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Neutrosophic Analysis of Experimental Data Using Neutrosophic Graeco-Latin Square Design

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Abstract: Experimental designs are commonly used to produce valid, defensible, and supportable conclusions. Among commonly used block designs, the class of Latin square designs is used to study factors or treatment levels expressed as Latin letters and applying two blocking factors in rows and columns to simultaneously control two sources of nuisance variability. Another block design in which the error can be controlled by blocking three nuisance factors is obtained by simply using two superimposed Latin square designs, with one using the Latin letters and the other using the Greek letters. Such a design is termed as a Graeco-Latin square (GLS) design. While observing or measuring data in field or lab experiments, it is often noted to have vague, incomplete, and imprecise data for whatsoever reasons. In this regard, researchers have proposed various emerging approaches, which are based on fuzzy, intuitionistic fuzzy, and neutrosophic logic, and provide deeper understanding, analysis, and interpretations of the data. In this paper, we provide a brief review of the history of GLS designs and propose a neutrosophic Graeco-Latin square design, its model, and the analysis. To illustrate this, we have considered an experimental study which analyzes the effects of different formulations of a rocket propellant, which are used in aircrew escape systems, on the observed burning rate.

Keywords: imprecise data; neutrosophic statistics; block designs; Graeco-Latin square design

MSC: 62K10; 62K86

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1. Introduction

These methods of experimental design and analysis are important tools available to scientists and engineers to problem solve using statistical principles and techniques at the data collection stage. These are useful for making valid and supportable conclusions. Further, experimental design methods result in resource optimization, which results in saving time and money. Euler [1] suggested the method of construction of a Graeco-Latin square (GLS). In his paper, however, we acknowledge that Euler did not use the term orthogonal Latin square or GLS for a pair of orthogonal Latin squares, although this was derived from his work. He discussed the 4×4 square and considered two orthogonal Latin squares. Each of the two squares had the property that each diagonal also contains all the symbols. Euler discussed the Latin squares beginning with the famous 36-army-officers problem and presented the paper in St. Petersburg in 1779. In the officers problem, he denoted the regiments by the Latin letters a, b, c, d, e, and f and the ranks by the Greek letters α, β, γ, δ, ε, and ζ. Euler explained the arrangement of 36 squares of Latin and Greek letters in a 6×6 array, such that each row and column contains each Latin and Greek letter only once. Further, he showed that a GLS of order $4n + 2$, where n is a non-negative integer, is not possible. Thus, GL squares of the order 2, 6, 10, 14 ... are not possible. This conjecture, known as Euler's conjecture, remained a conjecture for several years. Klyve, D. and Lee

