{\varsigma-1} & e^{\sum\limits_{i\in M, f\in\psi(i)} -\varepsilon_i^*(\varsigma_{f+1}-s-1)-N_{\sigma,i}(\varsigma_f,s) ln(\mu_i\theta_i)} z^T(s)z(s) \\ & \leq \gamma \sum_{s=\varsigma_0}^{\varsigma-1} e^{\sum\limits_{i\in M, f\in\psi(i)} -\varepsilon_i^*(\varsigma_{f+1}-s-1)-N_{\sigma,i}(\varsigma_f,s) ln(\mu_i\theta_i)} \varpi^T(s)\varpi(s).\end{aligned} \tag{46}$$

Note $N_{\sigma,i}(\varsigma_f, s) < \frac{s-\varsigma_f}{\tau_{ai}}$. From (38), we have

$$N_{\sigma,i}(\varsigma_f, s) ln(\mu_i\theta_i) \leq \varepsilon_i^*(s-\varsigma_f). \tag{47}$$

It follows from (46) and (47) that

$$\sum_{s=\varsigma_0}^{\varsigma-1} e^{\sum\limits_{i\in M, f\in\psi(i)} -\varepsilon_i^*(\varsigma_{f+1}-\varsigma_f-1)} z^T(s)z(s) \leq \gamma \sum_{s=\varsigma_0}^{\varsigma-1} e^{\sum\limits_{i\in M, f\in\psi(i)} -\varepsilon_i^*(\varsigma_{f+1}-s-1)} \varpi^T(s)\varpi(s). \tag{48}$$

Thus,

$$\sum_{s=\varsigma_0}^{\varsigma-1} e^{\sum\limits_{i\in M, f\in\psi(i)} -\varepsilon_i^*(s-\varsigma_f)} z^T(s)z(s) \leq \gamma \sum_{s=\varsigma_0}^{\varsigma-1} \varpi^T(s)\varpi(s). \tag{49}$$

This implies

$$\sum_{s=\varsigma_0}^{\infty} e^{\sum\limits_{i\in M, f\in\psi(i)} -\varepsilon_i^*(s-\varsigma_f)} z^T(s)z(s) \leq \gamma \sum_{s=\varsigma_0}^{\infty} \varpi^T(s)\varpi(s). \tag{50}$$

According to Definition 3, System (3) is exponentially stable and has H_∞ performance index $\bar{\gamma} = \sqrt{\gamma}$.

The following theorem provides sufficient conditions for resolving the controller gain of System (3) based on Theorem 3. □

Theorem 4. *For System (3), given the parameters $\alpha_i > 0$, $\beta_i > 0$, $0 < \varepsilon_i^* < \beta_i$, $\mu_i > 1$, and $\gamma > 0$, if there exist matrices $X_i > 0$, $Q_i > 0$, $R_i > 0$, and $Y_i > 0$ satisfying*

$$\begin{bmatrix} \Sigma_1 & 0 & 0 & \Sigma_2 & \Sigma_3 & \Sigma_4 & X_j \\ * & \Sigma_5 & 0 & X_iA_{1i}^T & X_iA_{1i}^T & 0 & 0 \\ * & * & -\gamma I & B_{1i}^T & B_{1i}^T & 0 & 0 \\ * & * & * & -d^{-1}R_i' & 0 & 0 & 0 \\ * & * & * & * & -X_i & 0 & 0 \\ * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & -Q_i' \end{bmatrix} < 0, \tag{51}$$

$$\begin{bmatrix} \Sigma_1^1 & 0 & 0 & \Sigma_2^1 & \Sigma_3^1 & \Sigma_4^1 & X_i \\ * & \Sigma_5^1 & 0 & X_iA_{1i}^T & X_iA_{1i}^T & 0 & 0 \\ * & * & -\gamma I & B_{1i}^T & B_{1i}^T & 0 & 0 \\ * & * & * & -d^{-1}R_i' & 0 & 0 & 0 \\ * & * & * & * & -X_i & 0 & 0 \\ * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & -Q_i' \end{bmatrix} < 0, \tag{52}$$

