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Article

Generalization of Fermatean Fuzzy Set and Implementation of Fermatean Fuzzy PROMETHEE II Method for Decision Making via PROMETHEE GAIA

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Abstract: The Fermatean fuzzy set, in contrast to other generalizations of fuzzy sets like PFS and IFS, has a wide range of acceptance for both MF and NMF. In light of this, the Fermatean fuzzy set performs as an efficient, flexible, and comprehensive representation in situations that lack certainty. Here, the weaker forms of Fermatean fuzzy sets are introduced, and their traits are analyzed. Decomposition and continuity of the Fermatean fuzzy α-open set are also accustomed. With the goal of safeguarding our green environment, hiring the best supplier is of the utmost significance in the construction industry. Using outranking techniques, Visual PROMETHEE Academic Edition 1.4 is a live multi-criteria decision aid software program. It runs virtual analysis through GAIA and applies selected criteria to contrast parameters. It also saves them for possible export and editing. In this article, the PROMETHEE II method is applied for Fermatean fuzzy numbers with $FF(\alpha, \beta)$-level for selecting the optimal green supplier for a construction company. Because of its ability to handle vagueness, the FF PROMETHEE II method emerges as a valuable tool in Multi-criteria decision making. Furthermore, this study assesses the efficacy of the proposed technique by comparing its results with those obtained through other established methods.

Keywords: Fermatean fuzzy pre-open sets; Fermatean fuzzy semi-open sets; Fermatean fuzzy α-open sets; Fermatean fuzzy (α, β)-level; Fermatean fuzzy α-continuous function; green supplier selection; Fermatean fuzzy PROMETHEE II method; Visual PROMETHEE GAIA software

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1. Introduction

Although boundaries are accurate and clearly defined in classical set theory, they are frequently uncertain and hazy in practical situations. In 1965, Zadeh L A [1] defined the fuzzy set (FS) which is an extension of classical set theory, a potential tool to handle impressions and uncertainties using the membership function (MF), and a replacement of the characteristic function that accepts values ranging from 0 to 1. FS theory can deal only with MF, but it fails to address the non-membership function (NMF).

The paradigm of IFS was first explored by Atanassov K T [2] in 1986, which deals with both MF and NMF. In IFS theory, both MF and NMF take values in the range 0 to 1, individually with the restriction that their sum is between 0 and 1, which need not be in all cases of real-life situations. For such circumstances, the Pythagorean fuzzy set (PFS) introduced in 2013 by Yager R R [3] can be applied, in which the sum of the squares of MF and the NMF value is between 0 and 1, which extends the range of MF and NMF and he [4] also studied Pythagorean membership grades, complex numbers, decision making. Also the properties and applications of PFSs are investigated in [5].

The conception of the Fermatean fuzzy set (FFS) was naturalized in 2019 by Senapati T and Yager R R [6] in which the sum of the cubes of MF and NMF lies between 0 and 1, which extends the region of acceptance even more than IFS and PFS; hence, the uncertainty in a decision maker's opinion can be handled precisely.

The topological structures of FS, IFS, PFS, and FFS were introduced by Chang C L [7], Coker D [8], Olgun M [9], and Ibrahim H Z [10] in 1968, 1997, 2019, and 2022, respectively. Jeon J K et al. [11] studied alpha and pre-continuous functions in IFT. Ajay D [12] investigated alpha continuity in PFT. Njastad O [13] investigated some classes of nearly open sets. In this article, FF α-open set(FFαOS) and α-continuous functions (FFαCF) are introduced and their associated characteristics are addressed.

A predominant area of decision theory is MCDM. The MCDM frequently deals with deciding an enormous number of evident alternatives owing to several contradiction criteria, from best to worst. The ideas and techniques that can tackle complex challenges in the business department, science and engineering, management, and various other sectors of human attempts are under the supervision of the MCDM. Brans J P [14] developed a partial and complete ranking of the alternatives using PROMETHEE I and PROMETHEE II methods in 1984. By adopting PROMETHEE parameters, each decision maker is able to choose what specifications they like most. Without any valuable data loss, the PROMETHEE partial ranking will provide a calculation of positive and negative flows. PROMETHEE II has been applied in the context of dissimilarity to perform a thorough rating. Xu D [15] introduced new method based on PROMETHEE and TODIM for multi-attribute decision making with single valued neutrosophic sets, Arcidiacono [16] implemented GAIA-SMAA-PROMETHEE, Mareschal B [17] applied developments of the PROMETHEE and GAIA MCDM, and Rehman [18] used Visual PROMETHEE in solar panel cooling system evaluation. Janusonis [19] used IF PROMETHEE for MCDM. Durna [20] applied the treatment method by PROMETHEE and Cankaya [21] used PROMETHEE to choose the most suitable option for MFCs that have many electron acceptor designs.

The exploration and utilization of FS theories, including FFSs, in the context of MCDM offers promising avenues for addressing uncertainty and confusion. Kalaichelvan K [22] used fuzzy theory and machine learning for optimizing the economic order quantity. Farid H M A [23] applied the FF CODAS (Combinative Distance Based Assessment) approach to sustainable supplier selection, Gul M [24] used the FF TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) approach for occupational risk assessment in manufacturing, Mishra A R [25] used FF copula aggregation operators and similarity measures based complex proportional assessment approach for renewable energy source selection, Moreira M P [26] used PROMETHEE and fuzzy PROMETHEE multi criteria methods for ranking equipment failure modes, Zeng S [27] introduced integrated FF EDAS (Evaluation Based on Distance from Average Solution) in green supplier selection, and Akram M [28] used the Extended COPRAS method with linguistic FFSs and Hamy mean operators. Regarding other currently available multi-criteria analysis techniques for implementation, the PROMETHEE II method is a substantially simpler concept and is more user-friendly to apply to real-life problems. However, despite the advancements in fuzzy set theories and MCDM techniques, there exists a research gap in the integration and application of FFSs within PROMETHEE II method frameworks, particularly in the context of supplier selection and green construction evaluation. We employ the FF PROMETHEE II method because of its primary strengths in minimizing scaling impacts between criteria and its simplicity of usage for mathematical computations compared to other MCDM methods.

Before this study, the authors have gone through the various case studies conducted in decision making using the FF Bonferroni mean operator [29], the FF t-Norm and t-Conorm operators [30], and the FF Similarity Measure [31]. Kuppusamy V [32] applied a bipolar Pythagorean fuzzy approach to the decision problem. Keshavarz-Ghorabaee M et al. [33] used the FF WASPAS method for green construction supplier evaluation and have compared their results with methods like TOPSIS [34], EDAS [35], COPRAS, and Classic WASPAS [36]. Identifying the optimum supplier among the available suppliers is discussed in [33]. In order to successfully control vagueness and unpredictability, particularly when it comes to the evaluation of suppliers for sustainable construction, this research has been motivated by the need to address the shortcomings of the current approaches and important aspects of the PROMETHEE II method approach. It is of great interest to explore the integration of FFS in MCDM frameworks due to its development as an effective tool for expressing ambiguity and uncertainty. By integrating FFS into MCDM models, it becomes possible to capture and represent decision makers' subjective judgments more accurately, leading to improved decision outcomes and better-informed selections of green construction suppliers.

This research contributes to the corpus of knowledge by attempting to put forth a novel approach for MCDM in the selection of green construction suppliers that integrates FFS into the PROMETHEE II procedure. The key contributions of this research include the following:

- FFS generalized forms like FF pre-open, FF semi-open, FFα-open, and FF(α, β)-level are introduced and their relations are investigated. The FFα continuous function is also defined and its properties are studied.
- Development of a comprehensive framework for utilizing Fermatean fuzzy sets within the PROMETHEE II method, addressing the limitations of traditional FS theories in handling uncertainty.
- Investigation of the effectiveness and applicability of the FF PROMETHEE II method in the context of green construction supplier evaluation, considering both environmental sustainability and performance criteria.
- Evaluation of the proposed approach through comparative analysis with existing MCDM methods and Visual PROMETHEE Software Academic Edition 1.4, and demonstrating its advantages in addressing uncertainty and improving decision outcomes.
- Identification of practical implications and recommendations for implementing the FF PROMETHEE II method in decision-making scenarios, particularly in the instances of sustainable construction practices.

Overall, this research contributes to advancing the understanding and application of FFS in MCDM, offering valuable insights for researchers, practitioners, and decision makers involved in green construction supplier selection and sustainability assessment.

The flow of the article is as follows: The forthcoming section gives preliminary ideas. FFαOS and FFαCF are discussed in Section 3. The FF PROMETHEE II method is established in Section 4. Green construction supplier evaluation is explained in Section 5 with sensitivity analysis and comparison analysis. Finally, the article ends with Section 6, which gives a conclusion, limitations, and future work.

2. Preliminaries

Some of the key ideas utilized in this study are provided in this section.

Definition 1 ([1])**.** *A set* $\Gamma = \{\langle x, \mu_\Gamma(x)\rangle : x \in X\}$ *in the universe of discourse* X *is called FS where* $\mu_\Gamma(x) : X \to [0,1]$ *is a MF of* x *in* Γ.

Definition 2 ([6])**.** *A set* $\Gamma = \{\langle x, \mu_\Gamma(x), \nu_\Gamma(x)\rangle : x \in X\}$ *in the universe of discourse* X *is called FFS if* $0 \leq (\mu_\Gamma(x))^3 + (\nu_\Gamma(x))^3 \leq 1$. $\mu_\Gamma(x) : X \to [0,1]$, $\nu_\Gamma(x) : X \to [0,1]$, *and*

$\pi = \sqrt[3]{1 - (\mu_\Gamma(x))^3 - (\nu_\Gamma(x))^3}$ *are MF, NMF, and indeterminacy of x in* Γ. *The pair* $\Gamma = (\mu_\Gamma, \nu_\Gamma)$ *is called the FF number (FFN). The complement and the score of* Γ *are defined as* $\Gamma^c = (\nu_\Gamma, \mu_\Gamma)$ *and* $Score(\Gamma) = \mu_\Gamma^3 - \nu_\Gamma^3$.

Definition 3 ([6]). *If* $\Gamma_1 = (\mu_{\Gamma_1}, \nu_{\Gamma_1})$ *and* $\Gamma_2 = (\mu_{\Gamma_2}, \nu_{\Gamma_2})$ *are two FFSs then* $\Gamma_1 \cup \Gamma_2 = (max\{\mu_{\Gamma_1}, \mu_{\Gamma_2}\}, min\{\nu_{\Gamma_1}, \nu_{\Gamma_2}\})$ *and* $\Gamma_1 \cap \Gamma_2 = (min\{\mu_{\Gamma_1}, \mu_{\Gamma_2}\}, max\{\nu_{\Gamma_1}, \nu_{\Gamma_2}\})$.

