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Solving a Multimodal Routing Problem with Pickup and Delivery Time Windows under LR Triangular Fuzzy Capacity Constraints

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Abstract: This study models a container routing problem using multimodal transportation to improve its economy, timeliness, and reliability. Pickup and delivery time windows are simultaneously formulated in optimization to provide the shipper and the receiver with time-efficient services, in which early pickup and delayed delivery can be avoided, and nonlinear storage periods at the origin and the destination can be minimized. Furthermore, the capacity uncertainty of the multimodal network is incorporated into the advanced routing to enhance its reliability in practical transportation. The LR triangular fuzzy number is adopted to model the capacity uncertainty, in which its spread ratio is defined to measure the uncertainty level of the fuzzy capacity. Due to the nonlinearity introduced by the time windows and the fuzziness from the network capacity, this study establishes a fuzzy nonlinear optimization model for optimization problem. A chance-constrained linear reformulation equivalent to the proposed model is then generated based on the credibility measure, which makes the global optimum solution attainable by using Lingo software. A numerical case verification demonstrates that the proposed model can effectively solve the problem. The case analysis points out that the formulation of pickup and delivery time windows can improve the timeliness of the entire transportation process and help to achieve on-time transportation. Furthermore, improving the confidence level and the uncertainty level increases the total costs of the optimal route. Therefore, the shipper and the receiver must prepare more transportation budget to improve reliability and address the increasing uncertainty level. Further analysis draws some insights to help the shipper, receiver, and multimodal transport operator to organize a reliable and cost-efficient multimodal transportation under capacity uncertainty through confidence level balance and transportation service and transfer service selection.

Keywords: multimodal routing; pickup and delivery time windows; capacity uncertainty; LR triangular fuzzy number; uncertainty level; fuzzy nonlinear optimization model; chance-constrained linear reformulation

MSC: 90-80

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1. Introduction

The operational-level multimodal routing plays a crucial role in improving the operations and management of multimodal transportation systems [1] and is a promising approach to fully strengthen the advantages of multimodal transportation over unimodal transportation. In China, by enhancing the service level to attract more freight sources, multimodal routing enables multimodal transportation to replace road-dominated transportation that is low efficiency and energy-consuming to optimize the transportation structure [2]. Due to its great significance in practice, the multimodal routing problem (MRP) has been widely discussed in the research field of transportation planning [3] and is one of the six research topics in the decision support of multimodal transportation (policy support, terminal network design, multimodal service network design, multimodal routing, drayage operations, and ICT innovations) [4].

Modeling MRP provides quantitative support for the decision-making of the transportation actors and stakeholders. MRP with the lead time constraint can avoid delayed delivery of containers at the destination that would result in economic loss. Shahedi et al. [5] build a multi-objective mixed integer linear programming model to formulate the lead time-based multimodal freight routing problem considering the Physical Internet. Verma and Verter [6] explore a rail–truck multimodal routing problem of dangerous goods with lead time constraints, in which a bi-objective optimization model is proposed. Although the delayed delivery of containers can be avoided, the lead time constraint might result in early delivery that causes post-transportation storage. Therefore, increasing multimodal routing studies focus on the MRP with time window(s), in which the time window uses its lower bound and upper bound separately to reduce or avoid both lateness and earliness of the container delivery at the destination. Therefore, compared to the lead time constraint, the concept of a time window is more suitable for achieving time-efficient transportation, especially in the context that a just-in-time strategy is resorted by companies, and transportation at the right time is required by them to minimize storage [7,8].

Both hard and soft time windows are established by the MRP literature. Those with the hard time window force that the container delivery by the multimodal route at the destination must be accomplished within the time window and propose the upper and lower bound constraints. Currently, there are a few studies using such a kind of time window. Zhang et al. [9] consider a green multimodal service network design problem with hard time window constraints, construct an integer linear optimization model, and develop genetic and heuristic algorithms. However, a hard time window is too strict and cannot reflect the customers' attitudes that early and delayed transportation is acceptable to a certain degree. Consequently, the hard time window reduces the flexibility of the MRP. As a result, most MRP studies are interested in soft time windows that make the earliness and lateness allowable on condition that penalty or storage costs must be paid when the time window is violated, e.g., Zhang et al. [10], Fazayeli et al. [11], Yuan et al. [12], and Li et al. [13]. In this case, the penalty or storage period is calculated by the continuous piecewise linear function that introduces nonlinearity into the modeling of MRP. In transportation practice, delayed transportation could disrupt the further processing of the goods and could cause a long-term economic loss that is difficult to determine [14]. Therefore, the penalty costs in the soft time window cannot be given accurately. As a result, both hard and soft time windows yield disadvantages that reduce their feasibility when applied in practical transportation.

Furthermore, multimodal transportation starts with the container pickup from the shipper and ends with container delivery to the receiver. Therefore, both pickup and delivery determine the time efficiency of the entire multimodal transportation process [15]. However, the majority of relevant studies focus more on the delivery than the pickup when modeling the time window and assume that containers are released at a fixed time [13,16] or there is the earliest release time [17]. Currently, quite a few works simultaneously model the two types of time windows, in which Sun et al. [2] and Zhang et al. [18] use the soft time window. Although these studies discuss the pickup and delivery time windows, their modeling still has the disadvantage of the soft time windows indicated above.

In addition to the time window, uncertainty is a key direction of the MRP that seeks to improve the reliability of the multimodal route in practice [19] and has been highlighted by supply chain management [20]. The constantly changing status of the dynamic transportation network makes it difficult to accurately predict the real-time state of the parameters of the multimodal network in advanced transportation planning, which causes uncertainty. Formulating the uncertainty makes advanced multimodal routing more reliable in practical transportation where the real-time state of the multimodal network is eventually known. Currently, transportation planning studies pay attention to the capacity uncertainty of the transportation network [21]. Regarding MRP, the optimal route is infeasible if the capacities are overestimated and might not be the best solution if the capacities are underestimated. Compared to the MRP articles with deterministic capacity constraints, there are limited studies considering capacity uncertainty. Huang [22] assumes that the capacities of water and rail transportation obeys the normal distribution and designs a two-stage stochastic programming model for the multimodal routing problem in the stochastic environment. Uddin et al. [23,24] utilize the scenario-based approach to model the uncertain capacities of rail transportation and terminals and build stochastic programming models for the MRP. Contrary to stochastic programming, based on the fuzzy set theory [25], Sun et al. [8], Li et al. [13], and Lu et al. [26] use triangular fuzzy numbers to model capacity uncertainty and establish the fuzzy chance-constrained programming (FCCP) models to deal with their MRPs. A group of studies by Sun et al. [2,16,27] model trapezoidal fuzzy capacities and propose FCCP models for specific MRPs.