Stemkoski, L. [2] discussed the history of Euler's conjecture and repeated attempts at proving and disproving it. In addition, they surveyed mathematical techniques developed during the following two hundred years and which refuted Euler's conjecture. Dénes, J. and Keedwell, A.D. [3] prepared the first manuscript, devoted entirely to Latin squares, which provided an excellent account of the literature in three areas. Specifically, these areas include constructions of mutually orthogonal Latin squares (m.o.l.s.), geometric aspects of Latin squares, and algebraic aspects of Latin squares. Although statistical, algebraic, and geometric aspects were discussed, the major theme was the construction of orthogonal sets of Latin squares. Bose, R.C., Shrikhande, S.S., and Parker, E.T. [4], in their paper on the construction of m.o.l.s. and the falsity of Euler's conjecture, proved that if two orthogonal Latin squares of the order $v > 2$ do not exist, then 6 is the only Eulerian number. Fisher, R.A. and Yates, F. [5] provided the Graeco-Latin tables of order 3 up to order 12 (not including the order of six). Dodge, Y. and Shah, K.R. [6] showed that in an additive model with p-2 m.o.l.s., and if one omits up to p-1, observations from the same row, the same column, or that which corresponds to the same letter in any of the squares, they will all have effects that are estimable. Also, with only two missing observations not from the same row, the same column or that corresponding to the same letter in any of the squares, one degree of freedom is lost for each set of effects. Bose, R.C., Shrikhande, S.S., and Bhattacharya, K. [7] improved the method by obtaining better bounds on the maximum possible number of m.o.l.s. of the order v, denoted by $N(v)$. Euler's conjecture has been shown to be false for all $v = 4t + 2 > 6$. Bailey, R.A. [8] developed a general method based on groups for constructing quasi-complete Latin squares. This method provided a way to count the number of equivalent quasi-complete Latin squares of a side of, at most, 9. He discussed the randomization of such designs. He also provided an explicit construction for valid randomization sets of quasi-complete Latin squares whose side is an odd prime power. It was shown that randomization using a subset of all possible quasi-complete Latin squares may be valid while it may not be for the whole set. Hedayat, A. [9] stated that a Latin square design is called self-orthogonal if it is orthogonal to its transpose. He discussed such designs, outlining their applications, existence, and construction. Additional results on these designs are now available [10,11]. It is impossible to construct these designs for orders 2, 3, and 6. For other orders, they can be constructed. A table is given for orders 12, 14, 15, 16, 17, 18, 19, and 20. The table of such designs is available for $n \leq 20$, $n \neq 2, 3$, or 6. Martin, R.J. and Nadarajah, S. [12] showed that a Graeco-Latin square exists for all positive integers except for 1, 2, and 6. These GL squares, and the extension to sets of mutually orthogonal Latin squares, are useful in constructing experimental designs in several situations wherein there are four or more blocking or treatment structures. However, these are limited because, essentially, the numbers of treatments, blocks, or factor levels must all be the same.

Now, in what follows, we consider an example that an experimenter is interested in studying the effects of five different formulations (denoted by A, B, C, D, and E) of a rocket propellant used in aircrew escape systems on the observed burning rate [13]. Each formulation is mixed from a batch of raw material that is only large enough for five formulations to be tested. Further, several operators prepare formulations. It is anticipated that there may be substantial differences in the skills and experience of the operators. The experimenter stipulates that an additional factor, namely, test assemblies, could also be of importance as a contributing factor to the response variable burning rate. Let five test assemblies be denoted by the Greek letters α, β, γ, δ, and ε. Thus, it would seem that there are effects of three nuisance factors to be averaged out in the design. These nuisance factors are batches of raw material, operators, and test assemblies. The appropriate design for this problem consists of testing each formulation exactly once in each batch of raw material, for each formulation to be prepared and in each test assembly exactly once by each of the five operators. Thus, the experimenter considers a 5×5 Latin square design with treatments denoted by Latin letters A, B, C, D, and E, and superimposes on it a second 5×5 Latin square in which the test assemblies are denoted by Greek letters α, β, γ, δ, and ε. These two squares, when superimposed, have the property that each Greek letter appears once and only once with each Latin letter. Thus, we have the design obtained a Graeco-Latin square design. Since the data obtained from this experiment may have precise and/or imprecise values, our analysis needs to consider this fact. Smarandache [14] proposed neutrosophic logic, that each proposition has the degree of truth T, the degree of indeterminacy (neither true nor false) I, and the degree of falsity F. Here, T, I, and F are standard or non-standard real subsets of the non-standard unit interval $]^{-}0,1^{+}[$. The components T, I, and F are called neutrosophic components. Smarandache [15] generalized the intuitionistic fuzzy logic (IFL) and other logics to neutrosophic logic (NL). Further, the differences between IFL and NL (and the corresponding intuitionistic fuzzy set and neutrosophic set) are discussed.

