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Coincidence Point of Edelstein Type Mappings in Fuzzy Metric Spaces and Application to the Stability of Dynamic Markets

Satish Shukla ¹, Nikita Dubey ¹, Rahul Shukla ^{2,*} and Ivan Mezník ³

¹ Department of Mathematics, Shri Vaishnav Institute of Science, Shri Vaishnav Vidyapeeth Vishwavidyalaya, Gram Baroli Sanwer Road, Indore 453111, India; satishmathematics@yahoo.co.in (S.S.); dubey13nikita@gmail.com (N.D.)

² Department of Mathematical Sciences and Computing, Walter Sisulu University, Mthatha 5117, South Africa

³ Institute of Informatics, Faculty of Business and Management, Brno University of Technology, Kolejní 2906/4, 612 00 Brno, Czech Republic; meznik@vutbr.cz

* Correspondence: rshukla@wsu.ac.za

Abstract: In this paper, we prove a coincidence point result for a pair of mappings satisfying Edelstein-type contractive condition on fuzzy metric spaces. We describe the equilibrium of a simple demand–supply model of a dynamic market by the coincidence point of demand and supply functions. With the help of the coincidence point theorem in fuzzy metric spaces, it is showed that a dynamic market of a supply-sensitive nature (or demand-sensitive nature) always tends towards its equilibrium.

Keywords: Edelstein mapping; fuzzy metric space; coincidence point; dynamic market; demand and supply functions; sensitivity index; equilibrium point

MSC: 54H25; 47H10; 91B99



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1. Introduction

In this section, we provide some background information and terminology about the fixed point, coincidence point and dynamic markets, which will be helpful throughout the paper.

A mapping $\zeta: Y \rightarrow Y$ on a complete metric space (Y, δ) is called a contraction if for some fixed $\lambda \in [0, 1)$, the following condition is satisfied:

$$\delta(\zeta(b), \zeta(\ell)) \leq \lambda \delta(b, \ell) \text{ for all } b, \ell \in Y.$$

Banach [1] considered such types of mappings on metric spaces and showed that if the metric space is complete, then such mappings have a unique fixed point. A natural and interesting case in the above inequality arises when λ is taken to be equal to 1 and the inequality is taken to be strict, i.e., for ζ , the following strict inequality is satisfied:

$$\delta(\zeta(b), \zeta(\ell)) < \delta(b, \ell) \text{ for all } b, \ell \in Y.$$

In this case, ζ is called a contractive mapping. Rakotch [2] by way of example showed that contractive mapping (even compact contractive mapping in a complete metric space) may not have a fixed point. Edelstein [3] showed that a contractive mapping on a compact space must have a unique fixed point and this fixed point is the limit of an iterative sequence.

On the other hand, Kramosil and Michálek [4] introduced the notion of fuzzy metric spaces. Grabiec [5] extended the results of Banach [1] and Edelstein [3] in fuzzy metric spaces. The concept of Kramosil and Michálek [4] was further extended by George and Veeramani [6] so that they were able to obtain a Hausdorff induced topology by the fuzzy

metric. Miheţ [7] discussed the existence of fixed points of Edelstein fuzzy contractive mappings in the fuzzy metric spaces of George and Veeramani [6].

Consider the mappings $\xi, \zeta: Y \rightarrow Y$, where Y is a nonempty set. A point $b \in Y$ is a fixed point of ξ if $\xi(b) = b$, while b is called a coincidence point of ξ and ζ if $\xi(b) = \zeta(b)$ and, in this case, the common value $\ell = \xi(b) = \zeta(b)$ is called the point of coincidence of ξ and ζ . For some $b_0 \in Y$, the sequence $\{b_n\}$ given by $b_n = \xi(b_{n-1})$, $n \in \mathbb{N}$ is called a Picard sequence of ξ with an initial value b_0 , while the sequence $\{\ell_n\}$ given by $\ell_{n-1} = \xi(b_{n-1}) = \zeta(b_n)$, $n \in \mathbb{N}$ (provided it exists) is called a Jungck sequence (see [8]) of the ordered pair (ξ, ζ) with an initial value b_0 . Jungck first considered such sequences of ordered pairs of continuous mappings, and using this, he established the existence of fixed points of the mapping of the pair. Later, many authors used Jungck's approach and extended his work to various spaces and for various types of mappings, see, e.g., [9–14]. Jungck's sequence of an ordered pair of mappings plays an important role in establishing the existence of point of the coincidence of the mappings. We denote the class of all Jungck sequences of the ordered pair (ξ, ζ) with initial value in Y by $J_Y(\xi, \zeta)$.

In mathematical economics, a number of studies on economic models are subject to the analysis of the equilibrium states. A market that is continuously and rapidly changing (in terms of price of a commodity) over time is called a dynamic market, and the forces cause the change in price to represent the market dynamics. This dynamic is directly dependent on the variables (e.g., the supplied quantity of a commodity) associated with the market. The equilibrium of a dynamic market is a situation in which the market has a tendency to oppose the changes. The stability of a market is an important concept in the study of the equilibrium of a market. The price of the commodity at which the demanded and supplied quantities of the commodity becomes equal is called the equilibrium price, and at this stage, the market is said to be in equilibrium. A market can be temporarily at equilibrium without being stable (see, e.g., [15] and the references therein). If a market is not in equilibrium, then changes in variables (on which the dynamics depends) may force the market (i.e., price of the commodity) towards its equilibrium, and if this is the case, then the market is called stable. We will see that in a stable market, the competitive market forces always bring the price of the commodity towards its equilibrium value.

The fixed point theory has numerous applications in various disciplines like science, engineering and economics; see, e.g., [16–19]. In particular, the stability of dynamic markets can be identified by the equilibrium point (price), and the equilibrium point is considered as a fixed point of some particular function associated with the demand and supply at successive stages of the market (see, e.g., [20–22] and the references therein). Mezník [22] showed that in a simple demand and supply model where the model is completely characterized by the demand function $p = D(q)$ and the supply function $p = S(q)$ (where p stands for price and q for the quantity; demanded or supplied), the equilibrium price p_E can be considered as a fixed point of the function $D \circ S^{-1}$, where S^{-1} represents the inverse function of S . He proved that if $\left| \frac{D'}{S'} \right| = k < 1$ (where D' and S' are the derivatives of D and S), then the market is stable and the iterative sequence $p_k = D(S^{-1}(p_{k-1}))$, $k = 1, 2, \dots$ of the market prices converges to the equilibrium price p_E which is a fixed point of $D(S^{-1})$. Indeed, the equilibrium price p_E is the point at which the demand and supply functions becomes equal, i.e., $p_E = D(q_E) = S(q_E)$, where q_E is the equilibrium quantity. This state is called the equilibrium state of the market. Hence, we can say that the equilibrium quantity is a coincidence point of the demand and supply functions.

