axioms

MDPI

Article

Capital Asset Pricing Model and Ordered Weighted Average Operator for Selecting Investment Portfolios

Cristhian R. Uzeta-Obregon [1], Tanya S. Garcia-Gastelum [1], Pavel A. Alvarez [1], Cristhian Mellado-Cid [2], Fabio Blanco-Mesa [3] and Ernesto Leon-Castro [2,*]

[1] Unidad Culiacan, Universidad Autonoma de Occidente, Culiacán 80020, Sinaloa, Mexico; cristhian.uzeta@uadeo.mx (C.R.U.-O.); tanya.garcia@uadeo.mx (T.S.G.-G.); pavel.alvarez@uadeo.mx (P.A.A.)
[2] Faculty of Economics and Administrative Sciences, Universidad Católica de la Santísima Concepción, Concepción 4081393, Chile; cmellado@ucsc.cl
[3] Faculty of Economic and Administrative Sciences, School of Business Administration, Universidad Pedagogica y Tecnologica de Colombia, Tunja 150003, Colombia; fabio.blanco01@uptc.edu.co
* Correspondence: eleon@ucsc.cl

Abstract: The main objective of this article is to present the formulation of a Capital Asset Pricing Model ordered weighted average $CAPM_{OWA}$ and its extensions, called CAPM-induced OWA ($CAPM_{IOWA}$), CAPM Bonferroni OWA ($CAPM_{Bon\text{-}OWA}$), and CAPM Bonferroni-induced OWA $CAPM_{Bon\text{-}IOWA}$. A step-by-step process for applying this new proposal in a real case of formulating investment portfolios is generated. These methods show several scenarios, considering the attitude, preferences, and relationship of each argument, when underestimation or overestimation of the information by the decision maker may influence the decision-making process regarding portfolio investments. Finally, the complexity of the method and the incorporation of soft information into the modeling process lead to generating a greater number of scenarios and reflect the attitudes and preferences of decision makers.

Keywords: CAPM; OWA operator; Bonferroni OWA; portfolio investment

MSC: 03B52; 91B28; 90B50

Citation: Uzeta-Obregon, C.R.; Garcia-Gastelum, T.S.; Alvarez, P.A.; Mellado-Cid, C.; Blanco-Mesa, F.; Leon-Castro, E. Capital Asset Pricing Model and Ordered Weighted Average Operator for Selecting Investment Portfolios. *Axioms* **2024**, *13*, 660. https://doi.org/10.3390/axioms13100660

Academic Editors: Oscar Humberto Ross, Hui-Chin Tang and Shih-Chou Kao

Received: 23 August 2024
Revised: 21 September 2024
Accepted: 24 September 2024
Published: 25 September 2024

1. Introduction

Stock exchanges around the world are institutions that were created to support the sale and purchase of different values, such as shares, bonds, and other financial instruments. Companies need financial resources for their operation or expansion projects. In these cases, companies turn to stock exchanges. In this sense, stock markets and financial institutions pay tribute to channel investor savings in the neighborhood of productive investment, which are sources of growth and employment in countries around the world [1].

Risk is the main problem faced when investing in a stock exchange. In the context of stock exchanges, the aspects to consider when transferring risk are coverage, assurance, and diversification [2–4]. Within the diversification strategy, there is always the dilemma of selecting a company based on which criteria to invest in. Different studies have indicated relations between the price of action with aspects such as dividend policy, profit for action, return on capital and profit after taxes [5,6], estimated growth in the long term of the company [7], exchange rate [8], interest rates [9], and the value of futures [10], among others.

As can be seen, there are different criteria for selecting which asset and at what time. Because different criteria are considered, employing a Multi-Criteria Decision Analysis (MCDA) approach for this type of problem is adequate, where different opinions converge and unified results should be generated. The MCDA significantly improves the quality of the decision-making process by introducing transparency, analytical rigor, audibility, and conflict resolution for multidimensional decision problems [11]. The importance of the MCDA method is based on the opinions of those responsible for decision making. Although MCDA defines the "what" as being necessary to structure the decision problem, it does not support those responsible for decision making in the "how" to do so. Explicit consideration of the elements of the decision quality in MCDA can help the "how" and sensitize those responsible for making decisions about the criteria's importance, scope, and uncertainty. As can be seen, in investment decisions, specifically in portfolios, the influences of attitude and approximate reasoning on the content of information, rather than its measurement, are of considerable importance [12,13].

Another critical method for aggregating information within the decision-making process is the Ordered Weighted Averaging (OWA) operator [14,15]. This technique allows for information analysis through a parameterized study of maximum and minimum, generating different selection scenarios based on optimistic or pessimistic visions of the information used. The OWA operator has generated several extensions and has been used within various areas such as for suppliers [16], financial decisions [17], forecasting of the exchange rate [18], imports and exports [19], variance, and covariance [20], among others. Additionally, the use of OWA operators in conjunction with MCDA models has been studied, for example, with the OWA TOPSIS with MCDA [21], so combining both decision-making methodologies can considerably improve their quality.

Derived from the preceding, the use of MCDA models with OWA operator extensions would allow for generating new selection processes that best the profile of the investment portfolio through criteria selection based on the needs of the decision maker and the quantitative information of the company, allowing for the construction of an adaptable and flexible methodology suited to the needs of the financial environment characterized by uncertainty [22]. Given the current uncertainty of financial markets and the volatility of the price of shares within stock exchanges [23], it is necessary to generate adaptable and flexible portfolios that represent the realities of each of the investors based on their preferences and the characteristics of companies.

One of the ways to select the assets that will make up a portfolio is by considering the function of the expected yield according to the risk they have. To analyze this, the Capital Asset Pricing Model (CAPM) is used, whereby an expected return can be calculated by analyzing the market performance, the risk-free rate, and the expected performance of the specific asset. One of the weaknesses of the CAPM is that it only considers historical data within formulations, as can be seen through the means, variance, and covariance used, which would suggest that the markets have a cyclical behavior, which often does not happen. On the other hand, it is considered that all data are equally crucial through simple means, so the use of weights that allow for differentiating between the importance of the data when performing calculations would present necessary support for the decision maker and, in such a way, be able to generate diverse scenarios based on expectations, knowledge, and future visions of the financial market. In this sense, incorporating the OWA operator will allow for generating new formulations in the function of the expectations and knowledge of the financial market of a decision maker.

Based on the above, the objective of this article is to present an extended version of the CAPM through the incorporation of a weight vector, which will be used within the means, variances, and covariances, including the expectations of the decision maker within the formulation, and generate greater dynamism in the formulation, which, within its base structure, only allows a single result. At this point, the $CAPM_{OWA}$ and its extensions, called $CAPM_{IOWA}$, $CAPM_{Bon-OWA}$, and $CAPM_{Bon-IOWA}$, are presented. These methods can represent different scenarios, considering the attitude, preferences, and relationship of each argument. Also, a step-by-step process for the application of this new proposal in a real case for the formulation of investment portfolios is generated. The results show several scenarios, considering that underestimation or overestimation of the information by the decision maker may influence the decision-making process regarding portfolio investments. The document is structured as follows. Section 2 presents the preliminary formulations for the OWA, IOWA, Var-OWA, Cov_{OWA}, VarBonOWA, and BonOWACov operators and the CAPM, Section 3 presents a new proposition (CAPM) with Var-OWA and OWA-Cov combined with Bonferroni means and induced variables. In Section 4, the $CAPM_{OWA}$, $CAPM_{IOWA}$, $CAPM_{Bon-OWA}$, and $CAPM_{Bon-IOWA}$ are applied to create a portfolio using the Mexican Stock Exchange. Section 5 presents the main conclusions of the article.

2. Preliminaries

The following section briefly formulates and conceptualizes the OWA operator [15], IOWA operator [24], and Bonferroni means [25] combined with the variance and covariance extension [26,27] and CAPM formulation [28]. The first definition that will be presented is the OWA operator.

Definition 1. *An OWA operator of dimension n is a mapping* $OWA : R^n \rightarrow R$ *with an associated weight vector W of dimension n, such that* $\sum_{j=1}^{n} w_j = 1$ *and* $w_j \in [0,1]$*, according to the following formula:*

$$OWA(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j, \tag{1}$$

where b_j *is the jth largest element of the collection* a_i*.*

The OWA operator can be extended when the ordering process is not made based on the values of the attributes, but considering a specific order given by the expert or decision maker. This extension was named the induced OWA (IOWA) operator [24], which can be defined as follows.

Definition 2. *An IOWA operator of dimension n is mapping* $IOWA : R^n \times R^n \rightarrow R$ *with an associated weight vector W of dimension n, such that* $\sum_{j=1}^{n} w_j = 1$ *and* $w_j \in [0,1]$*, where an induced set of ordering variables are included* (u_i)*, such that the formula is:*

$$IOWA(\langle u_1, a_1\rangle, \langle u_2, a_2\rangle, \ldots, \langle u_n, a_n\rangle) = \sum_{j=1}^{n} w_j b_j, \tag{2}$$

where b_j *is the* a_i *value of the OWA pair* $< u_i, a_i >$ *with the jth largest* u_i*.* u_i *is the order-inducing variable and* a_i *is the argument variable.*

Another two interesting extensions are using the variance and covariance as a base with a combination of the OWA operator. These new operators are named OWA_{var} operator and OWA_{cov} [26,29] and are defined as follows.

Definition 3. *The Var-OWA* (${\sigma^2}_{OWA}$) *is defined as follows.*

$${\sigma^2}_{OWA}(a_1, \ldots, a_n) = \sum_{j=1}^{n} w_j D_j \tag{3}$$

where D_j *is the jth smallest of the* $(a_i - \mu)^2$*,* a_i *is the argument variable,* μ *is the average (in this case, the OWA operator),* $w_j \in [0,1]$*, and* $\sum_{j=1}^{n} w_j = 1$*.*

Definition 4. *In the case of the* OWA_{Cov}*, the definition is as follows:*

$$OWA_{Cov}(X, Y) = \sum_{j=1}^{n} w_j K_j, \tag{4}$$

where K_j *is the jth largest* $(x_i - \mu)(y_i - v)$*;* x_i *is the argument variable of the first set of elements* $X = \{x_1, \ldots, x_n\}$*;* y_i *is the argument variable of the second set of elements* $Y = \{y_1, \ldots, y_n\}$*;*

μ and v are the averages (or the OWA operator) of the sets X and Y, respectively; $w_j \in [0,1]$; and $\sum_{j=1}^{n} w_j = 1$.

