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Review

Analysis of Tire-Road Interaction: A Literature Review

Haniyeh Fathi [1,*,†], Zeinab El-Sayegh [1], Jing Ren [2] and Moustafa El-Gindy [1]

1 Department of Automotive and Mechatronics Engineering, Faculty of Engineering and Applied Science, Ontario Tech University, Oshawa, ON L1G 0C5, Canada; zeinab.el-sayegh@ontariotechu.ca (Z.E.-S.); moustafa.el-gindy@ontariotechu.ca (M.E.-G.)
2 Department of Electrical, Computer and Software Engineering, Faculty of Engineering and Applied Science, Ontario Tech University, Oshawa, ON L1G 0C5, Canada; jing.ren@ontariotechu.ca
* Correspondence: haniyeh.fathi@ontariotechu.net or haniyehfathi@vt.edu
† Current address: Center for Tire Research (CenTiRe), Department of Mechanical Engineering, Virginia Tech University, Blacksburg, VA 24061, USA.

Abstract: This paper presents a comprehensive literature review of the most popular and recent work on passenger and truck tires. Previous papers discuss a huge amount of work on the modeling of passenger car tires using finite element analysis. In addition, recent works on tire–road interaction and the validation of tires using experimental measurements have been described. Moreover, the history of the tire-road contact algorithms is explained. In addition, friction modeling that is implemented in tire–road interaction applications are discussed. Also, a summary of current state-of-the-art research work definitions and requirements of the tread rubber compound are covered from previous studies using various literature reviews and hyper-viscoelastic material models that are implemented for the tread top and the tread base rubber compound. Furthermore, the effect of tire temperature from previous works is presented here. Finally, this literature review also highlights the shortcomings of recent research work and describes the areas lacking in the literature.

Keywords: tire; finite element methods; rolling resistance; temperature; cornering; tire stiffness

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1. Introduction

Tires are the main component in vehicles as they have a significant effect on the vehicle's performance such as ride and handling. Moreover, they are directly in parallel with fuel consumption. Therefore, there is an essential need to examine and predict tire behavior and monitor tire performance under various operating conditions. In recent years, several researchers tried to predict tire behavior with different tools including numerical analysis and empirical analytical simulations in addition to the experimental tests.

Experimental tests such as cleat drum tests or cost-down tests are very time-consuming and expensive. As a result, many researchers put a huge effort into modeling tire performance with numerical approaches which are also considered a more effective solution for tire manufacturers and many research and development sections. Severe climate conditions have always been a source of concern for the safety of driving particularly in tire–snow interaction in the most difficult areas such as the Arctic region. The loss of traction in the tire–ice interaction is a main risk of collisions. For this reason, winter tires have been widely developed through the years after their introduction in the 1930s to protect passenger safety and provide a good ride. Three revolutionary changes have been made to the winter tires from that era including implementing studs, changing tread pattern design, and optimizing tire rubber properties. The application of studs is being shied away from today. This is due to the ever-increasing concentration of air pollution and environmental hazards caused by the stud's increased road wear. Therefore, it can be noted that tread pattern design and rubber definition are two main indicators for grip improvement in tires [1].

This literature review stands out due to its comprehensive analysis of tire–road interactions, focusing on both passenger and truck tires using finite element methods (FEM).

Unlike other reviews, it not only addresses tire performance modeling but also delves into experimental validation, temperature effects, and friction modeling. A key distinguishing feature is its emphasis on the gaps in research regarding winter tire performance, particularly under extreme conditions, which has been less explored in prior studies.

This review highlights recent advancements in tire modeling, with a specific focus on the impact of temperature on tire behavior, a critical factor for real-world applications. It also provides fresh insights into friction modeling and contact algorithms in tire–road interactions, while examining material definitions for tread rubber compounds, particularly through the use of hyper-viscoelastic models in current research. The primary audience includes automotive engineers, tire manufacturers, and researchers interested in tire performance and design, especially those involved in finite element analysis and vehicle dynamics. The review is systematically organized, starting with the history and evolution of tire models, followed by sections on friction modeling, thermal analysis, and material constitutive modeling. This structure, which integrates both historical context and state-of-the-art advancements, was chosen to address current challenges in the field while emphasizing real-world applicability, particularly for tire performance in extreme climates.