Definition 4 ([10]). *The pair* $(\mathcal{F}, \tau)$ *is said to be a FF topological space (FFTS) if*

1. $1_\mathcal{F} \in \tau$
2. $0_\mathcal{F} \in \tau$
3. *for any* $\Gamma_1, \Gamma_2 \in \tau$ *we have* $\Gamma_1 \cup \Gamma_2 \in \tau$
4. $\Gamma_1 \cap \Gamma_2 \in \tau$ *where* τ *is the family of FFS of non-empty set* $\mathcal{F}$.

The members of τ *and* τ^c *are called FFOS and FFCS. The union of all FFOSs contained in* $\Gamma \subseteq \mathcal{F}$ *and the intersection of all FFCSs containing* Γ *are called the FF interior of* Γ *(referred to by* $int(\Gamma)$*) and FF closure of* Γ *(referred by* $cl(\Gamma)$ *), respectively.*

Definition 5 ([10]). *A FFS* Γ *is a FFOS in a FFTS iff it contains a neighborhood of its each subset.*

Methodology: PROMETHEE II
Step 1: Determine the set of criteria with their weights w_i and alternatives to form a decision matrix.
Step 2: Normalize the decision matrix using the appropriate rule.
Step 3: Determine the pairwise comparison.
Step 4: Define the preference function.
Step 5: Determine the MCDM preference index.
Step 6: Calculate the net outranking flow using the entering outranking flow and the leaving outranking flow.
Step 7: Ranking of alternatives from higher to lower net flow value.

3. Fermatean Fuzzy α-Open Set and Fermatean Fuzzy α-Continuous Function

In this section, we introduce the generalization of FF set and $FF\alpha C$ function in FFTS.

Definition 6. *A FFS* $\Gamma = (\mu_\Gamma, \nu_\Gamma)$ *of a FFTS* $(\mathcal{F}, \tau)$ *is*

1. *a FF semi-open set* $(FFSOS)$ *if* $\Gamma \subseteq cl(int(\Gamma))$.
2. *a FF pre-open set* $(FFPOS)$ *if* $\Gamma \subseteq int(cl(\Gamma))$.
3. *a FF* α*-open set* $(FF\alpha OS)$ *if* $\Gamma \subseteq int(cl(int(\Gamma)))$.

Their complements are called FF semi-closed set $(FFSCS)$*, FF pre-closed set* $(FFPCS)$ *and FF* α *closed set* $(FF\alpha CS)$.

Definition 7. *Let* Γ *be a FFαOS in a FFTS* $(\mathcal{F}, \tau)$. *Then, the FF* α*-closure of* Γ *indicated by* $cl_\alpha(\Gamma)$ *is the intersection of all FF* α*-closed supersets of* Γ *and the FF* α*-interior of* Γ *indicated by* $int_\alpha(\Gamma)$ *is the union of all FF* α*-open subsets of* Γ.

Theorem 1. *The interior of a non-empty FFαOS of a FFTS is non-empty.*

Proof. Consider the FFαOS Γ of a FFTS with $int(\Gamma) = \emptyset$. By the definition of $FF\alpha OS$, $\Gamma \subseteq \emptyset$ which implies $\Gamma = \emptyset$ which is a contradiction. Therefore, $int(\Gamma) \neq \emptyset$. □

Theorem 2. *A FFS* Γ *of a FFTS* $(\mathcal{F}, \tau)$ *is a FFαOS iff* Γ *is FFSOS as well as FFPOS.*

Proof. The necessary part of the proof is obvious. Conversely, let Γ be both $FFSOS$ and $FFPOS$. Then, $\Gamma \subseteq cl(int(\Gamma))$ and $\Gamma \subseteq int(cl(\Gamma))$ confers $\Gamma \subseteq int(cl(int(\Gamma)))$. Hence, Γ is $FF\alpha OS$. □

Theorem 3. *Γ_2 is a FFαOS when Γ_1 and Γ_2 are two FFSs in a FFTS $(\mathcal{F}, \tau)$ such that $\Gamma_1 \subseteq \Gamma_2 \subseteq int(cl(int(\Gamma_1)))$.*

Proof. $\Gamma_1 \subseteq \Gamma_2$ imparts $int(cl(int(\Gamma_1))) \subseteq int(cl(int(\Gamma_2)))$. Hence, $\Gamma_2 \subseteq int(cl(int(\Gamma_2)))$. Therefore, Γ_2 is a *FFαOS*. □

Theorem 4. *Let Γ be a FFS in a FFTS $(\mathcal{F}, \tau)$. If Ω is a FFSOS such that $\Omega \subseteq \Gamma \subseteq int(cl(\Omega))$, then Γ is a FFαOS.*

Proof. The proof is obvious from the definition of *FFαOS* and *FFSOS*. □

Theorem 5. *If Γ is a FFS and Ω is a FFαOS in FFTS $(\mathcal{F}, \tau)$ then $\Omega \cap cl_\alpha(\Gamma) \subseteq cl_\alpha(\Gamma \cap \Omega)$.*

Lemma 1. *Any arbitrary union of FFαOS is a FFαOS.*

Proof. Let $\{\Gamma_i, i \in I\}$ be a set of *FFαOS*. Then, $\Gamma_i \subseteq int(cl(int(\Gamma_i)))$ implies $\bigcup \Gamma_i \subseteq \bigcup(int(cl(int(\Gamma_i)))) \subseteq (int(cl(int(\bigcup \Gamma_i))))$. Hence, $\bigcup \Gamma_i$ is a *FFαOS*. □

Theorem 6. *If Γ is a FFOS and Ω is a FFαOS in a FFTS $(\mathcal{F}, \tau)$, then $\Gamma \cap \Omega$ is a FFαOS in X.*

Proof. By the assumption, $\Gamma \cap \Omega \subseteq int(\Gamma) \cap int(cl(int(\Omega))) = int(cl(int(\Gamma)) \cap int(\Omega)) \subseteq int(cl(int(\Gamma) \cap int(\Omega))) = int(cl(int(\Gamma \cap \Omega)))$. Hence, $\Gamma \cap \Omega$ is *FFαOS* in X. □

Theorem 7. *A FFS Γ is FFαOS iff there is a FFOS Ω so that $\Omega \subseteq \Gamma \subseteq int(cl(\Omega))$.*

Proof. Since Γ is a *FFαOS*, $\Gamma \subseteq int(cl(int(\Gamma)))$. If $\Omega = int(\Gamma)$ then $\Omega \subseteq \Gamma \subseteq int(cl(\Omega))$. The converse is obvious. □

Theorem 8. *For any FFS Γ of a FFTS $(\mathcal{F}, \tau)$, the following are equivalent: 1. Γ is a FFαCS. 2. $cl(int(cl(\Gamma))) \subseteq \Gamma$. 3. There exists a FFCS K with $cl(int(K)) \subseteq \Gamma \subseteq K$.*

Proof. (1) $\implies$ (2). Since Γ is a *FFαCS*, Γ^c is a *FFαOS*. So, $\Gamma^c \subseteq int(cl(int(\Gamma^c)))$ and, hence, $cl(int(cl(\Gamma))) \subseteq \Gamma$.
(2) $\implies$ (3). $cl(int(cl(\Gamma))) \subseteq \Gamma$ implies $cl(int(cl(\Gamma))) \subseteq \Gamma \subseteq cl(\Gamma)$. Taking $K = cl(\Gamma)$ there exists a FFCS K such that $cl(int(K)) \subseteq \Gamma \subseteq K$.
(3) $\implies$ (1). Taking the complement for the assumption, $K^c \subseteq \Gamma^c \subseteq int(cl(K^c))$. Therefore, Γ is a *FFαCS*. □

Example 1. *Let $\mathcal{F} = \{1, 2\}$ and $\tau = \{0_\mathcal{F}, 1_\mathcal{F}, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4\}$ where $\Gamma_1 = \{(1, 0.7, 0.8), (2, 0.9, 0.6)\}$, $\Gamma_2 = \{(1, 0.6, 0.7), (2, 0.8, 0.6)\}$, $\Gamma_3 = \{(1, 0.6, 0.8), (2, 0.8, 0.6)\}$ and $\Gamma_4 = \{(1, 0.7, 0.7), (2, 0.9, 0.6)\}$ are FFS. Then, $(\mathcal{F}, \tau)$ is a FFTS on which the FFS $\Gamma = \{(1, 0.69, 0.63), (2, 0.9, 0.6)\}$ is FFSOS, FFPOS, and FFαOS.*

Example 2. *Let $\mathcal{F} = \{1, 2\}$ and $\tau = \{0_\mathcal{F}, 1_\mathcal{F}, \Gamma_1, \Gamma_2\}$ be the FFT with FFOS $\Gamma_1 = \{(1, 0.3, 0.7), (2, 0.4, 0.6)\}$ and $\Gamma_2 = \{(1, 0.9, 0.6), (2, 0.8, 0.3)\}$. Then, the FFS $\Gamma = \{(1, 0.4, 0.4), (2, 0.5, 0.5)\}$ is FFSOS but not FFPOS, FFαOS, and FFOS, and the FFS $G = \{(1, 0.8, 0.7), (2, 0.7, 0.5)\}$ is FFPOS but not FFSOS, FFαOS, and FFOS.*

Figure 1 and Example 2 demonstrate that *FFSOS* and *FFPOS* are independent of each other. *FFαOS* is the set of sets which are both *FFSOS* and *FFPOS*. The *FFOS* is a subset of *FFαOS*; that is, every *FFOS* is *FFαOS* but the converse is not possible.

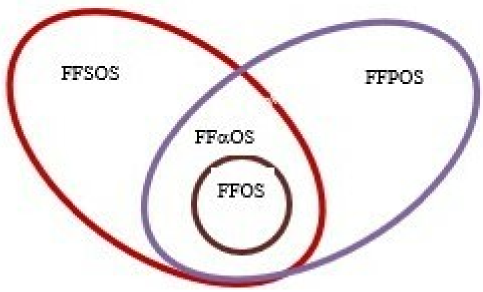


Figure 1. Decomposition of FFS.

