Hybrid near Algebra

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Abstract: The objective of this paper is to study the hybrid near algebra. It has been summarized with the proper definitions and theorems of hybrid near algebra, hybrid near algebra homomorphism and direct product of hybrid near algebra. It has been proved that a homomorphic image of a hybrid near algebra is a hybrid near algebra. It also investigated the intersection of two hybrid near algebras is a hybrid near algebra.

1. Introduction

An algebraic system with two binary operations, that satisfies all of a ring axioms, with the possible exception of one distributive law, is recognized as a near ring. The notion of a near ring was promoted and popularized by G. Pilz [1]. A near ring that admits the field as a right operator domain is referred to as a near algebra. Near algebra was familiarized by H. Brown [2]. P. Narasimha Swamy [3,4] and Rakshitha Deshmukh were developed the concepts in near algebra. The operators form only a near algebra in a quantum mechanical formalism of P. Jordan. In view of this, the study of near algebra is interesting not only as an axiomatic question but also for physical reasons. A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership function assigning a grade of membership to each object ranging between 0 and 1. The concept of fuzzy set was first established by Zadeh [5] in 1965. S. Nanda [6] and R. Biswas [7] were studied fuzzy field and fuzzy linear space. A hesitant fuzzy set is an excellent tool for expressing people's hesitancy in daily life and for dealing with uncertainty, which can be precisely and perfectly described in terms of decision makers' opinions. An extensive range of existing theories, including the theory of probability, fuzzy sets, vague sets, interval mathematics, rough sets, etc., are used to address a variety of problems in many different domains that require data with uncertainties. The concept of hesitant fuzzy sets was established by V. Torra [8]. All of these theories have their difficulties which are pointed out in soft set theory-first results [9]. To overcome these difficulties, Molodtsov [9] familiarized the soft set theory as a new mathematical tool for dealing with uncertainties that are free from problems. He successfully applied the soft set theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, etc.

As a parallel circuit of fuzzy sets, soft sets and hesitant fuzzy sets advocate the notion of hybrid structures in a group of parameters over an initial universe set illustrating several properties according to Jun, Song and Muhiuddin [10]. Using the idea, they proposed the concept of a hybrid subalgebra, a hybrid field and a hybrid linear space. B. Elavarasan [11] has studied hybrid structures applied to ideals in near-rings. Saima

Anis [12] has investigated hybrid ideals in semigroups. M. Himaya Jaleela Begum [13] were investigated has studied hybrid fuzzy bi-ideals in near-rings. Saima Anis [12] has investigated hybrid ideals in semigroups. M. Himaya Jaleela Begum [13] were investigated has studied hybrid fuzzy bi-ideals in near-rings. Srinivas et al. [14] introduced and studied the notion of fuzzy near algebra over a fuzzy filed.

In the present study, we introduce the notion of hybrid near algebra over a hybrid field and hybrid structure is used to analyze the structural statements of near algebras.

Throughout this paper, Y means a (right) near algebra over a field L.

2. Preliminaries

Definition 1 ([10]). *Let U be a universal set, $P(U)$ be the power set, L be the set of parameters, and I be the unit interval. A mapping $\widetilde{\zeta}_\lambda := \left(\widetilde{\zeta}, \lambda\right) : L \to P(U) X I$, $z \to \left(\widetilde{\zeta}(z), \lambda(z)\right)$, i.e., the image of z is designated by $\left(\widetilde{\zeta}(z), \lambda(z)\right)$ is entitled a hybrid structure in L over U, where $\widetilde{\zeta} : L \to P(U)$ and $\lambda : L \to I$ are the mappings.*

Definition 2 ([10]). *Let $\widetilde{\zeta}_\lambda$ be a hybrid structure in L over U. Then, the sets*

$$\widetilde{\zeta}_\lambda[\alpha, t] = \left\{z \in L / \widetilde{\zeta}(d) \supseteq \alpha, \lambda(d) \leq t\right\};$$
$$\widetilde{\zeta}_\lambda(\alpha, t] = \left\{z \in L / \widetilde{\zeta}(d) \supsetneq \alpha, \lambda(d) \leq t\right\};$$
$$\widetilde{\zeta}_\lambda[\alpha, t) = \left\{z \in L / \widetilde{\zeta}(d) \supseteq \alpha, \lambda(d) < t\right\};$$
$$\widetilde{\zeta}_\lambda(\alpha, t) = \left\{z \in L / \widetilde{f}(d) \supsetneq \alpha, \lambda(d) < t\right\};$$

are entitled the $[\alpha, t]$ hybrid cut, $(\alpha, t]$ hybrid cut, $[\alpha, t)$ hybrid cut, and (α, t) hybrid cut of $\widetilde{\zeta}_\lambda$, respectively, where $\alpha \in P(U)$ and $t \in I$. Obviously,

$$\widetilde{\zeta}_\lambda(\alpha, t) \subseteq \widetilde{\zeta}_\lambda(\alpha, t] \subseteq \widetilde{\zeta}_\lambda[\alpha, t] \text{ and } \widetilde{\zeta}_\lambda(\alpha, t) \subseteq \widetilde{\zeta}_\lambda[\alpha, t) \subseteq \widetilde{\zeta}_\lambda[\alpha, t].$$

Definition 3 ([10]). *Let L be a field. A hybrid structure $\widetilde{\zeta}_\lambda$ in L over U is entitled a hybrid field of L over U if*

(i) $\widetilde{\zeta}(d + a) \supseteq \widetilde{\zeta}(d) \cap \widetilde{\zeta}(a), \lambda(d + a) \leq \bigvee\{\lambda(d), \lambda(a)\}, \forall d, a \in L;$
(ii) $\widetilde{\zeta}(-d) \supseteq \widetilde{\zeta}(d), \lambda(-d) \leq \lambda(d), \forall d \in L;$
(iii) $\widetilde{\zeta}(da) \supseteq \widetilde{\zeta}(d) \cap \widetilde{\zeta}(a), \lambda(da) \leq \bigvee\{\lambda(d), \lambda(a)\}, \forall d, a \in L;$
(iv) *If $d \neq 0 \Rightarrow \widetilde{\zeta}(d^{-1}) \supseteq \widetilde{\zeta}(d), \lambda(d^{-1}) \leq \lambda(d), \forall d \in L.$*

Definition 4. *Let $\widetilde{\zeta}_\lambda$ be an $H F$ of a field L over U, Z be an algebra over L. A hybrid structure $\widetilde{k}_\delta$ in Z over U is entitled an $H A$ over the $H F$ $\left(\widetilde{\zeta}_\lambda, L\right)$ if the following conditions hold:*

(i) $\widetilde{k}(z + w) \supseteq \widetilde{k}(z) \cap \widetilde{k}(w), \delta(z + w) \leq \bigvee\{\delta(z), \delta(w)\}, \forall z, w \in Z;$
(ii) $\widetilde{k}(dz) \supseteq \widetilde{\zeta}(d) \cap \widetilde{k}(z), \delta(dz) \leq \bigvee\{\lambda(d), \delta(z)\}, \forall z \in Z, d \in L;$
(iii) $\widetilde{k}(zw) \supseteq \widetilde{k}(z) \cap \widetilde{k}(w), \delta(zw) \leq \bigvee\{\delta(z), \delta(w)\}, \forall z, w \in Z;$
(iv) $\widetilde{\zeta}(1) \supseteq \widetilde{k}(z), \lambda(1) \leq \delta(z), \forall z \in Z.$

Example 1. *Let $L = \{0, 1\}_{\oplus_2, \oplus_2}$ be a field. Then, the hybrid structure $\widetilde{\zeta}_\lambda$ in L over $U = \{u_1, u_2, u_3, u_4, u_5\}$, which is given as follows:*

L	$\widetilde{\zeta}$	λ
0	$\{u_1, u_2, u_3, u_4\}$	0.3
1	$\{u_2, u_3, u_4\}$	0.4

Therefore, $\widetilde{\zeta}_\lambda$ is an H F in L over U.