All these papers cited above state the significant influence of capacity uncertainty on the optimization of the MRP and promote the research progress on MRP under capacity uncertainty. However, the stochastic models taken by References [22–24] need to use large numbers of data to fit the probability distribution of the uncertain capacity, while there are not enough data in most cases [28]. The scenario-based approach to model the uncertain capacity as large numbers of scenarios challenges the computational efficiency of problem-solving [11]. Moreover, assuming that the capacity uncertainty obeys a certain distribution might not match practice. Under the above situation, fuzzy programming based on the use of fuzzy numbers to model uncertainty [29,30] is an alternative for MRP under capacity uncertainty. However, References [2,8,13,16,26,27] use regular fuzzy numbers when modeling uncertain capacity and only analyze the influence of the confidence level introduced by fuzzy programming approaches on the MRP. In these studies, the influence of the uncertainty level of the fuzzy capacity on the MRP is not considered. Additionally, apart from Li et al. [13], the studies cited above only formulate the uncertainty of the travel capacity of rail transportation, while taking the travel capacities of other transportation modes to be deterministic, together with the node transfer capacities between different transportation modes.

Consequently, there is a need to design time windows that can avoid inestimable economic loss and simultaneously maintain a certain flexibility. It is also important to use such time windows to improve the quality of both pickup and delivery services of multimodal transportation. Secondly, modeling the capacity uncertainty in MRP should cover the entire multimodal network, including the travel process on arcs and the transfer process at the nodes, which makes the modeling of capacity uncertainty complete and enhances the feasibility of the MRP under capacity uncertainty. Finally, modeling the capacity uncertainty should enable its uncertainty level to be measurable, so that we can analyze the influence of the uncertainty level on the MRP and draw insights to deal with uncertainty to organize reliable multimodal transportation. All of these three aspects are not considered comprehensively in the relevant MRP studies. Faced with the research gaps mentioned above, this study continues to investigate the multimodal routing problem with pickup and delivery time windows under capacity uncertainty (MRPPDTWCU) by making the following contributions. Specifically, this study is an extension of the study by Li et al. [13] in the following three aspects:

(1) Both the pickup and delivery time windows are incorporated into the MRP to improve the time efficiency of the entire transportation process, in which early pickup and delayed delivery are forbidden, and the storage caused by delayed pickup and early delivery is minimized. Through this setting, the weaknesses of both soft and hard time windows are fixed.

(2) The uncertainty of both travel capacities of transportation modes on the arcs and transfer capacities between different transportation modes at the nodes of the multimodal network is modeled by LR triangular fuzzy numbers (LRTFNs), in which we define the uncertainty level of the LR triangular fuzzy capacity (LRTFC).

(3) Based on the chance-constrained linear programming (CCLP) model using the credibility measure, we analyze the influence of the confidence level and the uncertainty level on the MRPPDTWCU and summarize the insights that help multimodal transportation to deal with the uncertain environment.

The remaining sections of this study are organized as follows. In Section 2, we introduce the problem scenario, the formulation of pickup and delivery time windows, and the modeling of uncertain capacity in the MRPPDTWCU. In Section 3, a fuzzy optimization model is initially constructed based on the specific problem description. In Section 4, we process the model in Section 3 to remove its fuzziness and nonlinearity to make its global optimum solution attainable for the shipper, receiver, and multimodal transport operator (MTO), which supports their decision-making. In Section 5, a numerical case based on a previous study is presented to verify the feasibility of the model, and further reveal some insights that help to organize a cost-efficient and reliable multimodal transportation. Finally, we summarize the conclusions in Section 6.

2. Problem Description

In multimodal transportation, the MTO is responsible for the shipper and the receiver (that is, the customers), and coordinates the carriers of rail, road, and water transportation to carry out the origin-to-destination transportation based on the customer demands [31]. Therefore, it is the work of the MTO that plans the multimodal route, and therefore the MRP is oriented on the MTO's viewpoint. The MTO needs to satisfy the customer demands by presenting an optimal route with high economy, timeliness, and reliability or a balanced state on these objectives. Among these objectives, improving the transportation economy by minimizing transportation-related costs is the most important consideration for customers to improve the profits of their products and maintain competition in the market. As a result, the objective of the MRP explored in this study is to minimize the total costs for multimodal transportation.

In this study, both the shipper and the receiver have a time window. For the container shipper, the lower bound of the pickup time window is the earliest release time of the containers at the origin. The container pickup time should not be earlier than that time. When the container pickup is delayed and later than the upper bound of the pickup time window, containers should be stored first before pickup. In this case, the shipper should pay for the storage. The receiver also has a fixed delivery time window. The upper bound is the latest delivery time, and the delivery of containers at the destination must be no later than that time to avoid disruption in the further processing of the containers and the resulting economic loss that is difficult to determine. However, container delivery is allowed to be accomplished earlier than the lower bound of the delivery time window, under the condition that the receiver needs to pay for the storage before the containers receive further processing. Under the above time window setting, improving the on-time pickup and delivery to optimize the timeliness of the entire multimodal transportation process can be achieved by reducing the storage costs covered in the optimization objective and following the time window constraints of the MRPPDTWCU. Under the above consideration, the MRPPDTWCU not only aims to give a physical path that connects the origin and the destination but also needs to plan the container pickup time at the origin.

