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Fuzzy-Set-Based Multi-Attribute Decision-Making, Its Computing Implementation, and Applications

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Abstract: This paper reflects the results of research analyzing models of multi-attribute decision-making based on fuzzy preference relations. Questions of constructing the corresponding multi-attribute models to deal with quantitative information concomitantly with qualitative information based on experts' knowledge are considered. Human preferences may be represented within the fuzzy preference relations and by applying diverse other preference formats. Considering this, so-called transformation functions reduce any preference format to fuzzy preference relations. This paper's results can be applied independently or as part of a general approach to solving a wide class of problems with fuzzy coefficients, as well as within the framework of a general scheme of multi-criteria decision-making under conditions of uncertainty. The considered techniques for fuzzy preference modeling are directed at assessing, comparing, choosing, prioritizing, and/or ordering alternatives. These techniques have served to develop a computing system for multi-attribute decision-making. It has been implemented in the C# programming language, utilizing the ".NET" framework. The computing system allows one to represent decision-makers' preferences in one of five preference formats. These formats and quantitative estimates are reduced to nonreciprocal fuzzy preference relations, providing homogeneous preference information for decision procedures. This paper's results have a general character and were applied to analyze power engineering problems.

Keywords: multi-attribute decision-making; fuzzy sets; fuzzy preference relations; transformation functions; computing system; power engineering problems

MSC: 03B52; 68U35; 68T37; 90C70

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1. Introduction

Diverse types of uncertainty are often encountered in a wide range of problems related to the design, planning, operation, and control of complex systems [1,2]. Taking into account the uncertainty factor in constructing mathematical models is a means to increase their adequacy and, as a result, the credibility and factual efficiency of decisions based on their analysis [3–5].

The internal (subjective) uncertainties associated with qualitative information [6–8] and external (objective) uncertainties associated with quantitative information [9–11] are the most explored in the literature. There are still uncertainties associated with selecting experts [12,13] and other areas, such as statistics [14]. Finally, but even less explored in the literature, there is the uncertainty associated with objectives [15–18].

The uncertainty of goals is an important kind of uncertainty related to the multi-criteria character of many optimization problems [1]. From a general point of view, researchers in the operations research, decision-making, and systems analysis fields agree that the uncertainty of goals is the most challenging to overcome [1,19]. This challenge is associated with the decision-makers' difficulty in establishing what they want [20].

In short, this type of uncertainty occurs when the objectives associated with the attributes of the decision-making problem have opposite directions; for example, when the decision (e.g., what type of alternative energy source to deploy) involves a minimization (e.g., environmental risk) and a maximization (e.g., power-plant lifetime) simultaneously. In these situations, the decision-maker faces the dilemma: Choosing the alternative with lower environmental risk, or choosing the alternative with a higher power-plant lifetime?

Therefore, considering the uncertainty of goals can be understood as the search for a solution that generates a compromise between the requirements of the attributes related to the decision-making problem. This is a considerable challenge because, in real-world problems, the uncertainty of goals cannot be effectively captured solely based on applying formal models, as decision-makers' knowledge, experience, and intuition are often the only sources of information for decision-making [18,21].

Considering the above, the general objective of this research is to improve and develop the techniques of multi-attribute decision-making for assessing, comparing, choosing, prioritizing, and/or ordering alternatives, considering the uncertainty of objectives. This research also includes developing a system of multi-criteria decision-making to assist decision-makers from different areas in solving decision problems in complex scenarios.

The system offers an important advance over existing systems, for two reasons. Existing systems ignore objective uncertainty [22,23], do not simultaneously deal with quantitative and qualitative information [24,25], and do not offer flexibility during the process evaluation of criteria and alternatives [26,27]. In particular, the process of evaluating criteria and alternatives in existing multi-criteria decision-making systems is carried out through a unique format, mostly through paired comparison [28,29].

The analysis of the applicability and universality of the system is demonstrated through the problem of choosing the best alternative energy source to be implemented in a mine far from the public energy network. The results obtained in the research offer contributions with a high degree of applicability and universality. In particular, these results can be applied independently, or as part of a general approach to solving a wide class of problems with fuzzy coefficients [30] within the framework of a general scheme of multi-criteria decision-making under conditions of uncertainty [31], as well as for improving models and methods for decision-making in conditions of uncertainty [32–36].

Among the possible examples of the application of the multi-criteria decision-making system are the choice of renewable energy portfolios [37], the selection of the most profitable investments [38], the selection of the most beneficial agricultural product suppliers [39], optimizing solid waste management [40], prioritizing infection prevention [41], estimating flood disaster risk [42], and others [43,44].

The remainder of this article is organized as follows: Section 2 discusses the literature on the multi-criteria approach, highlighting the fuzzy preference modeling, multi-attribute models, preference formats, and format transformation functions. Section 3 details the multi-criteria decision-making techniques and their applications within the nonreciprocal fuzzy preference relations framework. The computing implementation of the multi-attribute decision-making system is presented in Section 4 and includes information about the functionalities incorporated in the system. The application of the system to an energy source choice problem is presented in Section 5. Finally, Section 6 presents the final considerations of the research, including limitations and lines of future investigation.

2. Multi-Criteria Approach: Multi-Attribute Problems

It is possible to identify two main situations requiring a multi-criteria approach [45]: The first class is associated with problems whose solution consequences cannot be estimated using a single criterion. These problems are associated with analyzing models that include economic and physical indices (when alternatives cannot be reduced to a comparable form) and the need to consider indices whose cost estimations are difficult or impossible [30]. The second class is related to problems that may be solved based on a single criterion or several criteria. However, it is possible to reduce these problems to multi-criteria decision-making when the information uncertainty does not permit obtaining a unique solution. The use of additional criteria, including criteria of qualitative character, applied to alternatives, which cannot be distinguished based on the initial criteria, can serve as a convincing means to contract the decision uncertainty regions [46].

Considering this, it is necessary to distinguish criteria that can be objectives and attributes. In such a manner, multi-criteria decision-making problems can be classified into multi-objective decision-making and multi-attribute decision-making.

On the one hand, multi-objective decision-making is recognized as a continuous type of multi-criteria decision-making [47,48]. Its main characteristics are that the decision-maker needs to achieve multiple objectives while these objectives are non-commensurable and conflict with one another [49]. Multi-objective decision models include a vector of decision variables that can be continuous and discrete, and objective functions that describe the objectives and constraints. The decision-maker attempts to maximize or minimize the objective functions.

On the other hand, multi-attribute decision-making is associated with making preference decisions. In short, it involves comparing, choosing, prioritizing, and/or ordering the available alternatives, characterized by multiple, usually conflicting attributes [50]. The primary particularity of multi-attribute problems is that there is generally a limited number of predetermined alternatives, which are associated with a level of achieving the attributes by which the decision is made.

These multi-objective and multi-attribute models are also known as $<X, F>$ and $<X, R>$ models, with only the latter being explored in this research. Although $<X, R>$ models are applied for comparing, choosing, prioritizing, and ordering the available alternatives, their combination with $<X, F>$ models allows for solving decision-making problems under conditions of uncertainty, through the generalization of the classical approach to the decision-making process [51]. In this case, qualitative information obtained from the expert's knowledge, experience, and intuition obtained from $<X, R>$ models is applied to reduce regions of decision uncertainty [31].

2.1. Fuzzy Preference Modeling

Fuzzy preference modeling is an approach that makes it possible to consider the subjective uncertainty of decision-makers through the processing of uncertain, imprecise, and vague preferences and information [52]. This type of preference modeling requires techniques and concepts capable of processing fuzzy data to generate fuzzy or nuanced results and allow the choice of solutions with the desired degree of confidence or compromise between reliability and discrimination [53].