By keeping the definitions of a fixed point and a coincidence point in mind, it is more convenient and natural to consider the equilibrium point for a dynamic market as a coincidence point of the demand function D and the supply function S , and in this case, the equilibrium quantity q_E will be the corresponding coincidence point and the equilibrium price $p_E = D(q_E) = S(q_E)$ will be the point of coincidence of D and S .

In this paper, we establish the existence of the equilibrium point of a market considered by Mezník [22] with the help of a coincidence point theorem for the mappings satisfying

Edelstein type contractive mappings in fuzzy metric spaces (in the sense of George and Veeramani [6]). With this new approach, we do not need to find the derivatives of the demand and supply functions nor the inverse of the supply function. The iterative sequence used by Mezník [22] was the Picard sequence, while we use the Jungck sequence to prove the existence of equilibrium prices and the stability of the market.

2. Preliminaries

Before going into the main discussion, in this section, we state some definitions and a result which will be helpful throughout the discussion.

Definition 1 (Jungck [8,10]). *“Let Y be a nonempty set and $\xi, \zeta: Y \rightarrow Y$. Then, an element $b \in Y$ is called a coincidence point of ξ and ζ if $\zeta(b) = \xi(b)$ and this common value is called the corresponding point of coincidence of ξ and ζ . We denote the set of all coincidence points of ξ and ζ by $Co(\xi, \zeta)$ and the set of all points of coincidence by $P(\xi, \zeta)$. If $b \in Co(\xi, \zeta)$ is such that the common value $\xi(b) = \zeta(b)$ at this point is equal to b itself (i.e., $\xi(b) = \zeta(b) = b$), then b is called a common fixed point of ξ and ζ . The mappings ξ and ζ are called weakly compatible if $\xi(\zeta(b)) = \zeta(\xi(b))$ for all $b \in Co(\xi, \zeta)$.”*

Jungck [23] and several other authors considered the periodic points of Picard sequence of a mapping. Here, we define the periodic points of a pair of mappings.

Definition 2. *Let Y be a nonempty set, $\xi, \zeta: Y \rightarrow Y$ and $\{\ell_n\} \in J_Y(\xi, \zeta)$ with initial value b_0 . A point ℓ_n is said to be a periodic point of mappings ξ and ζ of period p if p is the smallest natural number such that $\ell_{n+p} = \ell_n$.*

Proposition 1 (Abbas and Jungck [24]). *“Let Y be a nonempty set and $\xi, \zeta: Y \rightarrow Y$ be weakly compatible self-mappings. If ξ and ζ have a unique point of coincidence $\ell = \xi(b) = \zeta(b)$, then ℓ is the unique common fixed point of ξ and ζ .”*

Definition 3 (Schweizer and Sklar [25]). *“A binary operation $*$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is called a t -norm if the following conditions are satisfied: for all $a, b, c, d \in [0, 1]$*

- (i) $a * b = b * a$;
- (ii) $a * b \leq c * d$ for $a \leq c, b \leq d$;
- (iii) $(a * b) * c = a * (b * c)$;
- (iv) $a * 0 = 0, a * 1 = a$.”

Definition 4 (George and Veeramani [6]). *“A triple $(Y, F, *)$ is called a fuzzy metric space if Y is a nonempty set, $*$ is a continuous t -norm and $F: Y \times Y \times (0, \infty) \rightarrow [0, 1]$ is a fuzzy set satisfying following conditions:*

- (i) $F(b, \ell, \tau) > 0$;
- (ii) $F(b, \ell, \tau) = 1$ if and only if $b = \ell$;
- (iii) $F(b, \ell, \tau) = F(\ell, b, \tau)$;
- (iv) $F(b, \kappa, \tau + v) \geq F(b, \ell, \tau) * F(\ell, \kappa, v)$;
- (v) $F(b, \ell, \cdot): (0, \infty) \rightarrow [0, 1]$ is a continuous mapping;

for all $b, \ell, \kappa \in Y$ and $v, \tau > 0$.”

A sequence $\{b_n\}$ in a fuzzy metric space $(Y, F, *)$ is said to be convergent to $b \in Y$ (see [5,6]) if $\lim_{n \rightarrow \infty} F(b_n, b, \tau) = 1$ for all $\tau > 0$ and, in this case, we say that the point b is the limit of $\{b_n\}$. The space $(Y, F, *)$ is called compact if every sequence in Y has a convergent subsequence in Y . A subset C of Y is called compact if every sequence in C contains a convergent subsequence with limit in C . A mapping $\xi: Y \rightarrow Y$ is called continuous at $b \in Y$ if, for every convergent sequence $\{b_n\}$ in Y with limit b , the sequence $\{\xi(b_n)\}$ converges to $\xi(b)$, while ξ is called continuous on $C \subseteq Y$ if it is continuous at each

point of C . It is obvious that if $\xi: Y \rightarrow Y$ is continuous and C is compact, then $\xi(C)$ is also compact.

Definition 5 (Grabiec [5]). “Let $(Y, F, *)$ be a fuzzy metric space, $C \subseteq Y$ and $\xi: C \rightarrow C$ be a mapping. Then, ξ is called Edelstein contractive on C if:

$$F(b, \ell, \tau) < F(\xi(b), \xi(\ell), \tau) \text{ for all } b, \ell \in C, b \neq \ell, \tau > 0.” \tag{1}$$

For various properties of a fuzzy metric space, the reader is referred to [6]. The notion of fuzzy metric spaces and contractive mappings in fuzzy metric spaces have several interesting and useful extensions, and for a detailed study on it, we refer the reader to [9,26–30].

We now state our main findings.

3. Main Result

In this section, we first introduce Edelstein ζ -contractive mappings on fuzzy metric spaces and prove a coincidence point result, as well as a common fixed point result.