Let $Z = \{0, 1, 2, 3, 4\}$ be a set with two binary operations "$\oplus_5$", "$\oplus_5$" by

$\oplus_5$	0	1	2	3	4	$\oplus_5$	0	1	2	3	4
0	0	1	2	3	4	0	0	0	0	0	0
1	1	2	3	4	0	1	0	1	2	3	4
2	2	3	4	0	1	2	0	2	4	1	3
3	3	4	0	1	2	3	0	3	1	4	2
4	4	0	1	2	3	4	0	4	3	2	1

Clearly, Z forms an algebra over L. Then, the hybrid structure $\widetilde{k}_\delta$ in Z over $U = \{u_1, u_2, u_3, u_4, u_5\}$ is given by:

Z	$\widetilde{k}$	δ
0	$\{u_2, u_3, u_5\}$	0.6
1	$\{u_3, u_4\}$	0.7
2	$\{u_3, u_5\}$	0.8
3	$\{u_1, u_3\}$	0.9
4	$\{u_2, u_3\}$	0.9

Therefore, $\left(\widetilde{k}_\delta,\ Z\right)$ is an H A over $\left(\widetilde{\zeta}_\lambda, L\right)$.

3. Hybrid near Algebra

In this section, we familiarize hybrid near algebra (H N A) and acquire some of the properties of hybrid near algebra over a hybrid field (H F).

Definition 5. *Let $\widetilde{\zeta}_\lambda$ be an H F of a field L over U, and let Y be a near algebra over L. A hybrid structure $\widetilde{\varrho}_\gamma$ in Y over U is entitled an H N A over $\left(\widetilde{\zeta}_\lambda, L\right)$ if the following conditions hold:*

(i) $\widetilde{\varrho}(z + w) \supseteq \widetilde{\varrho}(z) \cap \widetilde{\varrho}(w),\ \gamma(z + w) \leq \bigvee\{\gamma(z), \gamma(w)\},\ \forall\, z, w \in Y$;

(ii) $\widetilde{\varrho}(dz) \supseteq \widetilde{\zeta}(d) \cap \widetilde{\varrho}(z),\ \gamma(dz) \leq \bigvee\{\lambda(d), \gamma(z)\},\ \forall\, z \in Y,\ d \in L$;

(iii) $\widetilde{\varrho}(zw) \supseteq \widetilde{\varrho}(z) \cap \widetilde{\varrho}(w),\ \gamma(zw) \leq \bigvee\{\gamma(z), \gamma(w)\},\ \forall\, z,\, w \in Y$;

(iv) $\widetilde{\zeta}(1) \supseteq \widetilde{\varrho}(z),\ \lambda(1) \leq \gamma(z), \forall\, z \in Y.$

Example 2. *Let $L = Z_2 = \{0, 1\}_{\oplus_2,\ \oplus_2}$ be a field. Then, the hybrid structure $\widetilde{\zeta}_\lambda$ in L over $U = \{u_1, u_2, u_3, u_4, u_5\}$ which is given below forms an H F in L over U:*

L	$\widetilde{\zeta}$	λ
0	$\{u_1, u_2, u_3, u_4\}$	0.3
1	$\{u_2, u_3, u_4\}$	0.4

Let $Y = \{0, h, p, e\}$ be a set with two binary operations "+" and "." By

+	0	h	p	e	.	0	h	p	e
0	0	h	p	e	0	0	0	0	0
h	h	0	e	p	h	h	h	h	h
p	p	e	0	h	p	p	p	p	p
e	e	p	h	0	e	e	e	e	e

Clearly, Y forms a near algebra over L.

Then, the hybrid structure $\widetilde{\varrho}_\gamma$ in Y over $U = \{u_1, u_2, u_3, u_4, u_5\}$, which is given as follows:

Y	$\widetilde{\rho}$	γ
0	$\{u_2, u_3, u_4\}$	0.4
h	$\{u_2, u_4\}$	0.6
p	$\{u_2, u_3\}$	0.8
e	$\{u_2\}$	0.8

Clearly, $(\widetilde{\varrho}_\gamma, Y)$ is an H N A over $\left(\widetilde{\zeta}_\lambda, L \right)$.

Theorem 1. *If $(\widetilde{\varrho}_\gamma, Y)$ is an H N A over $\left(\widetilde{\zeta}_\lambda, L \right)$, then $\widetilde{\zeta}(0) \supseteq \widetilde{\varrho}(z)$, $\lambda(0) \leq \gamma(z), \forall z \in Y$.*

Proof. By the definition, $\widetilde{\zeta}(0) \supseteq \widetilde{\zeta}(1)$, $\lambda(0) \leq \lambda(1)$ and $\widetilde{\zeta}(1) \supseteq \widetilde{\varrho}(z)$, $\lambda(1) \leq \gamma(z), \forall z \in Y$. Hence, $\widetilde{\zeta}(0) \supseteq \widetilde{\varrho}(z)$, $\lambda(0) \leq \gamma(z), \forall z \in Y$. $\square$

Theorem 2. *If $(\widetilde{\varrho}_\gamma, Y)$ is an H N A over $\left(\widetilde{\zeta}_\lambda, L \right)$, then $\widetilde{\varrho}(0) \supseteq \widetilde{\varrho}(z)$, $\gamma(0) \leq \gamma(z), \forall z \in Y$.*

Proof. Let $z \in Y$. Then,

$$
\begin{aligned}
\widetilde{\varrho}(0) \ &= \widetilde{\varrho}(z - z) = \widetilde{\varrho}(1.z - 1.z) = \widetilde{\varrho}(1.z + (-1).z); \\
&\supseteq \widetilde{\varrho}(1.z) \cap \widetilde{\varrho}((-1)z); \\
&\supseteq \left(\widetilde{\zeta}(1) \cap \widetilde{\varrho}(z) \right) \cap \left(\widetilde{\zeta}(-1) \cap \widetilde{\varrho}(z) \right); \\
&\supseteq \left(\widetilde{\zeta}(1) \cap \widetilde{\varrho}(z) \right) \cap \left(\widetilde{\zeta}(1) \cap \widetilde{\varrho}(z) \right); \\
&\supseteq (\widetilde{\varrho}(z) \cap \widetilde{\varrho}(z)) \cap (\widetilde{\varrho}(z) \cap \widetilde{\varrho}(z)); \\
&\supseteq (\widetilde{\varrho}(z) \cap \widetilde{\varrho}(z)) = \widetilde{\varrho}(z);
\end{aligned}
$$

and

$$
\begin{aligned}
\gamma(0) \ &= \gamma(z - z) = \gamma(1.z - 1.z) = \gamma(1.z + (-1)z) \leq \bigvee\{\gamma(1.z), \gamma((-1).z)\}; \\
&\leq \bigvee\{\bigvee\{\lambda(1), \gamma(z)\}, \bigvee\{\lambda(-1), \gamma(z)\}\}; \\
&\leq \bigvee\{\bigvee\{\lambda(1), \gamma(z)\}, \bigvee\{\lambda(1), \gamma(z)\}\}; \\
&\leq \bigvee\{\bigvee\{\gamma(z), \gamma(z)\}, \bigvee\{\gamma(z), \gamma(z)\}\}; \\
&\leq \bigvee\{\gamma(z), \gamma(z)\} = \gamma(z). \qquad \square
\end{aligned}
$$

Theorem 3. *Let $\widetilde{\zeta}_\lambda$ be an H F of a field L over U and Y be a near algebra over L. A hybrid structure $\widetilde{\varrho}_\gamma$ in Y over U is an H N A over $\left(\widetilde{\zeta}_\lambda, L \right)$ if and only if*

(i) $\widetilde{\varrho}(dz + aw) \supseteq \left(\widetilde{\zeta}(d) \cap \widetilde{\varrho}(z) \right) \cap \left(\widetilde{\zeta}(a) \cap \widetilde{\varrho}(w) \right) \gamma(dz + aw) \leq \bigvee\{\lambda(d), \gamma(z), \lambda(a), \gamma(w)\}, \forall z, w \in Y; d, a \in L;$

(ii) $\widetilde{\varrho}(zw) \supseteq \widetilde{\varrho}(z) \cap \widetilde{\varrho}(w)$, $\gamma(zw) \leq \bigvee\{\gamma(z), \gamma(w)\}, \forall z, w \in Y;$