Conventional approach techniques first dissolve imprecision and then process non-fuzzy data. Fuzzy approach techniques first process fuzzy data and then dissolve imprecision, generating multiple sets of results [53]. Within this last approach, several structures are found in the literature: fuzzy preference relation, intuitionistic preference relation, fuzzy subset, fuzzy hesitation, fuzzy interval, and fuzzy estimate [54–56]. The development of this research is specifically based on the fuzzy preference relations model, which can take the following forms [57,58]:

- Reciprocal fuzzy preference relation is a fuzzy preference relation that satisfies the property of additive reciprocity (see expression in [59]).
- Nonreciprocal fuzzy preference relations are related to the notion of non-strict fuzzy preference relations associated with fuzzy preference structures [60].

Adopting this modeling in decision-making problems is especially advantageous for dealing with the uncertainty and imprecision inherent in decision-makers' assessment, comparison, prioritization, or ordering of alternatives. This advantage can be seen in constructing $<X, R>$ models.

2.2. <X, R> Models and Their Construction

Let us assume a set of X alternatives coming from the decision uncertainty region or predetermined alternatives, which are to be examined by q criteria of a quantitative and/or qualitative nature. The problem of decision-making may be presented as a pair $<X, R>$, where $R = \{R_1, R_2, \ldots, R_p, \ldots, R_q\}$ is a vector of fuzzy preference relations [61], which can be presented as

$$R_p = [X \times X, \mu_{R_p}(X_k, X_l)],\ p = 1, 2, \ldots, q, X_k, X_l \in X, \tag{1}$$

where $\mu_{R_p}(X_k, X_l)$ is a membership function of the p-th fuzzy preference relation.

R_p is defined as a fuzzy set of all pairs of the Cartesian product $X \times X$, such that the membership function $\mu_{R_p}(X_k, X_l)$ represents the degree to which X_k weakly dominates X_l (for example, the degree to which X_k is not worse than X_l for the p-th criterion). In a somewhat loose sense, $\mu_{R_p}(X_k, X_l)$ also represents the degree of truth of the statement; for instance, "X_k" is preferred over X_l.

A natural and convincing approach to building nonreciprocal fuzzy preference relations R_p is based on the consideration of $F(X_k)$ and $F(X_l)$ as fuzzy sets reflecting assessments of the attribute F for alternatives X_k and X_l [30]. According to Orlovsky [62], the quantity $\eta\{\mu[F(X_k)], \mu[F(X_l)]\}$ corresponds to the preference degree $\mu[F(X_k)] \succcurlyeq \mu[F(X_l)]$, while $\eta\{\mu[F(X_l)], \mu[F(X_k)]\}$ is the preference degree $\mu[F(X_l)] \succcurlyeq \mu[F(X_k)]$. Then, the membership functions of the generalized preference relations $\eta\{\mu[F(X_k)], \mu[F(X_l)]\}$ and $\eta\{\mu[F(X_l)], \mu[F(X_k)]\}$ are formed by the following correlations:

$$\eta\{\mu[F(X_k)], \mu[F(X_l)]\} = \sup_{F(X_k), F(X_l) \in F} \min\{\mu[F(X_k)], \mu[F(X_l)], \mu_R[F(X_k), F(X_l)]\} \tag{2}$$

$$\eta\{\mu[F(X_l)], \mu[F(X_k)]\} = \sup_{F(X_k), F(X_l) \in F} \min\{\mu[F(X_l)], \mu[F(X_k)], \mu_R[F(X_l), F(X_k)]\} \tag{3}$$

where $\mu_R[F(X_k), F(X_l)]$ and $\mu_R[F(X_l), F(X_k)]$ are the membership functions of the corresponding fuzzy preference relations reflecting the essence of the preferences of X_k over X_l and of X_l over X_k—for instance, "more valuable", "more suitable", etc.

When F can be measured on a numerical scale and the essence of preference behind relation R is coherent with the natural order ($\leq$) along the axis of measured values of F, (2) and (3) are reduced to the following correlations:

$$\eta\{\mu[F(X_k)], \mu[F(X_l)]\} = \sup_{\substack{F(X_k), F(X_l) \in F \\ F(X_k) \leq F(X_l)}} \min\{\mu[F(X_k)], \mu[F(X_l)]\}, \tag{4}$$

$$\eta\{\mu[F(X_k)], \mu[F(X_l)]\} = \sup_{\substack{F(X_k), F(X_l) \in F \\ F(X_l) \leq F(X_k)}} \min\{\mu[F(X_k)], \mu[F(X_l)]\} \tag{5}$$

If F has a maximization character, Correlations (4) and (5) must be written for $F(X_k) \geq F(X_l)$ and $F(X_l) \geq F(X_k)$, respectively.

Correlations (4) and (5) are in harmony with some well-known fuzzy number ranking indices [61,63]. However, it is appropriate to highlight that there are cases when the fuzzy quantities $F(X_k)$ and $F(X_l)$ have trapezoidal membership functions [64]. In these cases, they can be located in such a manner that it is not possible to distinguish X_k and X_l, as shown by the alternatives X_1 and X_2 in Figure 1.

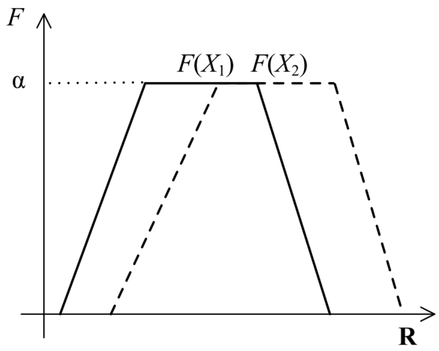

Figure 1. Comparison of alternatives with trapezoidal membership functions.

This impossibility of distinguishing alternative X_1 from alternative X_2 occurs because

$$\eta\{\mu[F(X_1)],\mu[F(X_2)]\} = \eta\{\mu[F(X_2)],\ \mu[F(X_1)]\} = \alpha \tag{6}$$

When building matrices R_p, the availability of fuzzy or linguistic estimates of alternatives $F_p(X_k)$, $p = 1, 2, ..., q, X_k \in X$ with the membership functions $\mu[F_p(X_k)]$, $p = 1, 2, ..., q, X_k \in X$ allows one to apply Correlations (4) and (5) to build R_p, p = 1, 2, . . ., q, applying the correlations presented by Ekel and Neto [65]:

$$\mu_{R_p}(X_k,\ X_l) = \sup_{\substack{X_k, X_l \in X \\ F_p(X_k) \leq F_p(X_l)}} \min\{\mu[F_p(X_k)],\ \mu[F_p(X_l)]\}, \tag{7}$$

$$\mu_{R_p}(X_k, X_l) = \sup_{\substack{X_k, X_l \in X \\ F_p(X_l) \leq F_p(X_k)}} \min\{\mu[F_p(X_k)],\ \mu[F_p(X_l)]\} \tag{8}$$

Other approaches to building R_p matrices are presented by Parreiras and Ekel [59] and Kokshenev et al. [66]. If estimates $F_p(X_k)$, p =1, 2,..., q, $X_k \in X$ are defined on a unit value scale, these approaches allow one to obtain $\mu_{R_p}(X_k, X_l)$, p =1, 2,..., q, X_k, $X_l \in X$ as follows:

$$\mu_{R_p}(X_k, X_l) = 1 - \delta_p(X_l,\ X_k), \tag{9}$$

where $\delta_p(X_l, X_k)$ corresponds to the volume of all positive differences between the worst outcomes of $F_p(X_l)$ and the best outcomes of $F_p(X_k)$.

The results presented by Kokshenev et al. [66] are appropriate for dealing with fuzzy and crisp estimates in the same domain while preserving the preference measures on an interval scale. Although <*X*, *R*> models are associated with constructing and analyzing fuzzy preference relations, many other preference formats can be employed in assessing, comparing, and/or ordering criteria or alternatives.

