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Reliability Assessment for a Spacecraft Landing Buffer Mechanism Based on Small Sample Data

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Abstract: The landing buffer mechanism (LBM) is a crucial component of aerospace landing explorations, determining the success of the entire mission. Reliability demonstration tests (RDTs) are required to ensure the LBM's performance meets the design index. However, the RDTs of the LBM often encounter limited samples and zero-failure data, making traditional binominal test programs and reliability assessment methods based on the maximum likelihood theory unsuitable. This paper introduces a novel small-sample test verification method for the LBM, which transforms traditional binominal tests into more informative metrological tests by measuring the buffer strokes of the LBM's subsystems. Furthermore, a confidence limit theory for the product of subsystem reliability is developed in conjunction with the reliability series model of the LBM. This theory can use the measurement data of subsystems to assess the confidence limit of the LBM's reliability. An actual engineering application demonstrates that the proposed method can effectively handle zero-failure data and verifies that the LBM's reliability exceeds 0.9999 with a confidence of 0.9.

Keywords: landing buffer mechanism; metrological test; buffer stroke; reliability assessment; small sample; zero-failure

1. Introduction

As deep space exploration advances, soft-landing exploration of extraterrestrial bodies has become one of the most important exploration methods. In recent years, several leading space powers have conducted soft landing explorations on the Moon and Mars, with notable examples being China's Chang'E lunar exploration [1,2] and America's Mars Science Laboratory [3,4]. The landing buffer mechanism (LBM), a core device in soft-landing exploration, plays a crucial role in these missions. According to aerospace exploration mission statistics, deep space probes are most likely to malfunction during the landing phase [5]. Therefore, the reliability of the LBM is directly related to the success of the whole exploration mission.

As a common method to quantify the failure risk in complex aerospace products, reliability assessment helps engineers make more informed decisions during the design phase [6]. The industrial sector demands the reliability assessment for the LBM and puts forward high-reliability index requirements. A reliability demonstration test (RDT) is essential to confirm that the actual reliability of the LBM meets these design indices [7,8]. The RDT has two key components: selecting the test program and determining the reliability assessment methodology. While the outcomes of the LBM's actions are typically binary (success or failure), traditional binomial tests are impractical for practical engineering due to their requirement for a lot of samples. For instance, validating a reliability index of 0.9997 (assessed at a confidence of 0.9) would necessitate at least 7675 zero-failure tests. As such, there is an urgent need to develop more efficient and feasible RDT programs and assessment methods.

Contrary to traditional count tests that only count successes and failures, metrological tests offer more comprehensive reliability data and require much smaller sample sizes. On

the other hand, the LBM with leg structure usually consists of subsystems such as primary and secondary struts. Considering these factors, this study proposes a metrological testing scheme to characterize the LBM's buffering performance by measuring the performance data from primary and secondary struts. This test program significantly reduces the number of buffer tests, saves test costs, and makes assessing the LBM's reliability possible.

During the landing buffer phase, the failure of buffering capability in either primary or secondary struts causes the entire LBM to fail [9,10]. Consequently, the LBM can be modeled as a series system composed of several subsystems. Such mathematical reliability models are typically presented in the form of block diagrams or fault trees [11]. Theoretically, the LBM's reliability can be assessed by combining the reliability series model and the statistical data of primary and secondary struts. However, the confidence limits assessment for series system reliability has always been challenging in reliability engineering [12,13]. In particular, buffer tests of the LBM face the problem of limited sample size and zero-failure data. On the one hand, aerospace machinery products are expensive and produced in limited batches, allowing only small-sample reliability tests [14–16]. On the other hand, RDTs typically do not permit sample failures; otherwise, they would be judged as unqualified [17].

The Lindstrom–Madden (LM) method [18] and the modified maximum likelihood (MML) method [19] are the main methods used in engineering to assess the lower confidence limit (LCL) for series systems reliability. The core idea behind these methods is to reduce the test results to a set of binomial tests with a pseudo-test number N and a pseudo-success number S (i.e., the system is considered to have passed S out of N tests). Then, the binomial distribution reliability theory is applied to estimate the confidence limits of the simplified system [20]. However, the traditional methods are based on the maximum likelihood theory and therefore have large errors in small sample cases [21]. These two methods are also inapplicable when zero-failure subsystems exist in a series system [22,23].

In recent years, the Bayesian and Bootstrap methods have been developed to estimate the confidence limit of system reliability in small sample cases [24]. The Bayesian approach combines pre-test reliability with field-test information, which is advantageous for small sample data [25,26]. However, the subjective selection of the prior distribution limits its application [27]. The Bootstrap method assumes that the empirical distribution fits the sample distributions well and extends the sample by resampling [28,29]. This method is intuitive and convenient, but the estimates are usually approximate.

Motivated by the realistic problems encountered in the reliability assessment of the LBM, this study makes two contributions: a novel small-sample test verification method for the LBM and a confidence limit theory for series systems. This small-sample test verification method transforms the traditional binominal tests into metrological tests by measuring the buffer strokes of the LBM's subsystems. Assuming that the buffer strokes obey the normal distribution, the reliability of zero-failure subsystems is assessed based on the normal distribution tolerance limit theory. On this basis, the LBM can be modeled as a series system composed of several subsystems. To estimate the reliability confidence limit for such a series system, a confidence limit theory for the product of subsystem reliability is developed. This theory can assess the LCL of the LBM's reliability with high confidence based on the reliability assessment results for primary and secondary struts.