In the MRPPDTWCU that should be conducted before the beginning of practical transportation, this study fully formulates the capacity uncertainty of the entire multimodal network. This study uses fuzzy set theory to build the uncertain capacity and constructs the fuzzy optimization model for the MRPPDTWCU. We adopt LR fuzzy numbers to model the fuzzy capacity. A fuzzy number $\tilde{g}$ is an LR fuzzy number if it has membership functions L

for left and R for right, and left hand and right hand spreads α and β that are both positive numbers [32,33]. The membership function of LR fuzzy number $\tilde{g}$ is as Equation (1).

$$\mu_{\tilde{g}}(x) = \begin{cases} L\left(\frac{g-x}{\alpha}\right) & \text{if } x \leq g \\ R\left(\frac{x-g}{\beta}\right) & \text{if } x \geq g \end{cases} \quad (1)$$

In Equation (1), g is the mean value of $\tilde{g}$. Symbolically, $\tilde{g}$ can be represented by $(g, \alpha, \beta)_{LR}$. $L\left(\frac{g-x}{\alpha}\right)$ and $R\left(\frac{x-g}{\beta}\right)$ are non-increasing functions, and there are $L(0) = 1$ and $R(0) = 1$. In this study, we consider the above two membership functions to be linear. Then, $\tilde{g} = (g, \alpha, \beta)_{LR}$ is an LRTFN [34,35] and can be used to express fuzzy capacity. In this expression, g is the mean value of the uncertain capacity, and α and β reflect the uncertainty levels that the multimodal network suffers capacity insufficiency (the worst condition) and is under capacity sufficiency (the best condition). Consequently, the utilization of the LRTFC enables the advanced MRPPDTWCU to consider all the possible capacity conditions that the multimodal network can face in practical transportation. Furthermore, we define $\frac{\alpha}{g}$ and $\frac{\beta}{g}$ as the left hand and right hand spread ratios of the LRTFN. It is obvious that bigger $\frac{\alpha}{g}$ and $\frac{\beta}{g}$ increase the uncertainty level of LRTFN $\tilde{g}$. Compared to the regular TFN, with the help of the LRTFN to model the fuzzy capacity, we can analyze the influence of the uncertainty level of the fuzzy capacity on the MRPPDTWCU.

3. Fuzzy Nonlinear Optimization Model for MRPPDTWCU

3.1. Symbols in the Model

(1) Sets, indices, and parameters:

N: set of nodes;
A: set of arcs;
S: set of transportation modes;
i, j, k: indices of transportation nodes, and $i, j, k \in N$;
Φ_j: set of predecessor nodes to node j, and $\Phi_j \subseteq N$;
Ψ_j: set of successor nodes to node j, and $\Psi_j \subseteq N$;
(i, j): arc from node i to node j, and $(i, j) \in A$;
S_{ij}: set of transportation modes on arc (i, j), and $S_{ij} \subseteq S$;
S_j: set of transportation modes linking node j, and $S_j \subseteq S$;
m, n: indices of transportation modes, and $m, n \in S$;
l_{ijm}: travel distance in km of transportation mode m on arc (i, j);
t_{ijm}: travel time in h of transportation mode m on arc (i, j);
t_j^{mn}: transfer time in h/TUE from transportation mode m to transportation mode n at node j;
$\tilde{g}_{ijm} = (g_{ijm}, \alpha_{ijm}, \beta_{ijm})$: LRTFC in TEU of transportation mode m on arc (i, j);
$\tilde{g}_j^{mn} = \left(g_j^{mn}, \alpha_j^{mn}, \beta_j^{mn}\right)$: LRTFC in TEU of node j to transfer containers from transportation mode m to transportation mode n;
c_{ijm}: travel costs in CNY/TEU of transportation mode m on arc (i, j);
c_j^{mn}: transfer costs in CNY/TEU from transportation mode m to transportation mode n at node j;
o: origin of container transportation, and $o \in N$;
d: destination of container transportation, and $d \in N$;
q: transportation volume in TEU of containers;
c_o^{store}: storage costs in CNY/TEU·h at the origin;
c_d^{store}: storage costs in CNY/TEU·h at the destination;
$[tw_o, tw_o^*]$: pickup time window of the shipper;
$[tw_d, tw_d^*]$: delivery time window of the receiver;

(2) Variables

x_{ijm}: 0–1 variable. If transportation mode m moves containers on arc (i, j), $x_{ijm} = 1$; otherwise, $x_{ijm} = 0$;

y_j^{mn}: 0–1 variable. If there is a transfer of containers from transportation mode m to transportation mode n at node j, $y_j^{mn} = 1$; otherwise, $y_j^{mn} = 0$;

z_o: non-negative variable meaning the container pickup time at the origin;

z_d: non-negative variable meaning the container delivery time at the destination;

w_o: non-negative variable in h meaning the storage period of the containers at the origin;

w_d: non-negative variable in h meaning the storage period of the containers at the destination.

3.2. Fuzzy Nonlinear Optimization Model

$$\min \sum_{(i,\,j)\in A} \sum_{m\in S_{ij}} \left(c_{ijm}\cdot q\cdot x_{ijm}\right) + \sum_{j\in N} \sum_{m\in S_j} \sum_{n\in S_j} \left(c_j^{mn}\cdot q\cdot y_j^{mn}\right) + \left[\left(c_o^{\text{store}}\cdot q\cdot w_o\right) + \left(c_d^{\text{store}}\cdot q\cdot w_d\right)\right] \tag{2}$$

s.t.

$$\sum_{i\in\Phi_j} \sum_{m\in S_{ij}} x_{ijm} - \sum_{k\in\Psi_j} \sum_{n\in S_{jk}} x_{jkn} = \begin{cases} -1 & j = o \\ 0 & \forall j \in N\backslash\{o,\ d\} \\ 1 & j = d \end{cases} \tag{3}$$

$$\sum_{m\in S_{ij}} x_{ijm} \le 1 \quad \forall (i,j) \in A \tag{4}$$

$$\sum_{m\in S_j} \sum_{n\in S_j} y_j^{mn} \le 1 \quad \forall j \in N\backslash\{o,\ d\} \tag{5}$$

$$\sum_{i\in\Phi_j} x_{ijm} = \sum_{n\in S_j} y_j^{mn} \quad \forall j \in N\backslash\{o,\ d\} \quad \forall m \in S_j \tag{6}$$