2.3. Preference Formats

For instance, the following formats are commonly used to establish alternative preferences: nonreciprocal fuzzy preference relations, additive reciprocal fuzzy preference relations, ordering of alternatives, utility values, fuzzy estimates, and multiplicative preference relations [67]. This list covers all practical situations of preference expression [60]. However, it is necessary to indicate that the number of existing preference formats is extensive [68]. For example, decision-makers can express their preferences by ratio bounds, intervals, or selected subsets and evaluate alternatives or criteria using values, order, fuzzy preference relations, or multiplicative preference relations [69,70].

The flexibility of using different preference expression formats has advantages and disadvantages for the assessment process [71]. On the one hand, experts can choose the most comfortable format to assess alternatives, reducing the cognitive stress of the assessment process. On the other hand, the heterogeneity of assessments carried out in different preference expression formats prevents information aggregation.

At this point, it is necessary to homogenize the assessments carried out in the different preference expression formats to take advantage of offering greater psychological comfort to specialists during the assessment process. In particular, Herrera-Viedma et al. [72,73] and Chiclana et al. [74,75] presented transformation functions that convert the different preference expression formats to other formats.

Therefore, we did not narrow the formulation of the problems of assessing, comparing, choosing, prioritizing, and/or ordering alternatives based merely on nonreciprocal fuzzy preference relations, since another format of representation of human preferences can be reduced into it by applying the so-called transformation functions [64].

2.4. Transformation Functions

In the multi-attribute decision-making system, transformation functions convert any preference format to nonreciprocal fuzzy preference relations. Applying these functions homogenizes the quantitative and qualitative information for applying decision-making techniques in a fuzzy environment.

However, it is valuable to point out that it is not always possible to perform direct conversion between preferred formats [20,60]. In the present case, it is necessary to apply transformation functions to convert the quantitative information into additive reciprocal fuzzy preference relations before converting them to nonreciprocal fuzzy preference relations. In particular, additive reciprocal fuzzy preference relations can be constructed by applying the following correlations:

$$\overline{\mu}_{R_p}(X_k, X_l) = \frac{F_p(X_k) - F_p(X_l)}{2\left[maxF_p(X) - minF_p(X)\right]} + 0.5 \tag{10}$$

$$\overline{\mu}_{R_p}(X_k, X_l) = \frac{F_p(X_l) - F_p(X_k)}{2\left[maxF_p(X) - minF_p(X)\right]} + 0.5 \tag{11}$$

where $F_p(X)$ is the objective function to be maximized (10) or minimized (11), and X_k and X_l correspond to the deterministic values for the alternatives k and l, respectively.

Then, all quantitative information converted into the additive reciprocal fuzzy preference relations format can be converted to nonreciprocal fuzzy preference relations by applying the following transformation function [20]:

$$\mu_{\mathrm{R_p}}(X_k, X_l) = \begin{cases} 1 + \overline{\mu}_{\mathrm{R_p}}(X_k, X_l) - \overline{\mu}_{\mathrm{R_p}}(X_l, X_k) & \text{if } \overline{\mu}_{\mathrm{R_p}}(X_k, X_l) < 0.5 \\ 1 & \text{if } \overline{\mu}_{\mathrm{R_p}}(X_k, X_l) \geq 0.5 \end{cases} \tag{12}$$

In addition to nonreciprocal fuzzy preference relations, three preference formats were considered in the system to represent qualitative information: order of alternatives, utility values, and multiplicative preference relations. These preference formats can be converted directly, not having to be converted into additive reciprocal fuzzy preference relations. In particular, the following transformation function can be applied to convert the vector of ordered alternatives to nonreciprocal fuzzy preference relations [64]:

$$\mu_{\mathrm{R_p}}(x_k, x_l) = \begin{cases} \frac{1}{2} + \frac{\mathrm{OA}(x_l) - \mathrm{OA}(x_k)}{2(n-1)} & \text{if } \mathrm{OA}(x_k) > \mathrm{OA}(x_l) \\ 1 & \text{if } \mathrm{OA}(x_k) \leq \mathrm{OA}(x_l) \end{cases} \tag{13}$$

where $\mathrm{OA}(x_l)$ and $\mathrm{OA}(x_k)$ correspond to the position of the alternatives k and l, respectively, in the importance vector of the alternatives related to the criterion X.

To convert the vector of values associated with each of the alternatives, that is, the homogenization of qualitative information in the utility values format to the nonreciprocal fuzzy preference relations format, the following transformation function is applied [64]:

$$\mu_{\mathrm{R_p}}(x_k, x_l) = \begin{cases} \frac{U_k}{U_l} & \text{if } U_k < U_l \\ 1 & \text{if } U_k \geq U_l \end{cases} \tag{14}$$

where U_k and U_l are the values assigned by decision-makers to the alternatives k and l, respectively.

Finally, the following transformation function is applied to convert the qualitative information in the multiplicative preference relation in nonreciprocal fuzzy preference relations:

$$\mu_{R_p}(x_k, x_l) = \begin{cases} 1 + \frac{1}{2}\log_m \frac{M(x_k,x_l)}{M(x_k,x_l)} & \text{if } \log_m M(x_k, x_l) < 0 \\ 1 & \text{if } \log_m M(x_k, x_l) \geq 0 \end{cases} \quad (15)$$

where m is the upper limit of a scale used in the analytic hierarchy and network processes [76], while $M(x_k, x_l)$ is a preference relation that reflects how much x_k is preferable to x_l.

In short, the application of these transformation functions allows for the following:

- Different formats are converted into a unique, single, and comparable format;
- Decision-makers choose the preferred format that they feel most comfortable with, offering psychological comfort in the evaluation process;
- Quantitative and qualitative information can be used concomitantly in the decision process through homogenization in the nonreciprocal fuzzy preference relations format.

Once homogenized, the quantitative and qualitative information in the nonreciprocal fuzzy preference relations format can be processed by different decision-making techniques, as shown in Section 3.

3. Multi-Criteria Decision-Making Techniques and Their Applications

Considering the advantages and rationality of applying nonreciprocal fuzzy preference relations, the situation of setting up a single fuzzy non-strict preference relation R can be processed to build a fuzzy strict preference relation as follows [62]:

$$R^S = R \backslash R^{-1} \quad (16)$$

where R^{-1} is the inverse relation.

The membership function corresponding to Correlation (16) is the following:

$$\mu_R^S(X_k, X_l) = \max\{\mu_R(X_k, X_l) - \mu_R(X_l, X_k), 0\} \quad (17)$$

This serves as the basis for the choice procedures, and its properties and questions of its axiomatic characterization are discussed by Banerjee [77].

The utilization of Correlation (17) allows one to build a set of non-dominated alternatives with the following membership function:

$$\mu_R^{ND}(X_k) = \inf_{X_l \in X}[1 - \mu_R^S(X_l, X_k)] = 1 - \sup_{X_l \in X} \mu_R^S(X_l, X_k), \quad (18)$$

Note that (18) allows for the assessment of the level of non-dominance of each alternative X_k. Considering that it is natural to choose alternatives providing the highest level of non-dominance, one can choose alternatives X^{ND} as follows:

$$X^{ND} = \{X_k^{ND} | X_k^{ND} \in X, \mu_R^{ND}(X_k^{ND}) = \sup_{X_k \in X} \mu_R^{ND}(X_k)\}. \quad (19)$$

Correlations (17)–(19) are valuable in solving choice problems and other problems associated with assessing, comparing, prioritizing, or ordering alternatives with a single criterion. These correlations may also be applied when R is a vector of fuzzy preference relations under different approaches to the multi-attribute analysis.