In neutrosophic statistics [16,17], which presents the classical statistics using the neutrosophic logic, the data are represented in terms of neutrosophic values. That means the data have the form (T, I, F), where T expresses the truth, F represents the falsity, and I represents the indeterminacy. A neutrosophic datum x can be expressed as $x = d + i$, where d is the determinate or sure part of x, and i is the indeterminate or unsure part of x. As an example, a data value $x = 1 + i$, where $i \in [0, 0.7]$, is equivalent to $x \in [1, 1.7]$. That means, for sure, that $x \geq 1$ (meaning that the determinate part of x is 1). Meanwhile, the indeterminate part $i \in [0, 0.7]$ means the possibility for number x to be greater than or equal to 1 but less than or equal to 1.7. Further, any precise (crisp) data value, say $x = 5$, can be expressed as the neutrosophic datum $x = 5 + i$, where $i \in [0, 0] = [5, 5]$.

In Section 2, we present the neutrosophic Graeco-Latin square design, and its statistical model [11]. Then, we provide the sum of squares formula, the hypothesis tests for the treatments, and the row and column effects. This is followed by confidence intervals for the treatment mean differences. In Section 3, for the illustration of the neutrosophic Graeco-Latin square design analysis, we consider an example. This example is adapted from the textbook by Montgomery [13]. The example provides data which were collected using the Graeco-Latin square design for studying the effects of five different formulations (denoted by A, B, C, D, and E) of a rocket propellant used in aircrew escape systems on the observed burning rate by the operators (1, 2, 3, 4, and 5). In Section 4, we present the summary statistics and the results about hypothesis testing from the analysis of variance (ANOVA). In Section 5, we conclude the research findings. A list of references is given in the bibliography.

2. Methods Neutrosophic Graeco-Latin Square Design

To compare p treatment (factor) effects in an experiment, a $p \times p$ neutrosophic Graeco-Latin square design (NGLSD) is a $p \times p$ Latin square design with treatments written as the Latin letters A, B, C, D, and E, and superimposed on it is a second $p \times p$ Latin square in which the treatments are denoted by the Greek letters α, β, γ, δ, and ε. These two Latin squares have the property that each Greek letter appears once and only once with each Latin letter. Blocking factors and treatments are randomly assigned to the rows and columns of the squares with Latin letters and Greek letters. The treatment effects and/or data collected from such a design are neutrosophic data.

2.1. Neutrosophic Graeco-Latin Square Design Model

The linear model for the neutrosophic Graeco-Latin square design is described as

$$Y_{ijkl}^{N} = \mu^{N} + \theta_{i}^{N} + \rho_{j}^{N} + \varphi_{k}^{N} + \omega_{l}^{N} + \varepsilon_{ijkl}; i, j, k, l = 1, 2, \ldots p,$$

where $Y_{i,j,k,l}^{N}$ = the response of the l^{th} Latin letter treatment and the k^{th} Greek letter treatment from the (i, j) cell, μ^{N} = the general location parameter (overall mean),ω_{l}^{N} = the l^{th} Latin treatment effect, θ_{i}^{N} = the i^{th} row factor effect, ρ_{j}^{N} = the j^{th} column factor effect, φ_{k}^{N} = the k^{th} Greek letter treatment effect, and ε_{ijkl} = the experimental random error associated with the l^{th} Latin letter treatment and the k^{th} Greek letter treatment observation from the (i, j) cell. Random errors ε_{ijkl} are assumed to be independent following neutrosophic Normal distribution, i.e.,$N_N(0,\sigma^2)$. The number of treatments to be compared is denoted by p. The total number of runs (observations) in the NGLSD experiment is $N^N = p \times p$.