Based on Definitions 3 and 4, new extensions were made using the Bonferroni means as an important element to analyze and include the interrelationship of the arguments in the formulation. In this sense, the formulations of the VarBonOWA ($\sigma^2{}_{BONOWA}$) and the BONOWACov are defined as follows [26,27].

Definition 5. *The VarBonOWA ($\sigma^2{}_{BONOWA}$) formulation is as follows.*

$$\sigma^2{}_{BONOWA}(a_1, \ldots, a_n) = \left(\frac{1}{n}\sum_i a_i^r \sigma^2{}_{OWA}\left(L^i\right)\right)^{\frac{1}{r+q}} \tag{5}$$

where $\sigma^2{}_{OWA}\left(L^i\right) = \left(\frac{1}{n-1}\sum_{\substack{j=1 \\ j \neq i}}^{n} a_j^q\right)$, with $\left(L^i\right)$ being the vector of all $\left(a_j - \mu_j\right)^2$ except $\left(a_i - \mu_i\right)^2$, a_i and a_j being the argument variables, μ_j and μ_i being the averages (in this case, the OWA operator), and w being an $n-1$ vector W_i associated with α_i, whose components w_{ij} are the OWA weights. Let W be an OWA weighing vector of dimension $n-1$ with components $w_i \in [0,1]$ when $\sum_i w_i = 1$. Here, r and q represent parameters to compensate for possible errors. Then, we can define this aggregation as $\sigma^2{}_{OWA}\left(L^i\right) = \left(\sum_{j=1}^{n-1} w_i a_{\pi_k(j)}\right)$, where $a_{\pi_k(j)}$ is the largest element in L^i and $w_i = \frac{1}{n-1}$ for all i.

Definition 6. *The Bonferroni Ordered Weighted Average Covariance (BONOWACov) operator in the formulation is as follows.*

$$BONOWACov(X,Y) = \left(\frac{1}{n}\sum_i a_i^r OWACov_W\left(E^i\right)\right)^{\frac{1}{r+q}}, \tag{6}$$

where $OWACov_W\left(E^i\right) = \left(\frac{1}{n-1}\sum_{\substack{j=1 \\ j\neq i}}^{n} a_j^q\right)$, with $\left(E^i\right)$ being the vector of all $\left(x_i - \mu_{OWACov}\right)\left(y_i - v_{OWACov}\right)$ except the $\left(x_j - \mu_{OWACov}\right)\left(y_j - v_{OWACov}\right)$, x_i being the argument variable of the first set of elements $X = \{x_1, \ldots, x_n\}$, y_i being the argument variable of the second set of elements $Y = \{y_1, \ldots, y_n\}$, and w being an $n-1$ vector W_i associated with α_i, whose components w_{ij} are the OWACov weights. Let W be an OWACov weighing vector of dimension $n-1$ with components $w_i \in [0,1]$ when $\sum_i w_i = 1$. Then, we can define this aggregation as $OWACov_W\left(L^i\right) = \left(\sum_{j=1}^{n-1} w_i a_{\pi_k(j)}\right)$, where $a_{\pi_k(j)}$ is the largest element in L^i and $w_i = \frac{1}{n-1}$ for all i.

Finally, both definitions can be extended using the IOWA operator, with their definitions being as follows.

Definition 7. *In the VarBonIOWA, let W be an OWA weighing vector of dimension $n-1$ with components $w_i \in [0,1]$ when $\sum_i w_i = 1$, where the weights are associated according to the largest value of u_i and u_i is the order-inducing variable.*

$$\sigma^2{}_{BONIOWA}(\langle u_1, a_1\rangle, \ldots, \langle u_n, a_n\rangle) = \left(\frac{1}{n}\sum_i b_i^r \sigma^2{}_{IOWA}\left(L^i\right)\right)^{\frac{1}{r+q}}, \tag{7}$$

where b_i is the a_i value of the $\sigma^2{}_{BONIOWA}$ pair $< u_i, a_i >$ with the jth largest u_i and $\sigma^2{}_{IOWA}\left(L^i\right) = \left(\frac{1}{n-1}\sum_{\substack{j=1 \\ j\neq i}}^{n} b_j^q\right)$, with $\left(L^i\right)$ being the vector of all $\left(b_j - \mu_j\right)^2$ except $\left(b_i - \mu_i\right)^2$, b_i

and b_j being the argument variables, μ_j and μ_i being the averages (in this case, the OWA operator), and w being an $n-1$ vector W_i associated with α_i, whose components w_{ij} are the OWA weights.

Definition 8. *In the BonIOWACov, let W be an IOWACov weighing vector of dimension $n-1$ with the components $w_i \in [0,1]$ when $\sum_i w_i = 1$, where the weights are associated according to the largest value of u_i, and u_i is the order-inducing variable.*

$$BonIOWACov(U,X,Y) = \left(\frac{1}{n}\sum_i b_i^r IOWACov_W\left(E^i\right)\right)^{\frac{1}{r+q}}, \tag{8}$$

where b_i is the a_i value of the BonIOWACov pair $< u_i, a_i >$ with the jth largest u_i, $IOWACov_W\left(E^i\right) = \left(\frac{1}{n-1}\sum_{\substack{j=1 \\ j\neq i}}^n b_j^q\right)$, with $\left(E^i\right)$ is the vector of all $(x_i - \mu_{IOWAcov})(y_i - v_{IOWAcov})$ except $(x_j - \mu_{IOWAcov})(y_j - v_{IOWAcov})$, x_i being the argument variable of the first set of elements $X = \{x_1, \ldots, x_n\}$, y_i being the argument variable of the second set of elements $Y = \{y_1, \ldots, y_n\}$, and w being an $n-1$ vector W_i that is associated with α_i, whose components w_{ij} are the IOWACov weights.

Definition 9. *According to the CAPM, the expected excess return on a portfolio/asset is:*

$$CAPM = R_f + \beta_i\left(R_b - R_f\right) \tag{9}$$

where $CAPM$ is the expected return of the portfolio/asset, R_f is the risk-free rate of interest, β_i is the coefficient that is related to the systematic risk, and $\left(R_b - R_f\right)$ is the equity risk premium.

For the calculation of the beta value, the formulation of publicly traded companies using historical prices [30] will be considered as follows:

$$\beta_i = \frac{Cov(R_p, R_b)}{Var(R_b)} \tag{10}$$

where Cov is the covariance, R_p is the return of the asset, R_b is the return of the market, and Var is the variance.

3. Capital Asset Pricing Model (CAPM) with Var-OWA and OWA-Cov Combined with Bonferroni Means and Induced Variables

Considering these preceding proposals, a new extension of the Capital Asset Pricing Model (CAPM) is put forth, which combines the OWA operator with its variance and covariance extensions. Since Equation (9) incorporates the beta value and the variance and covariance are considered in its calculation (Equation (10)), it seems appropriate to propose a Capital Asset Pricing Model (CAPM) OWA operator. This would combine the CAPM with ${\sigma^2}_{OWA}$ and OWA_{Cov} to reflect the potential for underestimation or overestimation on the part of decision makers. Based on the above, it is possible to use the OWA operator to calculate the β_i, and in this sense, the formulation will be as follows:

Proposition 1. *The β_{OWA} formulation is as follows:*

$$\beta_{OWA} = \frac{OWA_{Cov}(R_p, R_b)}{{\sigma^2}_{OWA}(R_b)}, \tag{11}$$

where OWA_{Cov} is the OWA covariance (Equation (4)) (OWA_{Cov} characteristics are shown widely in Equation (4) and they are applied in β_{OWA}), R_p is the return of the asset, R_b is the return of the marketm and ${\sigma^2}_{OWA}$ is the OWA variance (Equation (3)) (${\sigma^2}_{OWA}$ characteristics are shown widely in Equation (3) and they are applied in β_{OWA}).

Also, some special cases of the β_{OWA} can be seen when the OWA operator is not used in the whole formulation, and such cases can be seen in Table 1. It is important to note that Case 3 is the traditional β.

Table 1. Special cases of the β_{OWA}.

Operator	Total	Case 1	Case 2	Case 3
β_{OWA}	Uses: OWA_{Cov} and $\sigma^2{}_{OWA}$	Uses: OWA_{Cov} and Var	Uses: Cov and $\sigma^2{}_{OWA}$	Uses: Cov and Var

With the definition of the β_{OWA}, it is possible to define the $CAPM_{OWA}$ as follows:

Proposition 2. *The $CAPM_{OWA}$ is calculated as follows:*

$$CAPM_{OWA} = R_f + \beta_{OWA}\left(R_b - R_f\right), \tag{12}$$

where $CAPM_{OWA}$ is the expected return of the portfolio/asset, R_f is the risk-free rate of interest, β_{owa} is the new coefficient that combines an objective assessment of the risk by the market and a subjective assessment of the market data by decision makers, and $\left(R_b - R_f\right)$ is the equity risk premium.

On the other hand, by associating the weights with induced variables, a reordering is obtained when considering the induced value μ (Equation (2)). Thus, the Capital Asset Pricing Model (CAPM)-Induced OWA operator is defined as $CAPM_{IOWA}$. In this sense, the formulations of the β_{IOWA} and $CAPM_{IOWA}$ values are as follows:

Proposition 3. *The β_{IOWA} formulation is defined as follows:*

$$\beta_{IOWA} = \frac{IOWA_{Cov}\left(R_p, R_b\right)}{\sigma^2{}_{IOWA}(R_b)}, \tag{13}$$

where $IOWA_{Cov}$ is the induced OWA covariance (Equations (2) and (4)) ($IOWA_{Cov}$ characteristics are shown widely in Equations (2) and (4) and they are applied in β_{IOWA}), R_p is the return of the asset, R_b is the return of the market, and $\sigma^2{}_{IOWA}$ is the induced OWA variance (Equations (2) and (3)) ($\sigma^2{}_{IOWA}$ characteristics are shown widely in Equations (2) and (3) and they are applied in β_{IOWA}). In both, the order-inducing variable is u_i.