Definition 8. *A FFS N of a FFTS $(\mathcal{F},\tau)$ is called a FFα-neighborhood of a FFS Γ of $\mathcal{F}$ if there exists a FFαOS, Ω such that $\Gamma \subseteq \Omega \subseteq N$.*

Definition 9. *A FF point $A^x_{(r_1,r_2)}$ of a FFTS $(\mathcal{F},\tau)$ is called a FFα-interior point of Γ if there is FFαOS, Ω containing $A^x_{(r_1,r_2)}$ such that $A^x_{(r_1,r_2)} \in \Omega \subseteq \Gamma$. The set of all FFα-interior points of Γ is said to be $int_\alpha(\Gamma)$.*

Theorem 9. *A FFS Γ in a FFTS $(\mathcal{F},\tau)$ is a FFαOS iff for every FF point $A^x_{(r_1,r_2)} \in \Gamma$ there is a FFαOS, Ω such that $A^x_{(r_1,r_2)} \in \Omega \subseteq \Gamma$.*

Proof. The necessary part is obvious by Definition 9. Conversely, if, for every FF point $A^x_{(r_1,r_2)} \in \Gamma$, there is a $FF\alpha OS$, Ω such that $A^x_{(r_1,r_2)} \in \Omega \subseteq \Gamma$ then, by Lemma 1, Γ is a $FF\alpha OS$. □

Proposition 1. *Let $(\mathcal{F},\tau)$ be a FFTS and Γ and Ω be any two FFS $\mathcal{F}$. Then,*

1. $int_\alpha(0_\mathcal{F}) = 0_\mathcal{F}$ *and* $int_\alpha(1_\mathcal{F}) = 1_\mathcal{F}$.
2. $int(\Gamma) \subseteq int_\alpha(\Gamma)$, *if* Γ *is a FFαOS.*
3. *If* $\Gamma \subseteq \Omega$ *then* $int_\alpha(\Gamma) \subseteq int_\alpha(\Omega)$.
4. $int_\alpha(\Gamma) \cup int_\alpha(\Omega) \subseteq int_\alpha(\Gamma \cup \Omega)$.
5. $int_\alpha(\Gamma \cap \Omega) \subseteq int_\alpha(\Gamma) \cap int_\alpha(\Omega)$.
6. $int_\alpha(\Gamma \setminus \Omega) \subseteq int_\alpha(\Gamma) \setminus int_\alpha(\Omega)$.

Proof. 1. Obvious.
2. We have $int(\Gamma) \subseteq \Gamma$. Since Γ is a $FF\alpha OS$, $\Gamma = int_\alpha(\Gamma)$. Therefore, $int(\Gamma) \subseteq int_\alpha(\Gamma)$.
3. Let $\Gamma \subseteq \Omega$ and $x \in int_\alpha(\Gamma)$. Then, Γ is a $FF\alpha$-neighborhood and so is Ω, which implies x is a $FF\alpha$-interior point of Ω. Therefore, $x \in int_\alpha(\Omega)$. Hence, $int_\alpha(\Gamma) \subseteq int_\alpha(\Omega)$.
4. $\Gamma \subseteq \Gamma \cup \Omega$ implies $int_\alpha(\Gamma) \subseteq int_\alpha(\Gamma \cup \Omega)$ and $\Omega \subseteq \Gamma \cup \Omega$ implies $int_\alpha(\Omega) \subseteq int_\alpha(\Gamma \cup \Omega)$. Therefore, $int_\alpha(\Gamma) \cup int_\alpha(\Omega) \subseteq int_\alpha(\Gamma \cup \Omega)$.
5. $\Gamma \cap \Omega \subseteq \Gamma$ implies $int_\alpha(\Gamma \cap \Omega) \subseteq int_\alpha(\Gamma)$ and $\Gamma \cap \Omega \subseteq \Omega$ implies $int_\alpha(\Gamma \cap \Omega) \subseteq int_\alpha(\Omega)$. Hence, $int_\alpha(\Gamma \cap \Omega) \subseteq int_\alpha(\Gamma) \cap int_\alpha(\Omega)$.
6. Let $x \in int_\alpha(\Gamma \setminus \Omega)$. There exists $FF\alpha OS$, U such that $x \in U \subseteq \Gamma \setminus \Omega$. That is, $U \subseteq \Gamma$ and $U \cap \Omega = \phi$ so $x \notin \Omega$. Hence, $x \in int_\alpha(\Gamma)$ and $x \notin int_\alpha(\Omega)$. Therefore, $int_\alpha(\Gamma) \setminus int_\alpha(\Omega)$.
□

Proposition 2. *Γ and Ω be two FFSs in a FFTS $(\mathcal{F},\tau)$. Then,*

1. $cl_\alpha(0_\mathcal{F}) = 0_\mathcal{F}$ *and* $cl_\alpha(1_\mathcal{F}) = 1_\mathcal{F}$.
2. *If* $\Gamma \subseteq \Omega$ *then* $cl_\alpha(\Gamma) \subseteq cl_\alpha(\Omega)$.
3. $cl_\alpha(\Gamma \cup \Omega) \subseteq cl_\alpha(\Gamma) \cup cl_\alpha(\Omega)$.
4. $cl_\alpha(\Gamma \cap \Omega) \subseteq cl_\alpha(\Gamma) \cap cl_\alpha(\Omega)$.

Proof. 1. Obvious.

2. We have $\Gamma \subseteq \Omega \subseteq cl_\alpha(\Omega)$. By the definition, $cl_\alpha(\Gamma)$ is the smallest $FF\alpha CS$ containing Γ and $\Gamma \subseteq cl_\alpha(\Omega)$. Therefore, $cl_\alpha(\Gamma) \subseteq cl_\alpha(\Omega)$.
3. $\Gamma \subseteq \Gamma \cup \Omega$ implies $cl_\alpha(\Gamma) \subseteq cl_\alpha(\Gamma \cup \Omega)$ and $\Omega \subseteq \Gamma \cup \Omega$ implies $cl_\alpha(\Omega) \subseteq cl_\alpha(\Gamma \cup \Omega)$. Therefore, $cl_\alpha(\Gamma) \cup cl_\alpha(\Omega) \subseteq cl_\alpha(\Gamma \cup \Omega)$.
4. $\Gamma \cap \Omega \subseteq \Gamma$ implies $cl_\alpha(\Gamma \cap \Omega) \subseteq cl_\alpha(\Gamma)$ and $\Gamma \cap \Omega \subseteq \Omega$ implies $cl_\alpha(\Gamma \cap \Omega) \subseteq cl_\alpha(\Omega)$. Hence, $cl_\alpha(\Gamma \cap \Omega) \subseteq cl_\alpha(\Gamma) \cap cl_\alpha(\Omega)$.

□

Definition 10. *Let Γ be a FFS of a universe set X. Then, the (α, β)-cut of Γ is a crisp subset $C_{\alpha,\beta}(\Gamma)$ defined as $C_{\alpha,\beta}(\Gamma) = \{x \in X : \mu_\Gamma(x) \geq \alpha, \nu_\Gamma(x) \leq \beta\}$, called the FF (α, β)-level subset of Γ, where $\alpha^3 + \beta^3 \leq 1$ and $\alpha, \beta \in [0, 1]$.*

Definition 11. *Let (χ, τ_χ) and (ξ, τ_Y) be FFTS. A mapping $f : \chi \to \xi$ is a FF α-continuous ($FF\alpha C$) if the inverse image of each FFOS in ξ is a $FF\alpha OS$ in χ.*

Example 3. *Let $\mathcal{F} = \{f_1, f_2\}$, $\tau_\mathcal{F} = \{0_\mathcal{F}, 1_\mathcal{F}, \Gamma_1\}$ where $\Gamma_1 = \{(f_1, 0.75, 0.69), (f_2, 0.54, 0.83)\}$ and $\mathcal{G} = \{g_1, g_2\}$, $\tau_\mathcal{G} = \{0_\mathcal{G}, 1_\mathcal{G}, \Omega_1\}$ where $\Omega_1 = \{(g_1, 0.32, 0.87), (g_2, 0.91, 0.55)\}$ are two FFTSs. Let us define a mapping $f : (\mathcal{F}, \tau_\mathcal{F}) \to (\mathcal{G}, \tau_\mathcal{G})$ such that $f(\gamma) = \begin{cases} g_1 & if\ \gamma = f_1 \\ g_2 & if\ \gamma = f_2 \end{cases}$. Since the inverse image of each FFOS in $\tau_\mathcal{G}$ is a $FF\alpha OS$ in $\tau_\mathcal{F}$, f is a $FF\alpha C$.*

Theorem 10. *Let $f : \chi \to \xi$ be a mapping from a FFTS (χ, τ_χ) to a FFTS (ξ, τ_ξ). Then, f is a $FF\alpha C$ function iff for each FF point $A^\gamma_{(r_1,r_2)}(\delta)$ in χ, for each FFOS Ω in ξ and $f\left(A^\gamma_{(r_1,r_2)}(\delta)\right) \subseteq \Omega$ there exists a $FF\alpha OS$ Γ in χ such that $A^\gamma_{(r_1,r_2)}(\delta) \subseteq \Gamma$ and $f(\Gamma) \subseteq \Omega$.*

Proof. Let us assume f is a $FF\alpha C$ function. Ω is FFOS in Y and $f\left(A^\gamma_{(r_1,r_2)}(\delta)\right) \in \Omega$ implies $A^\gamma_{(r_1,r_2)}(\delta) \in f^{-1}(\Omega)$ and $f^{-1}(\Omega)$ is $FF\alpha OS$ in χ. Let $\Gamma = f^{-1}(\Omega)$; then, $A^\gamma_{(r_1,r_2)}(\delta) \in \Gamma$ and $f(\Gamma) \subseteq \Omega$. Conversely, let Ω be a FFOS in ξ such that $A^\gamma_{(r_1,r_2)}(\delta) \in f^{-1}(\omega)$ and, thus, there exists a $FF\alpha OS$ $A^\gamma_{(r_1,r_2)}(y) \in \Gamma$ and $f(F) \subseteq \Omega$. Then, $A^\gamma_{(r_1,r_2)}(y) \in \Gamma \subseteq f^{-1}(\Omega) = \bigcup \Gamma$ which is a $FF\alpha OS$. Hence, f is a $FF\alpha C$ function. □

Theorem 11. *A function $f : \chi \to \xi$ is a $FF\alpha C$ function iff every FFCS in ξ has $FF\alpha CS$ as inverse image in χ.*

Proof. Let H be a FFCS in χ. Then, H^c is $FF\alpha OS$ in ξ. Since f is $FF\alpha C$ function, $f^{-1}(H^c)$ is a $FF\alpha CS$ in χ. Hence, $\left[f^{-1}(H^c)\right]^c = f^{-1}(H)$ is a $FF\alpha CS$ in χ. Conversely, let us assume the inverse image of each $FFCS$ in ξ is a $FF\alpha CS$ in χ. Let Γ be a $FFOS$ ξ. Then, Γ^c is FFCS in ξ and, hence, $f^{-1}(\Gamma^c)$ is $FFCS$ in χ. Therefore, $\left[f^{-1}(\Gamma^c)\right]^c = f^{-1}(\Gamma)$ is a $FF\alpha OS$ in χ implies f is $FF\alpha C$ function. □

Theorem 12. *Let $f : (\chi, \tau_\chi) \to (\xi, \tau_\xi)$ be a mapping from χ to ξ. If f is $FF\alpha C$, then*

1. *$f(cl(int(cl(\Gamma)))) \subseteq cl(f(\Gamma))$ for every FFS Γ in χ.*
2. *$cl\left(int\left(cl\left(f^{-1}(\Omega)\right)\right)\right) \subseteq f^{-1}(cl(\Omega))$ for every FFS Ω in ξ.*

Proof. 1. Since f is $FF\alpha CF$ and $cl(f(\Gamma))$ is a FFCS in ξ, $f^{-1}(cl(f(\Gamma)))$ is a $FF\alpha CS$ in χ. Thus, $cl(int(cl(\Gamma))) \subseteq cl\left(int\left(cl\left(f^{-1}(cl(f(\Gamma)))\right)\right)\right) \subseteq f^{-1}(cl(\Omega))$. Hence, $f(cl(int(cl(\Gamma)))) \subseteq cl(f(\Gamma))$.