(iii) $\widetilde{\zeta}(1) \supseteq \widetilde{\varrho}(z)$, $\lambda(1) \leq \gamma(z), \forall z \in Y.$

Proof. Let $z, w \in Y$ and $d, a \in L$. Then,

$$\widetilde{\varrho}(dz + aw) \supseteq \widetilde{\varrho}(dz) \cap \widetilde{\varrho}(aw) \supseteq \left(\widetilde{\zeta}(d) \cap \widetilde{\varrho}(z)\right) \cap \left(\widetilde{\zeta}(a) \cap \widetilde{\varrho}(w)\right) \text{ and}$$

$$\gamma(dz + aw) \leq \bigvee\{\gamma(dz), \gamma(aw)\};$$
$$\leq \bigvee\{\bigvee\{\lambda(d), \gamma(z)\}, \bigvee\{\lambda(a), \gamma(w)\}\};$$
$$\leq \bigvee\{\lambda(d), \gamma(z), \lambda(a), \gamma(w)\}.$$

(ii) and (iii) hold directly, as $\widetilde{\varrho}_\gamma$ is an H N A of Y over U.
Conversely, presume that the three conditions of the hypothesis hold.
Let $z, w \in Y$ and $d, a \in L$. Then,

$$\widetilde{\varrho}(z + w) = \widetilde{\varrho}(1.z + 1.w) \supseteq \left(\widetilde{\zeta}(1) \cap \widetilde{\varrho}(z)\right) \cap \left(\widetilde{\zeta}(1) \cap \widetilde{\varrho}(w)\right);$$
$$\supseteq (\widetilde{\varrho}(z) \cap \widetilde{\varrho}(z)) \cap (\widetilde{\varrho}(w) \cap \widetilde{\varrho}(w));$$
$$= \widetilde{\varrho}(z) \cap \widetilde{\varrho}(w);$$
$$\gamma(z + w) = \gamma(1.z + 1.w) \leq \bigvee\{\lambda(1), \gamma(z), \lambda(1), \gamma(w)\};$$
$$\leq \bigvee\{\gamma(z), \gamma(z), \gamma(w), \gamma(w)\};$$
$$= \bigvee\{\gamma(z), \gamma(w)\}.$$

For any, $z \in Y, d \in L$,

$$\widetilde{\varrho}(dz) = \widetilde{\varrho}(dz + 0.z);$$
$$\supseteq \left(\widetilde{\zeta}(d) \cap \widetilde{\varrho}(z)\right) \cap \left(\widetilde{\zeta}(0) \cap \widetilde{\varrho}(z)\right);$$
$$\supseteq \left(\widetilde{\zeta}(d) \cap \widetilde{\varrho}(z)\right) \cap \left(\widetilde{\zeta}(d) \cap \widetilde{\varrho}(z)\right);$$
$$= \widetilde{\zeta}(d) \cap \widetilde{\varrho}(z);$$
$$\gamma(dz) = \gamma(dz + 0.z) \leq \bigvee\{\lambda(d), \gamma(z), \lambda(0), \gamma(z)\};$$
$$\leq \bigvee\{\lambda(d), \gamma(z), \lambda(d), \gamma(z)\};$$
$$= \bigvee\{\lambda(d), \gamma(z)\}.$$

From the hypothesis, $\widetilde{\varrho}(zw) \supseteq \widetilde{\varrho}(z) \cap \widetilde{\varrho}(w)$, $\gamma(zw) \leq \bigvee\{\gamma(z), \gamma(w)\}, \forall z, w \in Y$ and $\widetilde{\zeta}(1) \supseteq \widetilde{\varrho}(z)$, $\lambda(1) \leq \gamma(z), \forall z \in Y$. Hence, $(\widetilde{\varrho}_\gamma, Y)$ is an H N A over $\left(\widetilde{\zeta}_\lambda, L\right)$. $\square$

Proposition 1. *If* $(\widetilde{\varrho}_\gamma, Y)$ *is an H N A over* $\left(\widetilde{\zeta}_\lambda, L\right)$*, then* $\widetilde{\varrho}(z - w) \supseteq \widetilde{\varrho}(z) \cap \widetilde{\varrho}(w)$ *and* $\gamma(z - w) \leq \bigvee\{\gamma(z), \gamma(w)\}, \forall z, w \in Y$.

Proof. Given, $(\widetilde{\varrho}_\gamma, Y)$ is an H N A over $\left(\widetilde{\zeta}_\lambda, L\right)$.
Let $z, w \in Y$ and 1 be the identity element in L.
Then,

$$\widetilde{\varrho}(z - w) = \widetilde{\varrho}(1.z + (-1).w);$$
$$\supseteq \widetilde{\varrho}(1.z) \cap \widetilde{\varrho}((-1)w);$$
$$\supseteq \left(\widetilde{\zeta}(1) \cap \widetilde{\varrho}(z)\right) \cap \left(\widetilde{\zeta}(-1) \cap \widetilde{\varrho}(w)\right);$$
$$\supseteq \left(\widetilde{\zeta}(1) \cap \widetilde{\varrho}(z)\right) \cap \left(\widetilde{\zeta}(1) \cap \widetilde{\varrho}(w)\right);$$
$$\supseteq (\widetilde{\varrho}(z) \cap \widetilde{\varrho}(z)) \cap (\widetilde{\varrho}(w) \cap \widetilde{\varrho}(w));$$
$$\supseteq (\widetilde{\varrho}(z) \cap \widetilde{\varrho}(w));$$

and

$$\gamma(z - w) = \gamma(1.z + (-1).w);$$
$$\leq \bigvee\{\gamma(1.z), \gamma((-1).w)\};$$
$$\leq \bigvee\{\bigvee\{\lambda(1), \gamma(z)\}, \bigvee\{\lambda(-1), \gamma(w)\}\};$$
$$\leq \bigvee\{\bigvee\{\lambda(1), \gamma(z)\}, \bigvee\{\lambda(1), \gamma(w)\}\};$$
$$\leq \bigvee\{\bigvee\{\gamma(z), \gamma(z)\}, \bigvee\{\gamma(w), \gamma(w)\}\};$$
$$\leq \bigvee\{\gamma(z), \gamma(w)\}. \qquad \square$$

Theorem 4. $\left(\widetilde{\varrho}_\gamma, Y\right)$ *is an H N A over H F* $\left(\widetilde{\zeta}_\lambda, L\right)$ *if and only if a nonempty set* $\widetilde{\varrho}_\gamma[\alpha, t]$ *is a sub near algebra over the field* $\widetilde{\zeta}_\lambda[\alpha, t] \ \forall \ t \in [0, 1]$ *and* $\alpha \in P(U)$.