Therefore, with β_{IOWA}, it is possible to define the $CAPM_{IOWA}$ as follows:

Proposition 4. *The $CAPM_{IOWA}$ is calculated as follows:*

$$CAPM_{IOWA} = R_f + \beta_{IOWA}\left(R_b - R_f\right), \tag{14}$$

where $CAPM_{IOWA}$, IOWA pair $< u_i, a_i >$ with the jth largest u_i. u_i is the order-inducing variable and a_i is the argument variable and the expected return of the portfolio/asset, R_f is the risk-free rate of interest, β_{IOWA} is the coefficient that is related to the systematic risk, and $\left(R_b - R_f\right)$ is the equity risk premium related to the reordering variable u_i.

It is important to note that the main properties of the OWA operator are applied to the $CAPM_{IOWA}$ and $CAPM_{OWA}$ operators. These are the following:

(a) It is monotonic, because if $a_i \geq d_i$, for all i, then $CAPM_{OWA}(a_1, \ldots, a_n) \geq CAPM_{OWA}$ $(b_1, \ldots, b_n)$; $CAPM_{IOWA}(\langle u_i, a_i\rangle, \ldots, \langle u_n, a_n\rangle) \geq CAPM_{IOWA}(\langle u_i, b_i\rangle, \ldots, \langle u_n, b_n\rangle)$. (b) Commutativity: Let $\left(a'_1, a'_2, \ldots, a'_n\right)$ *be any permutation of* $(a_1, a_2, \ldots, a_n)$, then: $CAPM_{OWA}$ $\left(a'_1, a'_2, \ldots, a'_n\right) = CAPM_{OWA}(b_1, b_2, \ldots, b_n)$, and $CAPM_{IOWA}(\langle u_i, a_i\rangle, \ldots, \langle u_n, a_n\rangle) = CAPM_{IOWA}(\langle u_i, b_i\rangle, \ldots, \langle u_n, b_n\rangle)$. (c) Idempotency: Let $a_i = a, i = 1, 2, \ldots, n,$ *then* $CAPM_{OWA}(a, a, \ldots, a) = a$, and Let $|u_i, a_i| = a, i = 1, 2, \ldots, n,$ *then* $CAPM_{OWA}$

$(\langle u_i, a_i\rangle, \ldots, \langle u_n, a_n\rangle) = a$. (d) Boundedness: The $CAPM_{OWA}$ and $CAPM_{IOWA}$ lie between the max and min operators $\min(a_1, a_2, \ldots, a_n) \leq CAPM_{OWA}(a_1, a_2, \ldots, a_n) \leq \max(a_1, a_2, \ldots, a_n)$ and $\min(\langle u_1, a_1\rangle, \ldots, \langle u_n, a_n\rangle) \leq CAPM_{IOWA}(\langle u_1, a_1\rangle, \ldots, \langle u_n, a_n\rangle) \leq \max(\langle u_1, a_1\rangle, \ldots, \langle u_n, a_n\rangle)$.

Now, considering the varying degrees of complexity inherent to decision-making processes, it becomes imperative to propose methodologies that facilitate the aggregation of disparate forms of information, encompassing both hard and soft datasets. This is achieved using the Bonferroni means extension, as presented in Equations (5)–(8).

Proposition 5. *The β_{BonOWA} formulation is defined as follows:*

$$\beta_{BonOWA} = \frac{BonOWACov(R_p, R_b)}{\sigma^2{}_{BonOWA}(R_b)}, \tag{15}$$

where BonOWACov is the Bonferroni OWA covariance (Equation (6)) (BonOWACov characteristics are shown widely in Equation (6) and they are applied in β_{BonOWA}), R_p is the return of the asset, R_b is the return of the market, and $\sigma^2{}_{BonOWA}$ is the Bonferroni OWA variance (Equation (5)) ($\sigma^2{}_{BonOWA}$ characteristics are shown widely in Equation (5) and they are applied in β_{BonOWA}).

With the definition of the β_{BonOWA}, it is possible to define the $CAPM_{BonOWA}$ as follows:

Proposition 6. *The $CAPM_{BonOWA}$ is calculated as follows:*

$$CAPM_{BonOWA} = R_f + \beta_{BonOWA}(R_M - R_f), \tag{16}$$

where $CAPM_{BonOWA}$ is the subjective return of the portfolio/asset, R_f is the risk-free rate of interest, B_{BonOWA} is the coefficient that is related to the interrelated and systematic risk simultaneously, and $(R_b - R_f)$ is the equity risk premium.

(a) Commutativity: Let $(a_1', a_2', \ldots, a_n')$ *be any permutation of* $(a_1, a_2, \ldots, a_n)$, then: $CAPM_{BonOWA}(a_1', a_2', \ldots, a_n') = CAPM_{BonOWA}(a_1, a_2, \ldots, a_n)$. (b) Idempotency: Let $a_j = a$, $j = 1, 2, \ldots, n$, *then* $CAPM_{BonOWA}(a, a, \ldots, a) = a$. (c) Monotonicity: Let $a_i (i = 1, 2, \ldots, n)$ *and* $b_i (i = 1, 2, \ldots, n)$ be two collections of crisp data *If* $a_i \geq b_i$ *for all* i, *then* $CAPM_{BonOWA}$ $(a_1, a_2, \ldots, a_n) \geq CAPM_{BonOWA}(b_1, b_2, \ldots, b_n)$. (d) Boundedness: The $CAPM_{BonOWA}$ lies between the max and min operators: $\min(a_1, a_2, \ldots, a_n) \leq CAPM_{BonOWA}(a_1, a_2, \ldots, a_n) \leq \max(a_1, a_2, \ldots, a_n)$.

Some special cases of the β_{BonOWA} operator are shown as follows:

If $r = 1$ and $q = 1$, then Equation (2) reduces to the following:

$$\beta_{BonOWA^{1,1}} = \frac{\left(\frac{1}{n}\sum_i a_i^1\left(\frac{1}{\text{n}-1}\sum_{\substack{j=1 \\ j\neq i}}^{\text{n}} a_j^1\right)\right)^{\frac{1}{2}}(R_p, R_b)}{\left(\frac{1}{n}\sum_i a_i^1\left(\frac{1}{\text{n}-1}\sum_{\substack{j=1 \\ j\neq i}}^{\text{n}} a_j^1\right)\right)^{\frac{1}{2}}(R_b)}, \tag{17}$$

If $q = 0$, then Equation (2) reduces to the following:

$$\beta_{BonOWA^{r,0}} = \frac{\left(\frac{1}{n}\sum_i a_i^r\right)^{\frac{1}{r+0}}(R_p, R_b)}{\left(\frac{1}{n}\sum_i a_i^r\right)^{\frac{1}{r+0}}(R_b)}, \tag{18}$$

If $r = 2$ and $q = 0$, then Equation (2) reduces to the square mean as follows:

$$\beta_{BonOWA^{2,0}} = \frac{\left(\frac{1}{n}\sum_i a_i^2\right)^{\frac{1}{2}}(R_p, R_b)}{\left(\frac{1}{n}\sum_i a_i^2\right)^{\frac{1}{2}}(R_b)}, \tag{19}$$

If $r = 1$ and $q = 0$, then Equation (2) reduces to the usual average as follows:

$$\beta_{BonOWA^{1,0}} = \frac{\frac{1}{\mathrm{n}}\sum_{k=1}^{n} a_i(R_p, R_b)}{\frac{1}{\mathrm{n}}\sum_{k=1}^{n} a_i(R_b)}, \tag{20}$$

If $r \to +\infty$ and $q = 0$, then Equation (2) reduces to the max operator as follows:

$$\lim_{r\to\infty}\beta_{BonOWA^{\infty,0}} = \frac{max\{a_i\}(R_p, R_b)}{max\{a_i\}(R_b)}, \tag{21}$$

If $r \to 0$ and $q = 0$, then Equation (2) reduces to the geometric mean operator as follows:

$$\lim_{r\to\infty}\beta_{BonOWA^{r,0}} = \frac{\left(\prod_{i=1}^{n} a_i\right)^{\frac{1}{n}}(R_p, R_b)}{\left(\prod_{i=1}^{n} a_i\right)^{\frac{1}{n}}(R_b)}, \tag{22}$$

Proposition 7. *The $\beta_{BonIOWA}$ formulation is defined as follows:*

$$\beta_{BonIOWA} = \frac{BonIOWACov(R_p, R_b)}{\sigma^2{}_{BonIOWA}(R_b)}, \tag{23}$$

where BonIOWACov is the Bonferroni-induced OWA covariance (Equation (8)) (BonIOWACov characteristics are shown widely in Equation (6) and they are applied in $\beta_{BonIOWA}$), R_p is the return of the asset, R_b is the return of the market, and $\sigma^2{}_{BonIOWA}$ is the Bonferroni-induced OWA variance (Equation (7)) ($\sigma^2{}_{BonIOWA}$ characteristics are shown widely in Equation (5) and they are applied in $\beta_{BonIOWA}$). In both, the order-inducing variable is u_i.

Proposition 8. *The $CAPM_{BonIOWA}$ is calculated as follows:*

$$CAPM_{BonIOWA} = R_f + \beta_{BonIOWA}(R_M - R_f), \tag{24}$$

where $CAPM_{BonIOWA}$ is the expected return of the portfolio/asset, R_f is the risk-free rate of interest, $\beta_{BonIOWA}$ is the coefficient that is related to the interrelated and systematic risk simultaneously, and $(R_b - R_f)$ is the equity risk premium related to the reordering variable u_i.

(a) Commutativity: Let $(a_1', a_2', \ldots, a_n')$ *be any permutation of* $(a_1, a_2, \ldots, a_n)$, then: $CAPM_{BonIOWA}(\langle u_i, a_i\rangle, \ldots, \langle u_n, a_n\rangle) = CAPM_{BonIOWA}(\langle u_i, b_i\rangle, \ldots, \langle u_n, b_n\rangle)$. (b) Idempotency: Let $|u_i, a_i| = a, i = 1, 2, \ldots, n,$ *then* $CAPM_{BonIOWA}(\langle u_i, a_i\rangle, \ldots, \langle u_n, a_n\rangle) = a$. (c) Monotonicity: Let $a_i(i = \langle u_1, 1\rangle, \ldots, \langle u_n, a_n\rangle)$ *and* $b_i(i = \langle u_i, b_1\rangle, \ldots, \langle u_n, b_n\rangle)$ be two collections of crisp data $If\ |u_i, a_i| \geq |u_i, b_i|\ for\ all\ i,\ then\ CAPM_{BonIOWA}(\langle u_1, 1\rangle, \ldots, \langle u_n, a_n\rangle) \geq CAPM_{BonIOWA}(\langle u_i, b_1\rangle, \ldots, \langle u_n, b_n\rangle)$. (d) Boundedness: The $CAPM_{BonIOWA}$ lies between the max and min operators: $\min(\langle u_i, a_i\rangle, \ldots, \langle u_n, a_n\rangle) \leq CAPM_{BonIOWA}(\langle u_i, a_i\rangle, \ldots, \langle u_n, a_n\rangle) \leq \max(\langle u_i, a_i\rangle, \ldots, \langle u_n, a_n\rangle)$.

