S4 from Dejan Brkić, Pavel Praks, Renáta Praksová, Tomáš Kozubek: Symbolic regression approaches for direct calculation of pipe diameter

EXAMPLES:

The following numerical examples illustrate the pipeline design process using the newly discovered approximations:

Numerical Example 1: Q=0.0324 m^3/s, Δh=1.37 m, the pipe is of asphalted cast-iron and absolute roughness of inner pipe surface is estimated to Ɛ=$0.12\cdot10^{-3}$ m, L=60.66 m, ν=$1.0974\cdot10^{-6}$ m^2/s (from in Moody as reported in Yetilmezso et al.). The diameter D=0.15 m as from Moody (reported in Yetilmezso et al.) where the relative error for the diameter D is compared with this value.

From Section 4.1.2.:

-Equation (4):

$$A=\sqrt{0.247\cdot\mathcal{E}}=\sqrt{0.247\cdot0.00012}=\sqrt{0.00002964}=0.00544426303552648$$

$$D=\sqrt{0.00080352+\sin(0.01318608)+Q\cdot A+\frac{0.00736\cdot Q+\sin(0.247\cdot Q\cdot A)+0.00615\cdot A+0.0000874}{\sqrt{\frac{1.37}{60.66}}}}=0.129\text{ m}$$

Relative error:

$$\frac{|0.15-0.129|}{0.15}\cdot100\%=14\%$$

-Equation (5):

$$B=\ln\left(\frac{\Delta h}{L}-0.0001\right)=\ln\left(\frac{1.37}{60.66}-0.0001\right)=-3.79491$$

$$D=(0.0007317507418+0.000062452944+0.000055992417-0.0108864+0.00002591164+0.017459621928)^{0.388}=0.0074493296708^{0.388}=0.1494\text{ m}$$

Relative error:

$$\frac{|0.15-0.1494|}{0.15}\cdot100\%=0.4\%$$

-Equation (6):

$$D=\left(\frac{\frac{0.0324}{\left(\frac{1.37}{60.66}\right)^{0.48744607}}+0.00012}{\ln(0.0324)-\frac{\ln(0.00012)}{0.39880937}}\right)^{0.39849746+0.00012\cdot\frac{\left(\frac{1.37}{60.66}\right)^{0.28922623}}{0.31922475}}$$

$$D=\left(\frac{0.205694800771456413}{19.207832187126}\right)^{0.3986230530835676}=0.1639\text{ m}$$

Relative error:

$$\frac{|0.15-0.1639|}{0.15}\cdot100\%=9.26\%$$

From Section 4.2.:

-Equation (7):

$$\left.\begin{array}{c}
\varepsilon^*=0.00012\cdot\left(\frac{9.81\cdot1.37}{60.66\cdot0.0324^2}\right)^{0.2}=0.000349994061\\
\nu^*=0.0000010974\cdot\left(\frac{9.81\cdot1.37\cdot0.0324^3}{60.66}\right)^{-0.2}=0.00001161289544698324\\
D_r^*=0.255+\frac{\ln(\varepsilon^*)}{425.025}-\frac{2.223}{\ln(\varepsilon^*)-3.421}=0.255-0.018722650+0.19536683770=0.431644187\\
D_s^*=0.3+\frac{\ln(\nu^*)}{311.526}-\frac{1.7}{\ln(\nu^*)}-5.06\cdot\nu^*=0.3-0.036476552+0.149603185=0.413126633\\
D^*=1.019\cdot(D_r^{*20}+1.9\cdot D_s^{*20.9})^{0.051}=0.4392990152\\
D=\frac{\varepsilon\cdot D^*}{\varepsilon^*}=0.150619\text{ m}
\end{array}\right\}$$

Relative error:
$\frac{|0.15-0.150619|}{0.15} \cdot 100\% = 0.412\%$

-Equation (8):

$$\left.\begin{array}{c}
N = -3.790473762 \\
Y = -13.72256681 \\
P = -3.429596856 \\
K = -9.028018815 \\
U = 0.2 \cdot N - 0.4 \cdot P + 0.4567 = 1.07044399 \\
M = Y - 0.2 \cdot N - 0.6 \cdot P - 0.4567 = -11.36341395 \\
Z = K + U = -7.957574825 \\
D_r^* = 0.255 + \frac{Z}{425.025} - \frac{2.223}{Z-3.421} = 0.431644569 \\
D_s^* = 0.3 + \frac{M}{311.526} - \frac{1.7}{M} - 5.06 \cdot e^M = 0.413067553 \\
V = 0.051 \cdot \ln\left(D_r^{*20} + 1.9 \cdot D_s^{*20.9}\right) + 0.0188 = -0.822636098 \\
D = e^{V-U} = 0.15060721
\end{array}\right\}$$

Relative error:
$\frac{|0.15-0.15060721|}{0.15} \cdot 100\% = 0.4048\%$

<u>From Section 4.3.:</u>

Suppose that ρ=1000 kg/m^3

$\Delta p = \rho \cdot g \cdot \Delta h = 1000 \cdot 9.81 \cdot 1.37 = 13439.7$ Pa

$$\left.\begin{array}{l}
\Pi_1 = \frac{128}{3.1415^3} \cdot \frac{13439.7 \cdot 0.0324^3}{1000 \cdot 60.66 \cdot 0.0000010974^5} = 1954599446478208271 5773345.4174 \\
\Pi_2 = \frac{\pi \cdot \nu \cdot \varepsilon}{4 \cdot Q} = \frac{3.1415 \cdot 0.0000010974 \cdot 0.00012}{4 \cdot 0.0324} = 3.19220720189 \cdot 10^{-9}
\end{array}\right\}$$