The rest of the paper is organized as follows. Section 2 describes the composition and working principle of the LBM and constructs the corresponding reliability model. Section 3 defines the reliability of primary and secondary struts and introduces the assessment method for subsystems. In Section 4, the reliability assessment methods for the LBM and its combinations are proposed, all including confidence limit calculation formulas. An engineering application is provided in Section 5 to illustrate the practicality and effectiveness of the proposed method. Finally, Section 6 summarizes the paper.

2. A Reliability Series Model for the LBM

Figure 1 shows the typical design structure of the LBM from a Chinese lunar lander, which includes a primary strut, two secondary struts, and a foot-pad [30,31]. The primary strut cushions the longitudinal impact load, while two secondary struts cushion the transverse impact load. The foot-pad serves as the main support component [32]. The primary and secondary struts are filled with aluminum honeycomb material as buffer elements [33,34], as shown in Figure 2.
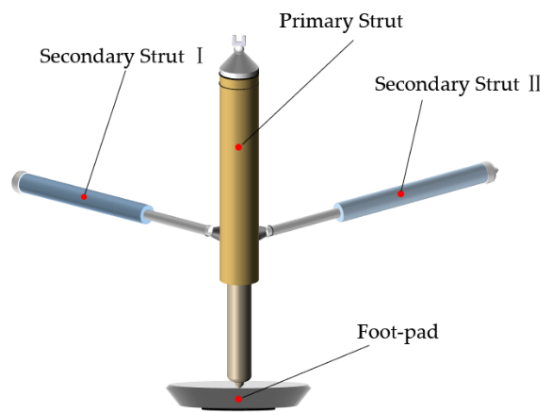

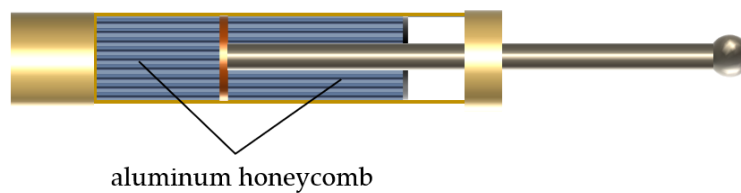**Figure 1.** The entire structure of a spacecraft LBM.

Figure 2. Schematic diagram of aluminum honeycomb element.

During the free-fall phase, the foot-pad first contacts the planetary surface to prevent the lander from sinking excessively. Simultaneously, the foot-pad transmits the impact load to the primary strut, pushing its inner cylinder to slide along the outer cylinder. This action deforms the aluminum honeycomb, absorbing the longitudinal impact load during a soft landing. The buffer elements of secondary struts provide a bi-directional buffer when the lateral load exceeds a certain threshold. The pull-rod element performs a tensile buffer, while the aluminum honeycomb element provides a compression buffer.

The composition and operation of the LBM reveal that its performance is determined by the primary strut, two secondary struts, and foot-pad. The foot-pad's primary challenge is its strength, which can be ensured by increasing its strength margin, rendering its reliability as 1. In other words, the foot-pad does not influence the system state (success or failure) and is not included in the LBM's reliability model. In addition, the inability of any one strut will lead to the failure of the LBM. Thus, the LBM can be regarded as a series system consisting of the primary strut, secondary strut I, and secondary strut II. Figure 3 provides the LBM's reliability block diagram.
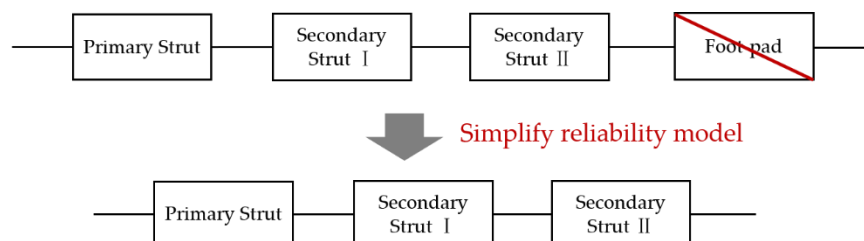

Figure 3. The reliability block diagram of LBM.

The buffer reliability model of a single LBM can be expressed as

$$R = R_1 \times R_2 \times R_3 \tag{1}$$

where R, R_1, R_2 and R_3 denote the reliability of a single LBM, primary strut, secondary strut I, and secondary strut II, respectively.

A spacecraft is equipped with four identical LBMs that collectively buffer and absorb energy upon landing [35]. Since the failure of any one LBM will fail the lander cushion, the four LBMs constitute a series system. This combination of LBMs is called the landing buffer system (LBS) for ease of description. Then, the buffer reliability R^* of the LBS can be expressed as

$$R^* = R^4 = R_1^4 \times R_2^4 \times R_3^4 \tag{2}$$

In engineering practice, it is not easy to use Equations (1) and (2) directly to calculate the LBM and LBS's buffer reliability because the actual value of R_i is unknown, and its LCL with the confidence γ can only be obtained via the corresponding reliability tests, which satisfies

$$P(R_i \geq R_{\mathrm{L}i}) = \gamma \quad i = 1, 2, 3 \tag{3}$$

where γ represents the probability that the actual value of reliability R_i is not less than its LCL $R_{\mathrm{L}i}$ estimated from the test sample is $\gamma\%$. Theoretically, the assessment results are safe and secure.

