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Editorial

Computational Algebra, Coding Theory, and Cryptography: Theory and Applications

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1. Introduction

The primary aim of this Special Issue is to explore innovative encoding and decoding procedures that leverage various algebraic structures to enhance error-control coding techniques. By examining the application of algebraic structures in error-correction codes, this Special Issue highlights the development of new algorithms designed to improve both the error-correction capacity and the efficiency of encoding and decoding processes. The algebraic structures covered include commutative algebras, computational algebras, ordered algebras, and hypercompositional algebras, with an emphasis on combinatorial aspects drawn from lattice theory, category theory, graph theory, and mathematical modeling.

This Special Issue contains 10 papers published in the journal *Axioms*. These papers explore various aspects of algebraic structures in the context of error-control coding, cryptography, and related fields. They present new theoretical developments and practical applications aimed at improving encoding and decoding processes. We hope that this issue will inspire further research and innovation at the intersections of algebraic structures, coding theory, and cryptography.

2. Overview of the Published Papers

Contribution 1 introduces Ω-ideals in Ω-algebras, linking them to Ω-congruences and Ω-homomorphisms, while exploring equation-solving in Ω-rings and Ω-fields. Ω-algebras, defined by lattice-valued Ω-equality, fulfill identities as lattice formulas.

Contribution 2 presents relative versions of Brauer's, Robinson's, and Olsson's conjectures on finite group direct products, proving that the anchor group of an irreducible character of a finite simple group with an odd prime degree is trivial.

Contribution 3 characterizes Sheffer stroke and Hilbert algebras using ideals and stabilizers, investigating their properties and minimal ideals, and defining stabilizers within these algebras, with illustrative examples provided.

Contribution 4 proves that sequences defined by prime and composite number conditions are bounded and periodic under certain conditions on the set K and the real number τ.

Contribution 5 classifies four-dimensional 3-Hom–Lie algebras with a nilpotent twisting map, analyzing their solvability and nilpotency and providing a classification up to Hom algebra isomorphism.

Contribution 6 explores coding results over Frobenius local rings, focusing on linear codes over $\mathbb{Z}_{p^4}[u]/(u^2 - p^3\beta, pu)$, and examining generator matrices and MacWilliams relations in error-correction.

Contribution 7 studies p-numerical semigroups of triples (W_i, W_{i+2}, W_{i+k}), defining the p-Frobenius number and p-genus, and exploring the structure of these semigroups.

Investigating the relationship between ring commutativity and multiplicative generalized derivations, Contribution 8 provides insights into semiprime ideals and their structural implications for rings.

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Contribution 9 presents an algebraic and geometric technique for constructing topological quantum codes using quotient lattices and geometric projections, including the introduction of new surface and color codes.

Contribution 10 introduces two-term differential $Leib_\infty$-conformal algebras, classifies certain subclasses, and explores non-Abelian extensions and the inducibility of automorphisms, deriving Wells exact sequences for differential Leibniz conformal algebras.

Conflicts of Interest: The author declares no conflicts of interest.

List of Contributions

1. Jimenez, J.; Serrano, M.L.; Šešelja, B.; Tepavčević, A. Omega Ideals in Omega Rings and Systems of Linear Equations over Omega Fields. *Axioms* **2023**, *12*, 757. https://doi.org/10.3390/axioms12080757.
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4. Dubickas, A. A Class of Bounded Iterative Sequences of Integers. *Axioms* **2024**, *13*, 107. https://doi.org/10.3390/axioms13020107.
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10. Hummdi, A.Y.; Sögütcü, E.K.; Gölbaşı, Ö.; Rehman, N.u. Some Identities Related to Semiprime Ideal of Rings with Multiplicative Generalized Derivations. *Axioms* **2024**, *13*, 669. https://doi.org/10.3390/axioms13100669.