Definition 6. Let $(Y, F, *)$ be a fuzzy metric space, $C \subseteq Y$ and $\xi, \zeta: C \rightarrow C$ be two mappings. Then, ξ is called Edelstein ζ -contractive on C if:

$$F(\zeta(b), \zeta(\ell), \tau) < F(\xi(b), \xi(\ell), \tau) \text{ for all } b, \ell \in C, \zeta(b) \neq \zeta(\ell), \tau > 0. \tag{2}$$

It is obvious that every Edelstein contractive mapping ξ on C is Edelstein ζ -contractive on C with $\zeta = I_C$ (the identity mapping of C).

Example 1. Let $Y = (0, \infty)$, $a * b = ab$ for all $a, b \in [0, 1]$ and the fuzzy metric $F: Y \times Y \times (0, \infty) \rightarrow [0, 1]$ be defined by:

$$F(b, \ell, \tau) = \frac{\min\{b, \ell\}}{\max\{b, \ell\}} \text{ for all } b, \ell \in Y.$$

Then $(Y, F, *)$ is a fuzzy metric space. Consider the mappings $\xi, \zeta: Y \rightarrow Y$ defined by $\xi(b) = b^2 + 2$ and $\zeta(b) = b^2 + 1$ for all $b, \ell \in Y$. If $b < \ell$, then $\frac{b^2+1}{\ell^2+1} < \frac{b^2+2}{\ell^2+2}$, i.e., $F(\zeta(b), \zeta(\ell), \tau) < F(\xi(b), \xi(\ell), \tau)$. Similar inequality holds if $\ell < b$. Hence ξ is Edelstein ζ -contractive on Y . On the other hand, ζ is Edelstein contractive on Y since (2) does not hold for $b = 1$ and $\ell = 2$.

From the above example, it is clear that $\xi(Y) \subset \zeta(Y)$; therefore, it is always possible to construct a Jungck sequence of the ordered pair (ξ, ζ) with some initial value $b_0 \in Y$ (e.g., one can choose $b_0 = 1$), while ξ and ζ have no coincidence point and no periodic point (since for any $\{\ell_n\} \in J_Y(\xi, \zeta)$, $\ell_{n+p} \neq \ell_n$ for all $p \in \mathbb{N}$). In the next proposition, we establish the fact that if ξ and ζ have no point of coincidence (therefore no coincidence point), then they have no periodic point. Further, we will establish the conditions under which ξ and ζ will have point of coincidence.

Proposition 2. Let $(Y, F, *)$ be a fuzzy metric space, $C \subseteq Y$ and $\xi, \zeta: C \rightarrow C$ be two mappings such that $J_Y(\xi, \zeta) \neq \emptyset$. If ξ is Edelstein ζ -contractive on C , then either ξ and ζ have a point of coincidence or they have no periodic point.

Proof. Suppose that $\{\ell_n\} \in J_Y(\xi, \zeta)$ with initial value b_0 . Note that any periodic point of order 1 is a point of coincidence of ξ and ζ . Hence, assume that there is no periodic point of order 1. We claim that there is no $n \in \mathbb{N}$ such that for a $p \geq 2$ we have $\ell_{n+p} = \ell_n$. On the

contrary, suppose that there exist $n \in \mathbb{N}$ and $p \geq 2$ such that $\ell_{n+p} = \ell_n$. Then, since there is no periodic point of order 1, by condition (2) we have: for all $\tau > 0$

$$\begin{aligned} F(\ell_{n+p}, \ell_{n+p-1}, \tau) &= F(\xi(b_{n+p}), \xi(b_{n+p-1}), \tau) \\ &> F(\zeta(b_{n+p}), \zeta(b_{n+p-1}), \tau) \\ &= F(\ell_{n+p-1}, \ell_{n+p-2}, \tau). \end{aligned}$$

Proceeding in similar way, we obtain

$$F(\ell_{n+p}, \ell_{n+p-1}, \tau) > F(\ell_{n+1}, \ell_n, \tau) \text{ for all } \tau > 0. \tag{3}$$

Again,

$$\begin{aligned} F(\ell_{n+1}, \ell_n, \tau) &= F(\ell_{n+1}, \ell_{n+p}, \tau) \\ &= F(\xi(b_{n+1}), \xi(b_{n+p}), \tau) \\ &> F(\zeta(b_{n+1}), \zeta(b_{n+p}), \tau) \\ &= F(\ell_n, \ell_{n+p-1}, \tau) \\ &= F(\ell_{n+p}, \ell_{n+p-1}, \tau). \end{aligned}$$

The above inequality with (3) yields a contradiction. This proves the claim. \square

Lemma 1. *Let $(Y, F, *)$ be a fuzzy metric space, $C \subseteq Y$ and $\xi, \zeta: C \rightarrow C$ be two mappings. If ξ is Edelstein ζ -contractive on C and ζ is continuous on C , then ξ is continuous on C .*

Proof. If $C = \emptyset$ then the result is trivial. Suppose that $b \in C$ and $\{b_n\}$ is a sequence in C and converging to b . Then, by continuity of ζ , the sequence $\{\zeta(b_n)\}$ converges to $\zeta(b)$, i.e., $\lim_{n \rightarrow \infty} F(\zeta(b_n), \zeta(b), \tau) = 1$ for all $\tau > 0$. Since ξ is Edelstein ζ -contractive on C by (2), we have:

$$\lim_{n \rightarrow \infty} F(\xi(b_n), \xi(b), \tau) \geq \lim_{n \rightarrow \infty} F(\zeta(b_n), \zeta(b), \tau) = 1$$

for all $\tau > 0$. Hence, $\{\xi(b_n)\}$ converges to $\xi(b)$, and so, ξ is also continuous at $b \in C$ for all $b \in C$. \square

Theorem 1. *Let $(Y, F, *)$ be a fuzzy metric space, C be a nonempty compact subset of Y and $\xi, \zeta: C \rightarrow C$ be two mappings such that $\xi(C) \subseteq \zeta(C)$. If ξ is an Edelstein ζ -contractive mapping and ζ is continuous, then $P(\xi, \zeta) \cap \zeta(C)$ is singleton and for each initial value $b_0 \in C$ there exists $\{\ell_n\} \in J_C(\xi, \zeta)$, $\ell_{n-1} = \xi(b_{n-1}) = \zeta(b_n)$, $n \in \mathbb{N}$ which converges to the unique element of $P(\xi, \zeta) \cap \zeta(C)$.*