Some special cases of the $\beta_{BonIOWA}$ operator are shown as follows:

If $r = 1$ and $q = 1$, then Equation (2) reduces to the following:

$$\beta_{BonIOWA^{1,1}} = \frac{\left(\frac{1}{n}\sum_i \langle u_i, a_i\rangle_i^1 \left(\frac{1}{\mathrm{n}-1}\sum_{\substack{j=1\\ j\neq i}}^{\mathrm{n}} \langle u_i, a_i\rangle_j^1\right)\right)^{\frac{1}{2}} (R_p, R_b)}{\left(\frac{1}{n}\sum_i \langle u_i, a_i\rangle_i^1 \left(\frac{1}{\mathrm{n}-1}\sum_{\substack{j=1\\ j\neq i}}^{\mathrm{n}} \langle u_i, a_i\rangle_j^1\right)\right)^{\frac{1}{2}} (R_b)}, \tag{25}$$

If $q = 0$, then Equation (2) reduces to the following:

$$\beta_{BonIOWA^{r,0}} = \frac{\left(\frac{1}{n}\sum_i \langle u_i, a_i\rangle_i^r\right)^{\frac{1}{r+0}} (R_p, R_b)}{\left(\frac{1}{n}\sum_i \langle u_i, a_i\rangle_i^r\right)^{\frac{1}{r+0}} (R_b)}, \tag{26}$$

If $r = 2$ and $q = 0$, then Equation (2) reduces to the square mean as follows:

$$\beta_{BonIOWA^{2,0}} = \frac{\left(\frac{1}{n}\sum_i \langle u_i, a_i\rangle_i^2\right)^{\frac{1}{2}} (R_p, R_b)}{\left(\frac{1}{n}\sum_i \langle u_i, a_i\rangle_i^2\right)^{\frac{1}{2}} (R_b)}, \tag{27}$$

If $r = 1$ and $q = 0$, then Equation (2) reduces to the usual average as follows:

$$\beta_{BonIOWA^{1,0}} = \frac{\frac{1}{\mathrm{n}}\sum_{k=1}^{n} \langle u_i, a_i\rangle (R_p, R_b)}{\frac{1}{\mathrm{n}}\sum_{k=1}^{n} \langle u_i, a_i\rangle (R_b)}, \tag{28}$$

If $r \to +\infty$ and $q = 0$, then Equation (2) reduces to the max operator as follows:

$$\lim_{r\to\infty} \beta_{BonIOWA^{\infty,0}} = \frac{max\{\langle u_i, a_i\rangle\} (R_p, R_b)}{max\{\langle u_i, a_i\rangle\} (R_b)}, \tag{29}$$

If $r \to 0$ and $q = 0$, then Equation (2) reduces to the geometric mean operator as follows:

$$\lim_{r\to\infty} \beta_{BonIOWA^{r,0}} = \frac{\left(\prod_{i=1}^{n} \langle u_i, a_i\rangle\right)^{\frac{1}{n}} (R_p, R_b)}{\left(\prod_{i=1}^{n} \langle u_i, a_i\rangle\right)^{\frac{1}{n}} (R_b)}, \tag{30}$$

A novel family of CAMP method extensions has been proposed, comprising the following four new approaches: $CAPM_{OWA}$, $CAPM_{IOWA}$, $CAPM_{BonOWA}$, and $CAPM_{BonIOWA}$. Each of these cases presents a different level of complexity. The $CAPM_{OWA}$ considers the attitude of the decision maker by the inclusion of the weighting vector and the reordering process based on the value of the arguments. The $CAPM_{IOWA}$ incorporates a new reordering process based on induced values, which allows the decision maker to select what will be the ordering relationship between the weights and arguments based on criteria that they consider to be the most appropriate based on their knowledge. The $CAPM_{BonOWA}$ not only incorporates the weighting vector, but through the use of Bonferroni means, it allows for the interrelatedness of these arguments, simultaneously allowing for a more complex analysis between the arguments. Finally, the $CAPM_{BonIOWA}$ considers the attitude of the decision maker, the ordering of arguments according to the induced variable, and the interrelatedness of these arguments simultaneously. Thus, each of the proposed methods can represent a different scenario, considering the attitude, preferences, and relationship of each argument to make a portfolio investment decision.

4. Application of the $CAPM_{OWA}$ and Its Extensions in Creating an Investment Portfolio

In this section, the use of the $CAPM_{OWA}$ and its extensions in the Mexican Stock Exchange is explored. Also, the step-by-step instructions for using this new operator are presented.

4.1. Step by Step

To incorporate the expectations and knowledge of the decision maker into the portfolio selection process with the use of the $CAPM_{OWA}$ and its extensions, it is necessary to consider the following steps:

1. Define the investor's profile. The purpose of this is to identify financial instruments that could be part of the investor's potential portfolio. This profile is related to knowledge of the different fixed income instruments and equity instruments, and it also considers the attitude of the investors to potential losses and aversion to risk. Many financial institutions obtain this information through different questionnaires provided to the person.
2. Define the value limits of the β. The objective is that the assets that make up the investment portfolio are directly related to the investor's profile. Therefore, those with a high result have a higher volatility than the market, so their possibility for profit or loss can be more significant. In this way, what is sought is that the companies that make up the portfolio meet the risk criteria allowed by the investor.
3. Define the assets that will make up the portfolio. The types of assets that will make up the investment portfolio must be identified. For this research, only stocks will be used.
4. Define the industries that will make up the portfolio. Each industry has different risks and volatilities; therefore, investors may feel more open to one type of industry than another, so the search for possible assets should be limited based on the industries defined as possible for investment.
5. Define the number of assets that will make up the portfolio.
6. Define the possible shares to integrate the portfolio through the β traditional.
7. Calculate the β_{OWA} and its extensions for defining shares that can make up the investment portfolio.
8. Identify which assets will be part of the portfolio.
9. Calculate the $CAPM$, $CAPM_{OWA}$, and its extensions for each of the assets.
10. Identify the weight of each asset within the portfolio.
11. Calculate the $CAPM$, $CAPM_{OWA}$, and its extensions for the investment portfolio and analyze the results.

4.2. Application in an Investment Portfolio

Within this section, the steps defined in the previous point will be used to identify the specific structures of investment portfolios through the use of the traditional $CAPM$, $CAPM_{OWA}$, and its extensions.

Step 1. In this case, the investor is male, 36 years old, with an investment horizon of 3 years, an average income of MXN 50,000 Mexican pesos per month, a mortgage loan with a monthly payment of MXN 10,500, without additional equity, with an investment capacity of MXN 1,000,000, a risk tolerance defined as moderate, and an investment objective of a return of more than two or three percent higher than Mexican Treasury Certificates. With all the above information, it can be defined that the investor's profile is moderate, indicating that it can make investments in different equity instruments, but it has a medium aversion to risk, and because of that, the possible investment options must consider limited potential losses.

Step 2. In this case, the limits established for the β value are 1 ± 0.30, that is, a minimum value of 0.70 and a maximum value of 1.30. These limit values are obtained considering the investor profile and, after some talk rounds, the main objective is to obtain a value that meets the risk aversion of the investor.

Step 3. The portfolio only considers stocks from the Mexican market, and a selection of 23 stocks is made (See Table 2), so all of them are used to form the portfolio. Yahoo Finance and programming in R are used to download the daily information of each stock from 1 January 2021 to 31 December 2023. The decision to only include the Mexican Stock market is because the investor that provided the information can only invest in these assets; because of this limitation, the proposed method is presented considering this. This is also an exciting approach, because even when an investor has limited asset options, it is always important to measure the risk of those assets and include in the portfolio only options that meet the requirements provided by the investor.

Table 2. Symbol, names, and β values.

Symbol	Name	Beta
GRUMAB.MX	Gruma, S.A.B. de C.V.	0.5582
GENTERA.MX	Gentera, S.A.B. de C.V.	0.6381
OMAB.MX	Grupo Aeroportuario del Centro Norte, S.A.B. de C.V.	1.0546
GAPB.MX	Grupo Aeroportuario del Pacífico, S.A.B. de C.V.	1.3007
FEMSAUBD.MX	Fomento Económico Mexicano, S.A.B. de C.V.	0.8007
AC.MX	Arca Continental, S.A.B. de C.V.	0.4906
GMEXICOB.MX	Grupo México, S.A.B. de C.V.	1.4889
CEMEXCPO.MX	CEMEX, S.A.B. de C.V.	1.5406
GCC.MX	GCC, S.A.B. de C.V.	0.7323
MEGACPO.MX	Megacable Holdings, S. A. B. de C. V.	0.5507
TLEVISACPO.MX	Grupo Televisa, S.A.B.	1.4779
GCARSOA1.MX	Grupo Carso, S.A.B. de C.V.	1.3786
GFNORTEO.MX	Grupo Financiero Banorte, S.A.B. de C.V.	0.825
BIMBOA.MX	Grupo Bimbo, S.A.B. de C.V.	0.4555
ALPEKA.MX	ALPEK, S.A.B. de C.V.	0.7325
ALSEA.MX	Alsea, S.A.B. de C.V.	0.6207
KIMBERA.MX	Kimberly-Clark de México, S. A. B. de C. V.	1.0612
ASURB.MX	Grupo Aeroportuario del Sureste, S. A. B. de C. V.	0.5046
CUERVO.MX	Becle, S.A.B. de C.V.	0.4502
LABB.MX	Genomma Lab Internacional, S.A.B. de C.V.	0.5263
BOLSAA.MX	Bolsa Mexicana de Valores, S.A.B. de C.V.	0.7592
BBAJIOO.MX	Banco del Bajío, S.A., Institución de Banca Múltiple	0.6921
PINFRA.MX	Promotora y Operadora de Infraestructura, S. A. B. de C. V.	1.1042

In addition, daily profitability is generated through the $\frac{Actual\ value}{Actual\ value_{-1}} - 1$. Subsequently, the omitted data are deleted, and the database is exported to Excel 365 version 2408 (The R 4.4.1 code is in Appendix A).