$\frac{1}{\sqrt{\Pi_1}} = 2.26 \cdot 10^{-13}$

$\mathrm{Re} = \frac{1.4446}{(2.26 \cdot 10^{-13})^{0.37972} \cdot (3.19220720189 \cdot 10^{-9} + (2.26 \cdot 10^{-13})^{0.7})^{0.051}} = 243620.3544$

$D = \frac{4 \cdot Q}{\nu \cdot \mathrm{Re} \cdot \pi} = \frac{4 \cdot 0.0324}{0.0000010974 \cdot 243620.3544 \cdot 3.1415} = 0.1543$ m

Relative error:
$\frac{|0.15-0.1543|}{0.15} \cdot 100\% = 2.86\%$

Numerical Example 2: Q=0.003 m^3/s, Δh=10 m, the pipe is of commercial steel and the absolute roughness of the inner pipe surface is estimated to ε=0.05·10^{-3} m, L=100 m, t=20°C (as reported in Yetilmezso et al.). The diameter D=0.0444 m as reported in Yetilmezso et al. where the relative error for the diameter D is compared with this value.

-From Equation (3):

$\nu = \frac{10^{-6}}{\frac{8.914}{100000} \cdot t^2 + \frac{2.04}{100} \cdot t + 0.555} = \frac{10^{-6}}{\frac{8.914}{100000} \cdot 400 + \frac{2.04}{100} \cdot 20 + 0.555} = \frac{10^{-6}}{0.035656 + 0.408 + 0.555} = 1.0013458 \cdot 10^{-6}\ \frac{m^2}{s}$

<u>From Section 4.1.2.:</u>

-Equation (4): D=0.04182 m with the relative error $\frac{|0.0444-0.04182|}{0.0444} \cdot 100\% = 5.81\%$

-Equation (5): D=0.04201 m with the relative error $\frac{|0.0444-0.04201|}{0.0444} \cdot 100\% = 5.38\%$

-Equation (6): D= 0.04783 m with the relative error $\frac{|0.0444-0.04783|}{0.0444} \cdot 100\% = 7.72\%$

<u>From Section 4.2.:</u>

-Equation (7): D=0.04435 m (ε^*=0.000508661, ν^*=3.28098·10^{-5}, D_r^*=0.439161066, D_s^*=0.431343765, D^*=0.451199822) with the relative error $\frac{|0.0444-0.04435|}{0.0444} \cdot 100\% = 0.11\%$

-Equation (8):

$$\left.\begin{array}{c}
N = \ln\left(\frac{\Delta h}{L}\right) = -2.302585093 \\
Y = \ln(\nu) = -13.81416566 \\
P = \ln(Q) = -5.80914299 \\
K = \ln(\mathcal{E}) = -9.903487553 \\
U = 0.2 \cdot N - 0.4 \cdot P + 0.4567 = 2.319840178 \\
M = Y - 0.2 \cdot N - 0.6 \cdot P - 0.4567 = -10.32486285 \\
Z = K + U = -7.583647375 \\
D_r^* = 0.255 + \frac{Z}{425.025} - \frac{2.223}{Z-3.421} = 0.439162737 \\
D_s^* = 0.3 + \frac{M}{311.526} - \frac{1.7}{M} - 5.06 \cdot e^M = 0.431342232 \\
V = 0.051 \cdot \ln\left(D_r^{*20} + 1.9 \cdot D_s^{*20.9}\right) + 0.0188 = -0.795865789 \\
D = e^{V-U} = 0.044347188
\end{array}\right\}$$

Relative error:

$\frac{|0.0444-0.044347188|}{0.15} \cdot 100\% = 0.1189\%$

From Section 4.3.:

Suppose that ρ=1000 kg/m[3]

$\Delta p = \rho \cdot g \cdot \Delta h = 1000 \cdot 9.81 \cdot 10 = 98100$ Pa

$$\left.\begin{array}{l}
\Pi_1 = \frac{128}{3.1415^3} \cdot \frac{98100 \cdot 0.003^3}{1000 \cdot 100 \cdot 0.0000010013458^5} = 10862033707049843056857 5.09 \\
\quad \Pi_2 = \frac{\pi \cdot \nu \cdot \mathcal{E}}{4 \cdot Q} = \frac{3.1415 \cdot 0.0000010013458 \cdot 0.00005}{4 \cdot 0.003} = 1.310719929 \cdot 10^{-8}
\end{array}\right\}$$

$\frac{1}{\sqrt{\Pi_1}} = 3.0342 \cdot 10^{-12}$

$Re = \frac{1.4446}{(3.0342 \cdot 10^{-12})^{0.37972} \cdot (1.310719929 \cdot 10^{-8} + (3.0342 \cdot 10^{-12})^{0.7})^{0.051}} = 83969.12338$

$D = \frac{4 \cdot Q}{\nu \cdot Re \cdot \pi} = \frac{4 \cdot 0.003}{0.0000010013458 \cdot 83969.12338 \cdot 3.1415} = 0.045429$ m

Relative error:

$\frac{|0.0444-0.045429|}{0.0444} \cdot 100\% = 2.32\%$