2. Tire Axis Terminology

The modeling and validation of tires from different categories including passenger car tires and truck tires are described according to previous research works. Firstly, to represent the forces on the actual tire, the Society of Automotive Engineering (SAE) and International Standard Organization (ISO) standards established the Reference Tire Axis System. The system typically consists of three main axes:

1. **X-axis (Longitudinal Axis):** This axis points forward in the direction of the vehicle's travel or the tire's rolling motion. Forces acting along this axis include longitudinal forces, such as braking or driving forces.
2. **Y-axis (Lateral Axis):** The Y-axis is perpendicular to the X-axis, pointing laterally across the tire. Lateral forces, such as those generated during cornering or side-slipping, act along this axis.
3. **Z-axis (Vertical Axis):** This axis is perpendicular to both the X and Y axes, pointing vertically upward. The vertical force acting along this axis is the normal force between the tire and the road surface, which is influenced by the vehicle's weight and dynamic conditions.

Each axis is associated with moments, denoted as Mx, My, and Mz, which describe the rotational forces around the respective axes. For instance:

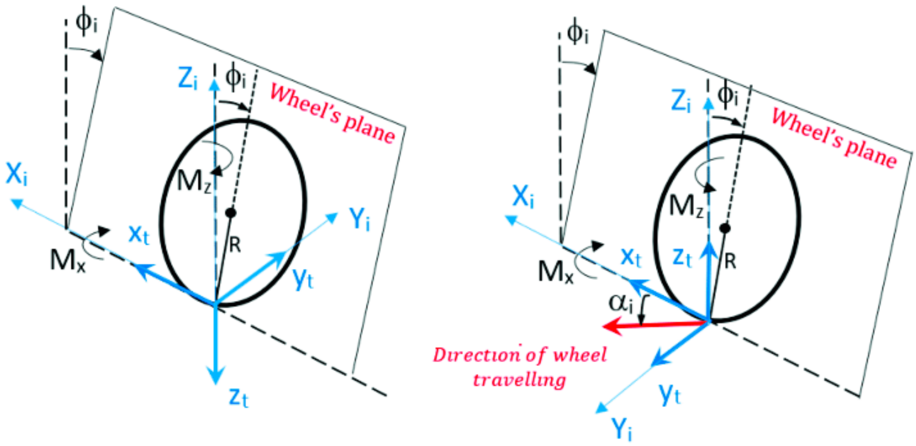
1. **Mx (Rolling Moment):** Rotation around the X-axis, related to tire deformation during rolling.
2. **My (Overturning Moment):** Rotation around the Y-axis, often associated with lateral load transfer during cornering.
3. **Mz (Aligning Moment):** Rotation around the Z-axis, describing the self-aligning torque that affects the tire's tendency to return to its straight-ahead position.

It should be noted that the main difference between the two below-mentioned axes is the positive direction of the Z-axis and Y-axis.

2.1. SAE Tire Standard Axis

According to the SAE standard shown in Figure 1a, tires in the rolling motion generate three forces that correspond to the longitudinal (X-axis), lateral (Y-axis), and vertical (Z-axis), respectively. During rolling, tires produce three moments along with forces around each axis that are represented as $M_x,\ M_y,\ M_z)$ [2].

The SAE tire standard is a set of guidelines created by the Society of Automotive Engineers (SAE) to ensure uniformity and reliability in tire testing, design, and performance evaluation. These standards define the key parameters for assessing tire characteristics such as load capacity, dimensions, rolling resistance, durability, and traction. The standards also establish a consistent tire axis system for measuring forces and moments acting on tires during vehicle operation, ensuring that tire behavior is accurately represented across different testing environments.



(**a**) (**b**)

Figure 1. Tire Axis Terminology (**a**) SAE standard and (**b**) ISO standard [2].

2.2. ISO Tire Standard Axis

The ISO tire coordinate system shown in Figure 1b is a standardized framework used to describe the forces and moments acting on a tire during vehicle operation. It is crucial in tire modeling and vehicle dynamics to represent tire behavior accurately. This coordinate system is used in various tire models, including those based on finite element analysis (FEA), to standardize how tire forces and moments are measured, simulated, and interpreted. It ensures consistency across different tire performance assessments and helps in analyzing tire behavior under different operating conditions.

3. Tire Modeling and Analysis

In this section, several tire modeling and analysis techniques are discussed in chronological order.

3.1. Semi-Empirical Tire Models

Semi-empirical tire models combine theoretical principles with empirical data to predict tire behavior under various conditions, offering a balance between accuracy and computational efficiency. The models integrate physical characteristics, such as contact patch deformation and tread elasticity, with real-world test data, including measurements of tire forces, slip angles, and moments. Adjustable parameters in these models allow for customization to fit specific tire types and conditions, enhancing their predictive accuracy within tested ranges. Examples include Pacejka's Magic Formula, which is widely used in motorsports for modeling lateral and longitudinal tire forces, and the Brush Model, which represents tire deformation through simplified contact patch elements. Semi-empirical models are essential in vehicle dynamics simulations, motorsports, and tire design, where they enable accurate predictions of tire performance without overly complex simulations.

