

## Article

# A Comparative Study on Multi-Criteria Decision-Making in Dressing Process for Internal Grinding

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**Abstract:** The MCDM problem is very important and often encountered in life and in engineering as it is used to determine the best solution among various possible alternatives. In this paper, the results of the MCDM problem in the dressing process for internal grinding are presented. To perform this work, an experiment with six input parameters, including the depth and the time of fine dressing, the depth and the time of coarse dressing, non-feeding dressing, and dressing feed rate, was conducted. The experiment was designed according to the Taguchi method with the use of L16 orthogonal arrays. In addition, TOPSIS, MARCOS, EAMR and MAIRCA methods were selected for the MCDM to obtain the minimum SR and the maximum MRR simultaneously. In addition, the weight determination for criteria was implemented by MEREC and entropy methods. From the results, the best solution to the multi-criteria problem for the dressing process in internal grinding has been proposed.

**Keywords:** dressing; internal grinding; multi-criteria decision making; MCDM; MARCOS; TOPSIS; EAMR; MAIRCA



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## 1. Introduction

MCDM is a problem for obtaining the best alternative among many different alternatives. This problem is very common in many fields such as in business [1,2], transport [3,4], medicine [5,6], military [7,8], and construction [9–11], etc. Recently, MCDM has been widely applied in mechanical manufacturing processes. This problem proves to be quite suitable for this field because a machining process often requires meeting many criteria simultaneously, such as maximum MMR, minimum SR, maximum tool life, or minimum machining cost. However, these criteria often contradict each other. For a small SR, it is necessary to reduce the depth of the cut and the feed rate, which in turn reduces the MMR. In addition, increasing the MMR will require an increase in the depth of cut and the feed rate, and it will increase the SR and reduce the tool life. Therefore, in this case, it is necessary to solve the MCDM problem to choose the best solution for a mechanical machining process.

Up to now, there have been numerous studies on MCDM for various mechanical machining processes such as milling, turning, grinding, and electrical discharge machining (EDM), etc. In addition, different MCDM methods such as MARCOS, TOPSIS, EAMR, and MAIRCA have been used for solving this problem. The studies on MCDM can use one or many methods to perform.

In fact, many studies use only one MCDM method to select the best option for mechanical processes. Saha, A. and Majumder, H. [12] reported MCDM results when turning ASTM A36 mild steel by using the COPRAS-G method. In their work, SR, the power consumption

and the tool vibration frequency were selected as criteria. Finally, an optimal set of input factors, including the depth of cut, the spindle speed and the feed rate, were presented. The Taguchi-DEAR method has been used for non-traditional machining processes such as in waterjet machining [13] and in electrical discharge machining [14,15]. The TOPSIS method has been applied for drilling [16,17], milling [18,19], turning [20,21], EDM [22–24], and abrasive waterjet machining [25], etc. The MARCOS method was used for turning, milling and drilling processes [26].

Until now, there have been numerous studies using a few different methods for solving the MCDM problem in mechanical machining processes. D.D. Trung and H.X. Think [27] used MAIRCA, TOPSIS, EAMR, and MARCOS methods for the turning process. TOPSIS and VIKOR methods were applied to the EDM process [28]. TOPSIS and COPRAS methods were used for the drilling process by Varatharajulu, M., et al. [29]. In [30], the use of eight methods containing TOPSIS, SAW, VIKOR, MOORA, WASPAS, COPRAS, PSI, and PIV for turning 150Cr14 steel was evaluated. The TOPSIS, MOORA, and GRA methods have been used for the PMEDM when processing SKD11 tool steel [31]. MCDM for hard turning using TOPSIS and PIV methods has been reported in [32].

In the grinding process as well as in internal grinding, the grinding wheel is gradually worn. In addition, some metal chips may adhere to the wheel surface. This results in the reduction in the cutting ability, the increase in the cutting forces and vibrations and, as a result, reduces the surface quality and the MRR. The dressing process aims to refresh the wheel surface to overcome the above disadvantages. Therefore, determining the best or the reasonable mode of the dressing process is very necessary. From the above analysis it can be seen that, although there have been several studies on MCDM in grinding processes, there is no research on applying MCDM methods to determine the best option for the dressing process so far.

This paper presents a study on MCDM for the dressing process for internal grinding. In this work, two criteria, including RS and MRR, were selected for the investigation based on their importance role in evaluating the effectiveness of the dressing process. In addition, four methods, including TOPSIS, MARCOS, EAMR, and MAIRCA, were employed for MCDM. In addition, MEREC and entropy methods were chosen to determine the weights of the criteria. The results of using the above-mentioned methods to determine the best alternative or to select the optimal input factors for the dressing process in internal grinding have been shown.

## 2. Methods of MCDM

### 2.1. TOPSIS Method

The TOPSIS method is performed according to the following steps [33]:

Step 1: Creating the initial matrix by the following equation:

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ x_{21} & \cdots & x_{2n} \\ \vdots & \cdots & \vdots \\ x_{mn} & \cdots & x_{mn} \end{bmatrix} \quad (1)$$

where  $m$  is the number of the alternatives;  $n$  is the number of criteria.