Proof. For a $\tau > 0$, we define a mapping $\varphi: C \rightarrow (0, 1]$ by:

$$\varphi(c)(\tau) = F(\zeta(c), \xi(c), \tau) \text{ for all } c \in C.$$

Since ζ is continuous on C , by Lemma 1, ξ is continuous on C . Since F is continuous on $Y \times Y \times (0, \infty)$ (see [31]), ζ and ξ are continuous, then the function φ is continuous and hence it attains its maximum value in C . Suppose that $\alpha \in C$ and for $\tau > 0$ there holds

$$\varphi(\alpha)(\tau) = \max_{c \in C} \varphi(c)(\tau).$$

We claim that $\alpha \in \text{Co}(\xi, \zeta)$. On the contrary, suppose that $\xi(\alpha) \neq \zeta(\alpha)$. As, $\xi(C) \subseteq \zeta(C)$, there exists $\beta \in C$ such that $\xi(\alpha) = \zeta(\beta) \neq \zeta(\alpha)$. Since ξ is Edelstein ζ -contractive, then for each $\tau > 0$, we obtain

$$\begin{aligned} \varphi(\beta)(\tau) &= F(\zeta(\beta), \xi(\beta), \tau) = F(\xi(\alpha), \zeta(\beta), \tau) \\ &> F(\zeta(\alpha), \zeta(\beta), \tau) = F(\zeta(\alpha), \xi(\alpha), \tau) \\ &= \varphi(\alpha)(\tau). \end{aligned}$$

This is a contradiction with the definition of α ; hence, we must have $\xi(\alpha) = \zeta(\alpha)$, i.e., $\alpha \in \text{Co}(\xi, \zeta)$ and $\zeta(\alpha) \in P(\xi, \zeta)$. The uniqueness of the point of coincidence $\zeta(\alpha)$ follows from the contractive condition (2); hence, $P(\xi, \zeta) \cap \zeta(C) = \{\zeta(\alpha)\}$.

Suppose that $b_0 \in C$. By the steps given below, we construct a sequence $\{\ell_n\} \in J_C(\xi, \zeta)$ with initial value b_0 such that $\{\ell_n\}$ converges to $\zeta(\alpha)$.

Since $\xi(b_0) \in \xi(C) \subseteq \zeta(C)$, there exists $b_1 \in C$ such that $f(b_0) = \zeta(b_1) = \ell_0$ (say). Again, since $\xi(b_1) \in \xi(C) \subseteq \zeta(C)$, there exists $b_2 \in C$ such that $\xi(b_1) = \zeta(b_2) = \ell_1$ (say). In a similar way, we obtain $\ell_{n-1} = \xi(b_{n-1}) = \zeta(b_n)$ for all $n \in \mathbb{N}$, i.e., $\{\ell_n\} \in J_C(\xi, \zeta)$ with an initial value b_0 . Now, we consider the following cases I and II:

Case I: If at any stage of the process a term of the sequence $\{\ell_n\}$ is equal to $\zeta(\alpha)$, i.e., if for an $n_0 \in \mathbb{N}$ we obtain $\ell_{n_0-1} = \xi(b_{n_0-1}) = \zeta(\alpha)$, then since $\alpha \in C$, the above process permits us to choose $b_{n_0} = \alpha$, so that $\ell_{n_0-1} = \xi(b_{n_0-1}) = \zeta(b_{n_0}) (= \zeta(\alpha))$. Again, since $\xi(b_{n_0}) (= \zeta(\alpha)) \in \xi(C) \subseteq \zeta(C)$ and $\alpha \in \text{Co}(\xi, \zeta)$ (i.e., $\zeta(\alpha) = \xi(\alpha)$) we choose $b_{n_0+1} = \alpha$, so that $\ell_{n_0} = \xi(b_{n_0}) = \zeta(b_{n_0+1}) = \xi(\alpha) = \zeta(\alpha)$. Proceeding in a similar way, we observe that the sequence $\{\ell_n\}$ is an eventually constant sequence (i.e., $\ell_{n_0+i} = \zeta(\alpha)$ for all $i \in \mathbb{N}$) converging to $\zeta(\alpha)$.

Case II: Suppose that $\ell_{n-1} = \xi(b_{n-1}) = \zeta(b_n) \neq \zeta(\alpha)$ for all $n \in \mathbb{N}$. Define $F_n(\tau) = F(\ell_n, \zeta(\alpha), \tau)$ for all $n \in \mathbb{N} \cup \{0\}$ and $\tau > 0$. Then, for each $\tau > 0$ we have

$$\begin{aligned} F_n(\tau) &= F(\ell_n, \zeta(\alpha), \tau) = F(\xi(b_n), \zeta(\alpha), \tau) \\ &> F(\zeta(b_n), \zeta(\alpha), \tau) \\ &= F(\ell_{n-1}, \zeta(\alpha), \tau) \\ &= F_{n-1}(\tau). \end{aligned}$$

Hence, $\{F_n(t)\}$ is an increasing sequence in $(0, 1]$, and it is convergent. Let us suppose

$$\lim_{n \rightarrow \infty} F_n(\tau) = \mathcal{M}(\tau) \in (0, 1] \text{ for each } \tau > 0.$$

Since $b_n \in C$ and C is compact, there exists a subsequence $\{b_{n_i}\}$ which converges to $b \in C$ and by continuity of ξ and ζ we obtain $\ell_{n_i-1} = \zeta(b_{n_i}) \rightarrow \zeta(b)$ and $\ell_{n_i} = \xi(b_{n_i}) \rightarrow \xi(b)$ as $i \rightarrow \infty$. We show that $\mathcal{M}(\tau) = 1$ for all $\tau > 0$. On the contrary, suppose that $\mathcal{M}(\tau_0) < 1$ for some $\tau_0 > 0$. Then, we have

$$1 > \mathcal{M}(\tau_0) = \lim_{i \rightarrow \infty} F_{n_i-1}(\tau_0) = \lim_{i \rightarrow \infty} F(\ell_{n_i-1}, \zeta(\alpha), \tau_0) = F(\zeta(b), \zeta(\alpha), \tau_0).$$

Since $\zeta(b) \neq \zeta(\alpha)$, by (2) we obtain

$$\begin{aligned} F(\zeta(b), \zeta(\alpha), \tau_0) &= \mathcal{M}(\tau_0) \\ &= \lim_{i \rightarrow \infty} F(\ell_{n_i}, \zeta(\alpha), \tau_0) = F(\xi(b), \zeta(\alpha), \tau_0) \\ &> F(\zeta(b), \zeta(\alpha), \tau_0). \end{aligned}$$

This contradiction shows that $\mathcal{M}(\tau) = \lim_{n \rightarrow \infty} F_n(\tau) = \lim_{n \rightarrow \infty} F(\ell_n, \zeta(\alpha), \tau) = 1$ for all $\tau > 0$, i.e., $\{\ell_n\}$ converges to $\zeta(\alpha)$. \square

Remark 1. If in the above theorem, the mappings ξ and ζ are weakly compatible, then by Proposition 1 the mappings ξ and ζ have a unique common fixed point.