Proof. Let $t \in [0, 1]$ be such that $\widetilde{\varrho}_\gamma[\alpha, t] \neq \varnothing$ and $z, w \in \widetilde{\varrho}_\gamma[\alpha, t]; d \in \widetilde{\zeta}_\lambda[\alpha, t]$. Then, $z, w \in Y$, $d \in L$ and $\widetilde{\varrho}(z) \supseteq \alpha$, $\gamma(z) \leq t$, $\widetilde{\varrho}(w) \supseteq \alpha$, $\gamma(w) \leq t$, $\widetilde{\zeta}(d) \supseteq \alpha$, $\lambda(d) \leq t$. Thus, $z - w \in Y$, $dz \in Y$. Now, $\widetilde{\varrho}(z - w) \supseteq \widetilde{\varrho}(z) \cap \widetilde{\varrho}(w) \supseteq \alpha \cap \alpha = \alpha$, $\gamma(z - w) \leq \bigvee\{\gamma(z), \gamma(w)\} \leq \bigvee\{t, t\} = t$, $\widetilde{\varrho}(dz) \supseteq \widetilde{\zeta}(d) \cap \widetilde{\varrho}(z) \supseteq \alpha \cap \alpha = \alpha$, $\gamma(dz) \leq \bigvee\{\lambda(d), \gamma(z)\} \leq \bigvee\{t, t\} = t$. Thus, $z - w \in \widetilde{\varrho}_\gamma[\alpha, t]$ and $dz \in \widetilde{\varrho}_\gamma[\alpha, t]$. Hence, $\widetilde{\varrho}_\gamma[\alpha, t]$ is a subspace of Y. Presume that $\widetilde{\zeta}_\lambda[\alpha, t] \neq \varnothing$. We know that $\widetilde{\zeta}_\lambda[\alpha, t]$ is a subfield of L. For $z, w \in \widetilde{\varrho}_\gamma[\alpha, t]$, $d \in \widetilde{\zeta}_\lambda[\alpha, t]$, we have $z, w \in Y$, $d \in L$, $\widetilde{\varrho}(z) \supseteq \alpha$, $\gamma(z) \leq t$, $\widetilde{\varrho}(w) \supseteq \alpha$, $\gamma(w) \leq t$, $\widetilde{\zeta}(d) \supseteq \alpha$, $\lambda(d) \leq t$. Thus, $z + w, zw, dz \in Y$, $d(zw) = (dz)w$. Since $(\widetilde{\varrho}_\gamma, Y)$ is an H N A over H F $\left(\widetilde{\zeta}_\lambda, L\right)$, we get $\widetilde{\varrho}(z + w) \supseteq \widetilde{\varrho}(z) \cap \widetilde{\varrho}(w) \supseteq \alpha \cap \alpha = \alpha$, $\gamma(z + w) \leq \bigvee\{\gamma(z), \gamma(w)\} \leq \bigvee\{t, t\} = t$. And $\widetilde{\varrho}(dz) \supseteq \widetilde{\zeta}(d) \cap \widetilde{\varrho}(z) \supseteq \alpha \cap \alpha = \alpha$, $\gamma(dz) \leq \bigvee\{\lambda(d), \gamma(z)\} \leq \bigvee\{t, t\} = t$, $\widetilde{\varrho}(zw) \supseteq \widetilde{\varrho}(z) \cap \widetilde{\varrho}(w) \supseteq \alpha \cap \alpha = \alpha$, $(zw) \leq \bigvee\{\gamma(z), \gamma(w)\} \leq \bigvee\{t, t\} = t$. $\widetilde{\zeta}(1) \supseteq \widetilde{\varrho}(z) \supseteq \alpha$, $\lambda(1) \leq \gamma(z) \leq t$ implies that $z + w, zw, dz \in \widetilde{\varrho}_\gamma[\alpha, t]$ and $d(zw) = (dz)w$. Hence, $\widetilde{\varrho}_\gamma[\alpha, t]$ is a sub near algebra of Y over a field $\widetilde{\zeta}_\lambda[\alpha, t]$.

Conversely, presume that $\widetilde{\varrho}_\gamma[\alpha, t] \neq \varnothing$. Hence, $\widetilde{\varrho}_\gamma[\alpha, t]$ is a sub near algebra over a field $\widetilde{\zeta}_\lambda[\alpha, t]$. If possible, presume that there exists $z, w \in Y$; $d, a \in L$ such that $\widetilde{\varrho}(dz + aw) \subseteq \left(\widetilde{\zeta}(d) \cap \widetilde{\varrho}(z)\right) \cap \left(\widetilde{\zeta}(a) \cap \widetilde{\varrho}(w)\right)$, $\gamma(dz + aw) \geq \bigvee\{\lambda(d), \gamma(z), \lambda(a), \gamma(w)\}$.

Put

$$u_0 = \tfrac{1}{2}\left\{ \widetilde{\varrho}(dz + aw) + \left(\widetilde{\zeta}(d) \cap \widetilde{\varrho}(z)\right) \cap (\zeta(a) \cap \widetilde{\varrho}(w)) \right\};$$
$$v_0 = \tfrac{1}{2}\left\{ \gamma(dz + aw) + \bigvee\{\lambda(d), \gamma(z), \lambda(a), \gamma(w)\} \right\}.$$

Then, we have $\widetilde{\varrho}(dz + aw) < u_0 < \left(\widetilde{\zeta}(d) \cap \widetilde{\varrho}(z)\right) \cap \left(\widetilde{\zeta}(a) \cap \widetilde{\varrho}(w)\right)$
and $\gamma(dz + aw) > v_0 > \bigvee\{\lambda(d), \gamma(z), \lambda(a), \gamma(w)\}$.
Hence, $\widetilde{\varrho}(dz + aw) < u_0$, $\left(\widetilde{\zeta}(d) \cap \widetilde{\varrho}(z)\right) \cap \left(\widetilde{\zeta}(a) \cap \widetilde{\varrho}(w)\right) > u_0$
and $\gamma(dz + aw) > v_0$, $\bigvee\{\lambda(d), \gamma(z), \lambda(a), \gamma(w)\} < v_0$. So that
$\widetilde{\varrho}(dz + aw) < u_0$, $\left(\widetilde{\zeta}(d) \cap \widetilde{\varrho}(z)\right) > u_0$ and $\left(\widetilde{\zeta}(a) \cap \widetilde{\varrho}(z)\right) > u_0$
and $\gamma(dz + aw) > v_0$, $\lambda(d) < v_0, \gamma(z) < v_0, \lambda(a) < v_0, \gamma(w) < v_0$.
Thus, $\widetilde{\varrho}(dz + aw) < u_0$, $\widetilde{\zeta}(d) > u_0$, $\widetilde{\varrho}(z) > u_0$, $\widetilde{\zeta}(a) > u_0$, $\widetilde{\varrho}(w) > u_0$
and $\gamma(dz + aw) > v_0$, $\lambda(d) < v_0, \gamma(z) < v_0, \lambda(a) < v_0, \gamma(w) < v_0$.
Since $z, w \in Y$; $d, a \in L$, we have $dz + aw \in Y$.
Hence, $\widetilde{\varrho}(dz + aw) < u_0$, $\widetilde{\zeta}(d) > u_0$, $\widetilde{\varrho}(z) > u_0$, $\widetilde{\zeta}(a) > u_0$, $\widetilde{\varrho}(w) > u_0$ and
$\gamma(dz + aw) > v_0$, $\lambda(d) < v_0, \gamma(z) < v_0, \lambda(a) < v_0, \gamma(w) < v_0$.
Therefore, $dz + aw \notin \widetilde{\varrho}_\gamma[u_0, t] \cap \widetilde{\varrho}_\gamma[v_0, t]$, $d, a \in \widetilde{\zeta}_\lambda[u_0, t] \cap \widetilde{\zeta}_\lambda[v_0, t]$ and
$z, w \in \widetilde{\varrho}_\gamma[u_0, t] \cap \widetilde{\varrho}_\gamma[v_0, t]$. Thus, $dz + aw \notin \widetilde{\varrho}_\gamma[u_0, t] \cap \widetilde{\varrho}_\gamma[v_0, t]$ and
$dz, aw \in \widetilde{\varrho}_\gamma[u_0, t] \cap \widetilde{\varrho}_\gamma[v_0, t]$, which is a contradiction to the fact that $\widetilde{\varrho}_\gamma[\alpha, t]$ is a sub near algebra over a field $\widetilde{\zeta}_\gamma[\alpha, t]$. Hence, $\widetilde{\varrho}(dz + aw) \supseteq \left(\widetilde{\zeta}(d) \cap \widetilde{\varrho}(z)\right) \cap \left(\widetilde{\zeta}(a) \cap \widetilde{\varrho}(w)\right)$, $\gamma(dz + aw) \leq \bigvee\{\lambda(d), \gamma(z), \lambda(a), \gamma(w)\}$. If possible, presume that there exists $z, w \in Y$ such that $\widetilde{\varrho}(zw) \subseteq \widetilde{\varrho}(z) \cap \widetilde{\varrho}(w)$, $\gamma(zw) \geq \bigvee\{\gamma(z), \gamma(w)\}$. Put $u_1 = \tfrac{1}{2}\{ \widetilde{\varrho}(zw) + \widetilde{\varrho}(z) \cap \widetilde{\varrho}(w)\}$, $v_1 = \tfrac{1}{2}\{ \gamma(zw) + \bigvee\{\gamma(z), \gamma(w)\}\}$. Then, we have $\widetilde{\varrho}(zw) < u_1 < \widetilde{\varrho}(z) \cap \widetilde{\varrho}(w)$ and $\gamma(zw) > v_1 > \bigvee\{\gamma(z), \gamma(w)\}$. Hence, $\widetilde{\varrho}(zw) < u_1$, $\widetilde{\varrho}(z) \cap \widetilde{\varrho}(w) > u_1$, $\gamma(zw) > v_1$, $\bigvee\{\gamma(z), \gamma(w)\} < v_1$.
So that $\widetilde{\varrho}(zw) < u_1$, $\widetilde{\varrho}(z) > u_1$ and $\widetilde{\varrho}(w) > u_1$ and $\gamma(zw) > v_1$, $\gamma(z) < v_1, \gamma(w) < v_1$.
Since $z, w \in Y$, we have $zw \in Y$. Thus, $\widetilde{\varrho}(zw) < u_1$, $\widetilde{\varrho}(z) > u_1$, $\widetilde{\varrho}(w) > u_1$ and $\gamma(zw) > v_1, \gamma(z) < v_1$, $\gamma(w) < v_1$. Therefore, $zw \notin \widetilde{\varrho}_\gamma[u_1, t] \cap \widetilde{\varrho}_\gamma[v_1, t]$, $z, w \in \widetilde{\varrho}_\gamma[u_1, t] \cap \widetilde{\varrho}_\gamma[v_1, t]$, which is a contradiction to the fact that $\widetilde{\varrho}_\gamma[\alpha, t]$ is a sub near-algebra over a field $\widetilde{\zeta}_\lambda[\alpha, t]$. Hence, $\widetilde{\varrho}(zw) \supseteq \widetilde{\varrho}(z) \cap \widetilde{\varrho}(w)$, $\gamma(zw) \leq \bigvee\{\gamma(z), \gamma(w)\}$. If possible, presume that there exists $z \in Y$ such that $\widetilde{\zeta}(1) \subseteq \widetilde{\varrho}(z)$, $\lambda(1) \geq \gamma(z)$. Put $u_2 = \tfrac{1}{2}\left\{ \widetilde{\zeta}(1) + \widetilde{\varrho}(z) \right\}$ and $v_2 = \tfrac{1}{2}\{ \lambda(1) + \gamma(z)\}$.