3.1. First Technique

The first technique aims to construct and analyze the membership function of a subset of non-dominated alternatives while considering all criteria. In particular, when R is a vector of fuzzy preference relations, Correlations (17)–(19) can be applied as a basis for the first technique for multi-attribute decision-making in a fuzzy environment, taking $\mathrm{R} = \cap_{p=1}^{q} R_p$:

$$\mu_{\mathrm{R}}(X_k, X_l) = \min_{1\leq\ p\leq\ q} \mu_{\mathrm{R_p}}(X_k, X_l),\ X_k,\ X_l \in\ X. \tag{20}$$

When applying Correlation (20), the set X^{ND} fulfills the role of a Pareto set [62]. Its contraction is possible based on differentiating the importance of R_p, $p = 1,\ 2, \ldots, q$ with the application of aggregating mono-objective fuzzy preference relations presented in the following correlation:

$$\mu_{\mathrm{T}}(X_k, X_l) = \sum_{p=1}^{q} \lambda_{\mathrm{p}} \mu_{\mathrm{R_p}}(X_k,\ X_l), X_k, X_l \in X, \tag{21}$$

where $\lambda_{\mathrm{p}} \geq\ 0$, $p = 1,\ 2, \ldots,\ q$ are weights or importance coefficients for the corresponding criteria, normalized as follows:

$$\sum_{p=1}^{q} \lambda_{\mathrm{p}} = 1. \tag{22}$$

The construction of $\mu_{\mathrm{T}}(X_k,\ X_l), X_k, X_l\ \in\ X$ allows one to obtain the membership function $\mu_{\mathrm{T}}^{ND}(X_k)$ of the non-dominated alternatives according to a correlation like (18). The intersection of $\mu_{\mathrm{R}}^{ND}(X_k)$ and $\mu_{\mathrm{T}}^{ND}(X_k)$ is defined as

$$\mu^{ND}(X_k) = \min\{\mu_{\mathrm{R}}^{ND}(X_k), \mu_{\mathrm{T}}^{ND}(X_k)\}, X_k \in X \tag{23}$$

providing us with

$$X^{\ ND} = \{X_k^{ND}\ |\ X_k^{ND} \in X, \mu^{ND}(X_k^{ND}) = \sup_{X_k \in\ X} \mu^{ND}(X_k)\} \tag{24}$$

3.2. Second Technique

The second technique has a lexicographic character and is associated with the step-by-step introduction of criteria for comparing alternatives. Correlations (18) and (19) are the basis of the second technique, which allows one to build a sequence $X^{\ 1}, X^{\ 2}, \ldots, X^{\mathrm{q}}$ so that $X\ \supseteq X^1 \supseteq\ X^2 \supseteq \ldots\ \supseteq X^{\mathrm{q}}$ by applying the following correlations:

$$\mu_{\mathrm{R_p}}^{ND}(X_k) = \inf_{X_l \in X^{p-1}} [1 - \mu_{\mathrm{R_p}}^{\mathrm{S}}(X_l, X_k)] = 1 - \sup_{X_l \in X^{\ p-1}} \mu_{\mathrm{R_p}}^{\mathrm{S}}(X_l, X_k),\ p = 1,\ 2, \ldots, q \tag{25}$$

$$X^{\ \mathrm{p}} = \{X_k^{ND,p} | X_k^{ND,p} \in X^{\mathrm{p}\ -1}, \mu_{\mathrm{R_p}}^{ND}(X_k^{ND,p}) = \sup_{X_l \in X^{\ \mathrm{p}-1}} \mu_{\mathrm{R_p}}^{ND}(X_k)\}. \tag{26}$$

Note that if R_p is transitive [73], it is possible to bypass the pairwise comparison of alternatives at the p-th step. In these circumstances, the comparison can be performed on a serial basis by applying Correlations (7) and (8) while memorizing the best alternatives.

3.3. Third Technique

The third technique is associated with building the membership functions of a subset of non-dominated alternatives for all criteria, generating a solution from their insertion. Correlation (18) can be represented in the following form:

$$\mu_{\mathrm{R_p}}^{ND}(X_k) = 1 - \sup_{X_l \in\ X} \mu_{\mathrm{R_p}}^{\mathrm{S}}(X_l, X_k),\ p = 1,\ 2, \ldots, q \tag{27}$$

This representation allows one to construct the membership functions of the set of non-dominated alternatives for each fuzzy preference relation. The membership functions $\mu^{ND}_{R_p}(X_k)$, $p = 1,\ 2, \ldots, q$ play a role identical to membership functions replacing objective functions $F_p(X)$, $p = 1,\ 2, \ldots, q$ in solving multi-objective problems [31], based on modifying the Bellman–Zadeh approach to decision-making in a fuzzy environment [78]. Therefore, it is possible to build

$$\mu^{ND}(X_k) = \min_{1 \leq p \leq q} \mu^{ND}_{R_p}(X_k) \tag{28}$$

to obtain X^{ND}.

If necessary to differentiate the importance of different preference relations, it is possible to transform Correlation (22) into

$$\mu^{ND}(X_k) = \min_{1 \leq p \leq q} [\mu^{ND}_{R_p}(X_k)]^{\lambda_p} \tag{29}$$

Note that the application of Correlation (23) does not require the normalization of λ_p, $p = 1,\ 2, \ldots, q$ like Correlation (22).

4. Multi-Criteria Decision-Making System (MDMS2) Implementation

The computational implementation of the multi-attribute decision-making system combines four key features: First, three techniques for solving multi-criteria decision-making problems. Second, the possibility of evaluating alternatives and criteria in different formats. Third, format transformation functions that allow for homogenizing qualitative information in different formats. Fourth, transformation functions that allow the use of quantitative data associated with the decision problem concomitantly with qualitative information. An overall scheme of the system's operation is illustrated in Figure 2.

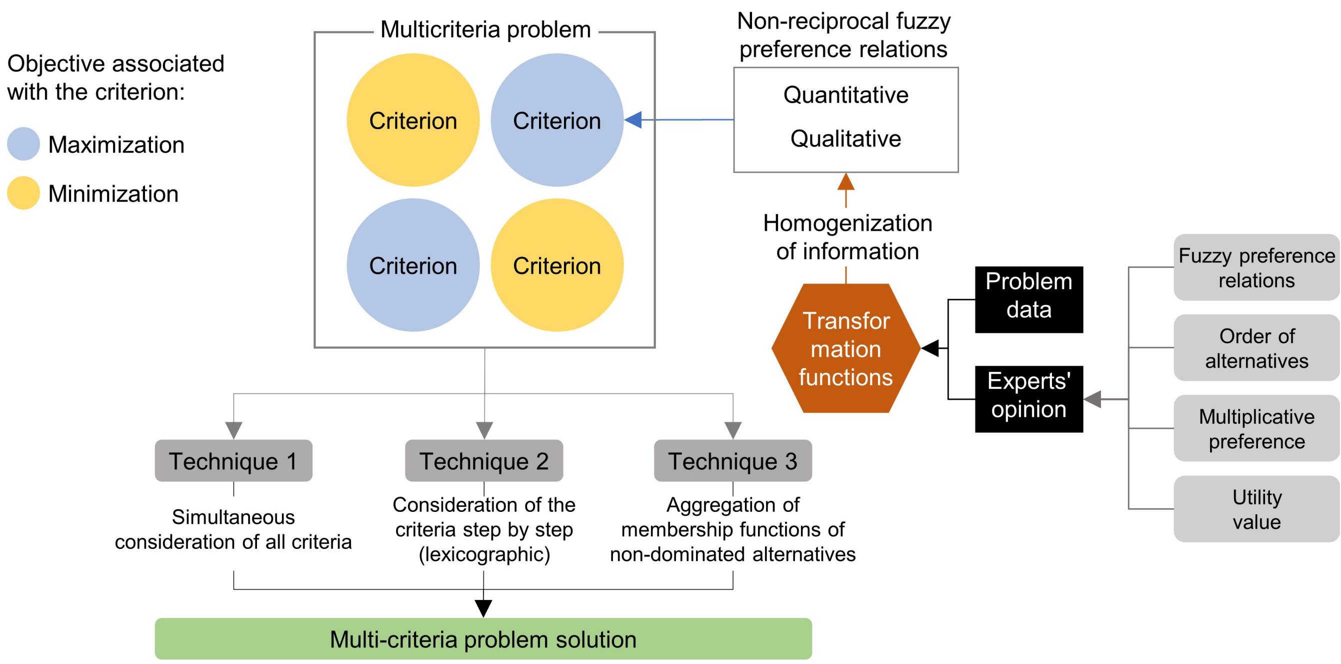

Figure 2. The overall scheme of the multi-attribute decision-making system.