Example 2. Let $Y = \{b_n : n \in \mathbb{N}\} \cup \{1\}$, where $b_n = 1 - \frac{1}{2^n}$ for all $n \in \mathbb{N}$, $a * b = ab$ for all $a, b \in [0, 1]$ and $F : Y \times Y \times (0, \infty) \rightarrow [0, 1]$ be defined by:

$$F(b, \ell, \tau) = \frac{\min\{b, \ell\}}{\max\{b, \ell\}} \text{ for all } b, \ell \in Y.$$

Then, $(Y, F, *)$ is a fuzzy metric space. Let $C = Y$, then C is a nonempty compact subset of Y . Consider the mappings $\xi, \zeta : Y \rightarrow Y$ defined by $\xi(b_n) = b_{n+2}$, $\zeta(b_n) = b_{n+1}$ for all $n \in \mathbb{N}$ and $\xi(1) = \zeta(1) = 1$. Then, it is clear that $\xi(C) \subseteq \zeta(C)$. If $n < m$, then we have

$$\begin{aligned} F(\xi(b_n), \zeta(b_m), t) &= F(b_{n+2}, b_{m+1}, t) = \frac{b_{n+1}}{b_{m+1}} = \frac{1 - \frac{1}{2^{n+1}}}{1 - \frac{1}{2^{m+1}}} \\ &= \frac{2^{n+1} - 1}{2^{m+1} - 1} \times 2^{m-n} < \frac{2^{n+2} - 1}{2^{m+2} - 1} \times 2^{m-n} \\ &= F(\xi(b_n), \zeta(b_m), t) \end{aligned}$$

for all $t > 0$. Also:

$$F(\zeta(1), \zeta(b_n), t) = F(1, \zeta(b_n), t) = 1 - \frac{1}{2^{n+1}} < 1 - \frac{1}{2^{n+2}} = F(\xi(1), \xi(b_n), t)$$

for all $t > 0$. Therefore, ξ is Edelstein ζ -contractive on Y . It is easy to verify that ζ is continuous. We see that $P(\xi, \zeta) \cap \zeta(C) = \{1\}$ is singleton and for each initial value $b_0 \in C$ the Jungck sequence $\{\ell_n\} \in J_C(\xi, \zeta)$, $\ell_{n-1} = \xi(b_{n-1}) = \zeta(b_n)$, $n \in \mathbb{N}$ converges to 1.

We now state an application of the theorems stated above.

4. Application to a Stable Market

In this section, we apply Theorem 1 to prove that under some suitable conditions, a dynamic market is stable and an equilibrium state of the market is obtained as a limiting state.

Consider a simple demand and supply model of a market. We represent various stages of the market graphically by taking the quantity (demanded or supplied) and the corresponding price of the commodity on the Q (horizontal) and P (vertical) axes, respectively. Such markets are completely characterized by the supply function (S) and the demand function (D) which express the price (p) in terms of the supplied quantity and demanded quantity (q), respectively. Under normal conditions of the market, the demand function $D(q)$ is decreasing, whereas the supply function $S(q)$ is increasing (see Figure 1).

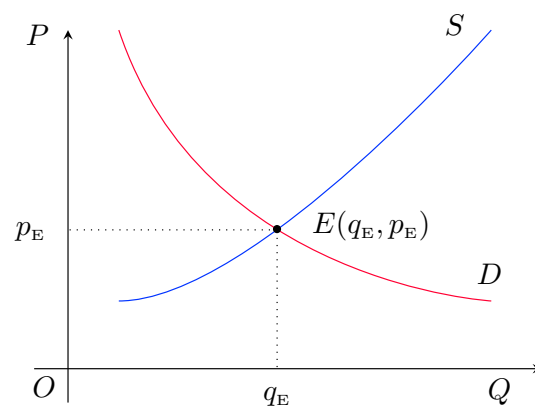


Figure 1. D: Demand curve, S: Supply curve, E: Equilibrium point.

In practice, we assume that the price of a commodity and its demanded and supplied quantities are nonnegative finite quantities and the demand and supply functions are

continuous functions of quantity q . We assume that the quantity (demanded or supplied) always lies in the interval $[0, a]$, whereas the corresponding prices varies in the range $[0, b]$ for some $a, b > 0$. We also assume that for each price $p \in [0, b]$, there is a definite supplied quantity $q \in [0, a]$, and similar is true for demanded quantity as well. For application purpose, with suitable choice of functions, we can assume that the demanded and supplied quantities and the corresponding prices all lies in the interval $[0, a]$ (see Remark 2 below), and in the rest of discussion, we denote this interval by $I = [0, a]$.

Definition 7. The sensitive index of a dynamic market in an interval $[q, r] \subseteq I$ is a positive number $i(q, r)$ and it is defined by

$$i(q, r) = \frac{|D(q) - D(r)|}{|S(q) - S(r)|}, q \neq r.$$

A dynamic market is said to be supply-sensitive if $i(q, r) < 1$ for all $q, r \in I$, i.e.,

$$|D(q) - D(r)| < |S(q) - S(r)| \text{ for all } q, r \in I \text{ with } q \neq r. \tag{4}$$

The above condition shows that corresponding to a definite change in the quantity, supply of commodity changes faster than the demand of commodity; hence, the market with such a nature will be called supply-sensitive.