Then, $\widetilde{\zeta}(1) < u_2 < \widetilde{\varrho}(z)$ and $\lambda(1) > v_2 > \gamma(z)$. Hence, $\widetilde{\zeta}(1) < u_2$, $\widetilde{\varrho}(z) > u_2$ and $\lambda(1) > v_2$, $\gamma(z) < v_2$. So that $1 \notin \widetilde{\zeta}_\lambda[u_2, t]$, $z \in \widetilde{\varrho}_\gamma[u_2, t]$, $1 \notin \widetilde{\zeta}_\lambda[v_2, t]$, $z \in \widetilde{\varrho}_\gamma[v_2, t]$. Therefore, $1 \notin \widetilde{\zeta}_\lambda[u_2, t] \cap \widetilde{\zeta}_\lambda[v_2, t]$ and $z \in \widetilde{\varrho}_\gamma[u_2, t] \cap z \in \widetilde{\varrho}_\gamma[u_2, t]$, which is a contradiction to the fact that $\widetilde{\varrho}_\gamma[\alpha, t]$ is a sub near algebra over a field $\widetilde{\zeta}_\lambda[\alpha, t]$. Thus, $\widetilde{\zeta}(1) \supseteq \widetilde{\varrho}(z)$, $\lambda(1) \le \gamma(z)$, $\forall z \in Y$. Hence, $(\widetilde{\varrho}_\gamma, Y)$ is an H N A over $\left(\widetilde{\zeta}_\lambda, L\right)$. $\square$

Definition 6. *Let $\widetilde{\varrho}_\gamma$ and $\widetilde{h}_\mu$ be hybrid structures in L over U. Then, the hybrid intersection of $\widetilde{\varrho}_\gamma$ and $\widetilde{h}_\mu$ is designated by $\widetilde{\varrho}_\gamma \widetilde{\cap} \widetilde{h}_\mu$ and is demarcated to be a hybrid structure $\widetilde{\varrho}_\gamma \widetilde{\cap} \widetilde{h}_\mu : L \to P(U) \, X \, I$, $z \to \left(\left(\widetilde{\varrho \cap h}\right)(z), (\gamma \vee \mu)(z)\right)$, for all $z \in L$, i.e., the image of z is $\left(\left(\widetilde{\varrho \cap h}\right)(z), (\gamma \vee \mu)(z)\right)$, where $\widetilde{\varrho \cap h} : L \to P(U), z \to \widetilde{\varrho}(z) \cap \widetilde{h}(z), \gamma \vee \mu : L \to I, z \to \vee\{\gamma(z), \mu(z)\}$.*

Theorem 5. *Let $\widetilde{\varrho}_\gamma, \widetilde{h}_\mu$ be two H N As of Y over an H F $\widetilde{\zeta}_\lambda$ of L. Then, $\widetilde{\varrho}_\gamma \widetilde{\cap} \widetilde{h}_\mu$ is an H N A of Y over an H F $\widetilde{\zeta}_\lambda$ of L.*

Proof. Let $\widetilde{\varrho}_\gamma$ and $\widetilde{h}_\mu$ be two H N As of Y over an H F $\widetilde{\zeta}_\lambda$ of L. For all $z, w \in Y$ and $d, a \in L$,

(i)
$$\left(\widetilde{\varrho}_\gamma \widetilde{\cap} \widetilde{h}_\mu\right)(dz + aw) = \left(\widetilde{\varrho \cap h}\right)(dz + aw)$$

$$= \widetilde{\varrho}(dz + aw) \widetilde{\cap} \widetilde{h}(dz + aw);$$
$$\supseteq \left(\left(\widetilde{\zeta}(d) \cap \widetilde{\varrho}(z)\right) \cap \left(\widetilde{\zeta}(a) \cap \widetilde{\varrho}(w)\right)\right) \widetilde{\cap} \left(\left(\widetilde{\zeta}(d) \cap \widetilde{h}(z)\right) \cap \left(\widetilde{\zeta}(a) \cap \widetilde{h}(w)\right)\right);$$
$$= \left(\left(\widetilde{\zeta}(d) \cap \widetilde{\varrho}(z)\right) \cap \left(\widetilde{\zeta}(d) \cap \widetilde{h}(z)\right)\right) \widetilde{\cap} \left(\left(\widetilde{\zeta}(a) \cap \widetilde{\varrho}(w)\right) \cap \left(\widetilde{\zeta}(a) \cap \widetilde{h}(w)\right)\right);$$
$$= \left(\widetilde{\zeta}(d) \cap \left(\widetilde{\varrho}(z) \widetilde{\cap} \widetilde{h}(z)\right)\right) \cap \left(\widetilde{\zeta}(a) \cap \left(\widetilde{\varrho}(w) \widetilde{\cap} \widetilde{h}(w)\right)\right);$$
$$= \left(\widetilde{\zeta}(d) \cap \left(\widetilde{\varrho \cap h}\right)(z)\right) \cap \left(\widetilde{\zeta}(a) \cap \left(\widetilde{\varrho \cap h}\right)(w)\right);$$

and

$$(\gamma \vee \mu)(dz + aw) = \vee\{\gamma(dz + aw), \mu(dz + aw)\};$$
$$\le \vee\{\vee\{\lambda(d), \gamma(z), \lambda(a), \gamma(w)\}, \vee\{\lambda(d), \mu(z), \lambda(a), \mu(w)\}\};$$
$$= \vee\{\vee\{\lambda(d), \gamma(z), \lambda(d), \mu(z)\}, \vee\{\lambda(a), \gamma(w), \lambda(a), \mu(w)\}\};$$
$$= \vee\{(\lambda(d), \vee\{\gamma(z), \mu(z)\}), (\lambda(a), \vee\{\gamma(w), \mu(w)\})\};$$
$$= \vee\{(\lambda(d), (\gamma \vee \mu)(z)), (\lambda(a), (\gamma \vee \mu)(w))\}.$$