A spacecraft LBM adopts a metrological test scheme to obtain quantitative information on product performance. Compared to traditional count tests, the metrological test provides more information and requires significantly smaller sample sizes, thus classifying it as a small sample test. This paper presents a novel small-sample test verification method for the LBM, addressing the challenge of reliability assessment in small-sample scenarios. For a clear description, Figure 4 illustrates the fundamental idea of the proposed method. Firstly, the buffer reliability of primary and secondary struts is assessed based on their performance data obtained from small sample buffer tests. Subsequently, the LBM's buffer reliability is assessed with the whole product's reliability series model.
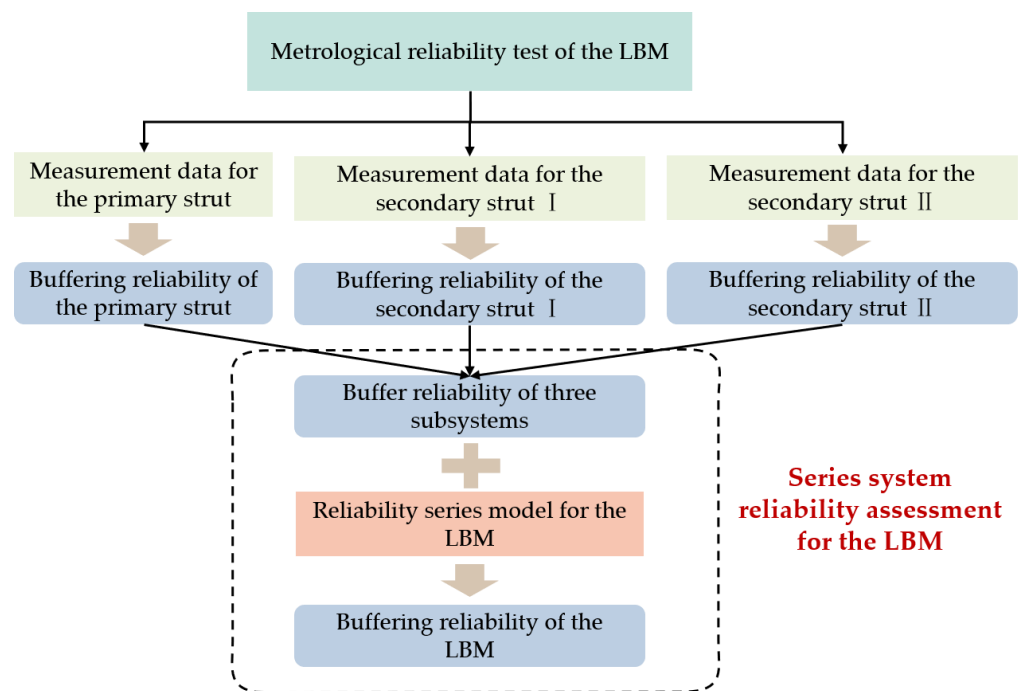

Figure 4. Flow chart of the fundamental idea of the small-sample test verification method.

3. Reliability Assessment Method for Subsystem

This section analyzes the reliability characteristic quantities (quantitative indicators that characterize the reliability level) of the primary and secondary struts according to the RDT scheme of the LBM. Then, the reliability assessment method for the subsystem is introduced.

3.1. Analysis and Determination of Reliability Characteristic Quantities

The buffer stroke is a performance parameter that characterizes the buffering capacity of primary and secondary struts. The buffering is deemed effective as long as the buffer strokes of struts remain within the maximum design value. Furthermore, the primary strut only has a one-way buffer capacity, while the two secondary struts can provide a bi-directional (tension or compression) buffer capacity. Consequently, the characteristic quantities of buffer test are determined as follows:

(1) The primary strut's buffer stroke X_1 (i.e., aluminum honeycomb compression);
(2) The secondary struts' buffer strokes X_2 and X_3 (i.e., aluminum honeycomb compression or pull-rod elongation).

Suppose the buffer stroke data obtained from buffer tests passes the normality examination. In that case, it can be inferred that the reliability characteristic quantities of primary and secondary struts all follow the normal distribution. In such a scenario, each subsystem's buffer reliability can be assessed using the normal distribution tolerance limit theory in conjunction with the tolerance limits of characteristic quantities.

3.2. Reliability Assessment of Subsystem Based on Normal Distribution Tolerance Limit Theory

The buffer reliability of each subsystem decreases as the buffer stroke increases. Once the buffer stroke surpasses the standard, it exceeds the subsystem's buffer capacity range. Therefore, the buffer strokes of primary strut, secondary strut I, and secondary strut II all have an upper limit.