On the other hand, if corresponding to a definite change in the quantity, demand of commodity changes faster than the supply of commodity, then the market is called demand-sensitive. Mathematically, the market is said to be a demand-sensitive if $i(q, r) > 1$ for all $q, r \in I$, i.e.,

$$|S(q) - S(r)| < |D(q) - D(r)| \text{ for all } q, r \in I \text{ with } q \neq r. \tag{5}$$

Theorem 2. A supply-sensitive market is always stable and there exists a Jungck sequence of prices $\{p_n\} \in J_I(D, S)$ which converges to the equilibrium price p_E .

Proof. Let $q \in I$, then consider the price corresponding to this demanded quantity, i.e., $p = D(q)$. Since for each price there is a definite supplied quantity, there exists $q' \in I$ such that $p = S(q')$. This shows that $D(I) \subseteq S(I)$.

Since the market is supply-sensitive, we have

$$|D(b) - D(\ell)| < |S(b) - S(\ell)| \text{ for all } b, \ell \in I \text{ with } b \neq \ell.$$

Note that the above inequality holds for all $b, \ell \in I$ with $S(b) \neq S(\ell)$.

Define a fuzzy set F on $\mathbb{R} \times \mathbb{R} \times (0, \infty)$ by:

$$F(b, \ell, \tau) = \frac{\tau}{\tau + |b - \ell|} \text{ for all } b, \ell \in I.$$

Then, it is easy to see that $(\mathbb{R}, F, *)$ is a fuzzy metric space, where $\tau_1 * \tau_2 = \tau_1 \tau_2$ for all $\tau_1, \tau_2 \in (0, \infty)$. Also, for $S(b) \neq S(\ell)$ since $|D(b) - D(\ell)| < |S(b) - S(\ell)|$, we obtain

$$F(S(b), S(\ell), \tau) = \frac{\tau}{\tau + |S(b) - S(\ell)|} < \frac{\tau}{\tau + |D(b) - D(\ell)|} = F(D(b), D(\ell), \tau).$$

Note that I is compact in the usual metric space $(\mathbb{R}, |\cdot|)$ and $\lim_{n \rightarrow \infty} F(q_n, q, \tau) = 1$ if and only if $\lim_{n \rightarrow \infty} |q_n - q| = 0$ for every sequence $\{q_n\}$ and q in \mathbb{R} . Therefore, I is a compact subset of the fuzzy metric space $(\mathbb{R}, F, *)$. Thus, with $Y = \mathbb{R}$, $C = I$, $\zeta \equiv D$ and $\xi \equiv S$, all the conditions of Theorem 1 are satisfied. Hence, by Theorem 1, for every initial value $q_0 \in I$ the Jungck sequence $\{p_n\} \in J_I(D, S)$, $p_{n-1} = D(q_{n-1}) = S(q_n)$, $n \in \mathbb{N}$ converges to the unique point of coincidence of D and S . Let $p_E = D(q_E) = S(q_E)$ be the unique point of coincidence of D and S . Obviously, p_E is the equilibrium price of the market and the market is stable. \square

Theorem 3. A demand-sensitive market is always stable and there exists a Jungck sequence of prices $\{p_n\} \in J_I(S, D)$ which converges to the equilibrium price p_E .

Proof. Consider $p = S(q)$ for $q \in I$. Since for each price there is a definite demanded quantity, there exists $q' \in I$ such that $p = D(q')$. This shows that $S(I) \subseteq D(I)$.

Since the market is demand-sensitive, we have

$$|S(b) - S(\ell)| < |D(b) - D(\ell)| \text{ for all } b, \ell \in I \text{ with } b \neq \ell.$$

Note that the above inequality holds for all $b, \ell \in I$ with $D(b) \neq D(\ell)$. Consider the fuzzy set F on $\mathbb{R} \times \mathbb{R} \times (0, \infty)$ and the t -norm $*$ defined in the proof of Theorem 2. Since $|S(b) - S(\ell)| < |D(b) - D(\ell)|$, for $D(b) \neq D(\ell)$ we obtain

$$F(D(b), D(\ell), \tau) = \frac{\tau}{\tau + |D(b) - D(\ell)|} < \frac{\tau}{\tau + |S(b) - S(\ell)|} = F(S(b), S(\ell), \tau).$$

Thus, with $Y = \mathbb{R}$, $C = I$, $\xi \equiv S$ and $\zeta \equiv D$, all the conditions of Theorem 1 are satisfied. Hence, by Theorem 1, for every initial value $q_0 \in I$, the Jungck sequence $\{p_n\} \in J_I(S, D)$, $p_{n-1} = S(q_{n-1}) = D(q_n)$, $n \in \mathbb{N}$ converges to the unique point of coincidence of D and S . Let $p_E = S(q_E) = D(q_E)$ is the unique point of coincidence of S and D . Obviously, p_E is the equilibrium price of the market and the market is stable. \square

Convergence for supply and demand-sensitive markets: First consider a supply-sensitive market. In practice, the price of a commodity is not equal to the equilibrium price. Suppose initially the price is $p_0 \neq p_E$, i.e., $p_0 > p_E$ (if $p_0 < p_E$, then a similar explanation will work). At this stage, suppose $p_0 = D(q_0)$. Since corresponding to this price p_0 there is a definite supplied quantity, say q_1 , the quantity q_1 can be obtained by drawing a line passing from the point $(0, p_0)$ and parallel to the Q axis, see Figure 2, i.e., $p_0 = D(q_0) = S(q_1)$.

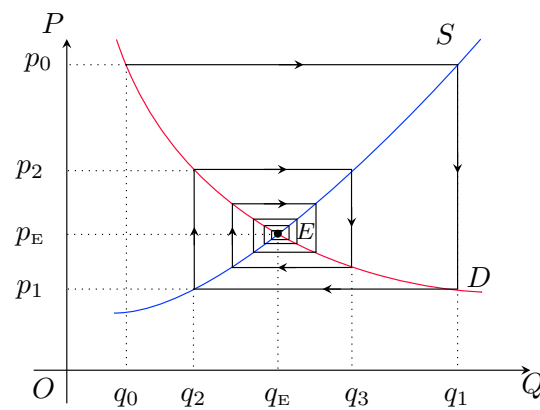


Figure 2. A supply-sensitive market.