Step 4. Due to the low number of shares with movements in the Mexican Stock Market, segmentation by industry is not carried out.

Step 5. Because the Mexican Stock Market does not have a lot of companies, the portfolio will consider all the stocks that meet the requirements.

Step 6. The asset selection process is carried out based on the β and β_{OWA}, where we analyze them separately. Initially, Table 2 shows the values using the β. In this regard, the following shares meet the requirements: OMAB. MX, FEMSAUBD. MX, GCC. MX, BIMBOA. MX, ALSEA. MX, ASURB. MX, BBAJIOO. MX, and GCARSOA1. MX.

Step 7. To calculate β_{OWA} and its extensions, it is necessary to calculate the $\overline{x}_{OWA}$ and $\overline{y}_{OWA}$, and a specific weight for each data piece must be determined to calculate both operations. To simplify the process, the decision maker is asked to give particular weight to each year analyzed, which are $2021 = 0.20,\ 2022 = 0.30,$ and $2023 = 0.50$ (To obtain this information, the investor that completed the information of step 1 is asked the following question: Considering the results obtained in the last three years, in your expertise, what is the significance of the results for each specific year, considering that the sum of the 3 years must be 1? One way to visualize this is the possibility that the behavior of the shares will be like that year. The investor gave all the information about each share, such as their volatility graphs and their average annual returns. The main purpose of this step is to obtain the economic knowledge and expectation of the investor.). It is worth mentioning that a more diffuse analysis could assign a different weight to the months or days of each year. To obtain the β_{IOWA}, the induced values are $I = (3, 2, 1)$, with the weighting vector and the induced values of the $\beta_{Bon-OWA}$ and $\beta_{Bon-IOWA}$ being obtained considering r = q = 1.

To make it clear how the calculation is made, let us take MXX as an example, which is the Mexican S&P, where $\overline{y} = 0.00037578$, and this result is the average of all the MXX values from 1 January 2021 to 31 December 2023. For the calculation of $\overline{y}_{OWA}$, the average will be identified for each year, where 2021 = 0.0007351, 2022 = −0.0003199, and 2023 = 0.0007135. Once the averages per year are obtained, the values are reordered and then multiplied by the weights so the final multiplication is $\overline{y}_{OWA} = (-0.0003199 \times 0.20) + (0.0007135 \times 0.30) + (0.0007351 \times 0.50) = 0.0005176$. Also, the process for the $\overline{y}_{IOWA} = (0.0007135 \times 0.20) + (-0.0003199 \times 0.30) + (0.00071351 \times 0.50) = 0.0000004143$, for the case of the $\overline{y}_{Bon-OWA}$ the calculation is the following.

$$\overline{y}_{Bon-OWA}(V^1) = 0.20 \times (|-0.0003199| + |0.0007135|) = 0.0002067$$

$$\overline{y}_{Bon-OWA}(V^2) = 0.30 \times (|0.0007135| + |0.0007351|) = 0.0004346$$

$$\overline{y}_{Bon-OWA}(V^3) = 0.50 \times (|-0.0003199| + |0.0007351|) = 0.0005275$$

$$\overline{y}_{Bon-OWA} = \Big(\tfrac{1}{3}[\{|-0.0003199| \times |0.0002067|\} + \{|0.0007135| \times |0.0004346|\} + \{|0.00073505| \times |0.0005275|\}\Big)^{\frac{1}{1+1}} = 0.0005046$$

Finally, in the case of the $\overline{y}_{Bon-IOWA}$, the process is

$$\overline{y}_{Bon-OWA}(V^1) = 0.20 \times (|0.0007351| + |-0.0003199|) = 0.0002067$$

$$\overline{y}_{Bon-OWA}(V^2) = 0.30 \times (|-0.0003199| + |0.0007351|) = 0.0003165$$

$$\overline{y}_{Bon-OWA}(V^3) = 0.50 \times (|0.0007351| + |0.0007135|) = 0.0007243$$

$$\overline{y}_{Bon-OWA} = \Big(\tfrac{1}{3}[\{|0.0007135| \times |0.0002067|\} + \{|0.0003199| \times |0.0003165|\} + \{|0.0007351| \times |0.0007243|\}]\Big)^{\frac{1}{1+1}} = 0.0004808$$

The average for each year and the results using different aggregation operators are presented in Table 3.

With these new averages, the Cov_{OWA} and Var_{OWA} are calculated. Assignments of monthly weights are enough for analysis purposes, and a weight related to each month is determined by the investor as $W = (0.10, 0.05, 0.5, 0.10, 0.10, 0.05, 0.05, 0.10, 0.10, 0.10, 0.05, 0.15)$ (To obtain the relative weights for the importance of each month, the guiding question for the investor is the following: Considering the impact that each month has on the volatility of the stock price, what weight would you assign to each of the months? Remember that the sum must equal 1. A simple way to understand the question is to consider the following: Which months have the highest volatility? and from there, assign weight. The main purpose is that the investors visualize if there are some months that are easier for the share to have the same pattern and assign it a higher weight, and for those where the changes are specific to some events that are hard to replicate in the future, assign them a lower weight.). Considering that the information that is analyzed is for three years (36 months), the weights will be divided by $\frac{1}{3}$ to make the sum equal to one, and the induced values are $I = (10, 9, 8, 5, 6, 4, 2, 3, 11, 12, 7, 1)$. The β results are presented in Table 4.

Step 8. In this step, the different suggestions of portfolios are analyzed based on the considerations of the investor. These portfolios are presented in Table 5.

Table 3. Averages daily stocks by year and by aggregation operator.

Symbol	2021	2022	2023	OWA	IOWA	Bon-OWA	Bon-IOWA
GRUMAB.MX	0.0005055	0.0002882	0.0008949	0.0006567	0.0005182	0.0005123	0.0004540
GENTERA.MX	0.0016087	0.0024034	0.0007947	0.0018432	0.0016843	0.0014331	0.0012836
OMAB.MX	0.0005700	0.0009790	0.0013587	0.0010870	0.0008504	0.0008512	0.0007393
GAPB.MX	0.0013762	0.0003898	0.0008959	0.0010348	0.0009842	0.0008012	0.0007687
FEMSAUBD.MX	0.0003877	−0.0000109	0.0016638	0.0009460	0.0005233	0.0007377	0.0005798
AC.MX	0.0015547	0.0010510	0.0008520	0.0012631	0.0012631	0.0010028	0.0009857
GMEXICOB.MX	0.0005546	−0.0004100	0.0016822	0.0009255	0.0004907	0.0008585	0.0006955
CEMEXCPO.MX	0.0014912	−0.0019151	0.0022475	0.0011881	0.0006206	0.0015974	0.0014990
GCC.MX	0.0012801	−0.0005963	0.0019400	0.0012348	0.0008492	0.0011432	0.0010341
MEGACPO.MX	0.0000122	−0.0007710	−0.0007888	−0.0003829	−0.0003829	0.0003795	0.0004038
TLEVISACPO.MX	0.0009566	−0.0027733	−0.0012319	−0.0004459	−0.0006001	0.0012668	0.0013730
GFNORTEO.MX	0.0011140	0.0008094	0.0013262	0.0011592	0.0010651	0.0009207	0.0008763
BIMBOA.MX	0.0017992	0.0014150	0.0003174	0.0013876	0.0013876	0.0010612	0.0009736
ALPEKA.MX	0.0010333	0.0011461	−0.0028989	0.0003033	0.0002807	0.0013258	0.0012524
ALSEA.MX	0.0016535	0.0000824	0.0023575	0.0016913	0.0013230	0.0012773	0.0011476
KIMBERA.MX	−0.0000676	0.0006687	0.0008574	0.0006158	0.0003383	0.0004874	0.0003833
ASURB.MX	0.0012203	0.0006121	0.0008623	0.0009913	0.0009663	0.0007826	0.0007633
CUERVO.MX	0.0002852	−0.0005637	−0.0007576	−0.0001780	−0.0001780	0.0004052	0.0004086
LABB.MX	0.0009686	−0.0005574	−0.0003678	0.0002624	0.0002435	0.0005743	0.0005750
BOLSAA.MX	−0.0005605	0.0002576	0.0000520	0.0000323	−0.0001926	0.0002442	0.0003054
BBAJIOO.MX	0.0017532	0.0028339	0.0002515	0.0019932	0.0017771	0.0015150	0.0012436
PINFRA.MX	−0.0002168	0.0004060	0.0008427	0.0004998	0.0001819	0.0004566	0.0003656
GCARSOA1.MX	0.0003556	0.0011733	0.0035974	0.0022218	0.0012493	0.0017222	0.0012481
MXX	0.0007351	−0.0003199	0.0007135	0.0005176	0.0004143	0.0005046	0.0004808

Table 4. Calculation of the β_{OWA} and its extensions.