2.2. Calculation of Sum of Squares

Latin-letter treatment sum of squares (LTrSS) = $\frac{1}{p}\sum_l y^2_{...l} - \frac{y^2_{....}}{p\times p}$, d.f. = $p-1$.
Greek-letter treatment sum of squares (GTrSS) = $\frac{1}{p}\sum_k y^2_{..k.} - \frac{y^2_{....}}{p\times p}$, d.f. = $p-1$.
Row sum of squares (RSS) = $\frac{1}{p}\sum_i y^2_{i...} - \frac{y^2_{....}}{p\times p}$, d.f. = $p-1$.
Column sum of squares (CSS) = $\frac{1}{p}\sum_j y^2_{.j..} - \frac{y^2_{....}}{p\times p}$, d.f. = $p-1$.
Total sum of squares (TSS) = $\sum_{ijkl} y^2_{ijkl} - \frac{y^2_{....}}{p\times p}$, d.f. = p^2-1.
Error sum of squares (ESS) = TSS − LTrSS − GTrSS − RSS − CSS, d.f. = $(p-1)(p-3)$.

2.3. Hypothesis Tests for the Treatments, Row, and Column Effects

Latin-Letter Treatment Effects
Null hypothesis: $\tau_1^N = \tau_2^N = \ldots = \tau_p^N = 0$.
Alternative hypothesis: at least one of $\tau_l^{N'}s$ is not equal to zero.

$$\text{Test statistics follow the F-distribution, i.e., } F(p-1,(p-1)(p-3)) \\ = (p-1)(p-3)LTrSS/(p-1)ESS.$$

Row Effects
Null hypothesis: $\alpha_1^N = \alpha_2^N = \ldots = \alpha_p^N = 0$.
Alternative hypothesis: at least one of $\alpha_i^{N'}s$ is not equal to zero.
Test statistics follow the F-distribution, i.e.,

$$F(p-1,(p-1)(p-3)) = (p-1)(p-3)RSS/(p-1)ESS.$$

Column Effects
Null hypothesis: $\beta_1^N = \beta_2^N = \ldots = \beta_p^N = 0$.
Alternative hypothesis: at least one of $\beta_j^{N'}s$ is not equal to zero.
Test statistics follow the F-distribution, i.e.,

$$F(p-1,(p-1)(p-3)) = (p-1)(p-3)CSS/(p-1)ESS.$$

Greek-Letter Treatment Effects
Null hypothesis: $\gamma_1^N = \gamma_2^N = \ldots = \gamma_p^N = 0$.
Alternative hypothesis: at least one of $\gamma_i^{N'}s$ is not equal to zero.
Test statistics follow the F-distribution, i.e.,

$$F(p-1,(p-1)(p-3)) = (p-1)(p-3)GTrSS/(p-1)ESS.$$

2.4. Confidence Intervals for the Treatment Mean Differences

The $100(1-\alpha)\%$ confidence interval for the difference between two means $\mu_A - \mu_B$:

$$\hat{\mu}_A - \hat{\mu}_B \pm t_{n_A+n_B-2,\frac{\alpha}{2}} \times \left[\frac{MESS}{n_A} + \frac{MESS}{n_B}\right]^{0.5},$$

where the mean error sum of squares is denoted by $MESS = \frac{ESS}{(p-1)(p-3)}$.

3. Illustration: Description of the Experiment

An experimenter is interested in comparing the effects of five different formulations (denoted by A, B, C, D, and E) of a rocket propellant used in aircrew escape systems on the observed burning rate [10]. Each formulation is mixed from a batch of raw material that is only large enough for five formulations to be tested. Further, several operators will prepare formulations. It is anticipated that there may be substantial differences in the skills and experience of the operators. The experimenter stipulates that an additional factor, test assemblies, could be of importance. Let there be five test assemblies denoted by the Greek letters α, β, γ, δ, and ε. Thus, it would seem that there are three nuisance factors to be averaged out in the design: batches of raw material, operators, and test assemblies. The appropriate design for this problem consists of testing each formulation exactly once in each batch of raw material type, for each formulation to be prepared, and in each test assembly exactly once by each of the five operators (1, 2, 3, 4, and 5).

Thus, the experimenter considers a 5 × 5 Latin square design with treatments denoted by the Latin letters A, B, C, D, and E as given in Table 1a.