(ii)

$$\left(\widetilde{\varrho}_\gamma \widetilde{\cap} \widetilde{h}_\mu\right)(zw) = \left(\widetilde{\varrho \cap h}\right)(zw) = \widetilde{\varrho}(zw) \widetilde{\cap} \widetilde{h}(zw) \supseteq (\widetilde{\varrho}(z) \cap \widetilde{\varrho}(w)) \widetilde{\cap} \left(\widetilde{h}(z) \cap \widetilde{h}(w)\right);$$
$$= \left(\widetilde{\varrho}(z) \cap \widetilde{h}(z)\right) \widetilde{\cap} \left(\widetilde{\varrho}(w) \cap \widetilde{h}(w)\right) = \left(\widetilde{\varrho \cap h}\right)(z) \cap \left(\widetilde{\varrho \cap h}\right)(w);$$

and

$$(\gamma \vee \mu)(zw) = \vee\{\gamma(zw), \mu(zw)\} \le \vee\{\vee\{\gamma(z), \gamma(w)\}, \vee\{\mu(z), \mu(w)\}\};$$
$$= \vee\{\vee\{\gamma(z), \mu(z)\}, \vee\{\gamma(w), \mu(w)\}\};$$
$$= \vee\{(\gamma \vee \mu)(z), (\gamma \vee \mu)(w)\}.$$

We have $\widetilde{\zeta}(1) \supseteq \widetilde{\varrho}(z)$, $\widetilde{\zeta}(1) \supseteq \widetilde{h}(z)$. Hence $\widetilde{\zeta}(1) \supseteq \widetilde{\varrho}(z) \widetilde{\cap} \widetilde{h}(z)$.
Therefore, $\widetilde{\zeta}(1) \supseteq \left(\widetilde{\varrho \cap h}\right)(z), \forall z \in Y$ and $\lambda(1) \le \gamma(z)$, $\lambda(1) \le \mu(z)$.
Thus, $\lambda(1) \le \gamma(z) \widetilde{\cap} \mu(z)$. Hence $\lambda(1) \le (\gamma \widetilde{\cap} \mu)(z) \forall z \in Y$. $\square$

Example 3. *Let $L = Z_2 = \{0, 1\}_{\oplus_2, \oplus_2}$ be a field. The hybrid structure $\widetilde{\zeta}_\lambda$ in L over $U = \{u_1, u_2, u_3, u_4, u_5\}$, which is given as follows:*

L	$\widetilde{\zeta}$	λ
0	$\{u_1, u_2, u_3, u_4\}$	0.3
1	$\{u_2, u_3, u_4\}$	0.4

Clearly, $\widetilde{\zeta}_\lambda$ is an H F in L over U. Let $Y = \{0, h, p, e\}$ be a set with two binary operations, "+" and "." By

+	**0**	**h**	**p**	**e**	**.**	**0**	**h**	**p**	**e**
0	0	h	p	e	0	0	0	0	0
h	h	0	e	p	h	h	h	h	h
p	p	e	0	h	p	p	p	p	p
e	e	p	h	0	e	e	e	e	e

Clearly, Y forms a near algebra over L. The hybrid structure $\widetilde{\varrho}_\gamma$, $\widetilde{h}_\mu$ in Y over $U = \{u_1, u_2, u_3, u_4, u_5\}$, which is given as follows:

Y	$\widetilde{\rho}$	γ	$\widetilde{h}$	μ
0	$\{u_2, u_3, u_4\}$	0.4	$\{u_1, u_3, u_4\}$	0.5
h	$\{u_2, u_4\}$	0.6	$\{u_1, u_4\}$	0.7
p	$\{u_2, u_3\}$	0.8	$\{u_1, u_5\}$	0.9
e	$\{u_2\}$	0.8	$\{u_1\}$	0.9

Hence, $\widetilde{\varrho}_\gamma \cap \widetilde{h}_\mu$ is an H N A of Y over an H F $\widetilde{\zeta}_\lambda$ of L.

Definition 7. *Let Y and Y' be two near algebras. Let $\omega : Y \to Y'$ be a mapping. Then,*

(i) *If $\widetilde{h}_\mu$ is a hybrid subset of Y', then the preimage of $\widetilde{h}_\mu$ under ω is the hybrid subset in Y over U defined by*

$$\omega^{-1}\left(\widetilde{h}_\mu\right)(z) = (\omega^{-1}\left(\widetilde{h}\right)(z), \omega^{-1}(\mu)(z)) = \left(\widetilde{h}(\omega)(z), \mu(\omega)(z)\right), \forall z \in Y.$$

(ii) *If $\widetilde{\varrho}_\gamma$ is a hybrid subset of Y, then the image of $\widetilde{\varrho}_\gamma$ under ω is the hybrid subset in Y' over U defined by*

$$\omega(\widetilde{\varrho})(w) = \begin{cases} \underset{z \in \omega^{-1}(w)}{\cup} \widetilde{\varrho}(z), & if\ \omega^{-1}(w) \neq \varnothing \\ \varnothing, & otherwise \end{cases}$$

and

$$\omega(\gamma)(w) = \begin{cases} \underset{z \in \omega^{-1}(w)}{\wedge} \gamma(z), & if\ \omega^{-1}(w) \neq \varnothing \\ 1, & otherwise, \end{cases}$$

for every $w \in Y$.

Theorem 6. *Let Y and Y' be two near algebras over a field L and $\omega : Y \to Y'$ be onto near algebra homomorphism. If $(\widetilde{\varrho}_\gamma, Y)$ is an H N A over $\left(\widetilde{\zeta}_\lambda, L\right)$, then $(\omega(\widetilde{\varrho}_\gamma), Y')$ is an H N A over $\left(\widetilde{\zeta}_\lambda, L\right)$.*

Proof. Let $z, w \in Y'$ and $d, a \in L$. Then, we have $\{q : q \in \omega^{-1}(dz + aw)\}$ $\supseteq \{ds + at : s \in \omega^{-1}(z)$ and $t \in \omega^{-1}(w)\}$ and $\{q : q \in \omega^{-1}(zw)\}$ $\supseteq \{st : s \in \omega^{-1}(z)$ and $t \in \omega^{-1}(w)\}$.

If $\omega^{-1}(z) \neq \varnothing, \omega^{-1}(w) \neq \varnothing$, then $\omega^{-1}(zw) \neq \varnothing$. Now

(i)　For all $z, w \in Y'$, $\exists\, s, t \in Y$ such that $z = \omega(s)$, $w = \omega(t)$

$$
\begin{aligned}
\omega(\widetilde{\varrho})(dz + aw) &= \bigcup_{q \in \omega^{-1}(dz+aw)} \widetilde{\varrho}(q); \\
&\supseteq \bigcup_{s \in \omega^{-1}(z),\, t \in \omega^{-1}(w)} \widetilde{\varrho}(ds + at); \\
&\supseteq \bigcup_{s \in \omega^{-1}(z),\, t \in \omega^{-1}(w)} \widetilde{\zeta}(d) \cap \widetilde{\varrho}(s) \cap \widetilde{\zeta}(a) \cap \widetilde{g}(t); \\
&= \left(\widetilde{\zeta}(d) \cap \bigcup_{s \in \omega^{-1}(z)} \widetilde{\varrho}(s) \right) \cap \left(\widetilde{\zeta}(a) \cap \bigcup_{t \in \omega^{-1}(w)} \widetilde{\varrho}(t) \right); \\
&= \left(\widetilde{\zeta}(d) \cap \omega(\widetilde{\varrho})(z) \right) \cap \left(\widetilde{\zeta}(a) \cap \omega(\widetilde{\varrho})(w) \right); \\
&= \widetilde{\zeta}(d) \cap \omega(\widetilde{\varrho})(z) \cap \widetilde{\zeta}(a) \cap \omega(\widetilde{\varrho})(y);
\end{aligned}
$$

and

$$
\begin{aligned}
\omega(\gamma)(dz + aw) &= \bigwedge_{q \in \omega^{-1}(dz+aw)} \gamma(q); \\
&\leq \bigwedge_{s \in \omega^{-1}(z),\, t \in \omega^{-1}(w)} \gamma(ds + at); \\
&\leq \bigwedge_{s \in \omega^{-1}(z),\, t \in \omega^{-1}(w)} \left(\vee\{\lambda(d), \gamma(s), \lambda(a), \gamma(t)\} \right); \\
&= \vee\left\{ \vee\left\{ \lambda(d), \bigwedge_{s \in \omega^{-1}(z)} \gamma(s) \right\}, \vee\left\{ \lambda(a), \bigwedge_{t \in \omega^{-1}(w)} \gamma(t) \right\} \right\}; \\
&= \vee\{\lambda(a), \omega(\gamma)(z), \lambda(b), \omega(\gamma)(w)\}.
\end{aligned}
$$