The fundamental difference between the model operationalized in the multi-attribute decision-making system illustrated in Figure 2 and other systems [23,25,27,29] is the possibility of considering quantitative and qualitative information (in different formats) and conflicting objectives (maximization and minimization) for the proper consideration of the uncertainty of objectives.

In such a manner, the techniques described in Section 3 and the preference format transformation functions presented in Section 2.4 were implemented within the multi-criteria decision-making system framework. The system was developed using the C# programming language and executed in the Microsoft Windows Operating System graphical environment.

An overview of the initial screen and the key features of the multi-attribute decision-making system is illustrated in Figure 3. Analyzing the initial screen allows us to identify the two main elements of the system: technique and dimension. The first element of the system is divided into three sub-elements, which correspond to the techniques discussed in Sections 3.1–3.3. The second element of the system is divided into two sub-elements, which correspond to the criteria and alternatives of the decision-making problem. Note that the system allows for the selection of only one technique. In turn, the number of criteria and alternatives of the dimension element can assume any value greater than or equal to two.

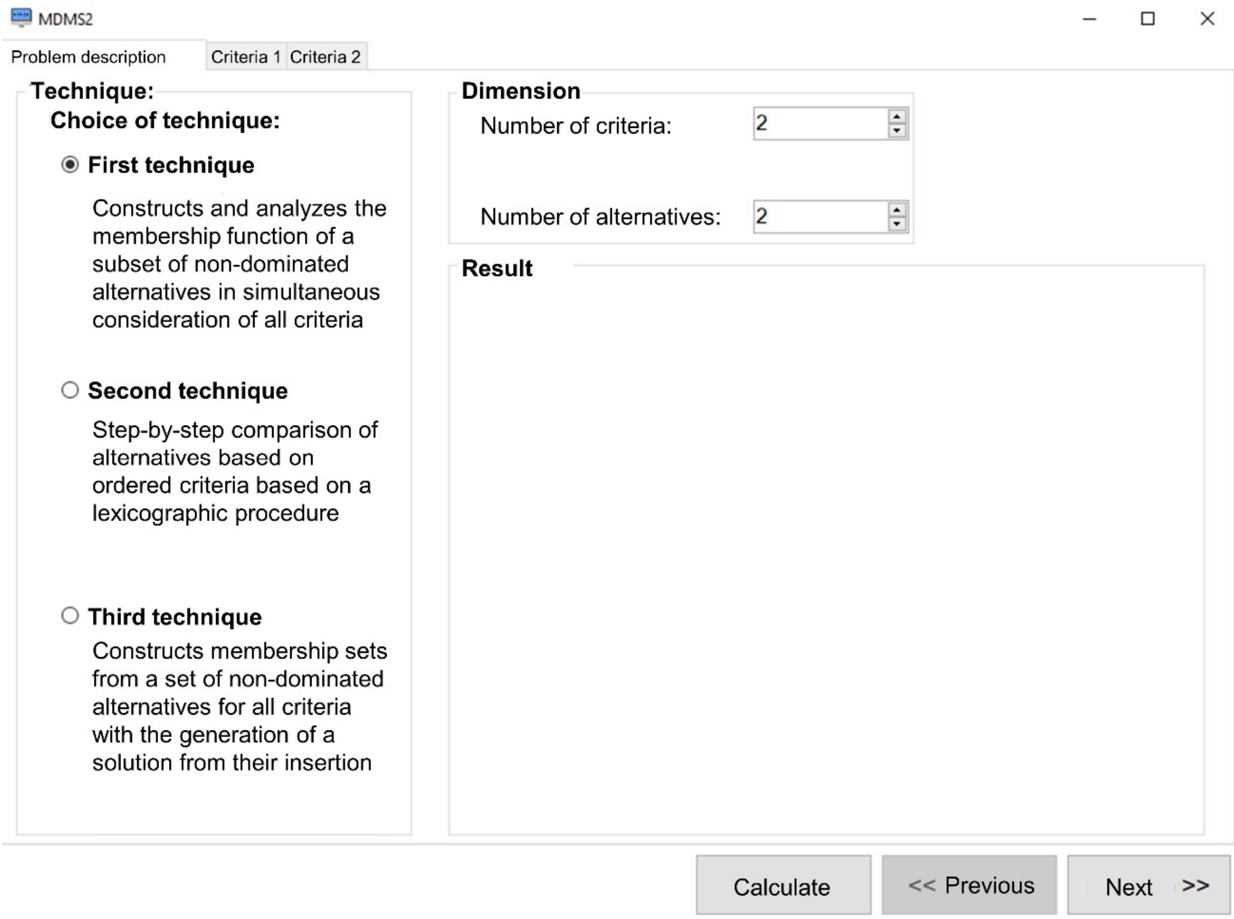

Figure 3. The initial screen of the multi-attribute decision-making system.

The first functionality presented on this screen is associated with the "problem description", in which the decision-maker must choose the technique to solve the problem. This choice determines which algorithm the system will use to solve the problem, such as the algorithm related to Technique 1 presented in Algorithm 1, Figure 4.

Algorithm 1: Calculates and generates the result of the multi-attribute problem according to the first technique.

```
public List<int> First-Technique()
    double[][] intersection = Util.Copy-Matrix(Preference-Relations[0].Relations);

1st Step:
    Perform the Intersection between all preference relations
    for (int i = 1; i < Quantity-of-Criteria; i++)
        for (int j = 0; j < intersection.Length; j++)
        for (int k = 0; k < intersection[j].Length; k++)
            if (Preference-Relations[i].Relacao[j][k] < intersection[j]k])
            intersection[j][k] = Preference-Relations[i]. Relation[j]k];;
        Console.WriteLine("\nIntersection:\n");
        Console.WriteLine(Util.Print-Matrix(intersection));

2nd Step:
    Transform to the strict preference relation double[][] strict = To-Strict(intersection);
        Console.WriteLine("\nStrict:\n");
        Console.WriteLine(Util.Print-Matrix(strict));

3rd Step:
    Generate the Non-Dominated Set of Alternatives double[] non-Dominated = Non-Dominated-Set(strict);
        Console.WriteLine("\nSet of Non-Dominated:\n");
        Console.WriteLine(Util.Print-Vector(Non-Dominated));

4th Step:
Generate the list of results with the indices of the alternatives return Result-ND(not-Dominated);
```