$$\sum_{m\in S_j} y_j^{mn} = \sum_{k\in\Psi_j} x_{jkn} \quad \forall j \in N\backslash\{o,\ d\} \quad \forall n \in S_j \tag{7}$$

$$z_o \ge tw_o \tag{8}$$

$$w_o = \begin{cases} z_o - tw_o^* & \text{if } z_o > t_o^* \\ 0 & \text{if } z_o \le t_o^* \end{cases} \tag{9}$$

$$z_d = z_o + \sum_{(i,\,j)\in A} \sum_{m\in S_{ij}} t_{ijm}\cdot x_{ijm} + \sum_{j\in N} \sum_{m\in S_j} \sum_{n\in S_j} t_j^{mn}\cdot q\cdot y_j^{mn} \tag{10}$$

$$z_d \le tw_d^* \tag{11}$$

$$w_d = \begin{cases} tw_d - z_d & \text{if } tw_d > z_d \\ 0 & \text{if } tw_d \le z_d \end{cases} \tag{12}$$

$$x_{ijm}\cdot q \le \widetilde{g}_{ijm} \quad \forall (i,j) \in A \quad \forall m \in S_{ij} \tag{13}$$

$$y_j^{mn}\cdot q \le \widetilde{g}_j^{mn} \quad \forall j \in N\backslash\{o,\ d\} \quad \forall m \in S_j \quad \forall n \in S_j \tag{14}$$

$$x_{ijm} \in \{0,1\} \quad \forall (i,j) \in A \quad \forall m \in S_{ij} \tag{15}$$

$$y_j^{mn} \in \{0,1\} \quad \forall j \in N\backslash\{o,\ d\} \quad \forall m \in S_j \quad \forall n \in S_j \tag{16}$$

$$z_o \ge 0 \tag{17}$$

$$z_d \ge 0 \tag{18}$$

$$w_o \ge 0 \tag{19}$$

$$w_d \ge 0 \tag{20}$$

- Equation (2) is the optimization objective of the MRPPDTWCU and aims to minimize the total costs of transportation that include travel costs, transfer costs, and storage costs at the origin and the destination.
- Equation (3) is the container flow equilibrium constraint.
- Equations (4) and (5) ensure that containers are unsplittable in the transportation process from the shipper to the receiver.
- Equations (6) and (7) ensure that the optimal route yields a smooth connection between the travel process on the selected arcs and the transfer process at the selected nodes. (Equations (3)–(7) are the general constraints of the MRP considering unsplittable flow [13]).
- Equation (8) ensures that container pickup time at the origin should be no earlier than the lower bound of the shipper's pickup time window.
- Equation (9) calculates the storage period of the containers at the origin using a continuous piecewise linear function.
- Equation (10) determines the container delivery time.
- Equation (11) ensures that the container delivery time at the destination should be no later than the upper bound of the receiver's delivery time window.
- Equation (12) uses the same function as Equation (9) to present the storage period of the containers at the destination.
- Equations (13) and (14) ensure that the container volume does not exceed the LRTFCs of the optimal route.
- Equations (15)–(20) are the variable domain constraints.

4. Model Defuzzification and Linearization

The fuzzy nonlinear optimization model proposed in Section 3.2 is not solvable since its capacity constraints contain imprecise information. Therefore, we should remove the fuzziness of the model to make its solution attainable. As a widely used defuzzification approach, FCCP constructs the chance constraints of its fuzzy constraints, in which the confidence level is introduced into the fuzzy optimization model [36]. The value of the confidence level represents the attitudes of decision-makers on the constraints. It should be predetermined before solving the FCCP model.

The establishment of an FCCP model should first determine the type of fuzzy measure. Compared to the possibility and necessity measures separately representing the extremely optimistic and pessimistic attitudes, the credibility measure is a compromised attitude. It shows the attitudes of decision-makers that are neither too optimistic nor too pessimistic and matches the practical decision-making situation. Additionally, the credibility measure is self-dual and can guarantee that the fuzzy event must hold when its credibility reaches 1.0, and must fail when the credibility is equal to 0 [37]. Under the above consideration, we utilize the credibility measure to construct the chance constraints of Equations (13) and (14) as follows:

$$Cr\left\{x_{ijm}\cdot q \le \tilde{g}_{ijm}\right\} \ge \delta \quad \forall (i,j) \in A \quad \forall m \in S_{ij} \tag{21}$$

$$Cr\left\{y_j^{mn}\cdot q \le \tilde{g}_j^{mn}\right\} \ge \delta \quad \forall j \in N \backslash \{o,\ d\} \quad \forall m \in S_j \quad \forall n \in S_j \tag{22}$$

As indicated by the above equations, the credibility (Cr) that the transportation volume of containers does not exceed the LRTFCs of the route should not be less than a minimum confidence level δ. Increasing the value of δ makes the optimal route more credible in practical transportation where the actual capacities emerge, as it improves the credibility that the optimal route satisfies the constraints of the actual capacities. As a result, with the increase in δ, the optimal route yields higher reliability in practical transportation. Although δ falls into a closed interval $[0,1]$, in practical routing decision-making, decision-makers prefer a δ that belongs to $[0.5,1]$ [38]. As an interactive parameter, δ should be set by the decision-makers, including the shipper, receiver, and MTO.

Suppose a positive LR triangular fuzzy $\tilde{g} = (g, \alpha, \beta)_{LR}$ where $g - \alpha > 0$ and a positive deterministic number τ. The distribution of the credibility that $\tilde{g} \geq \tau$ is as Equation (23) [39].

$$Cr\left\{\tilde{g} \geq \tau\right\} = \begin{cases} 1 & \text{if } g - \alpha \geq \tau \\ \frac{g+\alpha-\tau}{2\alpha} & \text{if } g - \alpha \leq \tau \leq g \\ \frac{g+\beta-\tau}{2\beta} & \text{if } g \leq \tau \leq g + \beta \\ 0 & \text{if } \tau \geq g + \beta \end{cases} \tag{23}$$

Therefore, given a confidence level $\delta \in [0.5, 1]$, the crisp reformulation of $Cr\left\{\tilde{g} \geq \tau\right\} \geq \delta$ can be realized by Equation (24).