Stocks	β	β_{OWA}	β_{IOWA}	$\beta_{Bon\text{-}OWA}$	$\beta_{Bon\text{-}IOWA}$
GRUMAB.MX	0.558	0.498	0.341	0.580	0.337
GENTERA.MX	0.638	0.923	0.892	0.995	0.874
OMAB.MX	1.054	1.150	1.177	1.184	1.164
GAPB.MX	1.300	1.241	1.197	1.276	1.190
FEMSAUBD.MX	0.800	0.667	0.594	0.740	0.593
AC.MX	0.490	0.658	0.538	0.607	0.531
GMEXICOB.MX	1.488	1.592	1.452	1.543	1.454
CEMEXCPO.MX	1.541	1.579	1.377	1.532	1.401
GCC.MX	0.732	0.817	0.827	0.862	0.831
MEGACPO.MX	0.551	1.008	0.755	1.051	0.772
TLEVISACPO.MX	1.478	1.985	1.858	2.227	1.921
GFNORTEO.MX	1.379	1.072	1.100	1.196	1.092
BIMBOA.MX	0.825	0.723	0.617	0.809	0.607
ALPEKA.MX	0.455	0.840	0.408	0.832	0.442
ALSEA.MX	0.733	0.929	0.873	1.082	0.868
KIMBERA.MX	0.621	0.792	0.645	0.787	0.643
ASURB.MX	1.061	0.826	0.816	0.895	0.808
CUERVO.MX	0.505	0.670	0.789	0.829	0.804
LABB.MX	0.450	0.940	0.563	0.742	0.570
BOLSAA.MX	0.526	0.936	0.825	0.907	0.834
BBAJIOO.MX	0.759	0.868	0.770	1.063	0.757
PINFRA.MX	0.692	0.966	0.919	0.945	0.917
GCARSOA1.MX	1.104	1.705	1.456	1.622	1.444

Step 9. For the calculation of the $CAPM$, $CAPM_{OWA}$, and its extensions, the risk-free rate that will be used is $R_f = 7.12\%$, which corresponds to the average of the 28-day Treasury Certificates (CETES) from 2 January 2020 to 28 December 2023, and the market risk rate is $r_m = 9.46\%$, which represents the average yield from 2020 to 2023. To better understand the calculations, the $CAPM$ and $CAPM_{OWA}$ for the GRUMAB.MX action will be performed. The results of all shares can be found in Table 6.

$$CAPM = 7.12\% + 0.5582380(9.46\% - 7.12\%) = 8.43\%$$

$$CAPM_{OWA} = 7.12\% + 0.4976700(9.46\% - 7.12\%) = 8.28\%$$

$$CAPM_{IOWA} = 7.12\% + 0.3410421(9.46\% - 7.12\%) = 7.92\%$$

$$CAPM_{Bon-OWA} = 7.12\% + 0.5800337(9.46\% - 7.12\%) = 8.48\%$$

$$CAPM_{Bon-IOWA} = 7.12\% + 0.3366573(9.46\% - 7.12\%) = 7.91\%$$

Table 5. Share selection for each portfolio.

Portfolio	Shares
Using β	OMAB. MX, FEMSAUBD. MX, GCC. MX, BIMBOA. MX, ALSEA. MX, ASURB. MX, BBAJIOO. MX and GCARSOA1. MX
Using β_{OWA}	GENTERA.MX, OMAB.MX, GAPB.MX, GCC.MX, MEGACPO.MX, GFNORTEO.MX, BIMBOA.MX, ALPEKA.MX, ALSEA.MX, KIMBERA.MX, ASURB.MX, LABB.MX, BOLSAA.MX, BBAJIOO.MX and PINFRA.MX
using β_{IOWA}	GENTERA.MX, OMAB.MX, GAPB.MX, GCC.MX, MEGACPO.MX, GFNORTEO.MX, ALSEA.MX, ASURB.MX, CUERVO.MX, BOLSAA.MX, BBAJIOO.MX and PINFRA.MX
using $\beta_{Bon\text{-}OWA}$	GENTERA.MX, OMAB.MX, GAPB.MX, FEMSAUBD.MX, GCC.MX, MEGACPO.MX, GFNORTEO.MX, BIMBOA.MX, ALPEKA.MX, ALSEA.MX, KIMBERA.MX, ASURB.MX, CUERVO.MX, LABB.MX, BOLSAA.MX, BBAJIOO.MX and PINFRA.MX
using $\beta_{Bon\text{-}IOWA}$	GENTERA.MX, OMAB.MX, GAPB.MX, GCC.MX, MEGACPO.MX, GFNORTEO.MX, ALSEA.MX, ASURB.MX, CUERVO.MX, BOLSAA.MX, BBAJIOO.MX and PINFRA.MX

Table 6. *CAPM* and $CAPM_{OWA}$ for the 23 shares.

Stocks	$CAPM$	$CAPM_{OWA}$	$CAPM_{IOWA}$	$CAPM_{Bon\text{-}OWA}$	$CAPM_{Bon\text{-}IOWA}$
GRUMAB.MX	8.43%	8.28%	7.92%	8.48%	7.91%
GENTERA.MX	8.61%	9.28%	9.21%	9.45%	9.17%
OMAB.MX	9.59%	9.81%	9.87%	9.89%	9.84%
GAPB.MX	10.16%	10.02%	9.92%	10.11%	9.90%
FEMSAUBD.MX	8.99%	8.68%	8.51%	8.85%	8.51%
AC.MX	8.27%	8.66%	8.38%	8.54%	8.36%
GMEXICOB.MX	10.60%	10.84%	10.52%	10.73%	10.52%
CEMEXCPO.MX	10.72%	10.82%	10.34%	10.70%	10.40%
GCC.MX	8.83%	9.03%	9.06%	9.14%	9.06%
MEGACPO.MX	8.41%	9.48%	8.89%	9.58%	8.93%
TLEVISACPO.MX	10.58%	11.77%	11.47%	12.33%	11.61%
GFNORTEO.MX	10.35%	9.63%	9.69%	9.92%	9.67%
BIMBOA.MX	9.05%	8.81%	8.56%	9.01%	8.54%
ALPEKA.MX	8.19%	9.08%	8.08%	9.07%	8.15%
ALSEA.MX	8.83%	9.29%	9.16%	9.65%	9.15%
KIMBERA.MX	8.57%	8.97%	8.63%	8.96%	8.62%
ASURB.MX	9.60%	9.05%	9.03%	9.21%	9.01%
CUERVO.MX	8.30%	8.69%	8.97%	9.06%	9.00%
LABB.MX	8.17%	9.32%	8.44%	8.86%	8.45%
BOLSAA.MX	8.35%	9.31%	9.05%	9.24%	9.07%
BBAJIOO.MX	8.90%	9.15%	8.92%	9.61%	8.89%
PINFRA.MX	8.74%	9.38%	9.27%	9.33%	9.27%
GCARSOA1.MX	9.70%	11.11%	10.53%	10.92%	10.50%

It is important to remember that not all stocks will be part of our portfolio, but only those specified in Step 8.

Step 10. Identification of the weight of each asset within the portfolio.

The selection of the weight that each asset will have within the portfolio depends on the types of assets that are integrated, whether they are bonds, stocks, currencies, metals, or any other kind of instrument; however, by considering only stocks in the portfolio, the decision is made that each of these has a weight equal to $\frac{1}{n}$.

Step 11. The portfolio performance calculation in Table 7 shows an example of how they are calculated based on the *CAPM*, and Table 8 presents the results using all the aggregation operators.

Table 7. Results for the portfolio *CAPM*.

Symbol	*CAPM*	Weight	Weighted Performance
OMAB.MX	9.59%	0.125	1.20%
FEMSAUBD.MX	8.99%	0.125	1.12%
GCC.MX	8.83%	0.125	1.10%
BIMBOA.MX	9.05%	0.125	1.13%
ALSEA.MX	8.83%	0.125	1.10%
ASURB.MX	9.60%	0.125	1.20%
BBAJIOO.MX	8.90%	0.125	1.11%
GCARSOA1.MX	9.70%	0.125	1.21%
		Total	9.19%

Table 8. Results of portfolio performance based on different aggregation operators.

Operator	Result
$CAPM$	9.19%
$CAPM_{OWA}$	9.31%
$CAPM_{IOWA}$	9.25%
$CAPM_{Bon\text{-}OWA}$	9.35%
$CAPM_{Bon\text{-}IOWA}$	9.25%

Based on the results obtained in Table 9, it can be seen that the ranking of portfolios should be $CAPM_{Bon-OWA} > CAPM_{OWA} > CAPM_{IOWA}\ or\ CAPM_{Bon-IOWA} > CAPM$. Even when the results show us an increase in the possible future profitability of the portfolio, the application is really in the selection of assets, since this decision must consider the investor's characteristics so that he does not face more or less risk than he is willing to. An interesting fact can be visualized in step 8, where we consider portfolios from 8 stocks to 17, which may meet or fail to meet the investor's criteria, depending on how the information is analyzed. Thus, this article aims to present an alternative way to calculate *CAPM* and β to identify the possible returns of assets considering different opinions on and future visions for the market of the investor or decision maker. It is essential to recognize that using the characteristics and expectations of each decision maker presents a significant limitation, since the parameters of acceptance and rejection are individual. Hence, the aim is to find a portfolio that considers the knowledge and expectations of the investor, and with that information, obtain the different risk possibilities and select the one that meets all the conditions expected and provides the highest expected return.

Table 9. Portfolios analysis.

Portfolio	Coefficient	*p*-Value	R-Square (adj)
$CAPM$	0.8849	0.000	61.71%
$CAPM_{OWA}$	0.7862	0.000	62.78%
$CAPM_{IOWA}$ *and* $CAPM_{Bon\text{-}IOWA}$	0.8287	0.000	60.08%
$CAPM_{Bon\text{-}OWA}$	0.7706	0.000	65.97%

To perform a better analysis of the portfolio generated, a linear regression will be made considering the expected returns of the different four portfolios' expected returns (dependent variable) with the market expected returns (R_M). Also, the $CAPM_{IOWA}$ and $CAPM_{Bon-IOWA}$ portfolio consider the same stocks, because one regression is conducted for the two operators. The hypotheses that are presented are the following: $H_0 = \beta_i = 0$ and $H_1 = 1.30 > \beta_i > 0.70$. We want to prove that the expected returns of the portfolio are still in the same risk range that the decision maker indicated in step 2 of Section 4.2. The complete results are presented in Appendix B and a summary of the results are presented in Table 9.

With the results obtained by the different models, it is possible to accept H_1, considering that all portfolios have a β between 0.70 and 1.30. Another interesting result is that all portfolios have a *p*-Value of 0.000, remembering that the variables are significant when the *p*-value is less than 0.050. These results demonstrate that the portfolios being proposed to the decision maker have the characteristics that the investor wants.

As seen with the different proposed portfolios, various scenarios can be generated. One of the main advantages is that these propositions include the information and expectations of the investor, remembering that the main idea is to create a portfolio that can meet the requirements that were settled in the initial stage. Because of that, with the addition of the weighting vector, induced values, and Bonferroni means, subjective information can be included in the formulation. One of the main limitations of this method is that the results are susceptible to changes in the weighting vector and will produce different portfolios based on the information provided by the investor. It is important to consider that this limitation is also the importance of the methods proposed because, in the end, what the investor wants is to have a portfolio that can meet all their requirements and not just a generic one that does not include the expectations and knowledge that they have about the past, present, and future of the market.