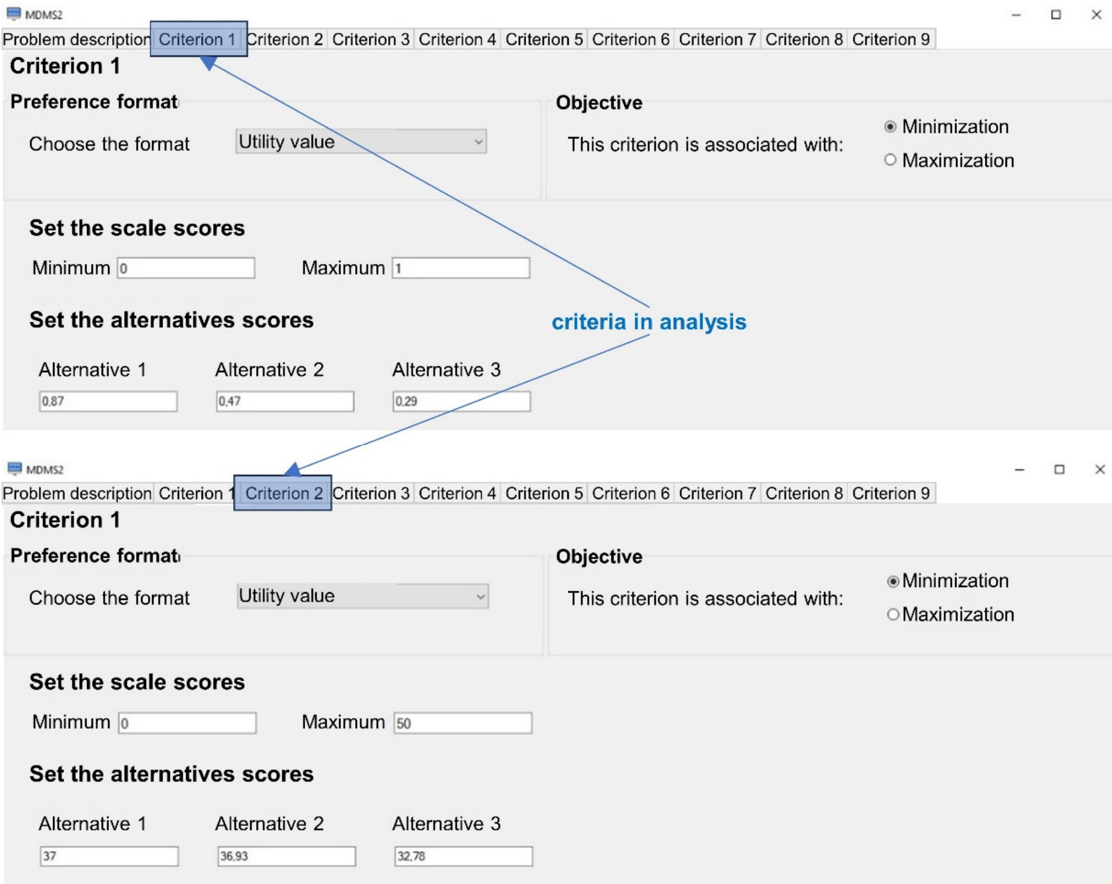

Figure 4. Criteria assessment functionality (e.g., capacity factor and levelized energy cost).

After choosing the appropriate technique for the problem, the decision-maker defines the numbers of alternatives and criteria for the problem. Once the decision-maker defines the number of criteria for the problem, the system creates a corresponding number of tabs. In other words, the system creates a tab for each criterion that the decision-maker defines.

Next, the decision-maker must inform which preferred format they feel most comfortable with for assessing the alternatives. The system allows the decision-maker to evaluate alternatives using the preference format nonreciprocal fuzzy preference relations, ordering of alternatives, utility values, fuzzy estimates, or multiplicative preference relations. Algorithm 2 shows how a particular format, chosen by the decision-maker, is transformed into nonreciprocal fuzzy preference relations. Note: algorithm that operationalizes the transformation function (order of alternatives format to nonreciprocal fuzzy preference relations).

Algorithm 2: Method that transforms an ordered array into the additive reciprocal fuzzy preference relation.

```
public static double[][] Ordered-To-Not-Reciprocal(int[] order-Alternatives)
    int num-Alternatives = order- Alternatives.Length; double[][]
    non-Reciprocal=new double[num-Alternatives][];
    for (int i = 0; i < num- Alternatives; i++)
        no-Reciprocal[i]=new double[num-Alternatives];
        for (int j = 0; j < num Alternatives; j++)
            if (order- Alternatives [i] > order- Alternatives [j])
            non-Reciprocal[i][j] = 0.5 + ((double)(order-Alternatives[j] - order-Alternatives[i]) / (2.0 * (double)(num-Alternatives - 1)));
        else
             non-Reciprocal[i][j] = 1;
     return non-Reciprocal;
```

Finally, information about each of the quantitative criteria related to each of the alternatives must be entered into the system.

5. Application Example: Choice of an Alternative Energy Source

The choice of alternative energy sources is a problem that has been widely explored in the literature [79–82]. However, studies in this area ignore or disregard the uncertainty of objectives in the search for a solution to the problem. This condition makes this application example quite pertinent and innovative.

5.1. Statement of the Decision Problem

The decision-making problem presented in this example is associated with the choice of an alternative energy source to be installed in an isolated location, which is disconnected from the energy utility system. The location of the power source installation is an iron ore exploration mine. The mine's total power demand is 20 megawatts (MW).

The problem under analysis is associated with choosing one among the available energy sources to meet the mine's energy demand. The available energy sources and their respective installed powers are presented below:

- Alternative 1: Diesel generation source with an installed capacity of 23.0 MW;
- Alternative 2: Wind energy source with an installed capacity of 42.6 MW;
- Alternative 3: Solar energy source with an installed capacity of 69.0 MW.

Nine criteria were considered to choose the alternatives. The six quantitative criteria in the decision-making model were the capacity factor, levelized energy cost, deployment time, space requirement, power-plant lifetime, and greenhouse gas emissions. The three qualitative criteria included in the decision-making model were environmental risk, corporate image risk, and technological maturity. The estimates of the alternatives from the point of view of the nine aforementioned criteria are given in Table 1.

Table 1. Assessing alternatives according to the criteria under consideration.

	Criteria	Scale	Diesel Generation	Wind Generation	Solar Generation
1	Capacity factor	%	87	47	29
2	Levelized cost of energy	USD/kW	37.00	36.93	0.08
3	Deployment time	Months	24	30	22
4	Space requirement	m^2/kW	4	43	23
5	Power-plant lifetime	Years	15	30	25
6	Greenhouse gas emissions	tCO_2/MWh	0.76	0.00	0.00
7	Environment risk		High	Low	High
8	Corporate image risk		High	Low	Low
9	Technological maturity		High	High	Medium

5.2. Steps to Solve the Problem

Firstly, it is fundamental to identify which normalization function should be applied to each criterion. Among the considered criteria, the levelized energy cost, deployment time, space requirement, greenhouse gas emissions, environmental risk, and corporate image risk have a negative relationship with the choice of energy source. Therefore, these criteria must be normalized by a minimization function. On the other hand, the capacity factor, power-plant lifetime, and technological maturity criteria are positively correlated with the choice of energy source and are normalized by the maximization function.

At this point, it is possible to summarize the decision-making problem as identifying one of the three energy sources based on nine criteria. From the point of view of operationalizing the solution, the decision-maker inserts the number of alternatives and criteria on the system's initial screen, as shown in Figure 2.

Secondly, the decision-maker defines the preferred format for assessing each of the eleven criteria associated with the problem. In the present example, the quantitative criteria, represented by numerical scales, were evaluated following the utility values preferences format. An example of using the system to evaluate the quantitative criteria capacity factor and levelized cost of energy using the utility value preference format is shown in Figure 4.

In short, this step involves defining the preference format, the scale associated with each criterion, the scores of the alternatives, and the normalization function to be applied (minimization and maximization) according to the objective of the problem.

Note that this part of the system operationalizes two concepts explored in the research. The first concept operationalized in the system is related to converting the utility value format to the multiplicative relationship format. This conversion is performed by applying the corresponding transformation function (14). The second concept operationalized in the system concerns considering the uncertainty of objectives in the decision-making process. This type of uncertainty is considered by considering a solution based on conflicting objectives, which simultaneously require the minimization of one objective and the maximization of another.