2. If Ω is a FFS in ξ, then $f^{-1}(\Omega)$ is in χ. Hence, $f\left(cl\left(int\left(cl\left(f^{-1}(\Omega)\right)\right)\right)\right) \subseteq cl\left(f\left(f^{-1}(\Omega)\right)\right) \subseteq cl(\Omega)$. Therefore, $cl\left(int\left(cl\left(f^{-1}(\Omega)\right)\right)\right) \subseteq f^{-1}(cl(\Omega))$.

□

4. Fermatean Fuzzy PROMETHEE II Method

This section presents the FF PROMETHEE II method framework, which includes a step-by-step approach and flowchart.

The standard of living has been made easy over the last few decades due to the rapid growth of technology. By this remarkable growth, natural resources are highly depleted. As far as the field of construction is considered, the usage of green raw material plays a prominent role in sustaining a green environment. Accordingly, the process of selection of green suppliers using MCDM methods has become far-reaching. The PROMETHEE II method is a simple, explicit, and balanced partial and complete outranking method that uses preference function for ranking. Here, we use the FF PROMETHEE II method to find the best supplier for the problem discussed in [33] and the ranks are compared with the other MCDM methods. Let $\mathcal{S}_m$, $m = 1,2,3,\ldots,i$ be the alternatives and $\mathcal{C}_n$, $n = 1,2,3,\ldots,j$ be the attributes with weights $\mathcal{W}_n$, $n = 1,2,3,\ldots,j$ and $\sum_{n=m}^{n} \mathcal{W}_n = 1$. Let x_{mn}, $m = 1,2,3,\ldots i$, $n = 1,2,3,\ldots,j$ illustrate the alternative's value, $\mathcal{S}_m$ with the attribute $\mathcal{C}_n$. Then, the FF PROMETHEE II method is developed in the following steps and it is visualized in the flowchart given in Figure 2.

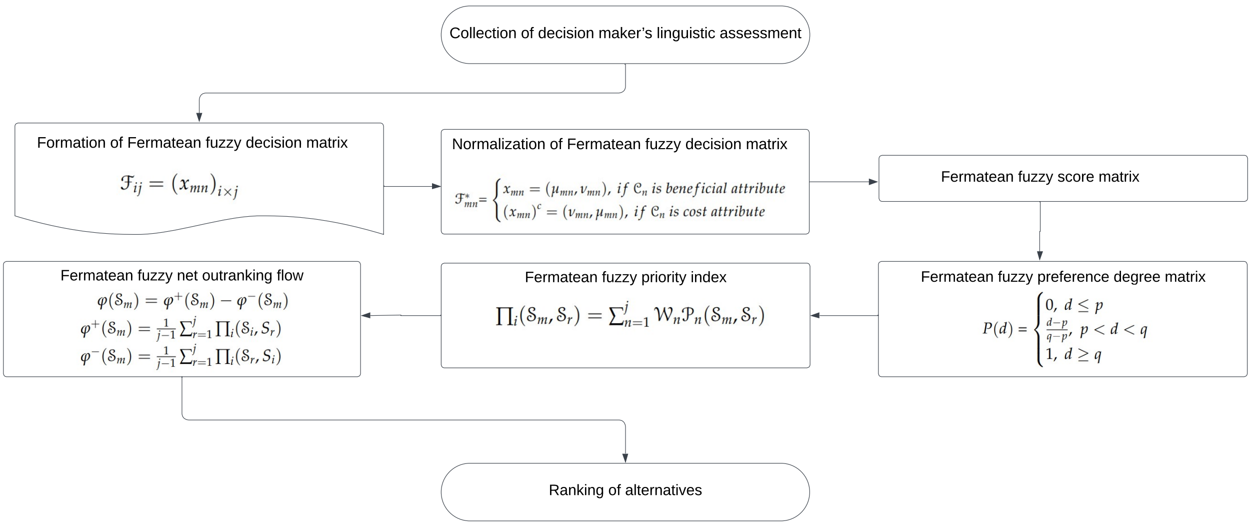


Figure 2. Flowchart of Fermatean fuzzy PROMETHEE II method.

Step 1: The FF decision matrix (FFDM) $\mathcal{F}_{ij} = (x_{mn})_{i\times j}$ is created using the data obtained from the decision makers pertaining to the $FF(\alpha, \beta)$-level.
Step 2: The normalized FF decision matrix (NFFDM) $\mathcal{F}^*_{mn}$ is determined from the FFDM using the rule

$$\mathcal{F}^*_{mn} = \begin{cases} x_{mn} = (\mu_{mn}, \nu_{mn}),\ if\ \mathcal{C}_n\ is\ beneficial\ attribute \\ (x_{mn})^c = (\nu_{mn}, \mu_{mn}),\ if\ \mathcal{C}_n\ is\ cost\ attribute \end{cases}$$

Step 3: The FF score matrix is obtained using the score stated in Definition 2.
Step 4: The FF preference degree matrix $\mathcal{P}_n(\mathcal{S}_m, \mathcal{S}_r)$ of alternatives $\mathcal{S}_m$ relative to $\mathcal{S}_r$ under the attribute $\mathcal{C}_n$ using the FF linear function is

$$P(d) = \begin{cases} 0,\ d \le p \\ \frac{d-p}{q-p},\ p < d < q \\ 1,\ d \ge q \end{cases}$$

where the difference between the $\mathcal{S}_m$ and $\mathcal{S}_r$ score values is represented by the parameter d of the FF priority function.

Step 5: The FF priority index is defined as $\prod_i(\mathcal{S}_m, \mathcal{S}_r) = \sum_{n=1}^{j} \mathcal{W}_n \mathcal{P}_n(\mathcal{S}_m, \mathcal{S}_r)$, where $\mathcal{W}_n$ is the weight of the attribute $\mathcal{C}_n$ and $\sum_{n=1}^{j} \mathcal{W}_n = 1$.
Step 6: Calculation of the entering outranking flow $\varphi^+(\mathcal{S}_m)$ and the leaving outranking flow $\varphi^-(\mathcal{S}_m)$ using
$\varphi^+(\mathcal{S}_m) = \frac{1}{j-1} \sum_{r=1}^{j} \prod_i(\mathcal{S}_i, S_r)$
$\varphi^-(\mathcal{S}_m) = \frac{1}{j-1} \sum_{r=1}^{j} \prod_i(\mathcal{S}_r, S_i)$
The FF net outranking flow $\varphi(\mathcal{S}_m) = \varphi^+(\mathcal{S}_m) - \varphi^-(\mathcal{S}_m)$.
Step 7: Ranking of alternatives is carried out from higher to lower FF net flow values.

5. Green Construction Supplier Evaluation Using Fermatean Fuzzy PROMETHEE II Method

In this section, we use the FF PROMETHEE II method for green construction supplier selection by considering the FF (α, β)-level value while taking the decision values of the decision makers.

Selecting suitable green logistics suppliers is essential for efficient and successful green supply chain management in the current circumstances of rising environmental consciousness and significant expectations from multiple stakeholders. Here, we consider the decision-making problem discussed in [33]. For the purpose of facilitating more complicated modeling of decision makers' preferences, PROMETHEE II method especially takes into consideration preference information provided by decision makers using preference functions. When decision makers desire to provide their specific choices for consideration when making decisions, this might prove advantageous. In contrast, WASPAS fails to employ explicit preference modeling; instead, it uses weighted aggregation. More in-depth examination of the ways in which criteria influence themselves is possible by means of PROMETHEE II's capability to capture relations among criteria via the outranking flows. WASPAS [33] may not explicitly record interactions, even if it can cope with them to some extent by weighted aggregation.

The set of 15 suppliers $\mathcal{S}_1$ to $\mathcal{S}_{15}$ were selected by the three experts from the departments of purchasing, projects, and manufacturing and process engineering by considering seven attributes: estimated cost $(\mathcal{C}_1)$, delivery efficiency $(\mathcal{C}_2)$, product flexibility $(\mathcal{C}_3)$, reputation and management level $(\mathcal{C}_4)$, eco-design $(\mathcal{C}_5)$, green image $(\mathcal{C}_6)$, and pollution $(\mathcal{C}_7)$. The FFDM is formed from the average of the three decision makers' nine linguistic values, namely very very high (0.9, 0.1), very high (0.8, 0.1), high (0.7, 0.2), medium high (0.6, 0.3), medium (0.5, 0.4), medium low (0.4, 0.5), low (0.25, 0.6), very low (0.1, 0.75), and very very low (0.1, 0.9), of the fifteen suppliers against the seven attributes. The FF number with considering (α, β)-level is assigned to the linguistic expressions of the decision experts and the average of all the FFN numbers are tabulated in Table 1.

The FF (α, β)-level values are fixed as (0.1, 0.9). That is, each FFN number that is used in MCDM to represent the decision must be greater than or equal to (0.1, 0.9) which indicates the MF must be greater than or equal to 0.1 and NMF must be less than or equal to 0.9. To effectively deal with uncertainty and ambiguity, the α level indicates how far the alternatives satisfy demands, whereas the β level describes the level of uncertainty and dissatisfaction. When taken together, they establish an exhaustive basis for right multifaceted, informed decision making. Both of these levels are present in FFS, which enhance decision making in challenging, uncertain situations by rendering choices more reliable and precise.

Table 1. FFDM representation.