Finally, another significant limitation is the complexity of using these methods in comparison to the traditional one, as can be seen in the results; in the end, the $CAPM_{IOWA}$ and $CAPM_{Bon-IOWA}$ present the same asset selection and, because of this, the same results. Considering that the process of using the $CAPM_{Bon-IOWA}$ is more complex, it may seem that, in the end, it has no benefit. The same could have happened with the other extensions if the asset or the weighting vector or induced values changed, and in the end, the asset selection remained the same for the different operators. However, an important point to consider is that, when the market presents, in general, a low volatility, it may be possible that the use of complex formulations and extensions of the CAPM is not needed because the results will remain similar, but considering that the data selection in this article was from 2021 to 2023, years where COVID-19 was prevalent and economic recovery was occurring, the markets presented a lot of speculation, and this is when subjective information can cause significant changes in portfolio selections. Another relevant fact to consider when deciding whether to use complex methods like the ones proposed here are the assets that can be included in the portfolio, because if the portfolio has an important percentage in fixed income instruments like government bonds, usually, the β will not change much, even when different equity instruments are selected, but if the portfolio is mainly composed of equity instruments, then the complexity of the portfolio increases and the use of more complex methodologies is required.

5. Conclusions

This article aims to present an improvement in the Capital Asset Pricing Model (CAPM) formulation using different aggregation methods based on the OWA operator. The proposed method is called the Capital Asset Pricing Model OWA operator ($CAPM_{OWA}$) operator. The main idea of this operator is that the ordering step considers the decision maker's attitude towards selecting assets considering their risk. Thus, this method can consider both the information derived from the stock market and the attitude of the decision makers in a Multi-Criteria Decision Analysis (MCDA). Moreover, the proposed method combines the IOWA operator and the Bonferroni OWA extension related to variance and covariance. This results in the introduction of three new extensions, as follows: the $CAPM_{IOWA}$, $CAPM_{Bon-OWA}$, and $CAPM_{Bon-IOWA}$ operators. In this sense, each operator offers a different level of complexity. The $CAPM_{IOWA}$ operator considers attitude and reordering according to the induced variable, the $CAPM_{Bon-OWA}$ operator considers attitude and the simultaneous interrelationship of the arguments, while the $CAPM_{Bon-IOWA}$ operator considers attitude, the ordered induced variable, and the simultaneous interrelationship of the arguments. Therefore, each method enables the generation of disparate scenarios and complexities, wherein an underestimation or overestimation of the information by the decision maker may influence the decision-making process regarding portfolio investments.

Among the main results, the proposed methods generate different selections of stocks, considering that the formulation of the β generates different possibilities of stocks that meet the investor's requirements. In the end, considering the final CAPM of the various portfolios, the $CAPM_{Bon-OWA}$ was the most profitable one, considering 17 stocks in contrast to the 8 stocks of the traditional CAPM. As can be seen from the results, using different methods can include or exclude the assets that can be included in the portfolio so that the investors' preferences continue to be considered. That focus is not lost within the portfolio. Indeed, incorporating soft information and investors' expectations into the modeling process generates a more significant number of scenarios. Furthermore, the complexity of the modeling itself is reflected in the way the attitudes and preferences of decision makers are represented.

These new propositions that include the perception, knowledge, and expectations of the investor in the asset selection process are essential, because sometimes the use of a generic portfolio does not consider important elements for the decision maker, and because of that, they do not meet the requirements and limitations that the portfolio must have, and in this specific article, this means having a higher risk than the investor is willing to face. In this sense, this custom-made portfolio, where the investor has more active participation in the asset selection, can incorporate different subjective elements and improve the portfolio quality.

Within future research directions, it is possible to visualize the use of different aggregation operators in other stages of analysis of the investment portfolio, as is the case of the relative weight of each asset within the portfolio, which, in the case of this article, was considered an equal weight; however, fuzzy or multi-criteria techniques could also be used to obtain such weights. Also, other options include the application of extensions using distance measures to make a comparison between the ideal and the real information [31,32], using experts to weigh the perceptions of decision makers [33,34], or incorporating penalty functions to consider the potential for disagreement among decision makers [35,36]. Similarly, further extensions can be made by utilizing other OWA extensions, such as Heavy Operators [37,38] and Prioritized Operators [39,40]. Finally, some interesting works have been developed with the use of extensions based on the Bonferroni means, such the normalized Bonferroni weighted mean operator [41,42] or its extension using distance measures [43], interval values [44], or linguistic information [43,45].

Author Contributions: C.R.U.-O. and T.S.G.-G. writing—original draft, E.L.-C. and F.B.-M. supervision, P.A.A. and C.M.-C. methodology, C.R.U.-O., T.S.G.-G. and P.A.A. conceptualization, C.M.-C., E.L.-C. and F.B.-M. formal analysis. All authors have read and agreed to the published version of the manuscript.

Funding: UCSC 2024.

Data Availability Statement: Data are obtained from Yahoo Finance! that is a free source of information. Also, in the Appendix A is the R code that was used to download the information.

Acknowledgments: This research was supported by Universidad Pedagogica y Tecnologica de Colombia, grant number SGI 3787 and Red Sistemas Inteligentes y Expertos Modelos Computacionales Iberoamericanos (SIEMCI), project number 522RT0130 in Programa Iberoamericano de Ciencia y Tecnología para el Desarrollo (CYTED).

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A

```
```{r}
tickers <- c("GRUMAB.MX", "GENTERA.MX", "OMAB.MX", "GAPB.MX", "FEMSAUBD.MX", "AC.MX", "GMEXICOB.MX", "CEMEXCPO.MX", "GCC.MX", "MEGACPO.MX", "TLEVISACPO.MX", "GFNORTEO.MX", "BIMBOA.MX", "ALPEKA.MX", "ALSEA.MX", "KIMBERA.MX", "ASURB.MX", "CUERVO.MX", "LABB.MX", "BOLSAA.MX", "BBAJIOO.MX", "PINFRA.MX", "GCARSOA1.MX", "^MXX")
```
```

```
start <- "2021-01-01"
end <- "2023-12-31"
n <- length(tickers)
p <- getSymbols(Symbols = tickers[1], src = "yahoo",
from = start, to = end,
auto.assign = F)[, 6]
```

```{r}
for (i in 2:n){
p = merge(p, getSymbols(Symbols = tickers[i], src = "yahoo",
from = start, to = end,
auto.assign=F)[, 6])
}
names(p) = gsub(".Adjusted", "", names(p))
head(p)
```

```{r}
ret = p/lag(p) - 1
ret = ret[-1, ]
tail(ret)
```

```{r}
apply(is.na(ret), 2, sum)
```

```{r}
ret2 = na.omit(ret)
```

```{r}
write.csv(as.data.frame(ret2),"base.csv")
```

Appendix B

1. Portfolio $CAPM$
Regression Equation

$$Portfolio\ CAPM = 0.000828 + 0.8849\ R_M$$

Coefficients

Term	Coef	SE Coef	95% CI	T-Value	*p*-Value	VIF
Constant	0.000828	0.000241	(0.000355, 0.001300)	3.44	0.001	
R_M	0.8849	0.0254	(0.8351, 0.9347)	34.88	0.000	1.00

Model Summary

S	R-sq	R-sq (adj)	PRESS	R-sq (pred)	AICc	BIC
0.0066091	61.76%	61.71%	0.0331086	61.51%	−5432.54	−5418.69

2. Portfolio $CAPM_{OWA}$
Regression Equation

$$Portfolio\ CAPM_{OWA} = 0.000403 + 0.7862R_M$$

Coefficients

Term	Coef	SE Coef	95% CI	T-Value	*p*-Value	VIF
Constant	0.000403	0.000209	(−0.000007, 0.000814)	1.93	0.054	
R_M	0.7862	0.0220	(0.7430, 0.8295)	35.68	0.000	1.00

Model Summary

S	R-sq	R-sq (adj)	PRESS	R-sq (pred)	AICc	BIC
0.0057404	62.83%	62.78%	0.0250087	62.54%	−5645.31	−5631.47

3. Portfolio $CAPM_{IOWA}$ and $CAPM_{Bon-IOWA}$
Regression Equation

$$Portfolio\ CAPM_{IOWA} \& CAPM_{Bon-IOWA} = 0.000413 + 0.8287R_M$$

Coefficients

Term	Coef	SE Coef	95% CI	T-Value	*p*-Value	VIF
Constant	0.000413	0.000233	(−0.000045, 0.000871)	1.77	0.077	
R_M	0.8287	0.0246	(0.7804, 0.8770)	33.70	0.000	1.00

Model Summary

S	R-sq	R-sq (adj)	PRESS	R-sq (pred)	AICc	BIC
0.0064054	60.13%	60.08%	0.0311633	59.78%	−5479.79	−5465.94

4. Portfolio $CAPM_{Bon-OWA}$
Regression Equation

$$Portfolio\ CAPM_{Bon-OWA} = 0.000346 + 0.7706R_M$$

Coefficients

Term	Coef	SE Coef	95% CI	T-Value	*p*-Value	VIF
Constant	0.000346	0.000191	(−0.000029, 0.000722)	1.81	0.070	
RM	0.7706	0.0202	(0.7311, 0.8102)	38.24	0.000	1.00

Model Summary

S	R-sq	R-sq (adj)	PRESS	R-sq (pred)	AICc	BIC
0.0052489	66.01%	65.97%	0.0209027	65.75%	−5780.48	−5766.63

References

1. BMV Grupo BMV Acerca De. Available online: https://www.bmv.com.mx/es/grupo-bmv/acerca-de (accessed on 21 May 2024).
2. Buchner, A.; Mohamed, A.; Schwienbacher, A. Diversification, Risk, and Returns in Venture Capital. *J. Bus. Ventur.* **2017**, *32*, 519–535. [CrossRef]
3. Geiger, M.A. Investor Views of Audit Assurance: Recent Evidence of the Expectation Gap. *J. Account.* **1994**, *18*, 60–66.
4. Xu, N.; Jiang, X.; Chan, K.C.; Yi, Z. Analyst Coverage, Optimism, and Stock Price Crash Risk: Evidence from China. *Pac.-Basin Financ. J.* **2013**, *25*, 217–239. [CrossRef]