Thirdly, the decision-maker assesses the qualitative criteria using the preferred format that offers the greatest psychological comfort. Figure 5 illustrates the assessment of the qualitative Criterion 9 using the fuzzy estimates format. Note that the decision-maker must provide the following information: type of membership function (triangular, trapezoidal, or Gaussian); level of importance of alternatives about the criterion; larger range (base of the trapezoid) and smaller range (top of the trapezoid) of uncertainty; objective associated with the criterion (minimization or maximization).

Figure 5 shows that the decision-maker evaluates alternatives based on Criterion 9 with the minimization objective. The graphs show that trapezoidal membership functions were constructed, and that the membership functions associated with Alternatives 1 and 2 were superimposed. The smaller [0.40; 0.60] and larger [0.25; 0.75] uncertainty ranges shown in Figure 5 and reflected in the trapezoid on the left are associated with evaluating Alternative 3. Ultimately, this figure demonstrates the system's usability and ease of use for fuzzy preference modeling.

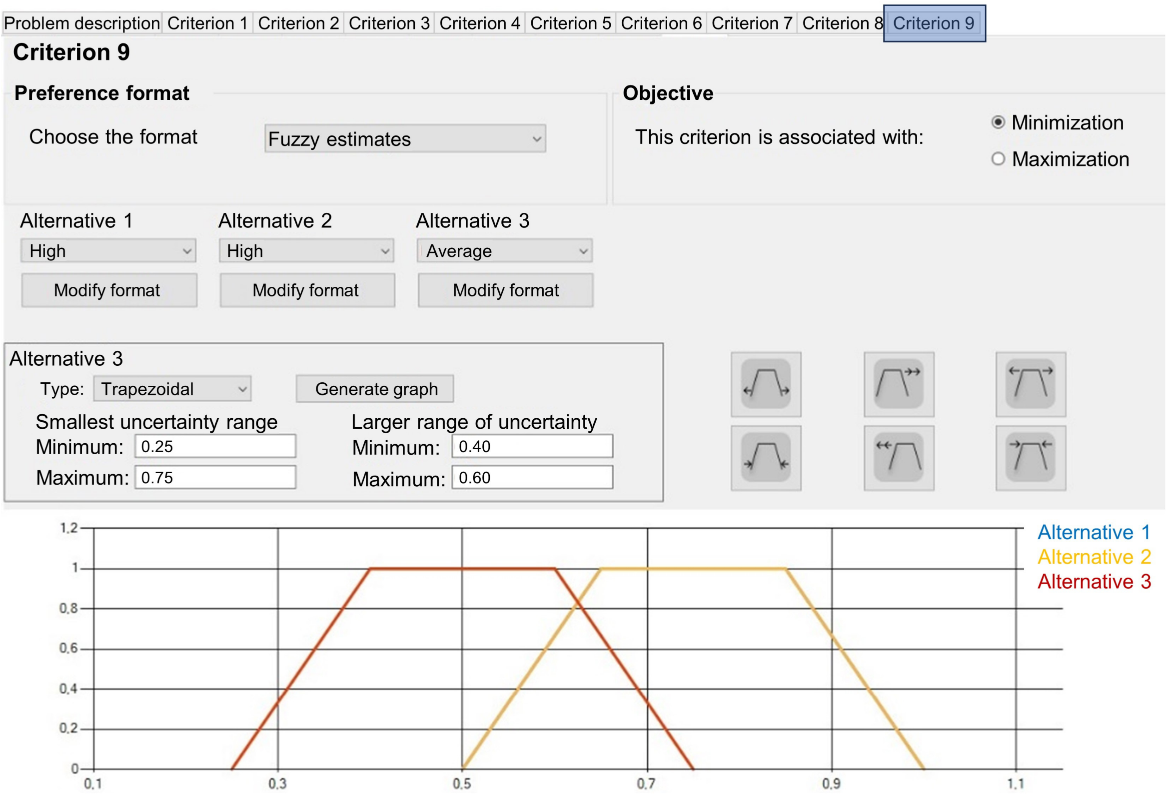

Figure 5. Fuzzy estimates for the technological maturity criterion (ninth criterion).

5.3. Processing and Solving the Decision Problem

The corresponding nonreciprocal fuzzy preference relations are constructed by inserting estimates related to quantitative and qualitative criteria into the multi-attribute decision-making system. The results of this procedure allow for obtaining nine matrices in the nonreciprocal fuzzy preference relationships format, as shown in Table 2. Note that each matrix corresponds to a criterion for the problem of choosing the energy source for the mine, with the alternatives (diesel, wind, and solar) represented by the lines of these matrices.

Table 2. Nonreciprocal fuzzy preference relationship matrices.

Criterion 1	Criterion 4	Criterion 7
[1.00 1.00 1.00] [0.60 1.00 1.00] [0.42 0.82 1.00]	[1.00 1.00 1.00] [0.22 1.00 0.60] [0.62 1.00 1.00]	[1.00 0.00 0.00] [1.00 1.00 1.00] [1.00 1.00 1.00]
Criterion 2	**Criterion 5**	**Criterion 8**
[1.00 0.99 0.92] [1.00 1.00 0.92] [1.00 1.00 1.00]	[1.00 0.57 0.86] [1.00 1.00 1.00] [1.00 0.71 1.00]	[1.00 0.00 0.00] [1.00 1.00 1.00] [1.00 1.00 1.00]
Criterion 3	**Criterion 6**	**Criterion 9**
[1.00 1.00 0.96] [1.00 1.00 0.96] [1.00 1.00 1.00]	[1.00 0.24 0.24] [1.00 1.00 1.00] [1.00 1.00 1.00]	[1.00 1.00 0.83] [1.00 1.00 0.83] [1.00 1.00 1.00]

Then, the results of applying the first technique to the nonreciprocal fuzzy preference relations are obtained by executing Correlations (14) and (11)–(13), respectively. These correlations allow us to prepare the data for executing the first technique, as presented in Table 3. In short, the intersection between fuzzy preference matrices, the strict fuzzy preference relation matrix construction, and obtaining the vector corresponds to the set of non-dominated alternatives. Note that the vector of the set of non-dominated alternatives corresponds to [0.58 0.89 1.00]. The highest value of this vector corresponds to the solution of the decision problem, Alternative 3 (solar energy), which reached a value equal to 1.00.

Table 3. Solution to the problem generated by the first technique.

Intersection between Fuzzy Preference Matrices	Strict Fuzzy Preference Relation of the Intersection	Set of Non-Dominated Alternatives
[1.00 0.00 0.00] [0.22 1.00 0.60] [0.42 0.71 1.00]	[0.00 0.00 0.00] [0.22 0.00 0.00] [0.42 0.11 0.00]	[0.58 0.89 1.00] Energy source chosen: Alternative 3 (solar)

In turn, applying the second technique implies the evaluation of the criteria using the order of alternatives format. This analysis is carried out using Correlations (11), (19), and (20). In short, decision-makers order the criteria that they consider most important in the decision. Based on the interactive criteria inclusion process, the system uses this ordering to process the lexicographic method. In our example, decision-makers consider the importance of criteria when choosing an alternative energy source in the following order: $6 \succ 7 \succ 8 \succ 9 \succ 2 \succ 3 \succ 4 \succ 5 \succ 1$.

Based on this order, applying Criterion 6 allows for cutting the first alternative. In particular, Table 4 shows that Criterion 6, "Greenhouse Gas Emissions", allows us to determine the levels of non-dominance of alternatives: [0.24 1.00 1.00]. This vector indicates the need to cut Alternative 1, diesel generation. So, Alternative 1 is removed from the matrices of the other following criteria. Note that it is impossible to reduce the decision uncertainty region regarding Alternatives 2 and 3 by Criterion 7, "Environmental Risk". Alternatives 2 and 3 present the same values for this criterion. This means that the "environmental risk" criterion does not have the power to distinguish the alternatives analyzed. Thus, Criterion 8, "Corporate Image Risk", is used to analyze Alternatives 2 and 3. The result of this operation is given by the vector [1.00 0.00]. This vector indicates that Alternative 2, wind energy, is the best energy source for the problem.