	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_5$	$\mathcal{C}_6$	$\mathcal{C}_7$
$\mathcal{S}_1$	(0.83, 0.10)	(0.10, 0.85)	(0.50, 0.40)	(0.15, 0.70)	(0.47, 0.43)	(0.10, 0.80)	(0.77, 0.13)
$\mathcal{S}_2$	(0.63, 0.27)	(0.38, 0.50)	(0.53, 0.37)	(0.47, 0.43)	(0.50, 0.40)	(0.50, 0.40)	(0.60, 0.30)
$\mathcal{S}_3$	(0.80, 0.10)	(0.10, 0.85)	(0.10, 0.80)	(0.10, 0.80)	(0.15, 0.75)	(0.10, 0.80)	(0.83, 0.10)
$\mathcal{S}_4$	(0.83, 0.13)	(0.10, 0.85)	(0.10, 0.75)	(0.30, 0.58)	(0.33, 0.53)	(0.47, 0.43)	(0.57, 0.33)
$\mathcal{S}_5$	(0.63, 0.27)	(0.50, 0.40)	(0.47, 0.43)	(0.43, 0.47)	(0.43, 0.47)	(0.43, 0.47)	(0.57, 0.33)
$\mathcal{S}_6$	(0.20, 0.65)	(0.50, 0.40)	(0.63, 0.27)	(0.47, 0.43)	(0.57, 0.33)	(0.38, 0.50)	(0.25, 0.62)
$\mathcal{S}_7$	(0.25, 0.62)	(0.87, 0.10)	(0.53, 0.37)	(0.80, 0.10)	(0.70, 0.20)	(0.73, 0.17)	(0.35, 0.53)
$\mathcal{S}_8$	(0.10, 0.75)	(0.83, 0.10)	(0.80, 0.10)	(0.73, 0.17)	(0.80, 0.10)	(0.87, 0.10)	(0.10, 0.90)
$\mathcal{S}_9$	(0.10, 0.85)	(0.73, 0.17)	(0.83, 0.13)	(0.73, 0.17)	(0.80, 0.10)	(0.73, 0.17)	(0.10, 0.80)
$\mathcal{S}_{10}$	(0.53, 0.37)	(0.30, 0.57)	(0.33, 0.53)	(0.35, 0.53)	(0.70, 0.20)	(0.80, 0.13)	(0.60, 0.30)
$\mathcal{S}_{11}$	(0.63, 0.27)	(0.33, 0.53)	(0.43, 0.47)	(0.50, 0.40)	(0.60, 0.30)	(0.60, 0.30)	(0.90, 0.10)
$\mathcal{S}_{12}$	(0.87, 0.10)	(0.10, 0.75)	(0.10, 0.80)	(0.10, 0.90)	(0.10, 0.75)	(0.20, 0.67)	(0.77, 0.13)
$\mathcal{S}_{13}$	(0.63, 0.27)	(0.63, 0.27)	(0.53, 0.37)	(0.50, 0.40)	(0.43, 0.47)	(0.43, 0.47)	(0.47, 0.43)
$\mathcal{S}_{14}$	(0.47, 0.43)	(0.10, 0.85)	(0.10, 0.85)	(0.25, 0.62)	(0.20, 0.70)	(0.50, 0.40)	(0.57, 0.33)
$\mathcal{S}_{15}$	(0.10, 0.85)	(0.83, 0.10)	(0.87, 0.10)	(0.70, 0.20)	(0.90, 0.10)	(0.87, 0.10)	(0.10, 0.85)

In order to aggregate criteria with numerical and comparable data, normalizing the decision matrix is one of the most essential stages in solving MCDM problems. The criteria are normalized using the formula stated in step 2. Here, the estimated cost $\mathcal{C}_1$ and the pollution $\mathcal{C}_7$ are non-beneficial criteria. The remaining delivery efficiency $(\mathcal{C}_2)$, product flexibility $(\mathcal{C}_3)$, reputation and management level $(\mathcal{C}_4)$, eco-design $(\mathcal{C}_5)$, and green image $(\mathcal{C}_6)$ are beneficial attributes. Therefore, the first and last column of the NFFDM given in Table 2 are different from FFDM.

Table 2. NFFDM representation.

	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_5$	$\mathcal{C}_6$	$\mathcal{C}_7$
$\mathcal{S}_1$	(0.10, 0.83)	(0.10, 0.85)	(0.50, 0.40)	(0.15, 0.70)	(0.47, 0.43)	(0.10, 0.80)	(0.13, 0.77)
$\mathcal{S}_2$	(0.27, 0.63)	(0.38, 0.50)	(0.53, 0.37)	(0.47, 0.43)	(0.50, 0.40)	(0.50, 0.40)	(0.30, 0.60)
$\mathcal{S}_3$	(0.10, 0.80)	(0.10, 0.85)	(0.10, 0.80)	(0.10, 0.80)	(0.15, 0.75)	(0.10, 0.80)	(0.10, 0.83)
$\mathcal{S}_4$	(0.13, 0.83)	(0.10, 0.85)	(0.10, 0.75)	(0.30, 0.58)	(0.33, 0.53)	(0.47, 0.43)	(0.33, 0.57)
$\mathcal{S}_5$	(0.27, 0.63)	(0.50, 0.40)	(0.47, 0.43)	(0.43, 0.47)	(0.43, 0.47)	(0.43, 0.47)	(0.33, 0.57)
$\mathcal{S}_6$	(0.65, 0.20)	(0.50, 0.40)	(0.63, 0.27)	(0.47, 0.43)	(0.57, 0.33)	(0.38, 0.50)	(0.62, 0.25)
$\mathcal{S}_7$	(0.62, 0.25)	(0.87, 0.10)	(0.53, 0.37)	(0.80, 0.10)	(0.70, 0.20)	(0.73, 0.17)	(0.53, 0.35)
$\mathcal{S}_8$	(0.75, 0.10)	(0.83, 0.10)	(0.80, 0.10)	(0.73, 0.17)	(0.80, 0.10)	(0.87, 0.10)	(0.90, 0.10)
$\mathcal{S}_9$	(0.85, 0.10)	(0.73, 0.17)	(0.83, 0.13)	(0.73, 0.17)	(0.80, 0.10)	(0.73, 0.17)	(0.80, 0.10)
$\mathcal{S}_{10}$	(0.37, 0.53)	(0.30, 0.57)	(0.33, 0.53)	(0.35, 0.53)	(0.70, 0.20)	(0.80, 0.13)	(0.30, 0.60)
$\mathcal{S}_{11}$	(0.27, 0.63)	(0.33, 0.53)	(0.43, 0.47)	(0.50, 0.40)	(0.60, 0.30)	(0.60, 0.30)	(0.10, 0.90)
$\mathcal{S}_{12}$	(0.10, 0.87)	(0.10, 0.75)	(0.10, 0.80)	(0.10, 0.90)	(0.10, 0.75)	(0.20, 0.67)	(0.13, 0.77)
$\mathcal{S}_{13}$	(0.27, 0.63)	(0.63, 0.27)	(0.53, 0.37)	(0.50, 0.40)	(0.43, 0.47)	(0.43, 0.47)	(0.43, 0.47)
$\mathcal{S}_{14}$	(0.43, 0.47)	(0.10, 0.85)	(0.10, 0.85)	(0.25, 0.62)	(0.20, 0.70)	(0.50, 0.40)	(0.33, 0.57)
$\mathcal{S}_{15}$	(0.85, 0.10)	(0.83, 0.10)	(0.87, 0.10)	(0.70, 0.20)	(0.90, 0.10)	(0.87, 0.10)	(0.85, 0.10)

The score matrix of the NFFDM is obtained by using the score value stated in Definition 2 and is tabulated in Table 3.

Table 3. The score matrix of the NFFDM.

	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_5$	$\mathcal{C}_6$	$\mathcal{C}_7$
$\mathcal{S}_1$	−0.57	−0.61	0.06	−0.34	0.02	−0.51	−0.45
$\mathcal{S}_2$	−0.23	−0.07	0.10	0.02	0.06	0.06	−0.19
$\mathcal{S}_3$	−0.51	−0.61	−0.51	−0.51	−0.42	−0.51	−0.57
$\mathcal{S}_4$	−0.57	−0.61	−0.42	−0.17	−0.11	0.02	−0.15
$\mathcal{S}_5$	−0.23	0.06	0.02	−0.02	−0.02	−0.02	−0.15
$\mathcal{S}_6$	0.27	0.06	0.23	0.02	0.15	−0.07	0.22
$\mathcal{S}_7$	0.22	0.66	0.10	0.51	0.34	0.39	0.11
$\mathcal{S}_8$	0.42	0.57	0.51	0.38	0.51	0.66	0.72
$\mathcal{S}_9$	0.61	0.38	0.57	0.38	0.51	0.38	0.51
$\mathcal{S}_{10}$	−0.10	−0.16	−0.11	−0.11	0.34	0.51	−0.19
$\mathcal{S}_{11}$	−0.23	−0.11	−0.02	0.06	0.19	0.19	−0.73
$\mathcal{S}_{12}$	−0.66	−0.42	−0.51	−0.73	−0.42	−0.29	−0.45
$\mathcal{S}_{13}$	−0.23	0.23	0.10	0.06	−0.02	−0.02	−0.02
$\mathcal{S}_{14}$	−0.02	−0.61	−0.61	−0.22	−0.34	0.06	−0.15
$\mathcal{S}_{15}$	0.61	0.57	0.66	0.34	0.73	0.66	0.61

The preference degree matrices $\mathcal{C}_1$ to $\mathcal{C}_7$ are obtained using the FF linear function. Using the SMART [33] method, the normalized weight of the attributes are calculated as $\mathcal{W}_1 = 0.31$, $\mathcal{W}_2 = 0.21$, $\mathcal{W}_3 = 0.11$, $\mathcal{W}_4 = 0.10$, $\mathcal{W}_5 = 0.04$, $\mathcal{W}_6 = 0.07$, and $\mathcal{W}_7 = 0.15$. Then, the FF priority index matrix $\prod_i(\mathcal{S}_i, \mathcal{S}_r)$ is obtained by using the formula defined in step 5 as tabulated in Table 4, As we can see, the diagonal values are 0, signifying that there is no preference for the same alternative.

Table 4. The FF priority index matrix $\prod_i(\mathcal{S}_i, \mathcal{S}_r)$.

	$\mathcal{S}_1$	$\mathcal{S}_2$	$\mathcal{S}_3$	$\mathcal{S}_4$	$\mathcal{S}_5$	$\mathcal{S}_6$	$\mathcal{S}_7$	$\mathcal{S}_8$	$\mathcal{S}_9$	$\mathcal{S}_{10}$	$\mathcal{S}_{11}$	$\mathcal{S}_{12}$	$\mathcal{S}_{13}$	$\mathcal{S}_{14}$	$\mathcal{S}_{15}$
$\mathcal{S}_1$	0.00	0.00	0.17	0.06	0.01	0.00	0.00	0.00	0.00	0.02	0.05	0.15	0.00	0.09	0.00
$\mathcal{S}_2$	0.34	0.00	0.44	0.31	0.02	0.01	0.00	0.00	0.00	0.06	0.10	0.10	0.47	0.23	0.00
$\mathcal{S}_3$	0.02	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.07	0.00	0.01	0.00
$\mathcal{S}_4$	0.10	0.01	0.16	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.09	0.17	0.00	0.04	0.00
$\mathcal{S}_5$	0.31	0.03	0.45	0.31	0.00	0.00	0.00	0.00	0.00	0.08	0.13	0.21	0.00	0.24	0.00
$\mathcal{S}_6$	0.60	0.26	0.69	0.56	0.24	0.00	0.05	0.00	0.00	0.27	0.36	0.69	0.21	0.33	0.00
$\mathcal{S}_7$	0.70	0.42	0.75	0.66	0.41	0.21	0.00	0.03	0.07	0.32	0.51	0.81	0.32	0.49	0.04
$\mathcal{S}_8$	0.88	0.62	0.96	0.88	0.60	0.36	0.23	0.00	0.09	0.44	0.63	0.98	0.53	0.73	0.02
$\mathcal{S}_9$	0.88	0.56	0.97	0.72	0.58	0.34	0.24	0.07	0.00	0.57	0.64	0.92	0.51	0.73	0.01
$\mathcal{S}_{10}$	0.39	0.08	0.47	0.33	0.09	0.05	0.01	0.00	0.01	0.00	0.15	0.46	0.09	0.22	0.00
$\mathcal{S}_{11}$	0.31	0.02	0.38	0.30	0.02	0.02	0.00	0.00	0.00	0.04	0.00	0.39	0.02	0.23	0.00
$\mathcal{S}_{12}$	0.06	0.00	0.07	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.00	0.05	0.00
$\mathcal{S}_{13}$	0.43	0.09	0.52	0.39	0.07	0.04	0.00	0.00	0.00	0.15	0.19	0.59	0.00	0.47	0.00
$\mathcal{S}_{14}$	0.27	0.07	0.29	0.17	0.07	0.01	0.00	0.00	0.00	0.03	0.15	0.32	0.07	0.00	0.00
$\mathcal{S}_{15}$	0.90	0.68	0.98	0.87	0.67	0.38	0.29	0.08	0.09	0.65	0.71	0.99	0.60	0.77	0.00

The correlations and significance among the alternatives of FF priority index matrix are calculated by the SPSS 27 software of IBM and are given in Table 5.

