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Editorial

Theory of Functions and Applications

Inna Kal'chuk

Faculty of Information Technologies and Mathematics, Lesya Ukrainka Volyn National University, 43025 Lutsk, Ukraine; k.inna.80@gmail.com

1. Introduction

In this editorial, we present "Theory of Functions and Applications", a Special Issue of *Axioms*. This Special Issue comprises 15 articles devoted to exploring current problems in both the theory of functions and in real and complex variables, as well as the applied applications of both. Though the featured articles are concerned with a variety of topics, issues related to the following fields take particular precedence in this Special Issue: function approximation, functional analysis, complex analysis, differential equations, numerical methods, and mathematical modeling. The main aim of this Special Issue is to share scholars' theories and methods relating to function theory, their significant, topical, and novel findings in this area, and applications and solutions for applied problems in related scientific fields.

2. Overview of the Published Papers

In contribution 1, focusing on oscillations for delay differential equations, investigated the asymptotic properties of solutions for the fourth-order delay differential equation with a non-canonical operator. They studied novel properties that contribute to achieving more effective terms in the oscillation of differential equations, established criteria that guarantee the exclusion of decreasing solutions and that ensure the oscillation of the studied equation, and also demonstrated the theoretical aspect of their work through the use of examples.

In contribution 2, extremal problems relating to the approximation theory were considered; for instance, the approximative properties of Favard sums on the Hölder classes of functions of one and two variables were studied. In this paper, it is proven that the value of the approximation of the class $H^{1,1}$ using the Favard method is greater than the value of the best approximation of this class using trigonometric polynomials. Moreover, the authors constructed classes for which these approximative characteristics are equal.

The problems considered in contribution 3 concern shifts in the wide class $\tilde{S}$ of L functions and the approximations of analytical functions relating to these. Using the continuous universality theorem, the authors proved that each set of the analytic non-vanishing functions in a strip can be approximated simultaneously by discrete shifts, which are defined by the Dirichlet series from the Selberg–Steuding class and the linearly independent multiset over the field of rational numbers. The probabilistic approach based on the weak convergence of probability measures in the space of analytic functions was also used in their research.

The contribution 4 focuses on nonparametric prediction in Hilbertian statistics; specifically, the authors discuss the Hilbert RE-regression for weak functional time series data. In addition to conducting an empirical study investigating the behavior of RE-regression estimation, they also obtained a new kernel estimation for RE-regression that improved the robustness of the classical regression in its minimizing of the effect of the largest variables.

In contribution 5, the authors studied the boundedness and compactness of the sum operator, as defined by the complex products of composition, multiplication, and the m-th-iterated radial derivative operators of Bloch-type spaces to weighted-type spaces on

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the unit ball. They also characterized the boundedness and compactness of all products of composition, multiplication, and m-th-iterated radial derivative operators.

In contribution 6 studied matrix representations of Sturm–Liouville problems with boundary- and interface-contained eigenparameters. In particular, they constructed a class of Sturm–Liouville problems based on a given matrix eigenvalue problem relating to a certain type and condition, depending on the eigenparameter. The authors proved that each Sturm–Liouville problem is equivalent to the original matrix's eigenvalue problem, using the method of characteristic-function factorization.

In contribution 7, the authors investigated two classes of normalized holomorphic and bi-univalent functions, which include bi-prestarlike functions. They found upper bounds for the first two coefficients $|a_2|$ and $|a_3|$ of the Taylor–Maclaurin series for the functions of each of these classes, and their findings can be used in the geometric theory of functions.

The authors of contribution 8 devoted their study to the class of twice-continuously differentiable functions (including the class of convex functions), and its characteristics, satisfying second-order differential inequalities; they and obtained new Hermite–Hadamard-type inequalities for the indicated functions.

The work in contribution 9 focuses on the issue of extending continuous functions, defined by subsets of metric spaces, to the entire space, so that the extended function preserves the basic properties of the original function. The authors proposed a new method that optimizes the integral p-mean instead of its maximum value. They also considered the more general theoretical approach based on measure-valued representations of metric spaces and the duality formula. In addition to this, they also discovered some explicit formulas relating to specific extensions that satisfy Lipschitz-type inequalities.

In contribution 10, the author compared two symmetries of different origins on the set of average functions and found the asymptotic series expansion for both of them in terms of a recursive algorithm for their coefficients, enabling them to perform a coefficient comparison. As a result, the author obtained the class of the means, which allowed for interpolating between those harmonic, geometric, and arithmetic.