$$Cr\left\{\tilde{g} \geq \tau\right\} \geq \delta \Longleftrightarrow g + (1 - 2\delta) \cdot \alpha \geq \tau \tag{24}$$

Accordingly, Equations (21) and (22) can be reformulated as Equations (25) and (26), respectively.

$$g_{ijm} + (1 - 2\delta) \cdot \alpha_{ijm} \geq x_{ijm} \cdot q \quad \forall (i, j) \in A \quad \forall m \in S_{ij} \tag{25}$$

$$g_j^{mn} + (1 - 2\delta) \cdot \alpha_j^{mn} \geq y_j^{mn} \cdot q \quad \forall j \in N \backslash \{o, d\} \quad \forall m \in S_j \quad \forall n \in S_j \tag{26}$$

Above all, we can obtain an FCCP model by replacing Equations (13) and (14) with Equations (25) and (26). This model is solvable since it removes the fuzziness. However, it is still a nonlinear optimization model due to the nonlinearity of Equations (9) and (10). To further improve the efficiency of finding the global optimum solution for the MRPPDTWCU, this study linearizes the two nonlinear constraints that use continuous piecewise linear functions as follows [13,16,27]:

$$w_o \geq z_o - tw_o^* \tag{27}$$

$$w_d \geq tw_d - z_d \tag{28}$$

Although the two equivalent linear equations are not straightforwardly based on the definitions of the variables, they can significantly improve the computational efficiency of problem-solving [40]. Finally, after the model processing, we present a CCLP model that takes Equation (2) as the objective and covers Equations (3)–(8), (10), (11), (15)–(20) and (25)–(28) as the constraints. The global optimum solution to the problem can be obtained by exact solution algorithms.

5. Numerical Case Verification

This section presents a numerical case in the Chinese scenario to verify the proposed model. The multimodal network presented in Sun and Lang's work [41] has been commonly modified and further used to carry out case verification (e.g., Li et al. [13] and Xiong and Wang [42]) and provide a foundation for us to design the case in this study. The multimodal network used to serve the container transportation from the shipper to the receiver is shown in Figure 1. The travel distances of the rail, road, and water transportation on the arcs are set as those given by Sun and Lang [41].

The travel costs and time of the three transportation modes are listed in Table 1, and the transfer time and costs between these transportation modes are presented in Table 2.

According to the specific MRPPDTWCU, we assume that the capacities in Sun and Lang [41] are the mean values of the LRTFCs of the multimodal network in this numerical case, which also makes the difference in the numerical case design between this study and Li et al. [13] more significant. The MTO serves a transportation order whose shipper is in Node 1 and receiver is in Node 35. The transportation volume of containers is 40 TEU. The pickup time window of the shipper is from 8:00 to 12:00 on day 1, and the delivery time window of the receiver is from 21:00 on day 2 to 3:00 on day 3. The storage costs at the origin and the destination are CNY 10/TEU·h and CNY 20 /TEU·h, respectively.

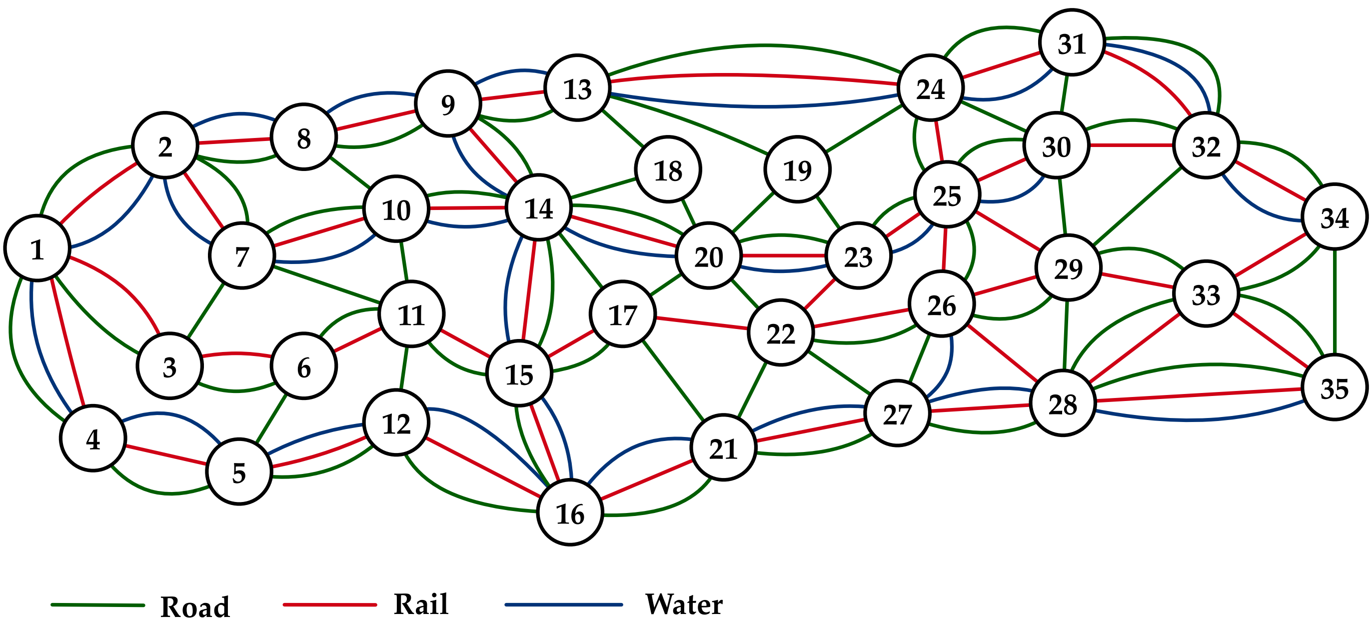

Figure 1. Multimodal network consisting of 35 nodes and 136 arcs [13,41,42].

Table 1. Travel costs and time of the transportation modes [13].

Transportation Modes	Travel Costs (CNY/TEU)	Travel Time (h)
Rail	$c_{ijm} = 500 + 2.03 \cdot d_{ijm}$	$t_{ijm} = d_{ijm}/60$
Road	$c_{ijm} = 15 + 8 \cdot d_{ijm}$	$t_{ijm} = d_{ijm}/80$
Water	$c_{ijm} = 950$	$t_{ijm} = d_{ijm}/30$

Table 2. Transfer time and costs between different transportation modes [13,41].