Suppose the buffer strokes $X_i (i = 1, 2, 3)$ of the primary strut, secondary strut I, and secondary strut II all obey the normal distribution. The tolerance limits of their characteristic quantities are defined as $X_1 < x_{1U}$, $X_2 < x_{2U}$, and $X_3 < x_{3U}$, where x_{1U}, x_{2U}, and x_{3U} denote the maximum buffer stroke of each subsystem, respectively. Then, their buffer reliability can be characterized by the probability that the buffer stroke is less than its allowable upper limit under the specified environmental conditions, that is

$$R_i = P(X_i < x_{iU}) = \Phi\left(\frac{x_{iU} - u_i}{\sigma_i}\right) \quad i = 1, 2, 3 \tag{4}$$

where μ_i, σ_i are the means and standard deviations of the buffer stroke X_i, respectively, and $\Phi(\cdot)$ is the standard normal distribution function.

Assuming that the LBM has carried out n buffer reliability tests, the measured buffer travel data for primary strut, secondary strut I, and secondary strut II are $x_i = (x_{i1}, x_{i2}, \cdots, x_{in})$, $i = 1, 2, 3$. According to these measurement data, the sample means $\overline{x}_i$ and sample standard deviations s_i of the three subsystems' buffer strokes can be calculated, respectively as follows:

$$\overline{x}_i = \frac{1}{n}\sum_{j=1}^{n} x_{ij} \quad i = 1, 2, 3 \tag{5}$$

$$s_i = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(x_{ij} - \overline{x}_i)^2} \quad i = 1, 2, 3 \tag{6}$$

Let

$$\psi_i = \frac{\sqrt{n}(x_{iU} - \overline{x}_i)}{s_i} \quad i = 1, 2, 3 \tag{7}$$

Based on the reliability theory of normal distribution complete data, the LCL $R_{\mathrm{L}i}$ of primary strut, secondary strut I, and secondary strut II with the confidence γ can be obtained using the following equation:

$$F_v\left(\sqrt{n}u_{R_{\mathrm{L}i}}, \psi_i\right) = \gamma \quad i = 1, 2, 3 \tag{8}$$

where $u_{R_{\mathrm{L}i}} = \Phi^{-1}(R_{\mathrm{L}i})$ is the $R_{\mathrm{L}i}$ quantile of the standard normal distribution and $F_v\left(\sqrt{n}u_{R_{\mathrm{L}i}}, \cdot\right)$ is the distribution function of a non-central t-distribution with degrees of freedom (the number of independent data in the sample) $v = n - 1$ and a non-central parameter $\delta = \sqrt{n}u_{R_{\mathrm{L}i}}$.

According to Equation (8), the LCL $R_{\mathrm{L}i}$ of each subsystem reliability with confidence γ is

$$R_{\mathrm{L}i} = \Phi\left\{F_v^{-1}(\gamma, \psi_i)/\sqrt{n}\right\} \quad i = 1, 2, 3 \tag{9}$$

where $F_v^{-1}(\gamma, \cdot)$ is the inverse function of $F_v\left(\sqrt{n}u_{R_{\mathrm{L}i}}, \cdot\right)$ obtained by Equation (8) for the non-central parameter $\delta = \sqrt{n}u_{R_{\mathrm{L}i}}$.

4. Reliability Assessment Methods for the LBM and LBS

It is well known that the reliability of a series system is the product of its individual subsystems' reliability. However, is the reliability LCL of the series system also the product of the individual subsystems' reliability LCL? Moreover, how does one calculate the confidence for the LCL of the system reliability in this scenario? Motivated by these questions, this section first discusses the LCL of a single LBM reliability with the confidence γ, and subsequently addresses the LCL of LBS when four sets of LBMs are typically combined.

4.1. Buffer Reliability Assessment Method for a Single LBM

As mentioned in Section 2, the buffer reliability $R = \prod_{i=1}^{3} R_i$ of a single LBM cannot be directly calculated. In this case, the product of $R_{\mathrm{L}i}(i = 1, 2, 3)$ could be taken as an approximate solution to the LCL of R with the confidence γ, and the approximate solution can be expressed as

$$R_{\mathrm{L}} = \prod_{i=1}^{3} R_{\mathrm{L}i} = \prod_{i=1}^{3} \Phi\left\{F_v^{-1}(\gamma, \psi_i)/\sqrt{n}\right\} \tag{10}$$

where $R_{\mathrm{L}1}$, $R_{\mathrm{L}2}$ and $R_{\mathrm{L}3}$ are the LCLs of the reliability for primary strut, secondary strut I, and secondary strut II with the confidence γ, respectively. $R_{\mathrm{L}1}$, $R_{\mathrm{L}2}$ and $R_{\mathrm{L}3}$ can be calculated by Equation (9) in Section 3.2.

Let γ' denote the actual confidence of R_{L}. It can be calculated by

$$\gamma' = P\{R \geq R_{\mathrm{L}}\} = P\left\{R_1 \geq \frac{R_{\mathrm{L}}}{R_2 \cdot R_3}\right\} \tag{11}$$

Although the actual value of $R_i(i = 1, 2, 3)$ in Equation (11) is unknown, their LCLs with any confidence γ_{k_i} can be determined from the test data $x_i(i = 1, 2, 3)$ of the primary and secondary struts; in other words, Fiducial distribution [36,37] of R_i can be constructed by Equation (9), which satisfies

$$P\left(R_i \geq R_{\mathrm{L}i, \gamma_{k_i}}\right) = \gamma_{k_i} \quad i = 1, 2, 3 \tag{12}$$