Table 6 displays the descriptive data that were calculated using SPSS software, including mean, median, mode, minimum, maximum, range, variance, standard deviation, skewness, kurtosis, standard error of mean, standard error of skewness, standard error of kurtosis, and sum.

Table 5. Frequency of alternatives' preference value.

Cor																
		s1	**s2**	**s3**	**s4**	**s5**	**s6**	**s7**	**s8**	**s9**	**s10**	**s11**	**s12**	**s13**	**s14**	**s15**
s1	PC	1	0.93 **	0.99 **	0.99 **	0.93 **	0.87 **	0.81 **	0.68 **	0.70 **	0.92 **	0.95 **	0.96 **	0.85 **	0.95 **	0.52 *
	Sig.		0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.05
	N	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
s2	PC	0.93 **	1	0.91 **	0.93 **	0.10 **	0.96 **	0.91 **	0.76 **	0.81 **	0.97 **	0.99 **	0.93 **	0.85 **	0.92 **	0.54 *
	Sig.	0.00		0.00	Cor	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04
	N	15	15	15			s1	s2	s3	s4	s5	s6	s7	s8	s9	s10
s3	PC	0.990 **	0.91 **	1	s1 **	PC **	1.00 **	0.93 **	0.99 **	0.99 **	0.93 **	0.87 **	0.81 **	0.68 **	0.70 **	0.92
	Sig.	0.00	0.00			Sig.		0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
	N	15	15	15		N	15	15	15	15	15	15	15	15	15	15
s4	PC	0.99 **	0.93 **	0.99 **	s2	PC **	0.93 **	1.00 **	0.91 *	0.93 **	0.99 **	0.96 **	0.91 **	0.76 **	0.81 **	0.97 *
	Sig.	0.00	0.00	0.00		Sig.	0.00		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	N	15	15	15		N	15	15	15	15	15	15	15	15	15	15
s5	PC	0.93 **	0.10 **	0.91 **	s3 **	PC	0.10 **	0.91 **	1.00 **	0.99 **	0.91 **	0.85 **	0.80 **	0.67 **	0.67 **	0.91 *
	Sig.	0.00	0.00	0.00		Sig.	0.00	0.00		0.00	0.00	0.00	0.00	0.01	0.01	0.00
	N	15	15	15		N	15	15	15	15	15	15	15	15	15	15
s6	PC	0.87 **	0.96 **	0.85 **	s4 **	PC **	1	0.93 **	0.99 **	1.00 **	0.92 **	0.87 **	0.80 **	0.64 **	0.75 **	0.92 *
	Sig.	0.00	0.00	0.00		Sig.	0.00	0.00	0.00		0.00	0.00	0.00	0.01	0.001	0.00
	N	15	15	15		N	15	15	15	15	15	15	15	15	15	15
s7	PC	0.81 **	0.91 **	0.80 **	s5 **	PC **	0.93 **	1	0.91 **	0.92 **	1.00 **	0.97 **	0.91 **	0.78 **	0.79 **	0.97
	Sig.	0.00	0.00	0.00		Sig.	0.00	0.00	0.00	0.00		0.00	0.00	0.00	0.00	0.00
	N	15	15	15		N	15	15	15	15	15	15	15	15	15	15
s8	PC	0.68 **	0.76 **	0.67 **	s6 *	PC **	0.87 **	0.96 **	1	0.87	0.97 **	1.000 **	0.92 **	0.79 **	0.80 **	0.93
	Sig.	0.01	0.00	0.01		Sig.	0.00	0.00	0.00	0.00	0.00		0.00	0.00	0.00	0.00
	N	15	15	15		N	15	15	15	15	15	15	15	15	15	15

Table 5. *Cont.*

Cor																
		s1	**s2**	**s3**	**s4**	**s5**	**s6**	**s7**	**s8**	**s9**	**s10**	**s11**	**s12**	**s13**	**s14**	**s15**
s9	PC	0.70 **	0.81 **	0.67 **	s7 **	PC **	0.81 **	0.91 **	0.80	1	0.91 **	0.92 **	1.00 **	0.77 **	0.648 **	0.91 *
	Sig.	0.00	0.00	0.01		Sig.	0.00	0.00	0.00	0.00	0.00	0.00		0.00	0.01	0.00
	N	15	15	15		N	15	15	15	15	15	15	15	15	15	15
s10	PC	0.92 **	0.97 **	0.91 **	s8 **	PC **	0.68 **	0.76 **	0.67 **	0.64 **	1	0.79 **	0.77 **	1.00 **	0.48 **	0.84
	Sig.	0.00	0.00	0.00		Sig.	0.01	0.01	0.01	0.01	0.00	0.01	0.01		0.07	0.00
	N	15	15	15		N	15	15	15	15	15	15	15	15	15	15
s11	PC	0.95 **	0.99 **	0.93 **	s9 **	PC **	0.70 **	0.81 **	0.67 **	0.75 **	0.79 **	1	0.65 **	0.48 **	1.000 **	0.698 *
	Sig.	0.00	0.00	0.00		Sig.	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.07		0.00
	N	15	15	15		N	15	15	15	15	15	15	15	15	15	15
s12	PC	0.96 **	0.93 **	0.95 **	s10 **	PC **	0.92 **	0.97 **	0.91 **	0.92 **	0.97 **	0.93 **	1	0.84 **	0.70 **	1.00 *
	Sig.	0.00	0.00	0.00		Sig.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
	N	15	15	15		N	15	15	15	15	15	15	15	15	15	15
s13	PC	0.84 **	0.85 **	0.83 **	s11 **	PC **	0.95 **	0.99 **	0.93 **	0.93 **	0.99 **	0.94 **	0.87 **	1	0.77 **	0.97
	Sig.	0.00	0.00	0.00		Sig.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.001	0.00
	N	15	15	15		N	15	15	15	15	15	15	15	15	15	15
s14	PC	0.95 **	0.92 **	0.95 **	s12 **	PC **	0.96 **	0.93 **	0.95 **	0.95 **	0.92 **	0.86 **	0.78 **	0.65 **	1	0.90
	Sig.	0.00	0.00	0.00		Sig.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
	N	15	15	15		N	15	15	15	15	15	15	15	15	15	15
s15	PC	0.515 *	0.543 *	0.48	s13 *	PC *	0.84 *	0.85	0.83	0.84 *	0.86	0.84 *	0.81 *	0.68	0.68	1
	Sig.	0.05	0.04	0.07		Sig.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00
	N	15	15	15		N	15	15	15	15	15	15	15	15	15	15

* correlation is significant at the 0.05 level (2-tailed). ** correlation is significant at the 0.01 level (2-tailed).

Table 6. Descriptive values of the alternatives.

Alternatives	**s1**	**s2**	**s3**	**s4**	**s5**	**s6**	**s7**	**s8**	**s9**	**s10**	**s11**	**s12**	**s13**	**s14**	**s15**
N	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
Mean	0.41	0.19	0.49	0.37	0.19	0.10	0.05	0.01	0.02	0.18	0.25	0.46	0.19	0.31	0.00
Std. Error of Mean	0.08	0.07	0.08	0.08	0.06	0.04	0.03	0.01	0.01	0.06	0.06	0.09	0.06	0.07	0.00
Median	0.34	0.07	0.45	0.31	0.07	0.01	0.00	0.00	0.00	0.06	0.15	0.39	0.07	0.23	0.00
Mode	0.31	0.00	0.00	0.31	0.00	0.00	0.00	0.00	0.00	0.00	0.15	0.00	0.00	0.23	0.00
SD	0.31	0.25	0.33	0.30	0.25	0.15	0.10	0.03	0.03	0.22	0.25	0.35	0.23	0.27	0.01
Var	0.10	0.06	0.11	0.09	0.06	0.02	0.01	0.00	0.00	0.05	0.06	0.12	0.05	0.07	0.00
Skewness	0.37	1.11	0.24	0.44	1.13	1.32	1.67	2.12	1.71	1.19	0.90	0.36	0.80	0.64	2.70
Std. Error of Skewness	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.58
Kurtosis	−1.10	−0.43	−1.03	−1.01	−0.41	−0.03	1.13	3.29	1.21	0.12	−0.82	−1.41	−1.16	−0.91	7.33
Std. Error of Kurtosis	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Range	0.90	0.68	0.98	0.88	0.67	0.38	0.29	0.08	0.09	0.65	0.71	0.99	0.60	0.77	0.04
Minimum	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Maximum	0.90	0.68	0.98	0.88	0.67	0.38	0.29	0.08	0.09	0.65	0.71	0.99	0.60	0.77	0.04
Sum	6.19	2.84	7.30	5.62	2.78	1.43	0.82	0.18	0.26	2.64	3.77	6.85	2.82	4.63	0.07

Based on the values of net flow, the alternative are ranked and tabulated as in Table 7. The entering outranking flow $\varphi^+(\mathcal{S}_m)$ and the leaving outranking flow $\varphi^-(\mathcal{S}_m)$ are measured using the formulae

$\varphi^+(\mathcal{S}_m) = \frac{1}{6}\sum_{r=1}^{7}\prod_i(\mathcal{S}_i, \mathcal{S}_r)$

$\varphi^-(\mathcal{S}_m) = \frac{1}{6}\sum_{r=1}^{7}\prod_i(\mathcal{S}_r, \mathcal{S}_i)$

The FF net outranking flow $\varphi(\mathcal{S}_m) = \varphi^+(\mathcal{S}_m) - \varphi^-(\mathcal{S}_m)$.