Table 1. (**a**) 5 × 5 Latin square design with treatments as Latin letters A, B, C, D, and E. (**b**) 5 × 5 Latin square design with treatments as Greek letters α, β, γ, δ, and ε. (**c**) 5 × 5 Graeco-Latin square design with treatments as Latin letters A, B, C, D, and E, with treatments as Greek letters α, β, γ, δ, and ε and operators denoted as 1, 2, 3, 4, and 5.

Operators→ Raw Material↓	**1**	**2**	**3**	**4**	**5**
			(**a**)		
1	A	B	C	D	E
2	B	C	D	E	A
3	C	D	E	A	B
4	D	E	A	B	C
5	E	A	B	C	D
			(**b**)		
1	α	γ	ε	β	δ
2	β	δ	α	γ	ε
3	γ	ε	β	δ	α
4	δ	α	γ	ε	β
5	ε	β	δ	α	γ
			(**c**)		
1	Aα	Bγ	Cε	Dβ	Eδ
2	Bβ	Cδ	Dα	Eγ	Aε
3	Cγ	Dε	Eβ	Aδ	Bα
4	Dδ	Eα	Aγ	Bε	Cβ
5	Eε	Aβ	Bδ	Cα	Dγ

And a 5 × 5 Latin square design with treatments as Greek letters α, β, γ, δ, and ε as given in Table 1b.

The two Latin squares in Table 1a,b are superimposed such that each Greek letter appears once and only once with each Latin letter. Two Latin squares are said to be orthogonal, and the design obtained is called a Graeco-Latin square design, as given in Table 1c.

In Table 2, we summarize the neutrosophic data after coding obtained by subtracting 25 from each observed burning rate value.

Table 2. Neutrosophic coded burning rate data in the rocket propellent experiment.

Operators→ Raw Material↓	**1**	**2**	**3**	**4**	**5**
1	Aα = [−0.99, −1.01]	Bγ = [−4.95, −5.05]	Cε = [−5.94, −6.06]	Dβ = [−0.99, −1.01]	Eδ = [−0.99, −1.01]
2	Bβ = [−7.92, −8.08]	Cδ = [−0.99, −1.01]	Dα = [4.95, 5.05]	Eγ = [1.98, 2.02]	Aε = [10.89, 11.11]
3	Cγ = [−6.93, −7.07]	Dε = [12.87, 13.13]	Eβ = [0.99, 1.01]	Aδ = [1.98, 2.02]	Bα = [−3.96, −4.04]
4	Dδ = [0.99, 1.01]	Eα = [5.94, 6.06]	Aγ = [0.99, 1.01]	Bε = [−1.98, −2.02]	Cβ = [−2.97, −3.03]
5	Eε = [−2.97, −3.03]	Aβ = [4.95, 5.05]	Bδ = [−4.95, −5.05]	Cα = [3.96, 4.04]	Dγ = [5.94, 6.06]
$\sum_l y_{.j..}$	[−17.82, −18.18]	[17.82, 18.18]	[−3.96, −4.04]	[4.95, 5.05]	[8.91, 9.09]

4. Results

The experimental data are analyzed using the 5 × 5 neutrosophic Graeco-Latin square design. The sums of squares of five different formulations of a rocket propellant (A, B, C, D, and E), five types of raw materials (RM1, RM2, RM3, RM4, and RM5), five operators (O1, O2, O3, O4, and O5), and five assemblies (α, β, γ, δ, and ε) are calculated. These calculations are presented in an ANOVA table. The results are discussed below.

4.1. Summary Statistics

The average burning rates and their effects due to different formulations are presented in Table 3.

Table 3. Neutrosophic summary statistics of average burning rate and effects due to formulations in rocket propellant experiment.