$$\begin{bmatrix} -\mu_i X_j & X_j \\ * & -X_i \end{bmatrix} \leq 0, \begin{bmatrix} -\mu_i Q_j' & Q_j' \\ * & -Q_i' \end{bmatrix} \leq 0, \begin{bmatrix} -\mu_i R_j' & R_j' \\ * & -R_i' \end{bmatrix} \leq 0, \tag{53}$$

where

$$\Sigma_1 = e^{\alpha_i}(X_i - 2X_j), \Sigma_2 = X_j A_i^T + Y_j^T B_i^T - X_j, \Sigma_3 = X_j A_i^T + Y_j^T B_i^T, \Sigma_4 = X_j C_i^T + Y_j^T D_i^T,$$
$$\Sigma_5 = e^{d\alpha_i}(Q_i' - 2X_i), \Sigma_1^1 = e^{-\beta_i} X_i, \Sigma_2^1 = X_i A_i^T + Y_i^T B_i^T - X_i, \Sigma_3^1 = X_i A_i^T + Y_i^T B_i^T,$$
$$\Sigma_4^1 = X_i C_i^T + Y_i^T D_i^T, \Sigma_5^1 = e^{-d\beta_i}(Q_i' - 2X_i).$$

Therefore, the corresponding state–feedback controller gain matrix $K_i = Y_i X_i^{-1}$ can be obtained.

Proof. Multiply both sides of (34) by $diag\{X_j, X_i, I, R_i', X_i, I\}$ at the same time, and multiply both sides of (35) by $diag\{X_i, X_i, I, R_i', X_i, I\}$ at the same time. Similarly, by Schur's complement, conditions (51)–(53) can be obtained. □

4. Numerical Example

Consider the discrete time delay switching System (1) with three subsystems, whose parameters are set as follows:

$$A_1 = \begin{bmatrix} -0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, B_1 = \begin{bmatrix} -0.4 & 0 \\ -1.8 & -0.6 \end{bmatrix}, A_{11} = \begin{bmatrix} -0.6 & 0 \\ 0.5 & 0.7 \end{bmatrix}, B_{11} = \begin{bmatrix} -0.07 & 0 \\ 0 & -0.2 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} -0.6 & 0.8 \\ 1.7 & 0.5 \end{bmatrix}, D_1 = \begin{bmatrix} -1.9 & 1 \\ 0.8 & 1.2 \end{bmatrix}, A_2 = \begin{bmatrix} 0.01 & 0 \\ 0 & -0.08 \end{bmatrix}, B_2 = \begin{bmatrix} -1.1 & -0.07 \\ 1.6 & 0.5 \end{bmatrix},$$
$$A_{12} = \begin{bmatrix} 0.1 & 0 \\ 0 & -0.2 \end{bmatrix}, B_{12} = \begin{bmatrix} -0.3 & 0 \\ 0 & -0.4 \end{bmatrix}, C_2 = \begin{bmatrix} 0.3 & -1.4 \\ -0.8 & -1 \end{bmatrix}, D_2 = \begin{bmatrix} 1 & 0.1 \\ -0.3 & -1.5 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} -0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, B_3 = \begin{bmatrix} -0.8 & 0.6 \\ 1.6 & 0.5 \end{bmatrix}, A_{13} = \begin{bmatrix} 0.1 & -0.1 \\ 0 & -0.8 \end{bmatrix}, B_{13} = \begin{bmatrix} -0.008 & 0 \\ 0 & -0.1 \end{bmatrix},$$
$$C_3 = \begin{bmatrix} -0.3 & -1.6 \\ -1.5 & 1.1 \end{bmatrix}, D_3 = \begin{bmatrix} -0.4 & 0.7 \\ 0.2 & -1.2 \end{bmatrix}, \varpi(\varsigma) = \begin{bmatrix} 2^{-\varsigma} sin(2\varsigma) & 2^{-\varsigma} cos(2\varsigma) \end{bmatrix}^T.$$