Step 2: Calculating the normalized values  $k_{ij}$  by:

$$k_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}} \quad (2)$$

Step 3: Determining the weighted normalized decision matrix by the following formula:

$$l_{ij} = w_j \times k_{ij} \quad (3)$$

Step 4: Calculating the best alternative  $A^+$  and the worst alternative  $A^-$  by:

$$A^+ = \{l_1^+, l_2^+, \dots, l_j^+, \dots, l_n^+\} \quad (4)$$

$$A^- = \{l_1^-, l_2^-, \dots, l_j^-, \dots, l_n^-\} \quad (5)$$

Wherein,  $l_j^+$  and  $l_j^-$  are the best and worst values of the  $j$  criterion ( $j = 1, 2, \dots, n$ ).

Step 5: Calculating  $D_i^+$  and  $D_i^-$  by:

$$D_i^+ = \sqrt{\sum_{j=1}^n (l_{ij} - l_j^+)^2} \quad i = 1, 2, \dots, m \quad (6)$$

$$D_i^- = \sqrt{\sum_{j=1}^n (l_{ij} - l_j^-)^2} \quad i = 1, 2, \dots, m \quad (7)$$

Step 6: Determining ratios  $R_i$  by:

$$R_i = \frac{D_i^-}{D_i^- + D_i^+} \quad i = 1, 2, \dots, m; 0 \leq R_i \leq 1 \quad (8)$$

Step 7: Ranking the order of alternatives by maximizing  $R$ .

## 2.2. MARCOS Method

MCDM using the MARCOS method is achieved by using the following steps [34]:

*Step 1:* Using step 1 of the TOPSIS methods.

*Step 2:* Building an extended initial matrix by adding the ideal (AI) and anti-ideal solution (AAI) into the initial decision-making matrix.

$$X = \begin{matrix} AAI \\ A_1 \\ A_2 \\ \vdots \\ A_m \\ AI \end{matrix} \begin{bmatrix} x_{aa1} & \cdots & x_{aan} \\ x_{11} & \cdots & x_{1n} \\ x_{21} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots \\ x_{m1} & \cdots & x_{mn} \\ x_{ai1} & \cdots & x_{ain} \end{bmatrix} \quad (9)$$

in which  $AAI = \min(x_{ij})$  and  $AI = \max(x_{ij})$  if the necessity set with criterion  $j$  is as large as possible;  $AAI = \max(x_{ij})$  and  $AI = \min(x_{ij})$  if the necessity set with criterion  $j$  is as small as possible;  $i = 1, 2, \dots, m; j = 1, 2, \dots, n$ .

*Step 3:* Normalizing the extended initial matrix (X). The elements of normalized matrix  $N = [n_{ij}]_{m \times n}$  are found by

$$n_{ij} = x_{AI} / x_{ij} \text{ if the criterion } j \text{ is as small as possible} \quad (10)$$

$$n_{ij} = x_{ij} / x_{AI} \text{ if criterion } j \text{ is as large as possible} \quad (11)$$

*Step 4:* Determining the weighted normalized matrix  $C = [c_{ij}]_{m \times n}$  by:

$$c_{ij} = n_{ij} \cdot w_j \quad (12)$$

in which  $w_j$  is the weight coefficient of the criterion  $j$ .

*Step 5:* Finding the utility degree of alternatives  $K_i^-$  and  $K_i^+$  by the following equation:

$$K_i^- = S_i / S_{AAI} \quad (13)$$

$$K_i^+ = S_i / S_{AI} \quad (14)$$

With

$$S_i = \sum_{i=1}^m c_{ij} \quad (15)$$

Step 6: Computing the utility function of alternatives  $f(K_i)$  by:

$$f(K_i) = \frac{K_i^+ + K_i^-}{1 + \frac{1-f(K_i^+)}{f(K_i^+)} + \frac{1-f(K_i^-)}{f(K_i^-)}} \quad (16)$$

where  $f(K_i^-)$  and  $f(K_i^+)$  are the utility functions related to the anti-ideal solution and the ideal solution. These functions are determined by:

$$f(K_i^-) = K_i^+ / (K_i^+ + K_i^i) \quad (17)$$

$$(K_i^+) = K_i^- / (K_i^+ + K_i^i) \quad (18)$$

Step 7: Determining the alternative with the highest possible value of the utility function by ranking the alternatives based on the final value of the utility functions  $f(K_i)$ .

### 2.3. The EAMR Method

The steps to complete the MCDM by the EAMR method are as follows [35].

Step 1: Constructing the decision matrix:

$$X_d = \begin{bmatrix} x_{11}^d & \cdots & x_{1n}^d \\ x_{21}^d & \cdots & x_{2n}^d \\ \vdots & \cdots & \vdots \\ x_{m1}^d & \cdots & x_{mn}^d \end{bmatrix} \quad (19)$$

in which  $d$  is the index demonstrating the decision maker;  $1 \leq d \leq k$  with  $k$  is the decision maker number.