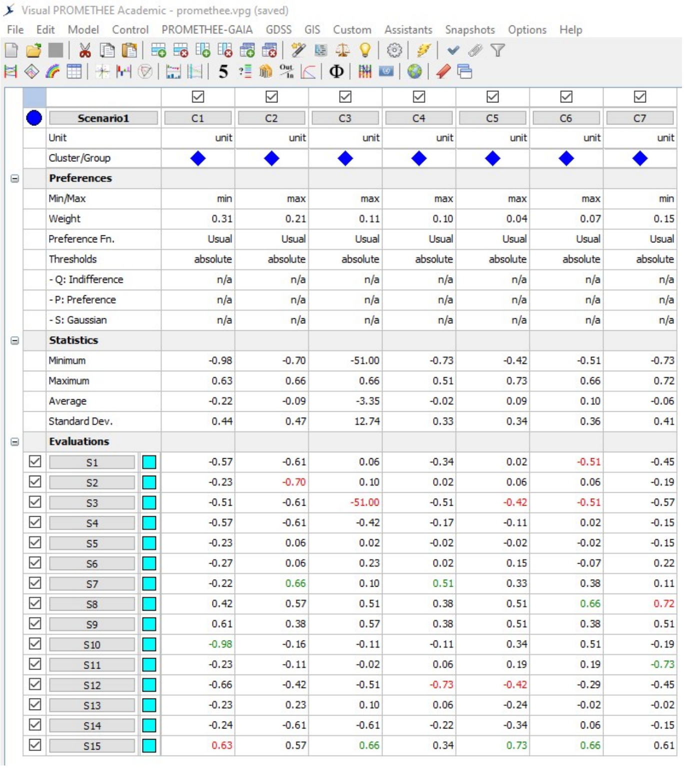
Table 7. Ranks of alternatives.

	$\varphi^+(S_m)$	$\varphi^-(S_m)$	φ	**FF PROMETHEE II Rank**
$\mathcal{S}_1$	0.026	0.441	−0.414	13
$\mathcal{S}_2$	0.149	0.203	−0.054	8
$\mathcal{S}_3$	0.010	0.508	−0.498	15
$\mathcal{S}_4$	0.041	0.401	−0.360	12
$\mathcal{S}_5$	0.126	0.199	−0.073	9
$\mathcal{S}_6$	0.304	0.102	0.202	5
$\mathcal{S}_7$	0.410	0.058	0.352	4
$\mathcal{S}_8$	0.567	0.013	0.554	2
$\mathcal{S}_9$	0.552	0.019	0.533	3
$\mathcal{S}_{10}$	0.168	0.187	−0.019	7
$\mathcal{S}_{11}$	0.123	0.270	−0.147	10
$\mathcal{S}_{12}$	0.019	0.489	0.008	14
$\mathcal{S}_{13}$	0.210	0.202	−0.414	6
$\mathcal{S}_{14}$	0.103	0.331	−0.228	11
$\mathcal{S}_{15}$	0.618	0.004	0.613	1

According to the score values from higher to lower, the alternatives are ranked. We can see that $S_{15} > S_8 > S_9 > S_7 > S_6 > S_{13} > S_{10} > S_2 > S_5 > S_{11} > S_{14} > S_4 > S_1 > S_{12} > S_3$.

5.1. Sensitivity Analysis

Visual PROMETHEE GAIA is a new tool for the visualization of rankings and information on spatial decision problems. The objective of Visual PROMETHEE GAIA is to provide decision makers simple but effective representations to improve the comprehension of convoluted scenarios concerning decision making, especially when dealing with imprecision and ambiguity. Figure 3 displays the data entered for PROMETHEE calculation. Criteria according to scenarios, preference of the criteria like max for beneficial and min for non-beneficial criteria, weight of the criteria, and preference function can be set up. The decision values obtained from experts considering $FF(\alpha, \beta)$-level are entered in the appropriate places which has been carried out using the score values of FFN. Visual PROMETHEE performs the automatic calculation of the statistical data.



Scenario1	C1	C2	C3	C4	C5	C6	C7
Unit	unit	unit	unit	unit	unit	unit	unit
Cluster/Group							
Preferences							
Min/Max	min	max	max	max	max	max	min
Weight	0.31	0.21	0.11	0.10	0.04	0.07	0.15
Preference Fn.	Usual	Usual	Usual	Usual	Usual	Usual	Usual
Thresholds	absolute	absolute	absolute	absolute	absolute	absolute	absolute
- Q: Indifference	n/a	n/a	n/a	n/a	n/a	n/a	n/a
- P: Preference	n/a	n/a	n/a	n/a	n/a	n/a	n/a
- S: Gaussian	n/a	n/a	n/a	n/a	n/a	n/a	n/a
Statistics							
Minimum	-0.98	-0.70	-51.00	-0.73	-0.42	-0.51	-0.73
Maximum	0.63	0.66	0.66	0.51	0.73	0.66	0.72
Average	-0.22	-0.09	-3.35	-0.02	0.09	0.10	-0.06
Standard Dev.	0.44	0.47	12.74	0.33	0.34	0.36	0.41
Evaluations							
S1	-0.57	-0.61	0.06	-0.34	0.02	-0.51	-0.45
S2	-0.23	-0.70	0.10	0.02	0.06	0.06	-0.19
S3	-0.51	-0.61	-51.00	-0.51	-0.42	-0.51	-0.57
S4	-0.57	-0.61	-0.42	-0.17	-0.11	0.02	-0.15
S5	-0.23	0.06	0.02	-0.02	-0.02	-0.02	-0.15
S6	-0.27	0.06	0.23	0.02	0.15	-0.07	0.22
S7	-0.22	0.66	0.10	0.51	0.33	0.38	0.11
S8	0.42	0.57	0.51	0.38	0.51	0.66	0.72
S9	0.61	0.38	0.57	0.38	0.51	0.38	0.51
S10	-0.98	-0.16	-0.11	-0.11	0.34	0.51	-0.19
S11	-0.23	-0.11	-0.02	0.06	0.19	0.19	-0.73
S12	-0.66	-0.42	-0.51	-0.73	-0.42	-0.29	-0.45
S13	-0.23	0.23	0.10	0.06	-0.24	-0.02	-0.02
S14	-0.24	-0.61	-0.61	-0.22	-0.34	0.06	-0.15
S15	0.63	0.57	0.66	0.34	0.73	0.66	0.61

Figure 3. Visual PROMETHEE data entities.

Figure 4 expresses the ranking of the alternatives by rainbow representation. Decision makers can instantly find the most preferred selections and comprehend the links between them based on their preference scores by visualizing PROMETHEE rankings in a rainbow graph form. In complicated MCDM situations, this can help with informed decision making.

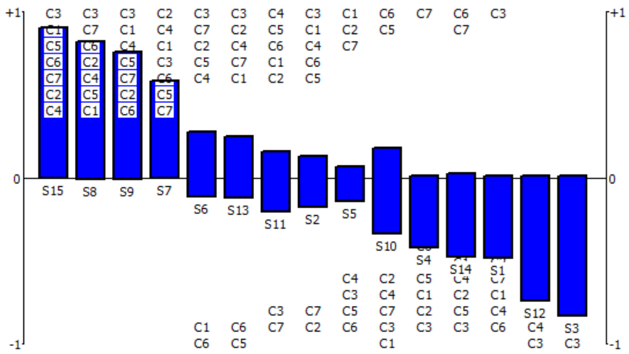


Figure 4. Visual PROMETHEE rainbow representation.

In PROMETHEE I method, preference functions are made use of to assess alternatives in relation to numerous criteria and determine the relative weight of each criterion. For every criterion, the procedure creates pairwise comparisons between the options, generating preference indices that show what choice has greater appeal over the others. A global preference assessment is developed through putting together the preference indices for every alternative across all criteria. PROMETHEE I method simply takes into account the direction of preferences, or whether one alternative is favored over another, without taking into account the intensities of preferences. It performs well in scenarios once the significance of each of the choices is the only aspect that is crucial, not the extent of preference differences. Figure 5 displays the ranking of the alternatives according to their score values which vary from −1 to +1. Red colour represents the negative outranking flow from 0 to −1 and the green colour represents the positive outranking flow from 0 to +1.

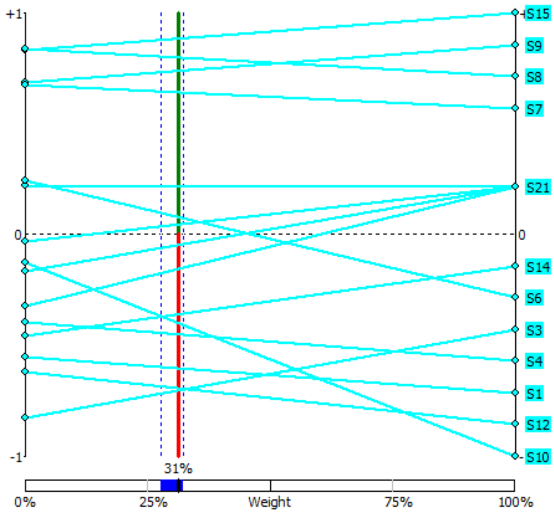


Figure 5. Visual PROMETHEE I.

By taking into consideration both the direction and the degree of preferences, PROMETHEE II improves upon PROMETHEE I. Preference functions are employed in PROMETHEE II to measure the degree of preference for each pairwise comparison while incorporating variations in performance between alternatives. The decision maker's choices can be described by mathematical functions such as Gaussian, linear, or other functions. A global preference ranking is produced by adding the preference values for every alternative across all criteria after they have been determined for each pairwise comparison. Compared to PROMETHEE I method, PROMETHEE II method yields broader results because it takes into consideration the degree of interest as well as the direction. It executes competently in scenarios where the relative ranking and effectiveness of preferences are both crucial factors in decision making. Figure 6 shows the ranking of the alternatives in the PROMETHEE II method.

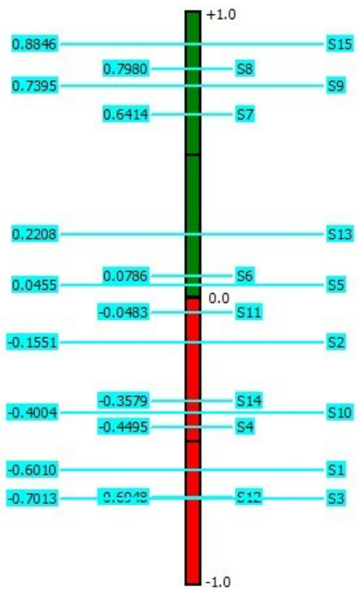


Figure 6. Visual PROMETHEE II.

The rank-reversal drawback that other PROMETHEE structures possess is circumvented by the Best–Worst PROMETHEE strategy. The Best–Worst PROMETHEE is a comprehensive methodology that is capable of helping in making decisions with multi-faceted outcomes in strategic domains of expertise. Figure 7 illustrates the better-worse rank of alternates.