We compare this switched system under the MDADT switching strategy and the ADT switching strategy. A set of appropriate data is chosen by contrasting the impact of each parameter on the system, as illustrated in Table 1. The MDADT switching strategy makes each subsystem have its own ADT; that is, the parameters of each subsystem are different. Choose $\alpha_1 = 1.7$, $\beta_1 = 2.5$, $\mu_1 = 1.1$, and $\varepsilon_1^* = 1.7$; then obtain $\tau_{a1} > 9.99$ s by solving (9); similarly, choose $\alpha_2 = 1.2$, $\beta_2 = 2.1$, $\mu_2 = 2.2$, and $\varepsilon_2^* = 1.4$ and obtain $\tau_{a2} > 10.26$ s; choose $\alpha_3 = 1.8$, $\beta_3 = 2$, $\mu_3 = 2.3$, and $\varepsilon_3^* = 1.3$ and obtain $\tau_{a3} > 12.97$ s. By solving (51)–(53), we can get the gain matrix of the controller:

$$K_1 = \begin{bmatrix} 0.4062 & 0.7363 \\ -1.0538 & -1.1525 \end{bmatrix}, K_2 = \begin{bmatrix} -0.2024 & -0.2399 \\ -0.6497 & 0.1711 \end{bmatrix}, K_3 = \begin{bmatrix} 0.4055 & -0.1867 \\ -0.7019 & -0.0834 \end{bmatrix},$$

The running time of each subsystem is the same when using the ADT switching strategy, so choose $\alpha = 1.8$, $\beta = 2.5$, $\mu = 1.03$, and $\varepsilon^* = 1.7$, and obtain $\tau_a > 10.15$ s. By solving (51)–(53), we can get the gain matrix of the controller:

$$K_1 = \begin{bmatrix} 0.3793 & 1.0235 \\ -1.0086 & -1.2475 \end{bmatrix}, K_2 = \begin{bmatrix} -0.1863 & -0.2506 \\ -0.7474 & 0.2720 \end{bmatrix}, K_3 = \begin{bmatrix} 0.5937 & -0.3481 \\ -0.8036 & 0.0012 \end{bmatrix},$$

To eliminate the impact of other variables, the MDADT switching strategy is chosen with the values $\alpha_1 = \alpha_2 = \alpha_3 = 1.8$, $\beta_1 = \beta_2 = \beta_3 = 2.5$, $\mu_1 = \mu_2 = \mu_3 = 1.03$, $\varepsilon_1^* = \varepsilon_2^* = \varepsilon_3^* = 1.7$; however, two sets of distinct average dwell times $\tau_{a1} = 11$ s, $\tau_{a2} = 12$ s, $\tau_{a3} = 14$ s and $\tau_{a1} = 11$ s, $\tau_{a2} = 13$ s, $\tau_{a3} = 15$ s are selected. Because the parameters of these two groups are the same as those under the ADT switching strategy (but the dwell time of each subsystem is different), the controller gain matrices of these two groups are the same as those under the ADT switching strategy.

Table 1. The parameters and calculation results of the system under the ADT switching signal and the MDADT switching signal.

Switching Schemes	ADT	MDADT	MDADT	MDADT
Parameters	$\alpha = 1.8$, $\beta = 2.5$, $\mu = 1.03$, $\varepsilon^* = 1.7$, $d = 4$	$\alpha_1 = \alpha_2 = 1.8$, $\alpha_3 = 1.8$, $\beta_1 = 2.5$, $\beta_2 = \beta_3 = 2.5$, $\mu_1 = \mu_2 = 1.03$, $\mu_3 = 1.03$, $\varepsilon_1^* = 1.7$, $\varepsilon_2^* = \varepsilon_3^* = 1.7$, $d = 4$	$\alpha_1 = \alpha_2 = 1.8$, $\alpha_3 = 1.8$, $\beta_1 = 2.5$, $\beta_2 = \beta_3 = 2.5$, $\mu_1 = \mu_2 = 1.03$, $\mu_3 = 1.03$, $\varepsilon_1^* = 1.7$, $\varepsilon_2^* = \varepsilon_3^* = 1.7$, $d = 4$	$\alpha_1 = 1.7$, $\alpha_2 = 1.2$, $\alpha_3 = 1.8$, $\beta_1 = 2.5$, $\beta_2 = 2.1$, $\beta_3 = 2$, $\mu_1 = 1.1$, $\mu_2 = 2.2$, $\mu_3 = 2.3$, $\varepsilon_1^* = 1.7$, $\varepsilon_2^* = 1.4$, $\varepsilon_3^* = 1.3$, $d = 4$
Dwell time	$\tau_a = 11$	$\tau_{a1} = 11$, $\tau_{a2} = 12$, $\tau_{a3} = 14$	$\tau_{a1} = 11$, $\tau_{a2} = 13$, $\tau_{a3} = 15$	$\tau_{a1} = 11$, $\tau_{a2} = 12$, $\tau_{a3} = 13$
H_∞ index	1.14	1.14	1.14	0.78