Transfer Types	Transfer Time (h/TEU)	Transfer Costs (CNY/TEU)
Rail~Road	0.067	5
Rail~Water	0.133	7
Road~Water	0.100	10

We assume that the left hand spread ratio is 20%, i.e., the spread is 20% of the mean values of the LRTFCs of the multimodal network. The shipper and the receiver prefer high reliability of the transportation of their containers and determine that the confidence level should be 0.9. In this case, we use Lingo software to run the branch-and-bound algorithm to solve the CCLP model. We obtain the optimal route "1-rail-3-road-6-rail-11-road-15-rail-16-road-21-rail-27-rail-28-road-35" and the container pickup time at Node 1 that is 11:33 on day 1. The total costs of using this route are CNY 289,341. The total costs consist of the travel costs of CNY 287,941 and transfer costs of CNY 1400, which means that the pickup and delivery of the containers are both within the corresponding time windows, and the optimal route can achieve on-time transportation. In this case, the pickup and delivery time windows have the same function as hard time windows. When the left hand spread ratio is 5% and the confidence level is 1.0, the optimal route with pickup and delivery time windows formulated in this study yields total costs of CNY 269,820, including the storage costs of CNY 6374. If the pickup and delivery time windows are the hard ones, the total costs of the optimal route are CNY 280,537 without any storage costs. Compared to the time windows designed in this study, the hard time windows increase the total costs of the optimal route by approximately 4%. Therefore, under this situation, the pickup and delivery time windows in this study can achieve a balance between the travel and transfer costs and the storage costs to minimize the total costs consisting of the above three parts.

In the numerical case, we further analyze how the spread ratio and confidence level influence the MRPPDTWCU. First of all, we set the left hand spread ratio as 25% and obtained the optimization results of the MRPPDTWCU under the values of the confidence level from 0.5 to 1.0 with a step of 0.1. The results are indicated in Figure 2.

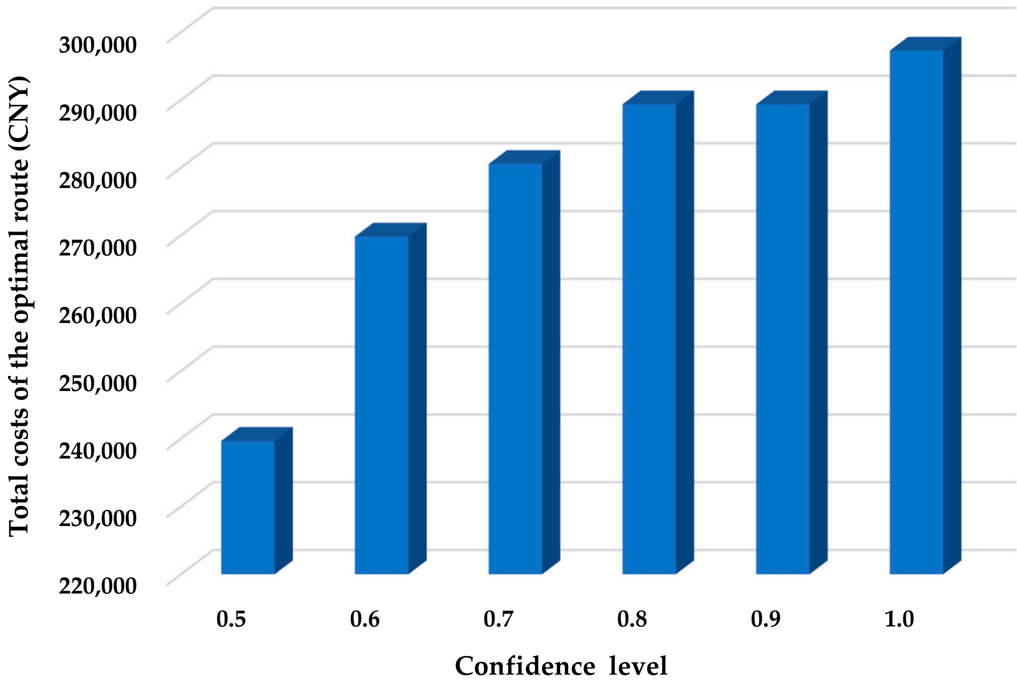

Figure 2. Sensitivity of the MRPPDTWCU with respect to the confidence level.

As we can see in Figure 2, with the enhancement of the confidence level, the total costs of the optimal route increase, because the increase in confidence level makes the chance constraints (i.e., Equations (25) and (26)) tighter, and further suppresses the feasible solution space of the MRPPDTWCU. Consequently, the best solution given under a smaller confidence level might be found to be infeasible by the model under a larger confidence level. Improving the confidence level helps to make the optimal route more reliable in practical transportation. Therefore, the economy and reliability of the MRPPDTWCU are conflicting. The decision-makers in multimodal transportation including the MTO, shipper, and receiver should make tradeoffs between the two objectives. The sensitivity shown in Figure 2 can provide a reference for them to make a decision.

The majority of the relevant studies consider multimodal routing under capacity certainty. These studies prefer to use the mean values of the LRTFNs that represent the most likely conditions to formulate the deterministic capacities [8]. In this case, the optimal route under capacity certainty in the numerical case yields total costs of CNY 239,668, which equals the one under fuzzy capacity constraints when the confidence level is 0.5. However, the modeling of MRP under capacity uncertainty using the methods proposed by this study can provide diverse solutions that depend on the shipper, receiver, and MTO's selection of the confidence level based on their attitudes. The solutions also include the one under capacity certainty that yields a low credibility, which means a low reliability in practical transportation. Consequently, compared to the deterministic modeling, modeling the MRP under capacity uncertainty can provide the shipper, receiver, and MTO with more optimal route schemes and thereby enable their decision-making to be more flexible, in which it is possible to make tradeoffs between lowering the costs and improving the reliability.

Then we analyze the influence of the left hand spread ratio on the MRPPDTWCU. In this case, we set the confidence level as 0.9 and present the optimization results of the

MRPPDTWCU under different left hand spread ratios from 5% to 30% with a step of 5%. The results are shown in Figure 3.