Fiducial statistical inference suggests that when the sample x_i is determined, the reliability R_i is regarded as a random variable and $R_{\mathrm{L}i, \gamma_{k_i}}$ is a function of the confidence γ_{k_i} only. In this case, Equation (12) can be considered as the distribution function of R_i. Therefore, γ' can be further calculated numerically as follows:

$$\gamma' = P\{R \geq R_L\} = \frac{1}{M^2}\sum_{k_2=1}^{M}\sum_{k_3=1}^{M} P\left\{R_1 \geq \frac{R_L}{R_{L2,\gamma_{k_2}} \cdot R_{L3,\gamma_{k_3}}}\right\}$$
$$= \frac{1}{M^2}\sum_{k_2=1}^{M}\sum_{k_3=1}^{M} F_v\left(\delta_{k_2 k_3}, \psi_1\right) \tag{13}$$

where

$$\delta_{k_2 k_3} = \sqrt{n}\Phi^{-1}\left\{R_L\bigg/\prod_{i=2}^{3}\Phi\left[F_v^{-1}(\gamma_{k_i},\psi_i)/\sqrt{n}\right]\right\} \tag{14}$$

$$\gamma_{k_i} = 1 - \frac{k_i - 0.5}{M} \quad k_i = 1, 2, \cdots, M, \ i = 2, 3 \tag{15}$$

A detailed explanation of the numerical algorithm of Equations (13)–(15) is summarized in Algorithm 1.

**Algorithm 1:** Numerical calculation of the confidence γ' for R_L

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(1) Select M according to the requirements of numerical calculation accuracy (e.g., $10^4, 10^5, 10^6$);
(2) Calculate the confidence γ_{k_2} and γ_{k_3} using Equation (15);
(3) Substitute γ_{k_2} and γ_{k_3} into Equation (14) to calculate $\delta_{k_2 k_3}$. If the value of $*$ in $\Phi^{-1}(*)$ is greater than 1, let the corresponding $\delta_{k_2 k_3}$ be negative infinity;
(4) The confidence γ' is obtained by Equation (13).

It is worth noting that there is a deviation between γ' and the required confidence γ. To obtain the exact LCL for the LBM with the confidence γ, it is necessary to continuously adjust the subsystem's confidence in Equation (10) and repeatedly use Algorithm 1 to calculate the γ' for R_L.

Note that Equation (11) is a monotonically increasing function of γ. Thus, the value of γ in Equation (10) can be continuously adjusted to γ'' by the dichotomy until the value of γ' calculated by Algorithm 1 is equal to the required value of γ. In other words, find the value of γ'' such that the confidence for R_L calculated by Algorithm 1 satisfies $\gamma' = \gamma$. At this time, the LCL of a single LBM reliability R with the confidence γ is provided by

$$R_{L,\gamma} = \prod_{i=1}^{3}\Phi\left\{F_v^{-1}(\gamma'',\psi_i)/\sqrt{n}\right\} \tag{16}$$

In summary, Figure 5 illustrates the main process of the LBM reliability assessment method.
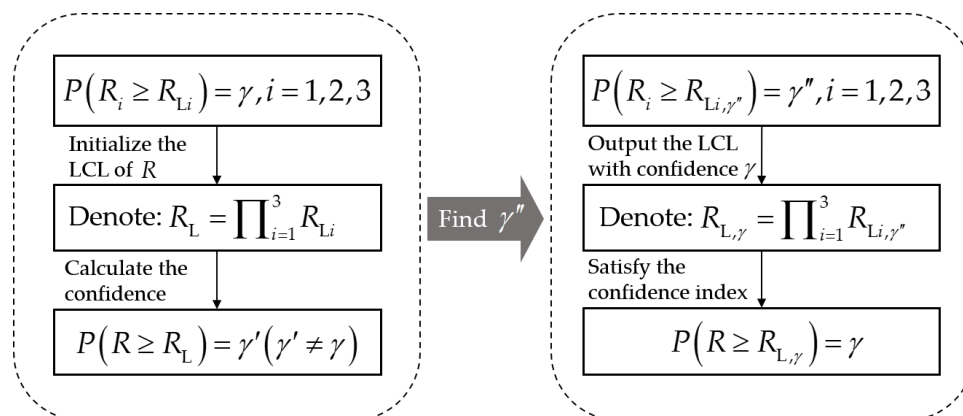
Figure 5. Flow chart of the reliability assessment method for a single LBM.

4.2. Buffer Reliability Assessment Method for the LBS

In engineering practice, four sets of identical LBMs are usually used in a combination, and the failure of one of them will lead to the failure of the entire combination, forming a series system called the LBS.

Although both the LBM and LBS can be modeled as series systems, their reliability assessment methods differ. The LBM's subsystems (primary and secondary struts) are different, hence the confidence limit theory in Section 4.1 is employed to calculate the LCL of the LBM's reliability. On the other hand, the LBS consists of four sets of identical LBMs, indicating that the subsystems of LBS are identical. At this time, the LCL of the LBS reliability R^* with the confidence γ is calculated using the following equation:

$$R^*_{\mathrm{L},\gamma} = R^4_{\mathrm{L},\gamma} \tag{17}$$

where $R_{\mathrm{L},\gamma}$ is the LCL of a single LBM reliability with the confidence γ, calculated by Equation (16).