Figures 1–4 describe the switching signals of the subsystem and controller. When $\varpi = 0$, let $x(0) = [0.3, -2.3]^T$. The motion trajectories of the system under the switching strategy of ADT and MDADT are illustrated in Figures 5–8, respectively. According to the graph, under the ADT switching strategy, the system gradually tends to be stable at 30 s, but it is still accompanied by fluctuations until it stabilizes at 60 s. The highest amplitude of the system is about 7.5, and the fluctuation is large before the system is stable. However, under the MDADT switching strategy, we can see that the system has stabilized around 30 s. Figure 6 has obvious fluctuations around 60 s and 96 s, Figure 7 also has obvious fluctuations around 62 s, and Figure 8 almost stabilizes after 30 s. Prior to achieving stability, it is noticeable that the vibration amplitude in Figure 5 is considerably greater than that in Figures 6–8 and exhibits a clear and dramatic variation. On the other hand, the paths of the systems depicted in Figures 6–8 exhibit comparatively smaller fluctuations within a specific range. Hence, we can observe that if the residence time of each subsystem is changed, the motion path of the system will be changed accordingly. In line with the MDADT switching strategy, we have the flexibility to select distinct parameters for each subsystem in order to modify its residence time, thereby facilitating rapid stabilization of the system.

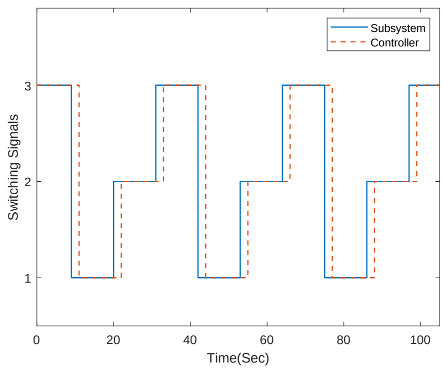

Figure 1. ADT switching signal.

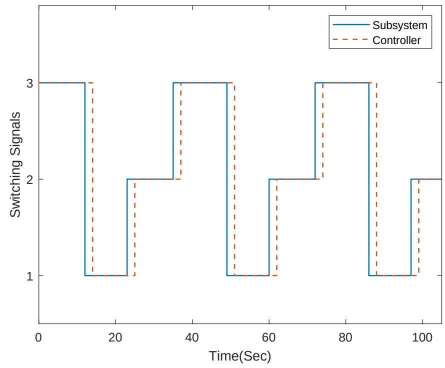

Figure 2. MDADT switching signal with $\tau_{a1} = 11$, $\tau_{a2} = 12$, and $\tau_{a3} = 14$.

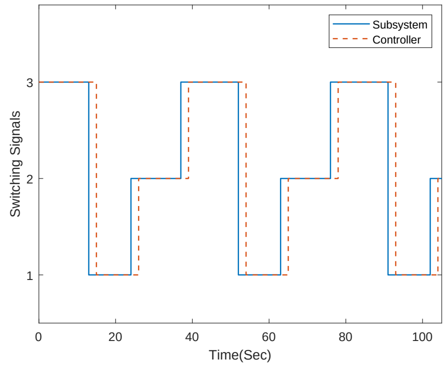

Figure 3. MDADT switching signal with $\tau_{a1} = 11$, $\tau_{a2} = 13$, and $\tau_{a3} = 15$.