Formulations	**A**	**B**	**C**	**D**	**E**
Mean	[−2.828, −2.772]	[−4.848, −4.752]	[−2.626, −2.574]	[4.752, 4.848]	[0.99, 1.01]
Effect	[−4.808, −4.792]	[−6.828, −6.772]	[−4.606, −4.594]	[2.732, 2.868]	[−1.03, −0.97]
Assemblies	α	β	γ	δ	E
Mean	[1.98, 2.02]	[−1.212, −1.188]	[−0.606, −0.594]	[−0.808, −0.792]	[2.574, 2.626]
Effect	[0, 0.04]	[−3.192, −3.168]	[−2.586, −2.574]	[−2.788, −2.772]	[0.594, 0.646]
Operators	O_1	O_2	O_3	O_4	O_5
Mean	[−3.636, −3.564]	[3.564, 3.636]	[−0.808, −0.782]	[0.99, 1.01]	[1.782, 1.818]
Effect	[−4.032, 3.968]	[3.16, 3.24]	[−1.204, −1.196]	[0.586, 0.614]	[1.378, 1.422]
Raw Material	RM_1	RM_2	RM_3	RM_4	RM_5
Mean	[−2.828, −2.772]	[1.782, 1.818]	[0.99, 1.01]	[0.594, 0.606]	[1.386, 1.414]
Effect	[−3.224, −3.176]	[1.378, 1.422]	[0.586, 0.614]	[0.19, 0.21]	[0.982, 1.018]

It is noted from Table 3 that the maximum average burning rate is between 4.752 and 4.848 due to Formulation D and the minimum is between −4.848 and −4.752 due to Formulation B. For Assembly ε, the maximum average burning rate is between 2.574 and 2.626, while for Assembly β, its minimum value is between −1.212 and −1.188. For the raw material RM2, the maximum average burning rate is between 1.782 and 1.818, while for the raw material RM1, its minimum value is between −2.828 and −2.772. The maximum average burning rate due to Operator O2 is between 3.564 and 3.636, while the minimum between −3.636 and −3.564 due to Operator O1.

In terms of effects, it may be noted that the maximum effect on burning rate is between 2.732 and 2.868 due to Formulation D, and that the minimum is between −6.828 and −6.772 due to Formulation B. Similarly, we can describe the neutrosophic effects due to assemblies, raw materials, and operators.

Charts for the means of burning rates due to raw materials, operators, formulations, and assemblies are given in Charts 1–4.

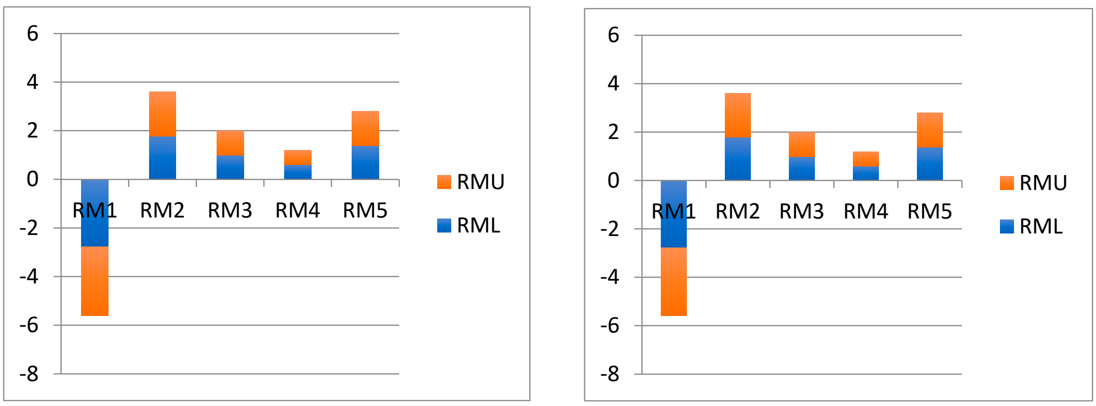

Chart 1. Average burning rates due to raw materials (RM1, RM2, RM3, RM4, and RM5).