Step 2: Finding the mean value of each alternative for each criterion by:

$$\bar{x}_{ij} = \frac{1}{k} (x_{ij}^1 + x_{ij}^2 + \cdots + x_{ij}^k) \quad (20)$$

where  $k$  is the decision maker index.

Step 3: Calculating the weights for the criteria.

Step 4: Finding the average weighted value for each criterion:

$$\bar{w}_j = \frac{1}{k} (w_j^1 + w_j^2 + \cdots + w_j^k) \quad (21)$$

Step 5: Determining  $n_{ij}$  values by the following equation:

$$n_{ij} = \frac{\bar{x}_{ij}}{e_j} \quad (22)$$

wherein  $e_j$  can be found by:

$$e_j = \max_{i \in \{1, \dots, m\}} (\bar{x}_{ij}) \quad (23)$$

Step 6: Determining the normalized weight values by:

$$v_{ij} = n_{ij} \cdot \bar{w}_j \quad (24)$$

Step 7: Finding the normalized score of the criteria.

$$G_i^+ = v_{i1}^+ + v_{i2}^+ + \cdots + v_{im}^+ \text{ if the criterion } j \text{ is as large as possible} \quad (25)$$

$$G_i^- = v_{i1}^- + v_{i2}^- + \dots + v_{im}^- \text{ if the criterion } j \text{ is as small as possible} \tag{26}$$

Step 8: Determining the ranking values (RV) from  $G_i^+$  and  $G_i^-$ .

Step 9: Finding the evaluation score of the alternatives by:

$$S_i = \frac{RV(G_i^+)}{RV(G_i^-)} \tag{27}$$

The best alternative is the one with the largest  $S_i$ .

### 2.4. MAIRCA Method

The steps required to conduct the MAIRCA method are as follows [36]:

Step 1: Creating the initial matrix as step 1 in the TOPSIS method.

Step 2: Determining preferences according to alternatives  $P_{A_j}$ . Assuming that the priority for each criterion is the same and it can be found as follows:

$$P_{A_j} = \frac{1}{m} \quad j = 1, 2, \dots, n \tag{28}$$

Step 3: Finding the elements  $t_{p_{ij}}$  by:

$$t_{p_{ij}} = P_{A_j} \cdot w_j \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots \tag{29}$$

in which  $w_j$  is the weight of the criterion  $j$ .

Step 4: Determining  $t_{r_{ij}}$  by:

$$t_{r_{ij}} = t_{p_{ij}} \cdot \left( \frac{x_{ij} - x_i^-}{x_i^+ - x_i^-} \right) \text{ if the criterion } j \text{ is as large as possible} \tag{30}$$

$$t_{r_{ij}} = t_{p_{ij}} \cdot \left( \frac{x_{ij} - x_i^+}{x_i^- - x_i^+} \right) \text{ if the criterion } j \text{ is as small as possible} \tag{31}$$

Step 5: Finding  $g_{ij}$  by:

$$g_{ij} = t_{p_{ij}} - t_{r_{ij}} \tag{32}$$

Step 6: Calculating the final values of the criterion functions (Qi) by the following formula:

$$Q_i = \sum_{i=1}^m g_{ij} \tag{33}$$

## 3. Weight Calculation of Criteria

In this study, the weight of the criteria is determined by two methods, MEREC and entropy. This section describes how to apply these methods.

### 3.1. The MEREC Method

The MEREC method can be applied by the following steps [37]:

Step 1: Establishing the initial matrix as in the TOPSIS method.

Step 2: Determining the normalized values by:

$$h_{ij} = \frac{\min x_{ij}}{x_{ij}} \text{ if the criterion } j \text{ is as large as possible} \tag{34}$$

$$h_{ij} = \frac{x_{ij}}{\max x_{ij}} \text{ if the criterion } j \text{ is as small as possible} \tag{35}$$

Step 3: Determining the alternative performance  $S_i$  by:

$$S_i = \ln \left[ 1 + \left( \frac{1}{n} \sum_j |\ln(h_{ij})| \right) \right] \tag{36}$$

Step 4: Determining the performance of  $i^{\text{th}}$  alternative  $S'_{ij}$  by:

$$S'_{ij} = Ln \left[ 1 + \left( \frac{1}{n} \sum_{k, k \neq j} |ln(h_{ij})| \right) \right] \quad (37)$$

Step 5: Determining the removal effect of the  $j^{\text{th}}$  criterion  $E_j$  by:

$$E_j = \sum_i |S'_{ij} - S_i| \quad (38)$$

Step 6: Determining the weight of the criteria by:

$$w_j = \frac{E_j}{\sum_k E_k} \quad (39)$$

### 3.2. The Entropy Method

The weights of the criteria can be found by the entropy method, which can be applied by the following steps [38]:

Step 1: Calculating the normalized values of indicators.

$$p_{ij} = \frac{x_{ij}}{m + \sum_{i=1}^m x_{ij}^2} \quad (40)$$

Step 2: Determining the entropy value for each indicator.

$$me_j = - \sum_{i=1}^m [p_{ij} \times ln(p_{ij})] - \left( 1 - \sum_{i=1}^m p_{ij} \right) \times ln \left( 1 - \sum_{i=1}^m p_{ij} \right) \quad (41)$$

Step 3: Finding the weight of each indicator.

$$w_j = \frac{1 - me_j}{\sum_{j=1}^m (1 - me_j)} \quad (42)$$

## 4. Experimental Setup

To perform the MCDM problem, an experiment was performed. This experiment was designed according to the Taguchi method with the design L16 orthogonal array ( $4^4 \times 2^2$ ). Table 1 shows the input factors and their levels. The experimental setup is depicted in Figure 1 with the dressing and grinding parameters shown in Tables 2 and 3, respectively. The experiment was carried out as follows: conducting the dressing process according to the plan as shown in Table 4. After dressing, the grinding wheel was used to grind test samples in keeping with the grinding mode, as shown in Table 3. After conducting experiments, the SR (in this case,  $R_a$  ( $\mu\text{m}$ )) was measured and the MMR ( $\text{mm}^3$ ) was calculated. The experimental plan and the responses (RS (the average result of three measurements) and MMR) are given in Table 4.

**Table 1.** Input parameters of the dressing process.

No.	Input Factors	Symbol	Unit	Levels			
				1	2	3	4
1	Coarse dressing depth	$a_r$	mm	0.025	0.03	-	-
2	Coarse dressing time	$n_r$	times	1	2	3	4
3	Fine dressing depth	$a_f$	mm	0.005	0.01	0.015	0.02
4	Fine dressing time	$n_f$	times	0	1	2	3
5	Non-feeding dressing	$n_0$	times	0	1	2	3
6	Dressing feed rate	$S_d$	m/min	1	1.2	-	-

**Table 2.** Input parameters of the grinding process.

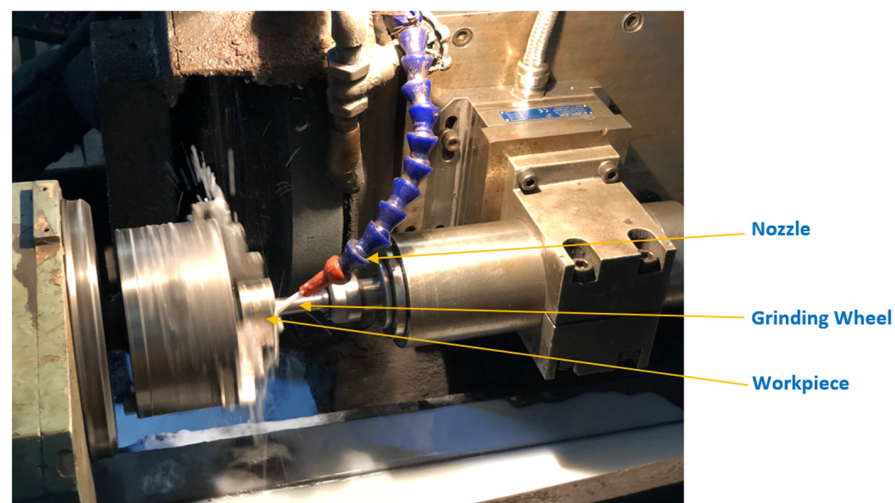
No.	Input Factors	Symbol	Unit	VALUE
1	Wheel speed	$n_s$	rpm	12,000
2	Workpiece speed	$n_w$	rpm	150
3	Radial wheel feed	$f_r$	mm/stroke	0.0025
4	Axial feed speed	$v_{fa}$	mm/min	1

**Table 3.** Specifications of the experimental setup.

Parameters	Specification
Grinding machine	Minakuchi MGU-65-26T (Japan)
Workpiece material	SKD11 too steel
Workpiece size	$\phi 25 \times \phi 36 \times 22$ (mm)
Grinding wheel	19A 120L 8 ASI T S 1A (Japan)
Grinding wheel size	$\phi 23 \times \phi 25 \times 8$ (mm)
Diamond dresser	DKB3E002110
Surface roughness tester	Mitutoyo SV-3100
Coolant material	Caltex Aquatex 3180 (3.9%; 2.87 L/min)