(ii)

$$
\begin{aligned}
\omega(\widetilde{\varrho})(zw) &= \bigcup_{q \in \omega^{-1}(zw)} \widetilde{\varrho}(q); \\
&\supseteq \bigcup_{s \in \omega^{-1}(z),\, t \in \omega^{-1}(w)} \widetilde{\varrho}(st); \\
&\supseteq \bigcup_{s \in \omega^{-1}(z),\, t \in \omega^{-1}(w)} \widetilde{\varrho}(s) \cap \widetilde{\varrho}(t); \\
&= \left(\bigcup_{s \in \omega^{-1}(z)} \widetilde{\varrho}(s) \right) \cap \left(\bigcup_{t \in \omega^{-1}(w)} \widetilde{\varrho}(t) \right); \\
&= \omega(\widetilde{\varrho})(z) \cap \omega(\widetilde{\varrho})(w);
\end{aligned}
$$

and

$$
\begin{aligned}
\omega(\gamma)(zw) &= \bigwedge_{q \in \omega^{-1}(zw)} \gamma(q); \\
&\leq \bigwedge_{s \in \omega^{-1}(z),\, t \in \omega^{-1}(w)} \gamma(st); \\
&\leq \bigwedge_{s \in \omega^{-1}(z),\, t \in \omega^{-1}(w)} \left(\vee\{\gamma(s), \gamma(t)\} \right); \\
&= \vee\left\{ \bigwedge_{s \in \omega^{-1}(z)} \gamma(s), \bigwedge_{t \in \omega^{-1}(w)} \gamma(t) \right\}; \\
&= \vee\{\omega(\gamma)(z), \omega(\gamma)(w)\}.
\end{aligned}
$$

(iii)　$\widetilde{\zeta}(1) \supseteq \omega(\widetilde{\varrho})(z)$ and $\lambda(1) \leq \omega(\gamma)(z)$, $\forall z \in Y'$. Therefore, $(\omega(\widetilde{\varrho}_\gamma), Y')$ is an H N A over $\left(\widetilde{\zeta}_\lambda, L \right)$. $\square$

Theorem 7. *Let Y, Y' be two near algebras over a field L and $\omega : Y \to Y'$ be onto near algebra homomorphism. If $\widetilde{h}_\mu$ is a hybrid near algebra in Y' over $\left(\widetilde{\zeta}_\lambda, L \right)$, then $\omega^{-1}\left(\widetilde{h}_\mu \right) := \left(\omega^{-1}\left(\widetilde{h} \right), \omega^{-1}(\mu) \right)$ is an H N A in Y over $\left(\widetilde{\zeta}_\lambda, L \right)$.*

Proof. Let $z, w \in Y$ and $d \in L$.

(i)

$$\omega^{-1}\left(\widetilde{h}\right)(dz + aw) = \widetilde{h}(\omega(dz + aw));$$
$$= \widetilde{h}(d\,\omega(z) + a\,\omega(w));$$
$$\supseteq \widetilde{\zeta}(d) \cap \widetilde{h}(\omega(z)) \cap \widetilde{\zeta}(a) \cap \widetilde{h}(\omega(w));$$
$$= \widetilde{\zeta}(d) \cap \omega^{-1}\left(\widetilde{h}\right)(z) \cap \widetilde{\zeta}(a) \cap \omega^{-1}\left(\widetilde{h}\right)(w);$$

and

$$\omega^{-1}(\mu)(dz + aw) = \mu(\omega(dz + aw));$$
$$= \mu(d\,\omega(z) + a\omega(w));$$
$$\leq \vee\{\lambda(d), \mu(\omega(z)), \lambda(a), \mu(\omega(w))\};$$
$$= \vee\{\lambda(d), \omega^{-1}(\mu)(z), \lambda(a), \omega^{-1}(\mu)(w)\}.$$

(ii)

$$\omega^{-1}\left(\widetilde{h}\right)(zw) = \widetilde{h}(\omega(zw)) = \widetilde{h}(\omega(z)\omega(w)) \supseteq \widetilde{h}(\omega(z)) \cap \widetilde{h}(\omega(w));$$
$$= \omega^{-1}\left(\widetilde{h}\right)(z) \cap \omega^{-1}\left(\widetilde{h}\right)(w);$$

and

$$\omega^{-1}(\mu)(zw) = \mu(\omega(zw)) = \mu(\omega(z)\omega(w)) \leq \vee\{\mu(\omega(z)), \mu(\omega(w))\};$$
$$= \vee\{\omega^{-1}(\mu)(z), \omega^{-1}(\mu)(w)\}.$$

(iii)

$$\widetilde{\zeta}(1) \supseteq \omega^{-1}\left(\widetilde{h}\right)(z) \text{ and } \lambda(1) \leq \omega^{-1}(\mu)(z),\ \forall z \in Y.$$

Therefore, $\omega^{-1}\left(\widetilde{h}_\mu\right) = \left(\omega^{-1}\left(\widetilde{h}\right), \omega^{-1}(\mu)\right)$ is an H N A in Y over $\left(\widetilde{\zeta}_\lambda, L\right)$. $\square$

Definition 8. *Let $\widetilde{\varrho}_\gamma$, $\widetilde{h}_\mu$ be two hybrid near algebras of Y, Y', respectively. Then, the direct product of $\widetilde{\varrho}_\gamma$ and $\widetilde{h}_\mu$ is denoted by $\widetilde{\varrho}X\widetilde{h}$ and $\gamma\ X\ \mu$ is the function demarcated by*

$$\left(\widetilde{\varrho}X\widetilde{h}\right)(z, z') = \widetilde{\varrho}(z) \cap \widetilde{h}(z') \text{ and } (\gamma\ X\ \mu)(z, z') = \vee\{\gamma(z),\ \mu(z')\}\ \forall z \in Y \text{ and } z' \in Y'.$$

Theorem 8. *Let Y,Y' be two near algebras over a field L and $\widetilde{\varrho}_\gamma$, $\widetilde{h}_\mu$ be two hybrid near algebras of Y, Y', respectively, over an $H\,F\left(\widetilde{\zeta}_\lambda, L\right)$. Then, $\widetilde{\varrho}_\gamma X\widetilde{h}_\mu$ is an $H\,N\,A$ of YXY' over an $H\,F\left(\widetilde{\zeta}_\lambda, L\right)$.*

Proof. We have $\left(\widetilde{\varrho}X\widetilde{h}\right)(z, z') = \widetilde{\varrho}(z) \cap \widetilde{h}(z')$, $(\gamma\ X\ \mu)(z, z') = \vee\{\gamma(z), \mu(z')\}$, $\forall z \in Y$, $z' \in Y'$. Let $(z, z'), (w, w') \in YXY'$ and $d \in L$. Then,