In contribution 11 the author proved that some results of q-analyses and the partition theory can be obtained as specializations of fundamental relations between complete and elementary symmetric functions; specifically, he showed that Rothe's q-binomial theorem is a specialization of the generating function of elementary symmetric functions. He also obtained the Uchimura identity, which provides connections between partitions and divisors. All results are accompanied by combinatorial interpretations involving well-known functions in the partition theory.

In contribution 12, the authors found new expressions for the high-order derivatives of different symmetric and non-symmetric polynomials in terms of Euler polynomials, as well as obtaining connection formulas between different polynomials and Euler polynomials. They also proved some new definite integral formulas of the products of different symmetric and non-symmetric polynomials with the Euler polynomials.

Contribution 13 is devoted to the study of the fractional analogue of the Brusselator model. The authors proposed an effective hybrid method, which is based on a combination of the quasi-linearization approach and the matrix-collocation method for the approximate processing of fractional Brusselator equations; these methods were used to model the problem of an autocatalytic chemical reaction. The authors analyzed the convergence and error of this method and also presented some numerical models to test its accuracy.

In contribution 14, the authors proved new theorems that simplify the calculation of improper integrals. Their results allow us to establish many examples of improper integral formulas and solve them directly, without complex calculations or the use of computer software. They presented some applications related to finding Green's function, one-dimensional vibrating-string problems, wave motion in elastic solids, and computing Fourier.

In contribution 15, the authors studied the hypothesis of Kurepa's function distribution and performed the analysis using PARI/GP software (version 2.13.4).

Conflicts of Interest: The author declares no conflicts of interest.

List of Contributions:

1. Nabih, A.; Cesarano, C.; Moaaz, O.; Anis, M.; Elabbasy, E.M. Non-Canonical Functional Differential Equation of Fourth-Order: New Monotonic Properties and Their Applications in Oscillation Theory. *Axioms* **2022**, *11*, 636.
2. Bushev, D.; Kal'chuk, I. On the Realization of Exact Upper Bounds of the Best Approximations on the Classes $H^{1,1}$ by Favard Sums. *Axioms* **2023**, *12*, 763.
3. Kačinskaitė, R.; Laurinčikas, A.; Žemaitienė, B. Joint Discrete Universality in the Selberg–Steuding Class. *Axioms* **2023**, *12*, 674.
4. Chikr Elmezouar, Z.; Alshahrani, F.; Almanjahie, I.M.; Kaid, Z.; Laksaci, A.; Rachdi, M. Scalar-on-Function Relative Error Regression for Weak Dependent Case. *Axioms* **2023**, *12*, 613.
5. Huang, C.-S.; Jiang, Z.-J. On a Sum of More Complex Product-Type Operators from Bloch-Type Spaces to the Weighted-Type Spaces. *Axioms* **2023**, *12*, 566.
6. Li, S.; Cai, J.; Li, K. Matrix Representations for a Class of Eigenparameter Dependent Sturm–Liouville Problems with Discontinuity. *Axioms* **2023**, *12*, 479.
7. Hamadneh, T.; Abu Falahah, I.; AL-Khassawneh, Y.A.; Al-Husban, A.; Wanas, A.K.; Bulboacă, T. Initial Coefficients Upper Bounds for Certain Subclasses of Bi-Prestarlike Functions. *Axioms* **2023**, *12*, 453.
8. Aldawish, I.; Jleli, M.; Samet, B. On Hermite–Hadamard-Type Inequalities for Functions Satisfying Second-Order Differential Inequalities. *Axioms* **2023**, *12*, 443.
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10. Mihoković, L. Coinciding Mean of the Two Symmetries on the Set of Mean Functions. *Axioms* **2023**, *12*, 238.
11. Merca, M. From Symmetric Functions to Partition Identities. *Axioms* **2023**, *12*, 126.
12. Abd-Elhameed, W.M.; Amin, A.K. New Formulas and Connections Involving Euler Polynomials. *Axioms* **2022**, *11*, 743.
13. Izadi, M.; Srivastava, H.M. Fractional Clique Collocation Technique for Numerical Simulations of Fractional-Order Brusselator Chemical Model. *Axioms* **2022**, *11*, 654.
14. Abu-Ghuwaleh, M.; Saadeh, R.; Qazza, A. A Novel Approach in Solving Improper Integrals. *Axioms* **2022**, *11*, 572.
15. Fabiano, N.; Gardašević-Filipović, M.; Mirkov, N.; Todorčević, V.; Radenović, S. On the Distribution of Kurepa's Function. *Axioms* **2022**, *11*, 388.