**Table 4.** Experimental plan and output results.

No.	Input Parameters					Output Results		
	$a_r$ (mm)	$n_r$ (times)	$a_f$ (mm)	$n_f$ (times)	$n_0$ (times)	$S_d$ (m/min)	Ra ( $\mu\text{m}$ )	MRR ( $\text{mm}^3/\text{s}$ )
1	0.025	1	0.005	0	0	1	0.3652	0.9186
2	0.03	1	0.01	1	1	1	0.2137	1.0413
3	0.025	1	0.015	2	2	1.2	0.1948	1.1215
4	0.03	1	0.02	3	3	1.2	0.2417	1.1111
5	0.03	2	0.005	1	2	1.2	0.1850	1.2459
6	0.025	2	0.01	0	3	1.2	0.2477	1.2667
7	0.03	2	0.015	3	0	1	0.2520	1.1782
8	0.025	2	0.02	2	1	1	0.2167	1.2251
9	0.03	3	0.005	2	3	1	0.3064	1.4878
10	0.025	3	0.01	3	2	1	0.3239	1.3724
11	0.03	3	0.015	0	1	1.2	0.3406	1.2709
12	0.025	3	0.02	1	0	1.2	0.3541	1.1988
13	0.025	4	0.005	3	1	1.2	0.3179	1.2273
14	0.03	4	0.01	2	0	1.2	0.3126	1.2647
15	0.025	4	0.015	1	3	1	0.3259	1.1247
16	0.03	4	0.02	0	2	1	0.3634	1.1898

**Figure 1.** Experimental setup.

### 5. MCDM Using the MEREC Method for Calculating the Weights of Criteria

This section deals with MCDM when using the TOPSIS, MARCOS, EAMR, and MAIRCA methods, and the weights of the criteria are calculated by the MEREC method.

#### 5.1. Determining the Weights for the Criteria

The calculation of the weights for the criteria when using the MEREC method can be completed by the following steps (see Section 3.1): (1) determining the normalized values  $h_{ij}$  by Equations (34) and (35), (2) calculating  $S_i$  and  $S'_{ij}$  by Equations (36) and (37); (3) finding the criterion removal effect by Equation (38), (4) determining the weight of the criteria  $w_j$  by Equation (39). It has been found that the weights of Ra and MRR were 0.5003 and 0.4997, respectively.

#### 5.2. Using TOPSIS Method

The steps to achieve MCDM using the TOPSIS method are as follows (see Section 2.1): The normalized values of  $k_{ij}$  are determined by the Formula (2). Furthermore, the normalized weighted values  $l_{ij}$  are found using Formula (3). In addition, the  $A^+$  and  $A^-$  values of Ra and MRS are calculated according to Equations (4) and (5). It is found that SR and MRR are equal to 0.0583 and 0.2058 for  $A^+$  and 0.2681 and 0.0212 for  $A^-$ . In addition, the values  $D_i^+$  and  $D_i^-$  have been determined according to Formulas (6) and (7). Finally, the ratio  $R_i$  is identified using Equation (8). The calculated results and ranking of alternatives by the TOSIS method are given in Table 5.

Table 5. Calculated parameters and ranking by the TOPSIS method.

Trial.	$k_{ij}$		$l_{ij}$		$D_i^+$	$D_i^-$	$R_i$	Rank
	RS	MRR	RS	MRR				
A1	0.3133	0.1899	0.1568	0.0949	0.0972	0.0000	0.0000	16
A2	0.1833	0.2153	0.0917	0.1076	0.0477	0.0663	0.5813	5
A3	0.1671	0.2318	0.0836	0.1158	0.0381	0.0761	0.6665	3
A4	0.2073	0.2297	0.1037	0.1148	0.0459	0.0566	0.5524	7
A5	0.1587	0.2575	0.0794	0.1287	0.0250	0.0844	0.7716	1
A6	0.2125	0.2618	0.1063	0.1308	0.0353	0.0620	0.6371	4
A7	0.2162	0.2436	0.1082	0.1217	0.0430	0.0555	0.5634	6
A8	0.1859	0.2532	0.0930	0.1265	0.0304	0.0712	0.7011	2
A9	0.2629	0.3076	0.1315	0.1537	0.0521	0.0640	0.5510	8
A10	0.2779	0.2837	0.1390	0.1418	0.0608	0.0501	0.4519	9
A11	0.2922	0.2627	0.1462	0.1313	0.0704	0.0379	0.3499	12
A12	0.3038	0.2478	0.1520	0.1238	0.0785	0.0293	0.2720	14
A13	0.2727	0.2537	0.1364	0.1268	0.0631	0.0378	0.3748	11
A14	0.2682	0.2614	0.1342	0.1306	0.0594	0.0423	0.4159	10
A15	0.2796	0.2325	0.1399	0.1162	0.0712	0.0272	0.2763	13
A16	0.3118	0.2460	0.1560	0.1229	0.0825	0.0280	0.2534	15