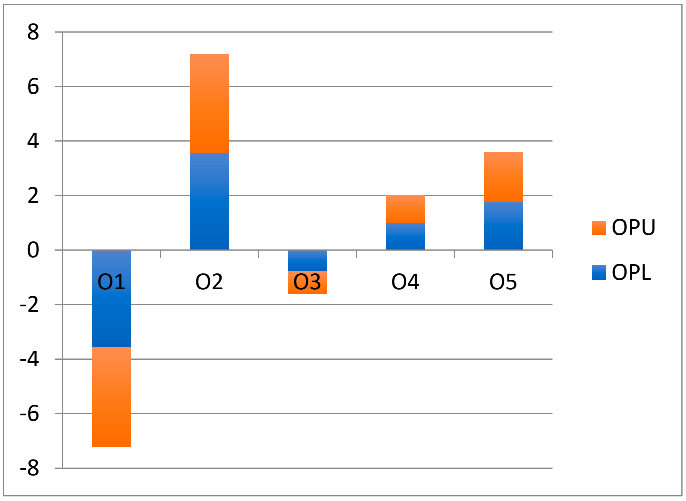

Chart 2. Average burning rates due to operators (O1, O2, O3, O4, and O5).

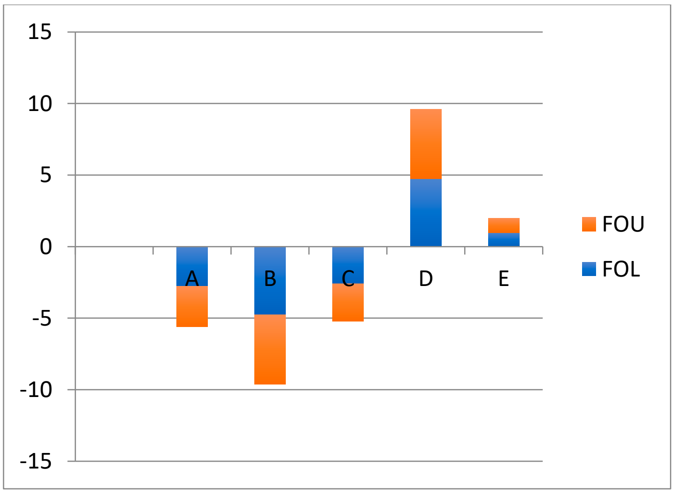

Chart 3. Average burning rates due to formulations (A, B, C, D, and E).

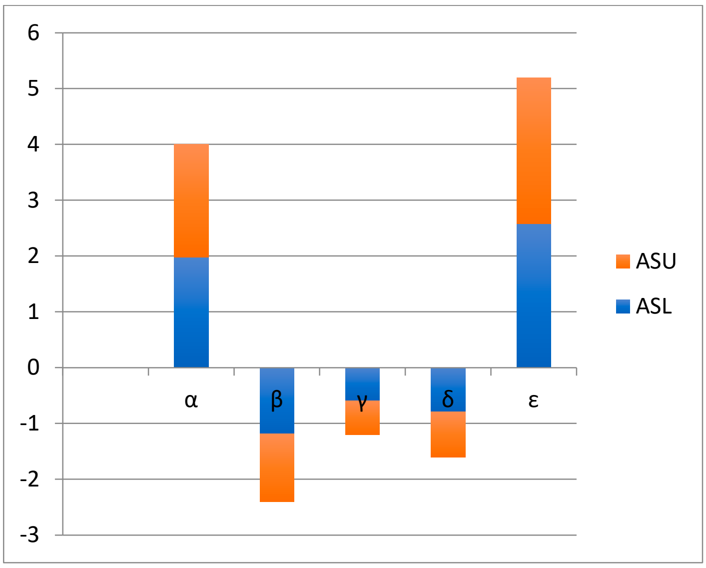

Chart 4. Average burning rates due to assemblies (α, β, γ, δ, and ε).