Equation (17) is proved as follows: since the four sets of LBMs are identical and the reliability of a single LBM is R, then the buffer reliability of the LBS is

$$R^* = R^4 \tag{18}$$

Also, because

$$P\big(R \geq R_{\mathrm{L},\gamma}\big) = \gamma \tag{19}$$

Furthermore, the following can be derived:

$$P\big\{R^* \geq R^*_{\mathrm{L},\gamma}\big\} = P\big\{R^4 \geq R^4_{\mathrm{L},\gamma}\big\} = P\big\{R \geq R_{\mathrm{L},\gamma}\big\} = \gamma \tag{20}$$

5. Application and Analysis of Small-Sample Reliability Assessment Method for the LBM

This section provides an implementation of the proposed small-sample reliability assessment method, carried out in an actual engineering project of the LBM. The industrial sector requires the RDTs to quantitatively verify that the LBM's reliability meets the index of 0.9997, as assessed by the confidence of $\gamma = 0.9$. After reliability index allocation, the buffer reliability of primary strut, secondary strut I, and secondary strut II must be no less than 0.9999. The LBM's buffer tests are completed on a specialized test bench, as shown in Figure 6.
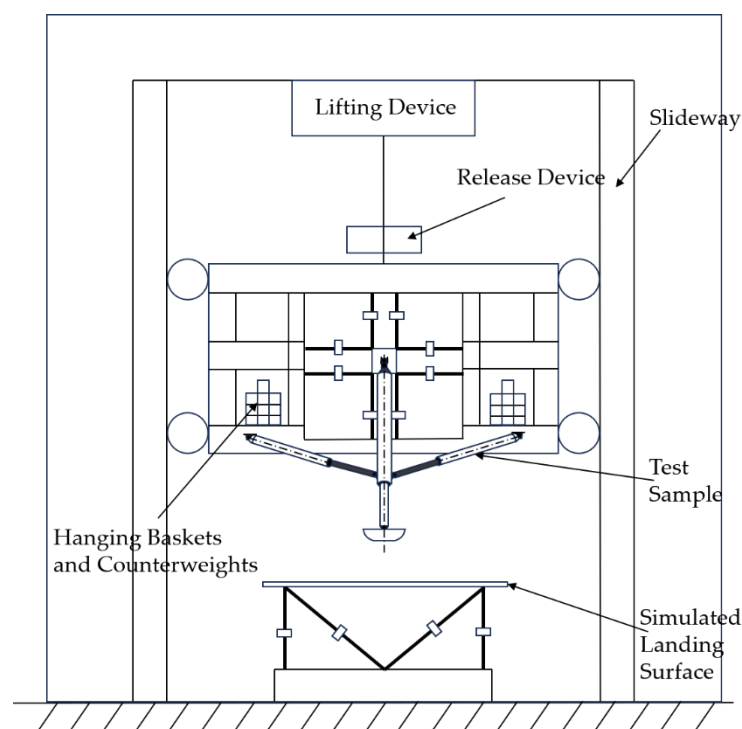
Figure 6. Schematic diagram of the buffer reliability tests.

5.1. Reliability Test Data

The data utilized in this analysis originates from the buffer reliability tests conducted by a Chinese spacecraft LBM. Given that the landing state of the spacecraft cannot be determined prior to landing, buffer reliability tests were carried out under various working conditions [38]. Three sets of LBMs are put into operation under each working condition, and each set of LBMs is tested once. Following the buffer tests, the primary strut and two secondary struts were disassembled and measured to obtain their buffer strokes.

Since the reliability of any landing state is not lower than that of the worst case, the minimum value of the LBM buffer reliability can be obtained via the buffer test data under the most severe conditions. The sample means and standard deviations of the primary strut, secondary strut I, and secondary strut II were calculated based on three groups of buffer strokes under the most severe conditions. The results are presented in Table 1.

Table 1. The sample means and standard deviations of the subsystems' buffer strokes under the most severe working condition (mm).

Performance Parameter	Small Size n	$\bar{x} \pm s$
Primary strut compression stroke	3	132.0 ± 8.2
Secondary strut I tension stroke	3	43.3 ± 6.1
Secondary strut II tension stroke	3	40.0 ± 3.0

In addition, based on the statistical analysis of previous buffer stroke data (obtained from subsystem-level testing) and engineering experience, it is determined that the buffer strokes of primary and secondary struts are subject to the normal distribution, and their maximum buffer design stroke are $X_1 < x_{1U}$, $X_2 < x_{2U}$ and $X_3 < x_{2U}$, where $x_{1U} = 423$ mm and $x_{2U} = 150$ mm. The maximum buffer design stroke of secondary strut I and secondary strut II is identical.

5.2. Buffer Reliability of Subsystems

Applying the normal distribute tolerance limit theory in Section 3.2, the sample means and standard deviations in Table 1, along with the allowable upper limits of buffer stroke are substituted into Equation (7) to calculate $\psi_i(i = 1, 2, 3)$. Then, according to Equation (9), the LCLs of the reliability for primary strut, secondary strut I, and secondary strut II with the confidence of $\gamma = 0.9$ are calculated as follows:

$$R_{L1} = \Phi\left\{ F_v^{-1}(\gamma, \psi_1) / \sqrt{n} \right\} > 0.9999999999 \tag{21}$$

$$R_{L2} = \Phi\left\{ F_v^{-1}(\gamma, \psi_2) / \sqrt{n} \right\} = 0.9999999922 \tag{22}$$

$$R_{L3} = \Phi\left\{ F_v^{-1}(\gamma, \psi_3) / \sqrt{n} \right\} > 0.9999999999 \tag{23}$$

The reliability assessment results show that the reliability LCLs for primary strut, secondary strut I, and secondary strut II have reached above 0.9999, which can meet the buffer reliability index requirements of subsystems.