#### 5.3. Using MARCOS Method

According to the MARCOS method, the steps for multi-objective decision making are implemented as in Section 2.2. First, the ideal solution (AI) and the anti-ideal solution (AAI) are calculated according to Formula (9). The obtained calculation results of Ra and MRR are 0.185 ( $\mu\text{m}$ ) and 1.4878 ( $\text{mm}/\text{h}$ ) with AI, and 0.3652 ( $\mu\text{m}$ ) and 0.9186 ( $\text{mm}^3/\text{s}$ ) with AAI. The next step is to calculate the normalized values  $u_{ij}$  according to the Formulas (10) and (11). Then, the normalized values taking into account the weight  $c_{ij}$  are determined by the Formula (12). In addition, the coefficients  $K_i^-$  and  $K_i^+$  are found by Equations (13) and (14). The values of  $f(K_i^-)$  and  $f(K_i^+)$  are achieved by Equations (17) and (18). It is found that  $f(K_i^-) = 0.4342$  and  $f(K_i^+) = 0.5658$ . Finally, the values of  $f(K_i)$  are calculated using Formula (16). The calculation results of some parameters and the ranking of the alternatives are shown in Table 6.



**Table 6.** Calculated parameters and ranking by the MARCOS method.

Trial.	$K^-$	$K^+$	$f(K^-)$	$f(K^+)$	$f(K_i)$	Rank
A1	0.335931	0.437706	0.565777	0.434223	0.2520	16
A2	0.468012	0.609803	0.565777	0.434223	0.3510	6
A3	0.509223	0.6635	0.565777	0.434223	0.3819	2
A4	0.452015	0.588959	0.565777	0.434223	0.3390	8
A5	0.549206	0.715596	0.565777	0.434223	0.4119	1
A6	0.477707	0.622436	0.565777	0.434223	0.3583	5
A7	0.456109	0.594294	0.565777	0.434223	0.3421	7
A8	0.501322	0.653204	0.565777	0.434223	0.3760	3
A9	0.479265	0.624465	0.565777	0.434223	0.3595	4
A10	0.446363	0.581596	0.565777	0.434223	0.3348	9
A11	0.417635	0.544164	0.565777	0.434223	0.3132	12
A12	0.396927	0.517182	0.565777	0.434223	0.2977	13
A13	0.420461	0.547846	0.565777	0.434223	0.3154	11
A14	0.430944	0.561505	0.565777	0.434223	0.3232	10
A15	0.39558	0.515426	0.565777	0.434223	0.2967	14
A16	0.39112	0.509615	0.565777	0.434223	0.2934	15

5.4. Using EAMR Method

MCDM by the EAMR method is carried out in the following steps (see Section 2.3): First, the decision matrix is built according to Formula (19) with the attention that since there is only one set of results,  $k = 1$ . Next, the mean of the alternatives for each criterion is gained by Equation (20) with the note that since  $k$  equals 1,  $\bar{x}_{ij} = x_{ij}$ . After that, the weights for the criteria are determined (see Section 3). Then, the average weighted values are found using Formula (21) with the note that since  $k$  is 1,  $\bar{w}_j = w_j$ . The  $n_{ij}$  values are calculated by Equation (22) with  $e_j$  determined by (23). Furthermore, Equation (24) is used to calculate  $v_{ij}$ . Equations (25) and (26) are applied to determine the respective values  $G_i$ . Finally,  $S_i$  values are calculated according to Formula (27). The calculated results and the ratings of the alternatives using the EAMR method are given in Table 7.

**Table 7.** Calculated parameters and ranking by the EAMR method.

Trial.	$n_{ij}$		$v_{ij}$		$G_i$		$S_i$	Rank
	Ra	MRR	Ra	MRR	Ra	MRR		
A1	1.0000	0.6174	0.5003	0.3085	0.5003	0.3085	0.6167	16
A2	0.5850	0.6999	0.2927	0.3497	0.2927	0.3497	1.1949	5
A3	0.5333	0.7538	0.2668	0.3767	0.2668	0.3767	1.4117	2
A4	0.6617	0.7468	0.3310	0.3732	0.3310	0.3732	1.1272	9
A5	0.5065	0.8374	0.2534	0.4184	0.2534	0.4184	1.6511	1
A6	0.6781	0.8513	0.3393	0.4254	0.3393	0.4254	1.2539	4
A7	0.6900	0.7919	0.3452	0.3957	0.3452	0.3957	1.1463	8
A8	0.5932	0.8234	0.2968	0.4115	0.2968	0.4115	1.3863	3
A9	0.8391	1.0000	0.4198	0.4997	0.4198	0.4997	1.1904	6
A10	0.8868	1.0000	0.4437	0.4997	0.4437	0.4997	1.1263	10
A11	0.9325	1.0000	0.4665	0.4997	0.4665	0.4997	1.0712	12
A12	0.9696	0.9478	0.4851	0.4736	0.4851	0.4736	0.9764	15
A13	0.8704	0.9704	0.4355	0.4849	0.4355	0.4849	1.1136	11
A14	0.8558	1.0000	0.4282	0.4997	0.4282	0.4997	1.1671	7
A15	0.8923	0.9452	0.4464	0.4723	0.4464	0.4723	1.0581	13
A16	0.9951	1.0000	0.4979	0.4997	0.4979	0.4997	1.0037	14