4.2. Hypotheses Tests

The analysis of variance (ANOVA) table is prepared for the neutrosophic Graeco-Latin square design and is shown in Table 4. The calculated *p*-values associated with the F-test statistics are, respectively, noted as [0.0004, 0.0101] for the formulations, [0.0594, 0.3443] for the assemblies, [0.0478, 0.3064] for the raw materials, and [0.0049, 0.0771] for the operators.

Table 4. Neutrosophic ANOVA table for the burning rate in rocket propellant experiment.

Source	**DF**	**SS**	**F(4, 8)**	***p*-Value**
Formulation	4	[323.273, 336.793]	[6.988, 18.939]	[0.0004, 0.0101]
Assemblies	4	[60.606, 63.406]	[1.310, 3.566]	[0.0594, 0.3443]
Raw Material	4	[66.487, 69.527]	[1.437, 3.910]	[0.0478, 0.3064]
Operator	4	[146.855, 157.095]	[3.174, 8.834]	[0.0049, 0.0771]
Error	8	[35.566, 92.527]		
Total	24	[662.388, 689.748]		

Since the maximum *p*-value for comparing formulations is 0.0101, which is smaller than the 5% significance level, the formulations have significantly different effects on the burning rates. The *p*-values for the assemblies, raw materials, and operators are, respectively, [0.0594, 0.3443], [0.0478, 0.3064], and [0.0049, 0.0771]. Thus, we may fail to conclude that their effects are significantly different at the 5% significance level. However, for operators, since *p*-values are in the interval [0.0049, 0.0771], this indicates that there are likely to be differences due to operators as well. Since the error degrees of freedom is 8 (low), tests may be less sensitive.

For the comparison of results using the neutrosophic approach and the classical approach based on the crisp data, we present the classical ANOVA results in Table 5.

Table 5. ANOVA table for the burning rate in rocket propellant experiment.

Source	**DF**	**SS**	**F(4, 8)**	***p*-Value**
Formulation	4	330.033	10.30604327	0.00303
Assemblies	4	62.006	1.936280671	0.19779
Raw Material	4	68.007	2.123675767	0.16933
Operator	4	151.975	4.745770651	0.02947
Error	8	64.0465		
Total	24	676.068		

It is noted that the *p*-values for the hypothesis tests of formulation, assemblies, raw materials, and operators from Table 4 are, respectively, [0.0004, 0.0101], [0.0594, 0.3443], [0.0478, 0.3064], and [0.0049, 0.0771], while the *p*-values from Table 5 are 0.00303, 0.19779, 0.16933, and 0.02947. Thus, it may be noted that the *p*-values for all factors from the classical ANOVA lie inside the neutrosphic *p*-value intervals from the neutrosophic ANOVA in Table 4.

5. Conclusions

Neutrosophic logic, neutrosophic math, and neutrosophic statistics provide useful tools to understand and study incomplete, vague, or imprecise information. We have considered the application of a neutrosophic Graeco-Latin square design. The experimental data analyzed using the neutrosophic statistics are for studying the effects of the formulations of a rocket propellant used in aircrew escape systems on the observed burning rate. The experiment considered three blocking factor operators, batches of raw material, and assemblies. We have compared the results from our proposed neutrosophic method with the classical crisp data method. It is noted that the proposed neutrosophic method results are in agreement with the classical analysis results. We have shown that the formulations are significantly different at the 1-percent significance level. We have also observed that the neutrosophic method resulted in estimates of error with lesser degrees of freedom.

Therefore, the replication of the experiment is recommended to increase the error degrees of freedom. Further, in terms of missing data for analysis and design, the proposed neutrosophic design can address problems under the MAR missing data mechanisms similar to these issues in the classical design [18]. A recent paper by Yang et al. [19] address the MAR missing data mechanism in artificial intelligence-enabled detection and the assessment of Parkinson's disease using nocturnal breathing signals. Thus, the proposed neutrosophic design does not need a priori statistical assumptions when handling incomplete missing information.

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