From Figure 2, it is clear that $q_0 < q_1$; hence, at this stage the supplied quantity is higher than the demanded quantity of commodity and there is a surplus of the commodity $q_1 - q_0$. The reaction of producers causes a decrement in the price of the commodity up to a value p_1 such that for this price, the corresponding demanded quantity is q_1 , i.e., $p_1 = D(q_1)$. This process leads the market to the next (first) stage.

The price p_1 at first stage of the market is lower than the equilibrium price, i.e., $p_1 < p_E$. At this stage, the supplied quantity is q_2 and the demanded quantity is q_1 and $q_2 < q_1$, $p_1 = D(q_1) = S(q_2)$; hence, there is shortage of commodity $q_1 - q_2$. Now producers increase the price up to a value p_2 such that for this price the corresponding demanded quantity is q_2 , i.e., $p_2 = D(q_2)$. Since there is a definite supplied quantity corresponding

to this price p_2 , say q_3 , i.e., $p_2 = D(q_2) = S(q_3)$. This process leads the market to the next (second) stage.

Similarly, for each stage of the market we obtain a sequence of prices $\{p_n\}$, where

$$p_n = D(q_n) = S(q_{n+1}) \text{ for all } n \geq 0.$$

Again, this sequence of prices is a Jungck sequence of the ordered pair (D, S) , i.e., $\{p_n\} \in J_1(D, S)$, and the market is supply-sensitive; hence, the Jungck sequence is forced by Theorem 2 to be convergent, and the limit of the Jungck sequence is the point of coincidence of the functions D and S which is actually the equilibrium price for the market. Hence, the market is stable.

On the other hand, for a demand-sensitive market, if the initial price is $p_0 \neq p_E$ and $p_0 > p_E$ (if $p_0 < p_E$, then a similar explanation will work). At this stage, suppose $p_0 = S(q_0)$. Since corresponding to this price p_0 there is a definite demanded quantity, say q_1 (the quantity q_1 can be obtained by drawing a line which is parallel to the P axis and passes through the point of intersection of: (i) demand curve; and (ii) the line parallel to Q axis and passing through the point $(0, p_0)$; see Figure 3), i.e., $p_0 = S(q_0) = D(q_1)$.

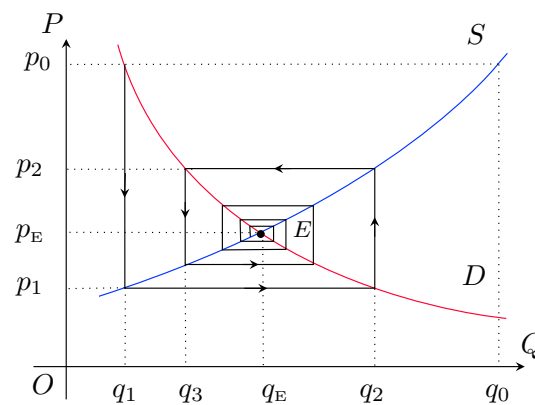


Figure 3. A demand-sensitive market.

From Figure 3 it is clear that $q_1 < q_0$; hence, at this stage there is a surplus of the commodity $q_0 - q_1$. The reaction of producers causes a decrement in the price of the commodity up to a value p_1 such that for this price the corresponding supplied quantity is q_1 , i.e., $p_1 = S(q_1)$. This process leads the market to the next (first) stage.

The price p_1 at first stage of the market is lower than the equilibrium price, i.e., $p_1 < p_E$. At this stage, the supplied quantity is q_1 and the demanded quantity is q_2 and $q_1 < q_2$, $p_1 = S(q_1) = D(q_2)$; hence, there is shortage of commodity $q_2 - q_1$. Now, producers increase the price up to a value p_2 such that for this price the corresponding supplied quantity is q_2 , i.e., $p_2 = S(q_2)$. Since there is a definite demanded quantity corresponding to this price p_2 , say q_3 , i.e., $p_2 = S(q_2) = D(q_3)$, this process leads the market to the next (second) stage.

Similarly, for each stage of the market, we obtain a sequence of prices $\{p_n\}$, where

$$p_n = S(q_n) = D(q_{n+1}) \text{ for all } n \geq 0.$$

Obviously, this sequence of prices is a Jungck sequence of the ordered pair (S, D) , i.e., $\{p_n\} \in J_1(S, D)$. Furthermore, since the market is demand-sensitive, the Jungck sequence is forced by Theorem 3 to be convergent, and the limit of the Jungck sequence is the point of coincidence of the functions D and S , which is actually the equilibrium price for the market. Hence, the market is stable.

Remark 2. In practical problems, it may happen that the demanded (supplied) quantities lie in an interval $[0, a]$ and the corresponding price of the commodity lies in the interval $[0, b]$ and $b \neq a$.

To overcome such situations, we use the method of “scaling”. We illustrate this method with the example of a supply-sensitive market as follows (the same can be conducted for a demand-sensitive market): consider the functions $D, S: [0, a] \rightarrow [0, b]$ satisfying $|D(b) - D(\ell)| < |S(b) - S(\ell)|$ for all $b, \ell \in [0, a]$ with $b \neq \ell$. Suppose that $D([0, a]) \subseteq S([0, a])$ and S is continuous. If $b \leq a$, i.e., $[0, b] \subseteq [0, a]$ then D, S are self-mappings of $[0, a]$; hence, by following a similar process to the proof of Theorem 2 one can ensure that D, S have a point of coincidence, i.e., there exists $\alpha (= q_E) \in [0, a]$ such that $D(\alpha) = S(\alpha)$. However, if $a < b$, i.e., $[0, b] \supset [0, a]$ then D, S are not self-mappings of $[0, a]$, and so, the same procedure is not applicable and the existence of point of coincidence cannot be concluded.