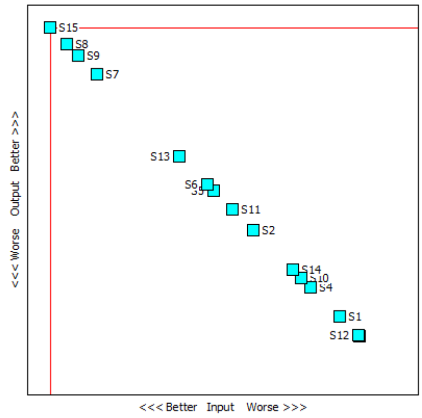


Figure 7. Visual PROMETHEE best-worst representation.

The weights allocated to the criterion apparently have an influence on the PROMETHEE I and II rankings. Individuals can customize the weights using the innovative software Visual PROMETHEE Academic Edition 1.4, "The Walking Weights" and can see how this influences the PROMETHEE II ranking. The walking weight representation is given in Figure 8 in which we can manipulate the results by giving preferable values to the criteria.

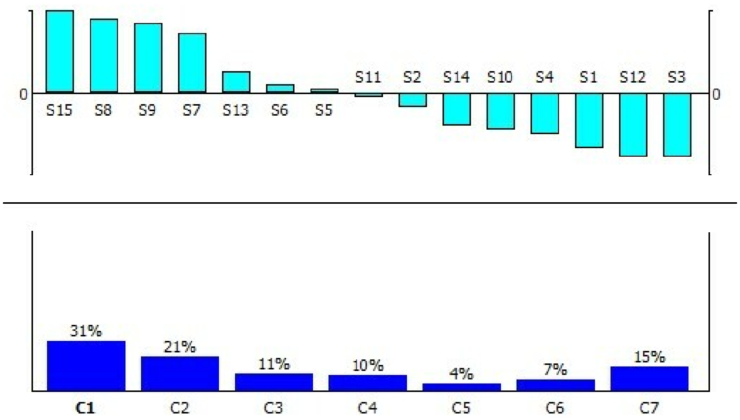


Figure 8. Visual PROMETHEE best–worst representation.

PROMETHEE GAIA enables decision makers with the capability to examine trade-offs, sensitivity analyses, and the influence of criteria weights on the final rankings. By integrating the analytical skills of the PROMETHEE technique with the visual insights presented by the Gaia visualization tool, PROMETHEE GAIA empowers decision makers to make more accessible and informed decisions in difficult decision-making scenarios. Figure 9 demonstrates the 3D representation of the preference of the alternatives considering its criterion weight. A k-dimensional space can be employed to describe the information associated with a decision problem that has k criteria. These data are projected onto a plane so that as little information as possible is lost, providing the GAIA plane. The red color line indicates the direction of the decision.

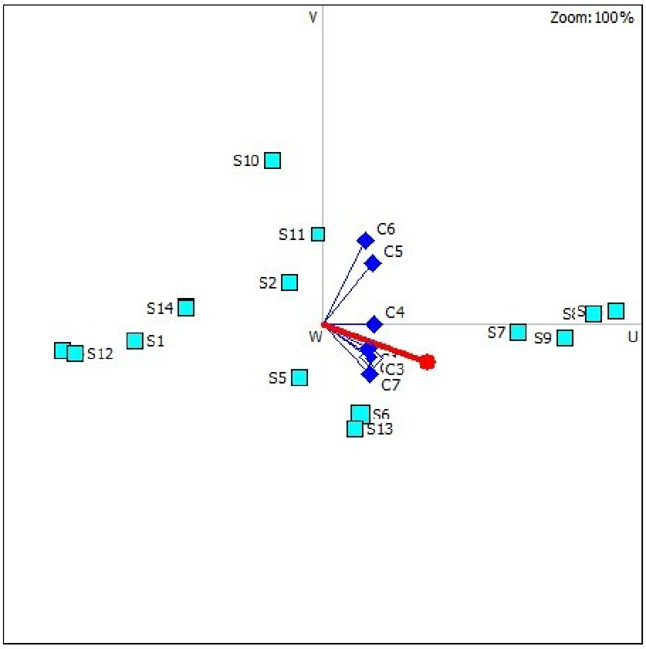


Figure 9. PROMETHEE GAIA plane.

5.2. Comparison Analysis

Table 8 presents a comparison between the FF PROMETHEE II approach and other methods that have been mentioned. The results indicate that FF PROMETHEE II is compatible with the other methods that are currently being used. Furthermore, the PROMETHEE II method can produce more detailed and precise outputs as it may be blended with other methods. It is not the only strategy that can be used to enhance decision making. Sensitivity analysis, which examines how well results adhere to modifications in preference functions or criteria weights, can be carried out with this method to get insight into the consistency of the selection rankings. The concept of transparency, versatility, and sensitivity to decisions are characteristics that are beneficial in a variety of decision-making scenarios, and PROMETHEE II delivers a systematic approach to MCDM.

Table 8. Comparison with existing method.

	TOPSIS	EDAS	COPRAS	Classic WASPAS	FF WASPAS	FF PROMETHEE II
$\mathcal{S}_1$	13	13	13	13	13	13
$\mathcal{S}_2$	9	9	9	9	9	8
$\mathcal{S}_3$	15	15	15	15	15	15
$\mathcal{S}_4$	12	12	12	12	12	12
$\mathcal{S}_5$	7	8	7	8	8	9
$\mathcal{S}_6$	5	5	5	5	5	5
$\mathcal{S}_7$	4	4	4	4	4	4
$\mathcal{S}_8$	2	2	3	2	2	2
$\mathcal{S}_9$	3	3	2	3	3	3
$\mathcal{S}_{10}$	8	7	8	7	7	7
$\mathcal{S}_{11}$	10	10	10	10	10	10
$\mathcal{S}_{12}$	14	14	14	14	14	14
$\mathcal{S}_{13}$	6	6	6	6	6	6
$\mathcal{S}_{14}$	11	11	11	11	11	11
$\mathcal{S}_{15}$	1	1	1	1	1	1

The results of the FF PROMETHEE II method and other methods discussed in [33] are compared using the graph given in Figure 10. The graph shows the PROMETHEE II methods is stable compared with existing methods.

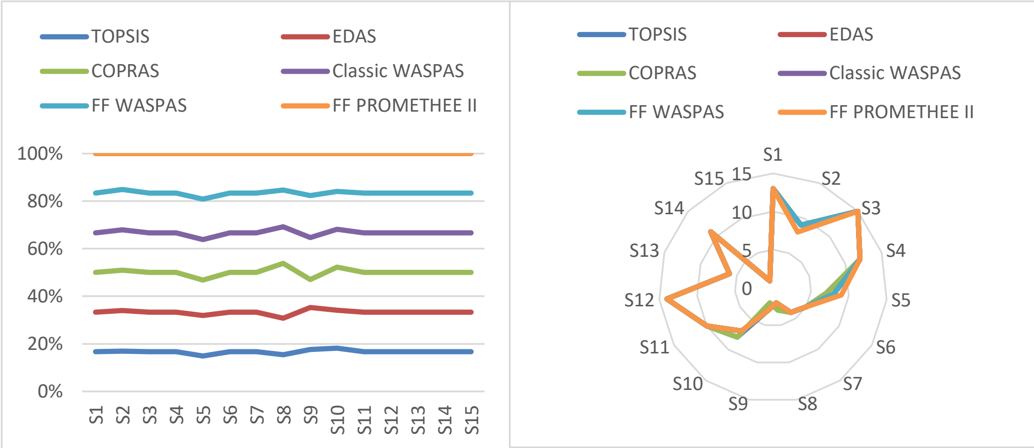


Figure 10. Comparison of various FF decision methods.

6. Conclusions, Limitations, and Future Work

In the present investigation, we have explored FF α continuous functions and generalizations of FFS in FFTS. We also have used the PROMETHEE II method for the selection of green suppliers. The FF PROMETHEE II approach guarantees a consistent ranking of the alternatives with outranking values, as evidenced by the visual indicators and sensitivity analysis. The PROMETHEE II method facilitates the creation of a systematic ranking among potential alternatives while employing outranking relations. The FF PROMETHEE II approach, which integrates the advantages of FF sets and the PROMETHEE II outranking method, provides an effective basis for decision making in an environment of ambiguity. When making uncertain choices in circumstances where there is intrinsic vagueness and ambiguity in the data, this strategy can especially effective. According to experimental findings, this method performs better than a number of conventional exploration tactics and can be employed to integrate numerous criteria. The method PROMETHEE II is important because compared to WASPAS this method is typically less sensitive to weight swings when there is uncertainty or inconsistency in the significance of the criterion. Also, the explanation of choosing the method is given at the start of Section 5 and highlighted in blue color. Due to this reason, the PROMETHEE II method is more important than WASPAS and many other methods like FF CODAS, FF CORPAS, FF TOPSIS, and FF EDAS, etc.

When dealing with challenges that have few criteria and choices, PROMETHEE performs successfully. Increasing the volume of criteria or alternates causes the procedures to become a little bit complicated and harder to execute. Notwithstanding these drawbacks, it is crucial that decision makers are aware of them and proceed with caution while using PROMETHEE, taking into account how effectively appropriate it is for the specific scenario being performed. By adopting PROMETHEE software academic edition 1.4, the limitation can be overcome. The advantage of the PROMETHEE II method is the results of this method can be used for GAIA plane presentation which gives visual support to decision makers. PROMCALC—PROMETHEE II CALCulation—is user-friendly software for outranking. In future work, the decision making can be performed by Visual PROMETHEE II, PROMCALC software, and GAIA for FF environment, and the analysis of alternatives can be carried out through the topological structure of the alternatives in connection with the criteria.

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Conflicts of Interest: All the authors hereby declare that they do not have any conflict of interests.

Abbreviations

The abbreviations utilized in this study are listed below.

Acronyms	Expansion
MF	Membership function
NMF	Non-membership function
IFS	Intuitionistic fuzzy set
IFT	Intuitionistic fuzzy topology
PFS	Pythagorean fuzzy set
PFT	Pythagorean fuzzy topology
FFS	Fermatean fuzzy set
FFTS	Fermatean fuzzy topological space
FFN	Fermatean fuzzy number
MCDM	Multi-criteria decision making
FFOS	Fermatean fuzzy open set
FFCS	Fermatean fuzzy closed set
FFαOS	Fermatean fuzzy α-open set
FFαCS	Fermatean fuzzy α-closed set
FFαCF	Fermatean fuzzy α-continuous function
FFDM	Fermatean fuzzy decision matrix
NFFDM	Normalized Fermatean fuzzy decision matrix
PROMETHEE	Preference ranking organization method for enrichment evaluations
GAIA	Geometrical analysis for interactive aid

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