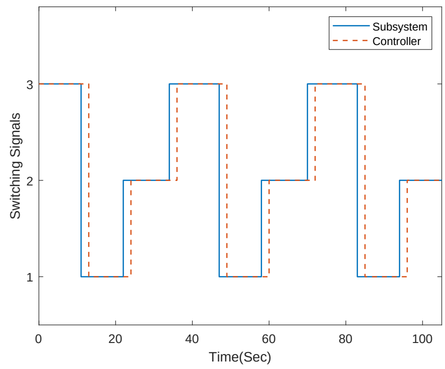

Figure 4. MDADT switching signal with $\tau_{a1} = 11$, $\tau_{a2} = 12$, and $\tau_{a3} = 13$.

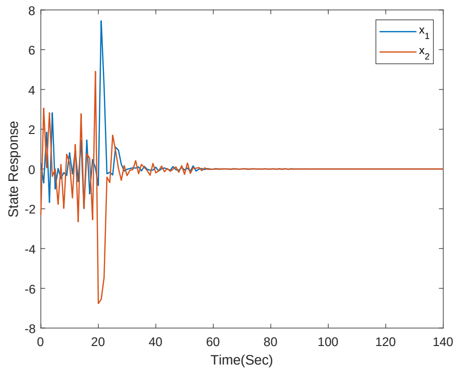

Figure 5. State response of System (4) under ADT switching signal.

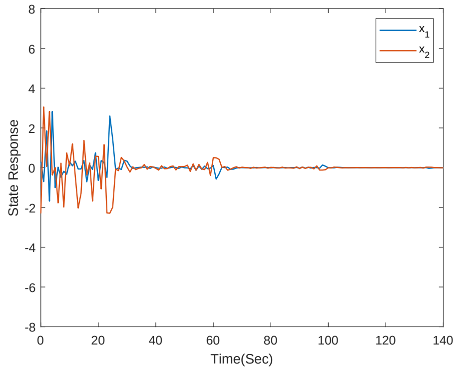

Figure 6. State response of System (4) under MDADT switching signal with $\tau_{a1} = 11$, $\tau_{a2} = 12$, $\tau_{a3} = 14$.

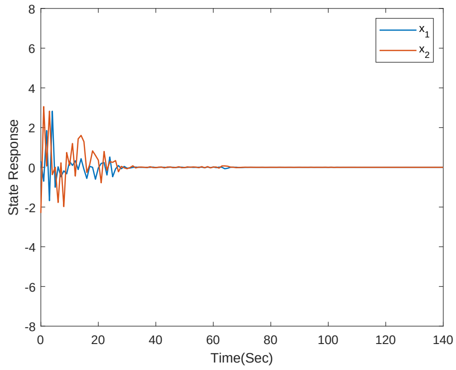

Figure 7. State response of System (4) under MDADT switching signal with $\tau_{a1} = 11$, $\tau_{a2} = 13$, and $\tau_{a3} = 15$.

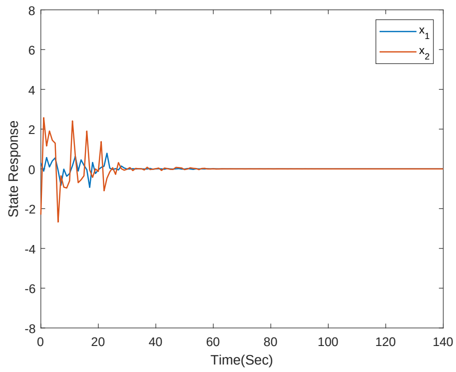

Figure 8. State response of System (4) under MDADT switching signal with $\tau_{a1} = 11$, $\tau_{a2} = 12$, and $\tau_{a3} = 13$.

Let $x(0) = [0, 0]^T$ when there is a disturbance. Figures 9–12 depict the switching system's movement trajectory under asynchronous switching based on the ADT switching strategy and the MDADT switching strategy. The figure shows that when using the ADT switching strategy, the system in Figure 9 experienced obvious drastic changes before it was stable, and it began to stabilize at about 30 s, but it was still accompanied by obvious fluctuations, and it was not completely stable until 57 s, with the largest amplitude being 0.038. In contrast, under the MDADT switching strategy, the system in Figure 10 tends to be stable at about 30 s, but there are still obvious fluctuations around 59 s and 97 s. The system in Figure 11 is generally stable around 33 s and has minimal fluctuations. The system in Figure 12 is nearly stable around 25 s, has significantly reduced fluctuation compared to Figure 9, and has a maximum amplitude of 0.021.