To construct self-mappings on $[0, a]$ with the desired properties, we apply the process of “scaling” on D, S such that the coincidence point can be concluded in a similar manner as we have conducted in the proof of Theorem 2. Suppose $a < b$, and consider the functions:

$$D_1(q) = \frac{a}{b}D(q), S_1(q) = \frac{a}{b}S(q) \text{ for all } q \in [0, a].$$

Then, we observe that $D_1, S_1: [0, a] \rightarrow [0, a]$. Also, for $b, \ell \in [0, a], b \neq \ell$ we have

$$\begin{aligned} |D_1(b) - D_1(\ell)| &= \left| \frac{a}{b}D(b) - \frac{a}{b}D(\ell) \right| < \frac{a}{b}|S(b) - S(\ell)| \\ &= \frac{a}{b} \left| \frac{b}{a}S_1(b) - \frac{b}{a}S_1(\ell) \right| \\ &= |S_1(b) - S_1(\ell)|. \end{aligned}$$

Since $D([0, a]) \subseteq S([0, a])$, we have

$$D_1([0, a]) = \left\{ \frac{a}{b}D(q) : q \in [0, a] \right\} \subseteq \left\{ \frac{a}{b}S(q) : q \in [0, a] \right\} = S_1([0, a]).$$

Thus, $D_1, S_1: [0, a] \rightarrow [0, a]$. Hence, again following a similar process to the proof of Theorem 2, one can ensure the existence of the coincidence point of S_1, D_1 . Let $\alpha (= q_E) \in \text{Co}(D_1, S_1)$. Note that $\alpha \in \text{Co}(D_1, S_1)$ if and only if $\alpha \in \text{Co}(D, S)$; hence, we are done.

5. Difference between the Two Strategies for Supply Sensitive and Demand-Sensitive Markets and Conclusions

In this section, we discuss two strategies for supply-sensitive and demand-sensitive markets and compare our approach with the approach of [22].

It is clear from the above discussion that to examine stability of a market, it is sufficient to consider the convergence of the Jungck sequence p_n of prices, and if it is convergent, then it converges to the equilibrium price, and in this situation, the market is stable. It is also clear that in a supply-sensitive market, when the price of the commodity is not equal to the equilibrium price, at any stage of the market, producers reduce (or increase) the price until the demanded quantity becomes equal to the supplied quantity of the previous stage. Such a strategy is called a “demand-based” strategy. Hence, to control a supply-sensitive market, the producers strategy should be demand-based. On the other hand, in a demand-sensitive market, when the price of the commodity is not equal to the equilibrium price, at any stage of the market, producers reduce (or increase) the price until the supplied quantity becomes equal to the demanded quantity of the previous stage. Such a strategy is called a “supply-based” strategy. Hence, to control a demand-sensitive market, the producers strategy should be supply-based. The consequences of these two strategies can be seen in Figures 2 and 3. In Figure 2, for a supply-sensitive market, the convergence of the sequence of prices p_n is represented by a rectangular spiral that converges clockwise towards the equilibrium price p_E , while in Figure 3, for a demand-sensitive market, it is represented by a rectangular spiral that converges anticlockwise towards the equilibrium price p_E . Also, if the producers do not follow the strategies suggested above, i.e., if a “demand-based” strategy is followed for a demand-sensitive market (or a “supply-based”

strategy is followed for a supply-sensitive market), then the rectangular spiral diverges and the equilibrium state of the market cannot be reached.

We see that if neither of the conditions (4) and (5) are satisfied for the market (i.e., if the market is neither supply-sensitive nor demand-sensitive), then the market may not be stable and the procedures discussed above (sequences of prices) may fail to converge and the equilibrium may be unreachable, e.g., at any stage, if $|D(q_{n-1}) - D(q_n)| = |S(q_{n-1}) - S(q_n)|$ for some $q_{n-1}, q_n \in I$ (i.e., the sensitivity index $\iota(q_{n-1}, q_n) = 1$) with $q_{n-1} \neq q_n$ then both the procedures discussed above fail to converge as shown in Figure 4. As we can see in Figure 4 for both strategies (supply-based and demand-based), the sequence of prices is represented by a rectangle through which we see repetition of price after each consecutive stage. Mathematically, in such cases, we see periodic points (prices) of the Jungck sequence, i.e., $p_0 = p_2 = \dots$, which causes the prevention of the convergence of procedures. On the other hand, if any one of the conditions (4) and (5) is satisfied, then by Proposition 2, such a prevention is not possible and the market is stable.

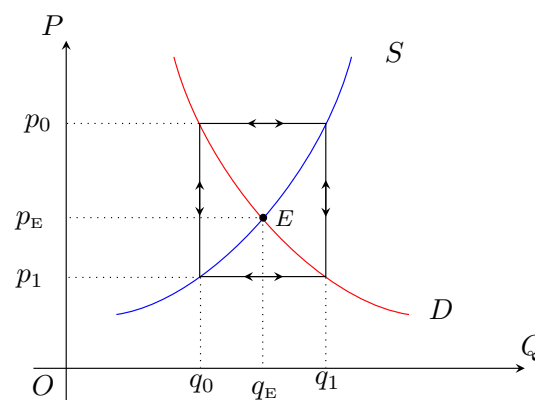


Figure 4. Failure of convergence.

Mezník’s [22] approach is based on the existence of the fixed point of the mapping DS^{-1} (see [22]) which involves the inverse of supply function S . Furthermore, the sufficient condition for convergence of sequence of prices involves the derivatives of the supply and demand functions. Further, the approach of Mezník [22] is rooted in the cobweb theory (see [32] and the references therein), which is based on the time lag between the supply and demand decision, and so, the demand of commodity is assumed to be a function of the current stage price and the supply of a commodity is a function of the previous stage. Hence, the suppliers control the demanded quantity (supplied quantity) by increasing or decreasing the price of the commodity for a current stage (e.g., in agricultural markets), i.e., by following the demand-based strategy. The drawback of this strategy is that in the case of a demand-sensitive market, the iteration procedure of Mezník [22] diverges. Here, we have presented a generalized approach of the coincidence point to establish the stability of a demand–supply model without demanding the calculation of inverse functions or a derivative of the supply and demand function. Also, we suggest the theoretical possibility of convergence of the iteration procedure of a sequence of prices in the case of a demand-sensitive market by applying the supply-based strategy for the markets of a favorable nature.

In Table 1, a comparison between these two approaches is provided.

Table 1. Comparison between two approaches.

Attribute	Mezník's [22] Approach	New Approach
Equilibrium price	Fixed point	Coincidence point
Nature of S and D	S and D must be differentiable	Derivatives are not involved
Existence of inverse (S^{-1}, D^{-1})	Requires the inverse	Inverse is not needed
Nature of sequence of price	Picard sequence	Jungck sequence
Sufficient condition for convergence	$\left \frac{D'(Q)}{S'(Q)} \right = \kappa < 1$ for all $Q \in I$	$\iota(q, r) < 1$ or $\iota(q, r) > 1$ for all $q, r \in I$

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