5.3. Buffer Reliability Assessment of the LBM and LBS

This section compares the proposed method with the commonly used reliability assessment method for engineering series systems to demonstrate the proposed approach's rationality and effectiveness. Since the scenario where all subsystems are zero-failure is beyond the application scope of the MML method [23], we select the LM method for comparison. The LM method is widely used for the reliability assessment of series systems in aerospace products due to its simplicity of calculation and easily comprehensible equations.

5.3.1. Buffer Reliability Assessment by the LM Method

Before using the LM method to assess the reliability of a series system, it is first necessary to convert the non-binomial data into binomial data. Let N_i and S_i be the pseudo-test and pseudo-success numbers of the ith subsystem converted from quantitative data to binomial data. Then, N_i and S_i can be obtained by solving the following equation

$$\begin{cases} S_i = N_i \cdot \widehat{R}_i \\ I_{R_{Li}}(S_i, N_i - S_i + 1) = 1 - \gamma \end{cases} \quad i = 1, 2, 3 \tag{24}$$

where $I(\cdot, \cdot)$ is the incomplete β-distribution function and $\widehat{R}_i = \Phi\{(x_{iU} - \overline{x}_i)/s_i\}$ is the point estimate of each subsystem's reliability. In particular, when $\widehat{R}_i \approx 1$, the zero-failure binomial data is obtained. At this time, Equation (24) is simplified as

$$N_i = S_i = \frac{\ln(1 - \gamma)}{\ln R_{Li}} \quad i = 1, 2, 3 \tag{25}$$

Substituting the reliability assessment results of each subsystem in Section 5.2 into Equation (25), the equivalent test results of the primary strut, secondary strut I, and secondary strut II can be obtained as $N_1 = S_1 > 2.302 \times 10^{10}$, $N_2 = S_2 = 2.952 \times 10^8$ and $N_3 = S_3 > 2.302 \times 10^{10}$.

After obtaining N_i and $S_i (i = 1, 2, 3)$, the pseudo-test number N and pseudo-failure number F of a single LBM can be obtained by the equation as follows:

$$\begin{cases} N = \min\{N_1, N_2, N_3\} = 2.952 \times 10^8 \\ F = N\left(1 - \prod\limits_{i=1}^{3} \frac{S_i}{N_i}\right) = 0 \end{cases} \tag{26}$$

Based on the binomial distribution reliability theory [17], the reliability LCL of a single LBM with the confidence of $\gamma = 0.9$ can be obtained as follows:

$$R_L = (1 - \gamma)^{\frac{1}{N}} = 0.9999999922 \tag{27}$$

The LBS consists of four identical LBMs connected in series. The pseudo-test number N^* and the pseudo-failure number F^* of the LBS can also be obtained by the LM method as

$$\begin{cases} N^* = \min\{N, N, N, N\} = 2.952 \times 10^8 \\ F^* = N^*\left[1 - \left(\frac{N-F}{N}\right)^4\right] = 0 \end{cases} \tag{28}$$

Similarly, the LCL R_L^* of the LBS buffer reliability with the confidence of $\gamma = 0.9$ is calculated as

$$R_L^* = (1 - \gamma)^{\frac{1}{N^*}} = 0.9999999922 \tag{29}$$

The calculation results are summarized in Table 2. As indicated in the table, when all subsystems have zero-failure data, the LM method equates the LCL of the LBM reliability to that of the secondary strut I. In other words, the LM method only utilizes test information from the secondary strut I, which is particularly wasteful in the case of small samples. Furthermore, the LM method gives the same LCL for the LBS reliability as that of a single LBM, assuming the reliability of the other LBMs is 1, which is dangerous and unreasonable in engineering applications.

Table 2. Buffer reliability assessment results of the LM method.

Results	Primary Strut	Secondary Strut I	Secondary Strut II	A Single LBM	Four Sets of LBMs (LBS)
Pseudo-test number	2.302×10^{10}	2.952×10^{8}	2.302×10^{10}	2.952×10^{8}	2.952×10^{8}
Pseudo-failure number	0	0	0	0	0
LCL for reliability	0.9999999999	0.9999999922	0.9999999999	0.9999999922	0.9999999922

5.3.2. Buffer Reliability Assessment by the Proposed Methodology

Substituting the reliability assessment results of each subsystem in Section 5.2 into Equation (10), the initial approximation for calculating the reliability LCL of the LBM is

$$R_{\mathrm{L}} = \prod_{i=1}^{3} R_{\mathrm{L}i}\big|_{\gamma=0.9} = \prod_{i=1}^{3} \Phi\left\{ F_2^{-1}(\gamma, \psi_i)/3 \right\} = 0.9999999922 \tag{30}$$

And according to Equation (13), the actual value of the confidence γ' of R_{L} is further obtained as

$$\gamma' = P(R \geq R_{\mathrm{L}}) = 0.8561 \tag{31}$$

Accordingly, using the method in Section 4.1, the reliability LCL of the LBM with the confidence of $\gamma = 0.9$ is obtained as

$$R_{\mathrm{L},\gamma} = \prod_{i=1}^{3} \Phi\left\{ F_2^{-1}(\gamma'', \psi_i)/3 \right\} = 0.9999984502 \tag{32}$$

where $\gamma'' = 0.9305$, which is calculated from Equations (12)–(15).