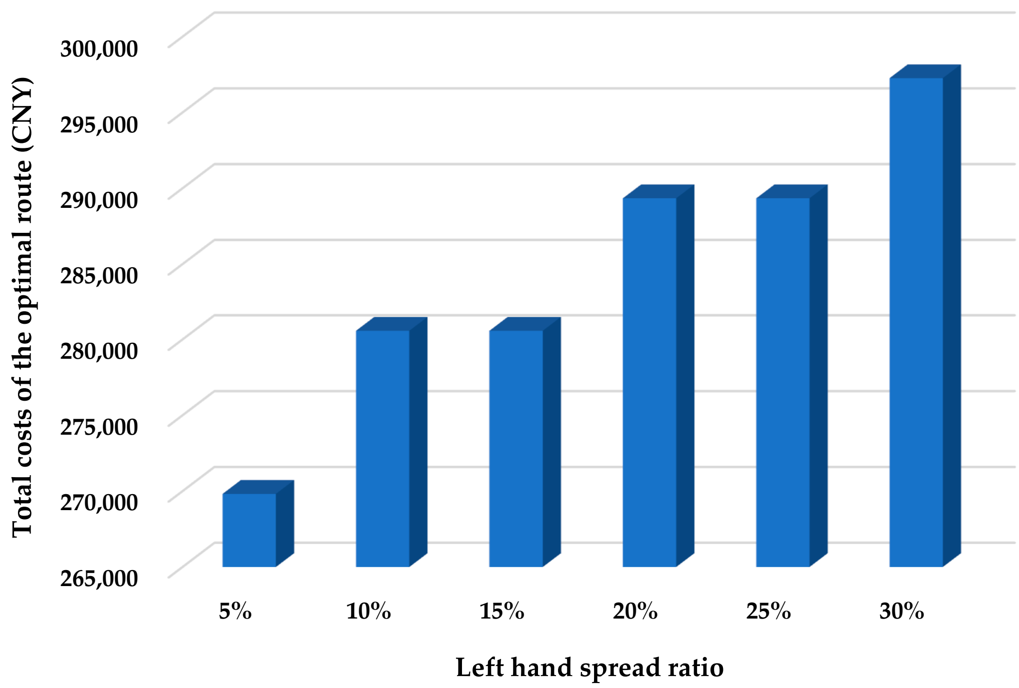

Figure 3. Sensitivity of the MRPPDTWCU with respect to the left hand spread ratio.

Figure 3 clarifies that improving the spread ratio increases the total costs of the optimal route, which shows a similar change to the variation of the total costs caused by the increase in the confidence level. The reason resulting in the sensitivity shown in Figure 3 is the same as that leading to the variation illustrated by Figure 2, i.e., improving the left hand spread ratio increases α_{ijm} and α_j^{mn}, thereby making the chance constraints tighter and the feasible solution space smaller, and finally increasing the objective values of the optimization model. Since the spread ratio indicates the uncertainty level of the LRTFCs of the multimodal network, Figure 3 demonstrates that the shipper and the receiver need to prepare more budget for a more costly multimodal route to deal with the increasing capacity uncertainty of the multimodal network, so that their containers can be transported successfully from origin to destination.

Additionally, we change the left hand spread ratio from 5% to 30% with a step of 5% and increase the confidence level from 0.5 to 1.0 with a step of 0.1. Then, we use the proposed model to generate the optimal routes under different combinations of the spread ratio and the confidence level. The results are illustrated in Figure 4. It should be noted that there is no feasible solution to the MRPPDTWCU when the left hand ratio is 30% and the confidence level is 1.0.

Figure 4 verifies the sensitivities shown in Figures 2 and 3. Furthermore, Figure 4 shows that the degree of increase in total costs of the optimal route caused by improving the confidence level is reduced when the left hand spread ratio becomes smaller, which is clearly illustrated by Figure 5. In Figure 5, the gap is calculated by Equation (29) where $\text{cost}_{1.0}$ and $\text{cost}_{0.5}$ are the total costs of the optimal route when the confidence level is 0.5 and 1.0, respectively.

$$gap = \frac{\text{cost}_{1.0} - \text{cost}_{0.5}}{\text{cost}_{0.5}} \cdot 100\% \quad (29)$$