5.5. Using MAIRCA Method

The MAIRCA method for MCDM is carried out according to the following steps (see Section 2.4): The initial matrix is set up according to Formula (1). The priority of criterion  $P_{A_j}$  is calculated using Formula (28). In this case, since the criteria are considered equal, the

priority for both SR and MRR is equal to  $1/16 = 0.0625$ . In addition, the value of parameter  $t_{p_{ij}}$  is found by Equation (29), with the note that the weight of the criterion is determined in Section 3. The value of  $t_{p_{ij}}$  of SR and the obtained MRS are 0.0316 and 0.0239, respectively. The values of  $t_{r_{ij}}$  are then calculated using Equations (30) and (31). The values of  $g_{ij}$  are identified using the Formula (32). The  $Q_i$  values are finally determined by Formula (33). The calculated parameters and the ranking of the alternatives when using the MAIRCA method are given in Table 8.

**Table 8.** Calculated results and ranking of alternatives by the MAIRCA method.

Trial.	$k_{ij}$		$l_{ij}$		$Q_i$	Rank
	Ra	MRR	Ra	MRR		
A1	0.0000	0.0000	0.0278	0.0278	0.0556	16
A2	0.0234	0.0060	0.0044	0.0218	0.0262	7
A3	0.0263	0.0099	0.0015	0.0179	0.0194	4
A4	0.0191	0.0094	0.0087	0.0184	0.0271	9
A5	0.0278	0.0160	0.0000	0.0118	0.0118	1
A6	0.0181	0.0170	0.0097	0.0108	0.0205	5
A7	0.0175	0.0127	0.0103	0.0151	0.0254	6
A8	0.0229	0.0149	0.0049	0.0128	0.0177	2
A9	0.0091	0.0278	0.0187	0.0000	0.0187	3
A10	0.0064	0.0221	0.0214	0.0056	0.0271	8
A11	0.0038	0.0172	0.0240	0.0106	0.0346	12
A12	0.0017	0.0137	0.0261	0.0141	0.0402	14
A13	0.0073	0.0151	0.0205	0.0127	0.0332	11
A14	0.0081	0.0169	0.0197	0.0109	0.0306	10
A15	0.0061	0.0100	0.0217	0.0177	0.0394	13
A16	0.0003	0.0132	0.0275	0.0145	0.0421	15

## 6. MCDM Using the Entropy Method for Calculating the Weights of Criteria

The weight calculation of criteria when using the entropy method is carried out by the following steps (see Section 3.2): Calculating the normalized values  $p_{ij}$  by Equation (39), finding the entropy value for each indicator  $me_j$  by Equation (40) and, calculating the weight of the criteria  $w_j$  by Equation (42). It is noted that the weights of Ra and MRR are 0.3897 and 0.6103, respectively.

MCDM according to TOPSIS, MARCOS, MAIRCA and EAMR methods when the weights are determined by the entropy method are performed similarly to those in Section 5. The results of the ranking of alternatives are presented in Table 8. In addition, this table also shows the ranking of the alternatives when the weights are determined by the MEREC method (summarized from Section 5).

## 7. Results and Remarks

Table 9 presents the ranking results of alternatives when applying four MCDM methods, including TOPSIS, MARCOS, EAMR and MAIRCA, with the weight calculation using the MERREC and the entropy method. The comparison of the results when using the four MCDM methods with the two mentioned weighting calculation methods is also shown in Figure 2. From this result, the following comments have been proposed:

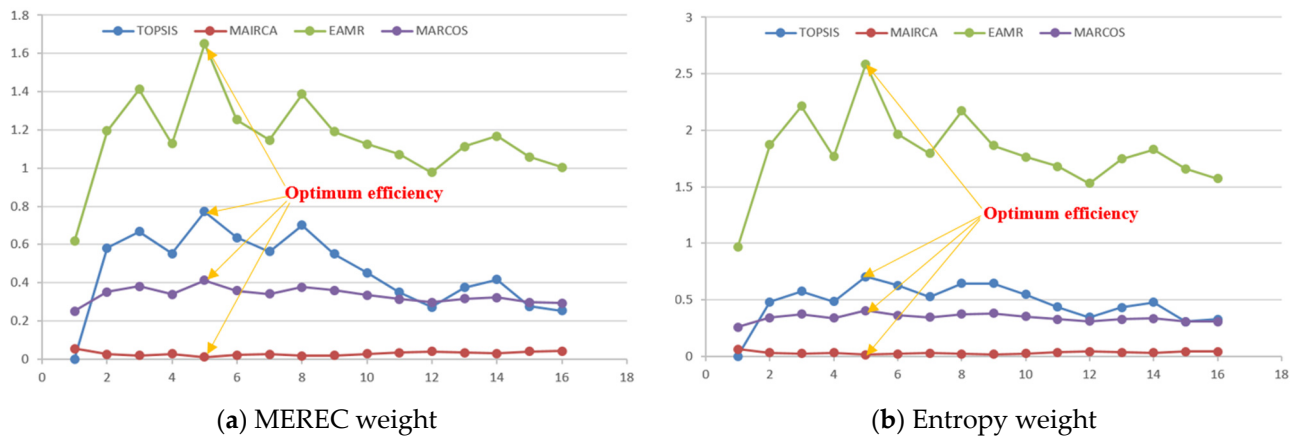
- MCDM when using TOPSIS, MARCOS, EAMR and MAIRCA methods with the weight calculation of criteria by MEREC and entropy will give different ranking results.
- MCDM when using the above four methods with the calculation of the weight of criteria by MEREC and entropy methods gives the same best alternative—A5. It is worth mentioning that the determination of the best alternative is independent of the MCDM method and the weighting calculation method used.
- The best alternative when internal grinding to achieve minimum SR and maximum MRR simultaneously is the one with the following input process parameters:

$a_r = 0.03$  (mm/L);  $n_r = 2$  (times);  $a_f = 0.005$  (mm);  $n_f = 1$  (times);  $n_0 = 2$  (times); and  $Sd = 1.2$  (m/min).

- TOPSIS, MARCOS, EAMR and MAIRCA methods can be used for MCDM when internal grinding. In addition, the weight calculation of criteria can be achieved using the MEREC method or the entropy method.

**Table 9.** Ranking of alternatives when using MEREC and the entropy method for weight calculation.

Trial.	MEREC Weight				Entropy Weight			
	TOPSIS	MARCOS	EAMR	MAIRCA	TOPSIS	MARCOS	EAMR	MAIRCA
A1	16	16	16	16	16	16	16	16
A2	5	6	5	7	9	8	5	10
A3	3	2	2	4	5	4	2	5
A4	7	8	9	9	8	9	9	9
A5	1	1	1	1	1	1	1	1
A6	4	5	4	5	4	5	4	4
A7	6	7	8	6	7	7	8	7
A8	2	3	3	2	3	3	3	3
A9	8	4	6	3	2	2	6	2
A10	9	9	10	8	6	6	10	6
A11	12	12	12	12	11	11	12	12
A12	14	13	15	14	13	13	15	13
A13	11	11	11	11	12	12	11	11
A14	10	10	7	10	10	10	7	8
A15	13	14	13	13	15	15	13	14
A16	15	15	14	15	14	14	14	15



**Figure 2.** Comparison graph of TOPSIS, MARCOS, EAMR and MAIRCA.

### 8. Conclusions

The article presents the results of a study on MCDM during the internal grinding SKD11 tool steel. In this work, six process factors, including the coarse dressing depth, the coarse dressing time, the fine dressing depth, the fine dressing time, the non-feeding dressing, and the dressing feed rate were selected for the investigation. Furthermore, the Taguchi method with L16 orthogonal array ( $4^4 \times 2^2$ ) design was chosen for the experimental design. In addition, four methods of MCDM, including TOPSIS, MARCOS, EAMR, and MAIRCA were applied. Moreover, the weight determination of criteria was carried out using the MEREC and the entropy method. Several following remarks can be:

- For the first time, the results of applying the four methods TOPSIS, MARCOS, EAMR, and MAIRCA when MCDM an internal grinding process have been reported.

- The use of the above methods and the weight determination for criteria according to the MEREC or entropy methods do not affect the results of choosing the best alternative.
- The above methods can be used for MCDM when internal grinding with the weight calculation of the criteria, which can be performed by MEREC or the entropy method.
- The following input factors  $a_r = 0.03$  (mm/l);  $n_r = 2$  (times);  $a_f = 0.005$  (mm);  $n_f = 1$  (times);  $n_0 = 2$  (times); and  $Sd = 1.2$  (m/min) was proposed for the best alternative for the dressing process when internal grinding to obtain a minimum SR and maximum MRR simultaneously.

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## Abbreviations

MCDM	Multi-Criteria Decision Making
TOPSIS	Technique for Order of Preference by Similarity to Ideal Solution
MARCOS	Measurement of Alternatives and Ranking according to Compromise Solution
EAMR	Area-based Method of Ranking
MAIRCA	Multi-Attributive Ideal–Real Comparative Analysis
SR	Surface Roughness
MRR	Material Removal Rate
MEREC	Method based on the Removal Effects of Criteria
VIKOR	Vlsekriterijumska optimizacija I KOmpromisno Resenje in Serbian
EDM	Electrical Discharge Machining
COPRAS	Complex Proportional Assessment
SAW	Simple Additive Weighting
MOORA	Multi-Objective Optimization on the basis of Ratio Analysis
WASPAS	Weighted Aggregates Sum Product Assessment
PSI	Preference Selection Index
PIV	Proximity Indexed Value
GRA	Grey Relational Analysis
PMEDM	Powder-Mixed Electrical Discharge Machining
COPRAS-G	COmprehensive Grey complex PROportional ASsessment

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