5. Hunjra, A.I.; Ijaz, M.; Chani, D.M.I.; Mustafa, U. Impact of Dividend Policy, Earning per Share, Return on Equity, Profit after Tax on Stock Prices. *Int. J. Econ. Empir. Res.* **2014**, *2*, 109–115.
6. Masum, A. Dividend Policy and Its Impact on Stock Price–A Study on Commercial Banks Listed in Dhaka Stock Exchange. *Glob. Discl. Econ. Bus.* **2014**, *3*, 7–16. [CrossRef]
7. Dechow, P.M.; Hutton, A.P.; Sloan, R.G. The Relation between Analysts' Forecasts of Long-term Earnings Growth and Stock Price Performance Following Equity Offerings. *Contemp. Account. Res.* **2000**, *17*, 1–32. [CrossRef]
8. Zhao, H. Dynamic Relationship between Exchange Rate and Stock Price: Evidence from China. *Res. Int. Bus. Financ.* **2010**, *24*, 103–112. [CrossRef]
9. Alam, M.M.; Uddin, G. Relationship between Interest Rate and Stock Price: Empirical Evidence from Developed and Developing Countries. *Int. J. Bus. Manag.* **2009**, *4*, 43–51. [CrossRef]
10. Chatrath, A.; Song, F.; Adrangi, B. Futures Trading Activity and Stock Price Volatility: Some Extensions. *Appl. Financ. Econ.* **2003**, *13*, 655–664. [CrossRef]
11. Kabir, G.; Sadiq, R.; Tesfamariam, S. A Review of Multi-Criteria Decision-Making Methods for Infrastructure Management. *Struct. Infrastruct. Eng.* **2014**, *10*, 1176–1210. [CrossRef]
12. Blanco-Mesa, F.; Merigó, J.M.; Gil-Lafuente, A.M. Fuzzy Decision Making: A Bibliometric-Based Review. *J. Intell. Fuzzy Syst.* **2017**, *32*, 2033–2050. [CrossRef]
13. Kacprzyk, J.; Yager, R.R.; Merigo, J.M. Towards Human-Centric Aggregation via Ordered Weighted Aggregation Operators and Linguistic Data Summaries: A New Perspective on Zadeh's Inspirations. *IEEE Comput. Intell. Mag.* **2019**, *14*, 16–30. [CrossRef]
14. Baez-Palencia, D.; Olazabal-Lugo, M.; Romero-Muñoz, J. Toma de Decisiones Empresariales a Través de La Media Ordenada Ponderada. *Inquietud Empres.* **2019**, *19*, 11–23. [CrossRef]
15. Yager, R.R. On Ordered Weighted Averaging Aggregation Operators in Multicriteria Decisionmaking. *IEEE Trans. Syst. Man Cybern.* **1988**, *18*, 183–190. [CrossRef]
16. Chang, S.-L.; Wang, R.-C.; Wang, S.-Y. Applying Fuzzy Linguistic Quantifier to Select Supply Chain Partners at Different Phases of Product Life Cycle. *Int. J. Prod. Econ.* **2006**, *100*, 348–359. [CrossRef]
17. Shouzhen, Z.; Su, C. Extended VIKOR Method Based on Induced Aggregation Operators for Intuitionistic Fuzzy Financial Decision Making. *Econ. Comput. Econ. Cybern. Stud. Res.* **2015**, *49*, 183–191.
18. León-Castro, E.; Avilés-Ochoa, E.; Gil Lafuente, A.M. Exchange Rate USD/MXN Forecast through Econometric Models, Time Series and HOWMA Operators. *Econ. Comput. Econ. Cybern. Stud. Res.* **2016**, *50*, 135–150.
19. Espinoza-Audelo, L.F.; Leon-Castro, E.; Mellado-Cid, C.; Merigo, J.M.; Blanco-Mesa, F. Uncertain Induced Prioritized Aggregation Operators in the Analysis of the Imports and Exports. *J. Mult.-Valued Log. Soft Comput.* **2021**, *36*, 543–568.
20. Merigo, J.M.; Guillen, M.; Sarabia, J.M. The Ordered Weighted Average in the Variance and the Covariance. *Int. J. Intell. Syst.* **2015**, *30*, 985–1005. [CrossRef]
21. Chen, Y.; Li, K.W.; Liu, S. An OWA-TOPSIS Method for Multiple Criteria Decision Analysis. *Expert Syst. Appl.* **2011**, *38*, 5205–5211. [CrossRef]
22. Gil-Lafuente, A.M. *Fuzzy Logic in Financial Analysis*; Springer: Berlin/Heidelberg, Germany, 2005; Volume 175.
23. Ahn, H.; Bae, K.; Chan, K. Limit Orders, Depth, and Volatility: Evidence from the Stock Exchange of Hong Kong. *J. Financ.* **2001**, *56*, 767–788. [CrossRef]
24. Yager, R.R. Induced Aggregation Operators. *Fuzzy Sets Syst.* **2003**, *137*, 59–69. [CrossRef]
25. Bonferroni, C. Sulle Medie Multiple Di Potenze. *Boll. dell'Unione Mat. Ital.* **1950**, *5*, 267–270.
26. Blanco-Mesa, F.; León-Castro, E.; Merigó, J.M. Covariances with OWA Operators and Bonferroni Means. *Soft Comput.* **2020**, *24*, 14999–15014. [CrossRef]
27. Blanco-Mesa, F.; León-Castro, E.; Merigó, J.M.; Herrera-Viedma, E. Variances with Bonferroni Means and Ordered Weighted Averages. *Int. J. Intell. Syst.* **2019**, *34*, 3020–3045. [CrossRef]
28. Baltes, L.P. Content Marketing-the Fundamental Tool of Digital Marketing. *Bull. Transilv. Univ. Brasov. Ser. V Econ. Sci.* **2015**, *8*, 111–118.
29. Yager, R.R. On the Inclusion of Variance in Decision Making under Uncertainty. *Int. J. Uncertain. Fuzziness Knowl.-Based Syst.* **1996**, *4*, 401–419. [CrossRef]
30. Jain, S.; Mitra, A.; Haidary, M.; Tiwari, A. Calculation of Beta of Private Company Using Modified CAPM Formula. *Int. J. Manag. Res. Bus. Strategy* **2017**, *6*, 115–120.
31. Alfaro-García, V.G.; Merigó, J.M.; Plata-Pérez, L.; Alfaro-Calderón, G.G.; Gil-Lafuente, A.M. Induced and Logarithmic Distances with Multi-Region Aggregation Operators. *Technol. Econ. Dev. Econ.* **2019**, *25*, 664–692. [CrossRef]
32. Juang, B.; Rabiner, L.R. A Probabilistic Distance Measure for Hidden Markov Models. *AT&T Tech. J.* **1985**, *64*, 391–408.
33. Kaufmann, A. Theory of Expertons and Fuzzy Logic. *Fuzzy Sets Syst.* **1988**, *28*, 295–304. [CrossRef]
34. Linares-Mustarós, S.; Ferrer-Comalat, J.C.; Corominas-Coll, D.; Merigó, J.M. The Ordered Weighted Average in the Theory of Expertons. *Int. J. Intell. Syst.* **2019**, *34*, 345–365. [CrossRef]
35. Beliakov, G.; Calvo, T.; James, S. On Penalty-Based Aggregation Functions and Consensus. In *Consensual Processes*; Springer: Cham, Switzerland, 2011; pp. 23–40.
36. Beliakov, G.; James, S. A Penalty-Based Aggregation Operator for Non-Convex Intervals. *Knowl.-Based Syst.* **2014**, *70*, 335–344. [CrossRef]
37. Espinoza-Audelo, L.; Aviles-Ochoa, E.; Leon-Castro, E.; Blanco-Mesa, F. Forecasting Performance of Exchange Rate Models with Heavy Moving Average Operators. *Fuzzy Econ. Rev.* **2019**, *24*, 3–21. [CrossRef]

38. Yager, R.R. Heavy OWA Operators. *Fuzzy Optim. Decis. Mak.* **2002**, *1*, 379–397. [CrossRef]
39. Perez-Arellano, L.A.; Blanco-Mesa, F.; Leon-Castro, E.; Alfaro-Garcia, V. Bonferroni Prioritized Aggregation Operators Applied to Government Transparency. *Mathematics* **2021**, *9*, 24. [CrossRef]
40. Yager, R.R. Modeling Prioritized Multicriteria Decision Making. *IEEE Trans. Syst. Man Cybern. Part B* **2004**, *34*, 2396–2404. [CrossRef]
41. Tian, Z.; Wang, J.; Zhang, H.; Chen, X.; Wang, J. Simplified Neutrosophic Linguistic Normalized Weighted Bonferroni Mean Operator and Its Application to Multi-Criteria Decision-Making Problems. *Filomat* **2016**, *30*, 3339–3360. [CrossRef]
42. Liu, P.; Wang, Y. Multiple Attribute Decision-Making Method Based on Single-Valued Neutrosophic Normalized Weighted Bonferroni Mean. *Neural Comput. Appl.* **2014**, *25*, 2001–2010. [CrossRef]
43. Zhou, J.; Baležentis, T.; Streimikiene, D. Normalized Weighted Bonferroni Harmonic Mean-Based Intuitionistic Fuzzy Operators and Their Application to the Sustainable Selection of Search and Rescue Robots. *Symmetry* **2019**, *11*, 218. [CrossRef]
44. Awang, A.; Aizam, N.A.H.; Ab Ghani, A.T.; Othman, M.; Abdullah, L. A Normalized Weighted Bonferroni Mean Aggregation Operator Considering Shapley Fuzzy Measure under Interval-Valued Neutrosophic Environment for Decision-Making. *Int. J. Fuzzy Syst.* **2020**, *22*, 321–336. [CrossRef]
45. Yang, L.; Li, B. Multiple-Valued Neutrosophic Uncertain Linguistic Sets with Dombi Normalized Weighted Bonferroni Mean Operator and Their Applications in Multiple Attribute Decision Making Problem. *IEEE Access* **2020**, *8*, 5906–5927. [CrossRef]