Four sets of identical and independent LBMs form a series system. It can be seen from Equation (17) that the reliability LCL of the LBM with the confidence of $\gamma = 0.9$ is

$$R_{\mathrm{L},\gamma}^{*} = R_{\mathrm{L},\gamma}^{4} = 0.9999938008 \tag{33}$$

The calculation results are summarized in Table 3. The table shows that the proposed method can estimate the reliability LCL of a single LBM using zero-failure data from all subsystems. When the required confidence is 0.9, the reliability LCLs of a single LBM and LBS are all not less than 0.9997, which meets the index requirements.

Table 3. Buffer reliability assessment results of the proposed method.

Results	Primary Strut	Secondary Strut I	Secondary Strut II	A Single LBM	LBS
LCL for reliability	0.9999999999	0.9999999922	0.9999999999	0.9999984502	0.9999938008

5.4. Comparison and Discussion

The LM method, based on maximum likelihood theory, is inapplicable to the instances where subsystems have zero-failure data. As evidenced by Equation (26), when the pseudo-test results of all subsystems exhibit no failure, the LM method equates the LCL of a single LBM's reliability to that of the subsystem with the fewest pseudo-test number. This implies that the LM method only utilizes test information from one subsystem and discards the valuable data of other zero-failure subsystems, resulting in a large error in assessment results. This issue persists when the LM method is employed to assess the buffer reliability of the LBS. The calculation error gradually accumulates during the bottom-up information synthesis, ultimately leading to a dangerous system reliability assessment.

Based on the confidence limit theory for the product of subsystem reliability, the proposed method can assess the reliability confidence limit of a single LBM using zero-failure measurement data from subsystems. This approach fully utilizes all test information and considers the failure possibility in each subsystem, yielding assessment results that satisfy high-reliability indices. This method is simple to calculate for the combination of

four LBMs, i.e., LBS. It considers the failure possibility in each LBM, ensuring that the reliability assessment results are sound and reliable.

It is worth noting that the proposed method employs the normal distribution tolerance limits theory, which restricts its applicability only to spacecraft mechanisms whose performance parameters obey the normal distribution. In principle, the fundamental idea of the small-sample test verification method is equally applicable to other distribution situations, but corresponding distribution models need to be established for further study.

6. Conclusions

A small-sample test verification method for the LBM is proposed to address the issue that the traditional binominal tests need a large sample size. This method transforms the RDTs of the LBM from binominal tests to metrological tests by measuring the performance data of primary and secondary struts. Given that the buffer stroke (compression amount or tension amount) is easily measurable and can visually express the buffer performance of the LBM, It is selected as the reliability characteristic quantity of primary and secondary struts. In conjunction with the maximum buffer design stroke, the small-sample reliability assessment of subsystems is realized using the normal distribution tolerance limit theory. This test method verifies that the reliability LCL for primary and secondary struts reaches more than 0.99999, which meets practical requirements.

By analyzing the operational principle of the LBM during soft landing, a reliability series model for the LBM is established. Subsequently, a confidence limit calculation theory for the product of subsystem reliability is proposed to assess the series system reliability. This theory can estimate the LCL of the LBM's reliability based on the reliability of primary and secondary struts. The proposed method is compared with the LM method via a practical engineering application. The comparison results indicate that the LM method overlooks the failure possibility of all zero-failure subsystems, resulting in a dangerous system reliability assessment. Conversely, the proposed method can utilize all zero-failure test information and consider the failure possibility in each subsystem. After applying this method, it is verified that the LCL of the LBM's reliability reaches above 0.9999 with a confidence of 0.9, satisfying the specified reliability Index of 0.9997.

In engineering practice, many aerospace mechanisms can be considered as a series system, including solid rockets, spacecraft connecting and separating mechanisms, and aero-engines with modular design. The proposed method can also be generalized to estimate the reliability confidence limits of such series systems. This approach saves test costs when samples are scarce and ensures the assessment results are safe and reliable.

Abbreviations

Acronym

LBM	landing buffer mechanism	LBS	landing buffer system
RDT	reliability demonstration test	LM	Lindstrom-Madden
LCL	lower confidence limit	MML	modified maximum likelihood

Notation

N	test number in binomial test	R_{Li}	LCL of R_i with the confidence γ
S	success number in binomial test	R	reliability of a single LBM
$\bar{x}$	sample mean	R_L	approximate solution to the LCL of R
s	sample standard deviation	γ'	actual confidence of R_L
X_i	buffer stroke of the i th subsystem	$R_{L,\gamma}$	LCL of R with the confidence γ
x_{iU}	maximum buffer design stroke of the i th subsystem	R^*	reliability of the LBS
γ	required confidence	$R^*_{L,\gamma}$	LCL of R^* with the confidence γ
R_i	reliability of the i th subsystem		

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