Table 4. Solution to the problem generated by the second technique.

	Criterion 6	Criterion 7	Criterion 8
Strict fuzzy preference relation	[0.00 0.00 0.00] [0.76 0.00 0.00] [0.76 0.00 0.00]	[0.00 0.00] [0.00 0.00]	[0.00 0.00] [1.00 0.00]
Set of non-dominated alternatives	[0.24 1.00 1.00]	[1.00 1.00]	[1.00 0.00]
Energy source	Alternative 2 (wind)		

Note: Strict fuzzy preference relation and non-dominated set of alternatives by adding the n-th criterion.

Finally, the third technique is used to choose the alternative energy source to be implemented in the mine by applying Correlations (11) and (21). These correlations permit one to construct the membership functions of the set of non-dominated alternatives for each criterion. In other words, a vector of a non-dominated set of alternatives is constructed based on each of the fuzzy strict preference relation matrices. The identification of the alternative to be chosen is obtained from the intersection of the non-dominated set of alternative vectors. Note in Table 5 that the result of this intersection, obtained through the application of Correlation (22), is given by the vector [0.00 0.22 0.42]. Based on the values of this vector, the diesel generation alternative has a value of [0.00], the wind energy alternative has a value of [0.22], and the solar energy alternative has a value of [0.42]. Therefore, it is possible to conclude that solar energy is the best alternative to the problem.

Table 5. Solution to the problem generated by the third technique.

	Criterion 1	**Criterion 4**	**Criterion 7**
Fuzzy strict preference relation	[0.00 0.40 0.58] [0.00 0.00 0.18] [0.00 0.00 0.00]	[0.00 0.78 0.38] [0.00 0.00 0.00] [0.00 0.40 0.00]	[0.00 0.00 0.00] [1.00 0.00 0.00] [1.00 0.00 0.00]
Non-dominated set of alternatives	[1.00 0.60 0.42]	[1.00 0.22 0.62]	[0.00 1.00 1.00]
	Criterion 2	**Criterion 5**	**Criterion 8**
Fuzzy strict preference relation	[0.00 0.00 0.00] [0.01 0.00 0.00] [0.08 0.08 0.00]	[0.00 0.00 0.00] [0.43 0.00 0.29] [0.14 0.00 0.00]	[0.00 0.00 0.00] [1.00 0.00 0.00] [1.00 0.00 0.00]
Non-dominated set of alternatives	[0.92 0.92 1.00]	[0.57 1.00 0.71]	[0.00 1.00 1.00]
	Criterion 3	**Criterion 6**	**Criterion 9**
Fuzzy strict preference relation	[0.00 0.00 0.00] [0.00 0.00 0.00] [0.04 0.04 0.00]	[0.00 0.00 0.00] [0.76 0.00 0.00] [0.76 0.00 0.00]	[0.00 0.00 0.00] [0.00 0.00 0.00] [0.17 1.17 0.00]
Non-dominated set of alternatives	[0.96 0.96 1.00]	[0.24 1.00 1.00]	[0.83 0.83 1.00]
Insertion of the non-dominated set of alternatives	[0.00 0.22 0.42]	Energy source chosen:	Alternative 3 (solar)

Applying the three techniques suggests the choice of Alternative 3, as this solution was obtained in the first and third techniques. However, the second technique indicates Alternative 2 as the problem solution. This contradiction can be considered a disadvantage of the system, as it generates another decision problem: which solution or technique to use?

5.4. Concluding Remarks

At this point, it is important to highlight the particularities of each technique and indicate possible limitations, advantages, and recommendations.

- Applying the second technique may lead to solutions different from the results obtained from the first technique.
- The first technique and the third share the same generic basis but may sometimes generate different solutions.
- The third technique is preferred from a substantial point of view.
- The first technique can lead to the choice of alternatives with a degree of non-dominance equal to one, which does not represent the best solution from the point of view of all preference relations.
- The third technique can generate alternatives with a degree of non-dominance equal to one only for alternatives that are the best solutions from the point of view of all fuzzy preference relations.

It should be stressed that the possibility of obtaining different solutions based on different approaches is natural and not necessarily negative. The presence of different solutions is an indication that more information must be considered to obtain a more coherent solution to the problem. Several strategies can be considered in this situation. The first strategy is the inclusion of other criteria of a quantitative or qualitative nature. The second strategy is the prior or subsequent choice of the technique used in the presence of different solutions. The third strategy is the choice of the alternative indicated by two techniques, with or without considering weights for these techniques. However, the advantage of choosing the second approach is evident when decision-makers are able to distinguish the order of importance of the criteria with greater confidence.

At this point, it is worth highlighting that the multi-attribute decision-making system developed in this research contributes to solving complex problems far beyond the definition of the best source for the generation of alternative energy (e.g., [83]). The usefulness of the multi-attribute decision-making system is not limited to solving problems in the engineering, mathematics, and operational research fields. The system has high applicability for social, urban planning, environmental, economic, and health researchers.

The appropriation of the system by researchers in these areas favors the understanding and elaboration of public policies that allow for reductions in social vulnerabilities [84], the equitable distribution of public infrastructure in cities [85], improvements in public healthcare services [86], making better decisions in custody hearings [87], promoting the digital maturation of companies, achieving opportunities and avoiding market threats [88], and improving accessibility to healthcare facilities [89], among others.

Finally, it is important to highlight that the system can be applied independently or as part of a general multi-criteria decision-making approach under conditions of uncertainty. On the one hand, the multi-attribute decision-making system can be applied independently when the original problem is multi-attribute, including problems of selecting the energy source to be installed in the mine. On the other hand, the multi-attribute decision-making system can be applied to reduce regions of decision uncertainty within the overall decision-making scheme, to construct robust solutions to mathematical programming problems with fuzzy coefficients in objective functions and constraints.

6. Conclusions

This paper's main focus is solving problems of multi-attribute decision-making based on fuzzy preference modeling to provide the possibility of aggregating quantitative and qualitative character information based on decision-makers' knowledge, experience, and intuition. The orientation on nonreciprocal fuzzy preference relations does not exclude the possibility of using other existing formats of representation of human preferences, because these formats can be reduced to nonreciprocal fuzzy preference relations. This can be done by applying transformation functions. Three different techniques for decision-making were discussed here. Sometimes, these techniques generate different results.

However, this is natural, and the choice of the technique is a prerogative of the decision-maker. Generally, the techniques can serve for assessing, comparing, choosing, prioritizing, or ordering alternatives. The techniques served for developing the computing system for multi-attribute decision-making. Its functioning was illustrated by solving the problem of choosing the energy source to meet the mine's energy demand.

All three techniques aimed at analyzing $<X, R>$ models require the explicit direct or indirect ordering of the criteria. Considering this, the future development of this paper's results is also associated with the representation of information related to the importance of the criteria as nonreciprocal fuzzy preference relations. Furthermore, another function of the computational system that will be developed and implemented is considering the uncertainty factor of quantitative information.

Finally, future developments should consider overcoming some limitations of the methodology integrated into the system. First, other techniques of multi-attribute decision-making exist, including a technique that presents information on the importance of criteria in the form of fuzzy preference relations, which were not implemented. Different decision-making techniques may generate different results. Although the choice of the technique for solving a concrete problem is a prerogative of the decision-maker, it is necessary to elaborate recommendations on interpreting and choosing the problem solution. Second, the multi-attribute decision-making system does not include a significant and increasingly frequent element in real situations, which is group decision-making [52,90,91]. Therefore, integrating group preference modeling is a key feature to consider in future developments.

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