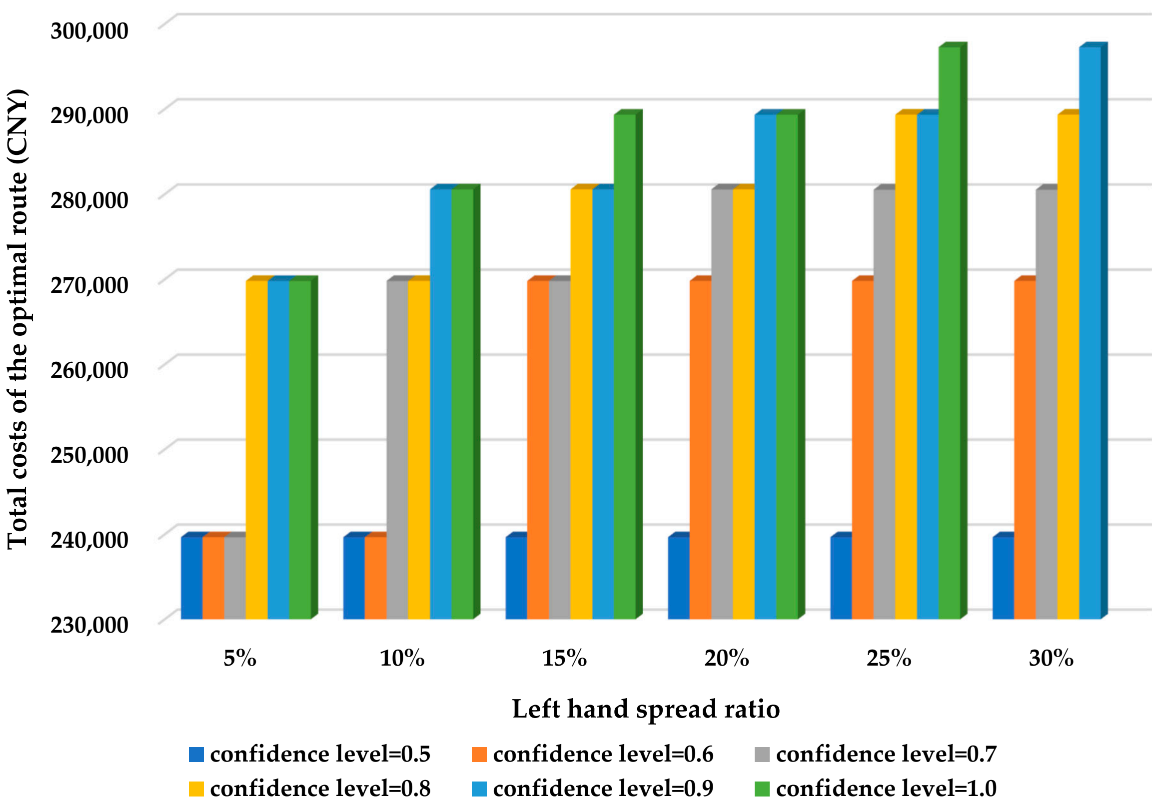

Figure 4. Sensitivity of the MRPPDTWCU with respect to the combination of the left hand spread ratio and the confidence level.

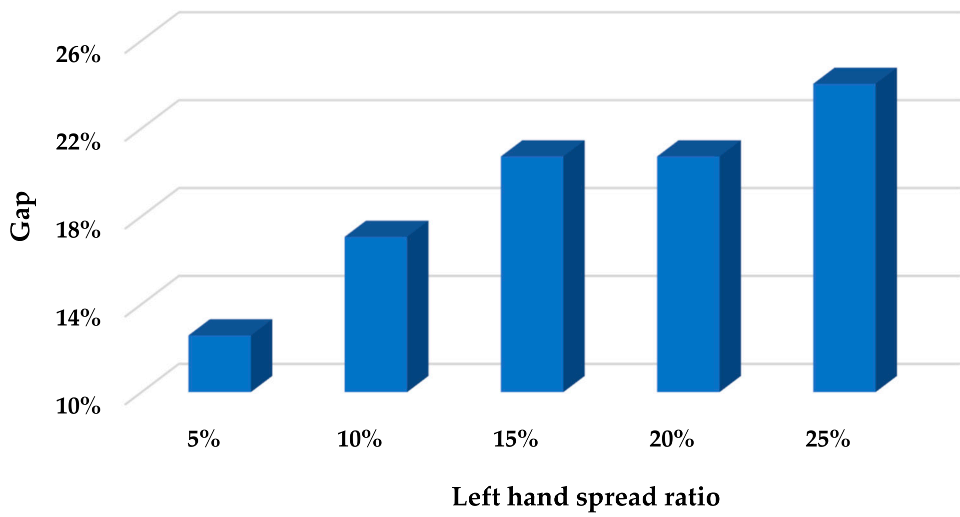

Figure 5. Gaps between the total costs of the optimal routes under confidence levels of 0.5 and 1.0.

Figures 4 and 5 indicate that if a more reliable multimodal route is needed and meanwhile the shipper and the receiver would like to reduce the budget prepared for the multimodal transportation, the MTO needs to select the transportation services and transfer services with smaller uncertainty levels of fuzzy capacities, i.e., the transportation services and transfer services with stable capacities.

6. Conclusions

This study explores a MRPPDTWCU. The modeling of pickup and delivery time windows aims to improve the timeliness of the entire multimodal transportation process from its beginning to its end. The capacity uncertainty is modeled by LRTFNs, which enables the uncertainty level of the fuzzy capacity of the multimodal network to be measurable. Under the above consideration, we propose a fuzzy nonlinear optimization model to address the MRPPDTWCU and design an equivalent CCLP reformulation based on the FCCP method and linearization processing. A numerical case verifies that the proposed model yields high efficiency in obtaining the global optimum solution to the MRPPDTWCU. The findings of the numerical case using the proposed model are:

(1) The modeling of pickup and delivery time windows is able to provide both the shipper and the receiver with on-time transportation services. Such a consideration also enables the shipper, receiver, and MTO to make a balance between the travel and transfer costs and the storage costs to realize the cost minimization.
(2) Compared to the deterministic modeling, the MRP under capacity uncertainty enables the shipper, receiver, and MTO to make more flexible decisions, in which they can make tradeoffs between the economy and reliability of transportation.
(3) The capacity uncertainty shows a significant influence on the MRPPDTWCU from two aspects. The first is the confidence level that is introduced into the MRPPDTWCU by the FCCP and reflects the reliability of transportation, and the second is the uncertainty level of the fuzzy capacities.
(4) Improving the confidence level to achieve a reliable multimodal route scarifies the transportation economy. Therefore, the shipper and the receiver should make tradeoffs between the reliability and economic objectives by determining a suitable confidence level, in which the sensitivity shown in Figure 2 can provide a solid reference. Then, the MTO can plan the optimal route based on their demand using the proposed model.
(5) To address the higher uncertainty level of the fuzzy capacity, the shipper and the receiver need to increase their transportation budget. To help them reduce the budget and meanwhile maintain a high confidence level to ensure reliable transportation, the MTO needs to use transportation services and transfer services with stable capacities.

In practical transportation, the shipper and the receiver first negotiate to determine their respective time windows and the transportation volume of the containers, and then propose their transportation order to the MTO. After receiving the transportation order, the MTO establishes the multimodal network for container transportation preferentially using the transportation services and transfer services with relatively stable capacities. The sensitivity analysis can help the shipper and the receiver to determine the confidence level based on their attitudes toward the reliability of transportation and the budget that they can afford for multimodal transportation. After setting the above issues, the MTO can use the proposed CCLP model to find the optimal route that yields the minimum costs to move the containers from the shipper to the receiver, in which the combination of economy and timeliness of multimodal transportation gets optimized.

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Data Availability Statement: All data used in this study are included in this paper and can be obtained from the cited reference [41].

Conflicts of Interest: The authors declare no conflicts of interest.

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