Therefore, under the MDADT switching strategy, we can set the dwell time for each subsystem so that the system can reach a stable state faster. However, the ADT switching strategy limits the dwell time for each subsystem, resulting in equal dwell times for each subsystem, which has certain limitations. Clearly, compared to the ADT switching strategy, the MDADT switching strategy can better maintain the robust performance of the system.

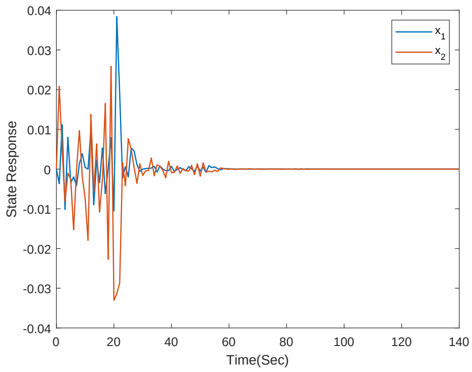

Figure 9. State response of System (3) under ADT switching signal.

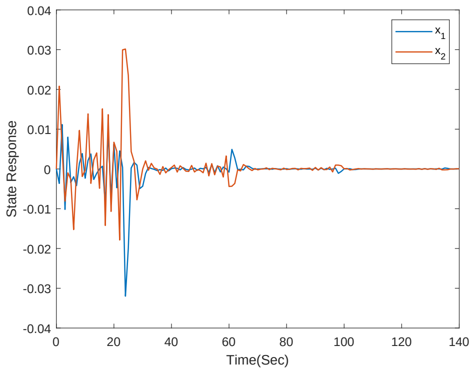

Figure 10. State response of system (3) under MDADT switching signal with $\tau_{a1} = 11$, $\tau_{a2} = 12$, and $\tau_{a3} = 14$.

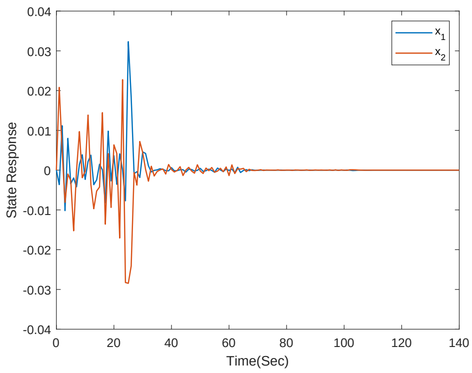

Figure 11. State response of System (3) under MDADT switching signal with $\tau_{a1} = 11$, $\tau_{a2} = 13$, and $\tau_{a3} = 15$.

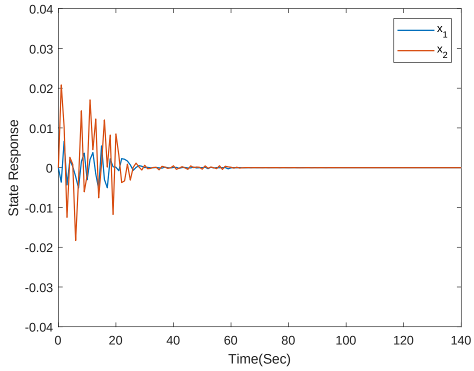

Figure 12. State response of System (3) under MDADT switching signal with $\tau_{a1} = 11$, $\tau_{a2} = 12$, and $\tau_{a3} = 13$.

5. Conclusions

This study examines the asynchronous control problem for discrete time delay switched linear systems based on MDADT. In order to address the independent switching delay of the sub-controller in relation to the subsystem, a classification analysis is conducted, and distinct Lyapunov functions are chosen for the matching and mismatching intervals between the subsystem and the controller. According to the MDADT technique, the stability of the asynchronous switching system can be achieved by modifying the proportion between the matching period and the mismatching period. Ultimately, the simulation of a discrete time delay switching system with three subsystems under the ADT technique and the MDADT technique is given. The analysis of the data confirms the effectiveness of the designed asynchronous control method.

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