(i)

$$\left(\widetilde{\varrho}X\widetilde{h}\right)[(z, z') + (w, w')] = \left(\widetilde{\varrho}X\widetilde{h}\right)(z + w,\ z' + w');$$
$$= \widetilde{\varrho}(z + w) \cap \widetilde{h}(z' + w');$$
$$\supseteq [\widetilde{\varrho}(z) \cap \widetilde{\varrho}(w)] \cap \left[\widetilde{h}(z') \cap \widetilde{h}(w')\right];$$
$$= \left[\widetilde{\varrho}(z) \cap \widetilde{h}(z')\right] \cap \left[\widetilde{\varrho}(w) \cap \widetilde{h}(w')\right];$$
$$= \left(\widetilde{\varrho}X\widetilde{h}\right)(z, z') \cap \left(\widetilde{\varrho}X\widetilde{h}\right)(w, w');$$
$$(\gamma\ X\ \mu)[(z, z') + (w, w')] = (\gamma\ X\ \mu)(z + w,\ z' + w');$$
$$= \vee\{\gamma(z + w),\ \mu(z' + w')\};$$
$$\leq \vee\{\vee\{\gamma(z),\ \gamma(w)\},\ \vee\{\mu(z'), \mu(w')\}\};$$
$$\leq \vee\{\vee\{\gamma(z),\ \mu(z')\},\ \vee\{\gamma(w), \mu(w')\}\};$$
$$= \vee\{(\gamma\ X\ \mu)(z, z'), (\gamma\ X\ \mu)(w, w')\}.$$

(ii)

$$\left(\widetilde{\varrho}\mathsf{X}\widetilde{h}\right)(d(z,z')) = \left(\widetilde{\varrho}\mathsf{X}\widetilde{h}\right)(dz,dz');$$
$$= \widetilde{\varrho}(dz) \cap \widetilde{h}(dz');$$
$$\supseteq \left[\widetilde{\zeta}(d) \cap \widetilde{\varrho}(z)\right] \cap \left[\widetilde{\zeta}(d) \cap \widetilde{h}(z')\right];$$
$$= \widetilde{\zeta}(d) \cap \left(\widetilde{\varrho}(z) \cap \widetilde{h}(z')\right);$$
$$= \widetilde{\zeta}(d) \cap \left(\widetilde{\varrho}\mathsf{X}\widetilde{h}\right)(z,z');$$
$$(\gamma\ \mathsf{X}\ \mu)[d(z,z')] = (\gamma\ \mathsf{X}\ \mu)(dz,dz');$$
$$= \vee\{\gamma(dz),\ \mu(dz')\};$$
$$\leq \vee\{\vee\{\lambda(d),\ \gamma(z)\},\ \vee\{\lambda(d),\mu(z')\}\};$$
$$\leq \vee\{\lambda(d),\ \vee\{\gamma(z),\mu(z')\}\};$$
$$= \vee\{\lambda(d),(\gamma\ \mathsf{X}\ \mu)(z,z')\}.$$

(iii)

$$\left(\widetilde{\varrho}\mathsf{X}\widetilde{h}\right)[(z,z')(w,w')] = \left(\widetilde{\varrho}\mathsf{X}\widetilde{h}\right)(zw,\ z'w');$$
$$= \widetilde{\varrho}(zw) \cap \widetilde{h}(z'w');$$
$$\supseteq [\widetilde{\varrho}(z) \cap \widetilde{\varrho}(w)] \cap \left[\widetilde{h}(z') \cap \widetilde{h}(w')\right];$$
$$= \left[\widetilde{\varrho}(z) \cap \widetilde{h}(z')\right] \cap \left[\widetilde{\varrho}(w) \cap \widetilde{h}(w')\right];$$
$$= \left(\widetilde{\varrho}\mathsf{X}\widetilde{h}\right)(z,z') \cap \left(\widetilde{\varrho}\mathsf{X}\widetilde{h}\right)(w,w');$$

and

$$(\gamma\ \mathsf{X}\ \mu)[(z,z')(w,w')] = (\gamma\ \mathsf{X}\ \mu)(zw,\ z'w');$$
$$= \vee\{\gamma(zw),\ \mu(z'W')\};$$
$$\leq \vee\{\vee\{\gamma(z),\ \gamma(w)\},\ \vee\{\mu(z'),\mu(w')\}\};$$
$$\leq \vee\{\vee\{\gamma(z),\ \mu(z')\},\ \vee\{\gamma(w),\mu(w')\}\};$$
$$= \vee\{(\gamma\ \mathsf{X}\ \mu)(z,z'),(\gamma\ \mathsf{X}\ \mu)(w,w')\}.$$

(iv)　Let '1' be the unity in L. Since $\left(\widetilde{\varrho}_\gamma, Y\right)$ and $\left(\widetilde{h}_\mu, Y'\right)$ are H N A over $\left(\widetilde{\zeta}_\lambda, L\right)$,

$$\widetilde{\zeta}(1) \supseteq \widetilde{\varrho}(z), \forall z \in Y,\ \lambda(1) \leq \gamma(z),\ \widetilde{\zeta}(1) \supseteq \widetilde{h}(z'), \forall z' \in Y' \text{ and } \lambda(1) \leq \mu(z\prime).$$

Then, $\widetilde{\zeta}(1) \supseteq \widetilde{\varrho}(z) \cap \widetilde{h}(z') = \left(\widetilde{\varrho}\mathsf{X}\widetilde{h}\right)(z,z')$ and $\lambda(1) \leq \vee\{\gamma(z),\mu(z')\} = (\gamma\ \mathsf{X}\ \mu)(z,z')$.

Hence, $\widetilde{\varrho}_\gamma \mathsf{X}\widetilde{h}_\mu$ is an H N A of $Y\mathsf{X}Y'$ over an H F $\left(\widetilde{\zeta}_\lambda, L\right)$. □

4. Conclusions

In this paper, we introduced the notion of H N A over an H F and investigated several properties. Using these notions, we introduced the concepts of H N A homomorphism and the direct product of H N A. Research can be extended to the ideal of an H N A and hybrid gamma near algebra.

References

1. Pilz, G. *Near-Ring*; North Holland Publishers: Amsterdam, The Netherlands, 1983.
2. Brown, H. Near algebras. *Ill. J. Math.* **1968**, *12*, 215–227. [CrossRef]
3. Swamy, P.N.; Kumar, K.V.; Saraswathi, C.; Srinivas, T. Bipolar fuzzy near algebra over bipolar fuzzy field. *Adv. Fuzzy Math.* **2017**, *12*, 945–955.
4. Swamy, P.N.; Deshmukh, R.; Rao, M.M.K.; Jyothi, B. Tripolar Fuzzy Near algebra. *AIP Conf. Proc.* **2020**, *2246*, 020098.
5. Zadeh, L.A. Fuzzy sets. *Inf. Control* **1965**, *8*, 338–353. Available online: https://www.sciencedirect.com/science/article/pii/S001 999586590241X (accessed on 8 September 2023). [CrossRef]
6. Nanda, S. Fuzzy fields and fuzzy linear spaces. *Fuzzy Sets Syst.* **1986**, *19*, 89–94.
7. Biswas, R. Fuzzy fields and fuzzy linear spaces redefined. *Fuzzy Sets Syst.* **1989**, *33*, 257–259. [CrossRef]
8. Torra, V. Hesitant fuzzy sets. *Int. J. Intell. Syst.* **2010**, *25*, 529–539. [CrossRef]
9. Molodtsov, D. Soft set theory—First results. *Comput. Math. Appl.* **1999**, *37*, 19–31. [CrossRef]
10. Jun, Y.B.; Song, S.Z.; Muhiuddin, G.G. Hybrid structures and applications. *Ann. Commun. Math.* **2018**, *1*, 11–25.
11. Elavarasan, B.; Muhiuddin, G.; Porselvi, K.; Jun, Y.B. Hybrid structures applied to ideals in near-rings. *Complex Intell. Syst.* **2021**, *7*, 1489–1498. [CrossRef]
12. Anis, S.; Khan, M.; Jun, Y.B. Hybrid ideals in semigroups. *Cogent Math.* **2017**, *4*, 1352117. [CrossRef]
13. Begum, M.H.J.; Rama, G. Hybrid fuzzy bi-ideals in near rings. *Turk. J. Comput. Math. Educ.* **2021**, *12*, 3291–3295. Available online: https://turcomat.org/index.php/turkbilmat/article/view/4407/3749 (accessed on 8 September 2023).
14. Srinivas, T.; Swamy, P.N. A note on fuzzy near algebra. *Int. J. Algebra* **2011**, *5*, 1085–1098. Available online: http://www.m-hikari.com/ija/ija-2011/ija-21-24-2011/index.html (accessed on 8 September 2023).