The Magic Formula is a mathematical model developed to describe the forces and moments generated by a tire as it interacts with the road surface. It is particularly effective at capturing the complex, nonlinear behavior of tires under varying conditions such as slip angle, load, and speed. First introduced by the Dutch researcher Hans B. Pacejka [3] in the 1980s, the Magic Formula became a fundamental tool in vehicle dynamics simulations due to its ability to model tire performance in a wide range of scenarios.

One of the key features of the Magic Formula is its ability to represent the nonlinear relationship between tire forces (such as lateral, longitudinal, and vertical forces) and the corresponding slip angles or slip ratios. Unlike purely physical models that rely on detailed knowledge of a tire's structure or materials, the Magic Formula is empirical, meaning it is based on fitting experimental data to a set of mathematical equations. This allows for highly accurate predictions of tire behavior without needing to know every detail of the tire's physical properties.

The general form of the Magic Formula is expressed as follows:

$$Y = D\sin(C\arctan(BX - E(BX - \arctan(BX)))) \quad (1)$$

where Y represents the force or moment being calculated (such as lateral or longitudinal force), and X represents the slip ratio or slip angle. The coefficients B, C, D, and E are derived from fitting the model to experimental tire data. Each coefficient plays a specific role: B is the stiffness factor, C is the shape factor, D determines the peak force, and E controls the curvature of the force–slip curve. These parameters allow the model to accurately represent the nonlinear nature of tire performance.

The Magic Formula is versatile and can be used to model a wide range of tire behaviors, including lateral forces during cornering (lateral slip), longitudinal forces during acceleration or braking (longitudinal slip), and aligning torque (self-aligning moment) during steering. This adaptability makes it suitable for use in various vehicle dynamics simulations, helping predict how tires will respond under different driving conditions.

Widely recognized for its accuracy, the Magic Formula has been validated across a variety of tire types and driving conditions, making it a reliable tool for vehicle dynamics simulation software like ADAMS and CarSim. Its detailed predictions of tire forces and moments are crucial for modeling vehicle handling characteristics, especially in applications like motorsport, where precise control over tire performance is critical. In addition to vehicle dynamics, the Magic Formula is extensively used by tire manufacturers during design and testing. It allows engineers to optimize tire performance by simulating how changes in design will affect tire behavior. Overall, the Magic Formula's combination of accuracy, versatility, and empirical reliability has made it indispensable in vehicle dynamics simulations and tire performance analysis.

Furthermore, the FTire (Flexible Ring Tire) model is an advanced, semi-physical tire model that simulates tire behavior with high precision, particularly for scenarios involving complex surfaces, high-frequency vibrations, and detailed road profiles. Representing the tire as a flexible ring supported by elastic and damping elements, FTire captures deformations and vibrations across the contact patch as the tire rolls over various terrains. This model's 3D contact patch simulation enables it to handle fine road details, like textures, bumps, and irregularities, ideal for off-road and rough surface conditions. FTire's ability to simulate high-frequency dynamics also makes it effective for noise, vibration, and harshness (NVH) analysis, capturing the effects of inflation, temperature, and wear on performance. Its design for real-time applications allows it to be used in hardware-in-the-loop (HIL) testing, which is valuable in automotive and motorsport settings for optimizing suspension, improving handling, and testing tire impacts on stability.

Beyond the commonly used semi-empirical tire models like Pacejka's Magic Formula and the Brush Model, there are several other types that offer valuable features for tire modeling. The LuGre Friction Model, for instance, was originally developed to model dynamic friction but has been adapted for tire–road interaction applications. It captures complex frictional behaviors, such as hysteresis and transient lag, making it especially useful in scenarios requiring real-time adjustments to changing road conditions. Another example is Dugoff's Model, which estimates tire forces under combined slip conditions by factoring in slip ratio and slip angle, providing accurate predictions for vehicle dynamics in maneuvers like skidding and cornering. Additionally, the UniTire Model is a comprehensive approach that unifies elements of both the Magic Formula and Brush models, making it adaptable for various tire types and operating conditions. The Fiala Model is also notable; it focuses on estimating lateral forces based on slip angle and vertical load and is often applied in vehicle stability and control systems due to its simplicity and computational efficiency. Each of these models integrates empirical data with theoretical frameworks, balancing accuracy with computational manageability, particularly in scenarios where fully physical or finite element models would be too resource-intensive.

3.2. Finite Element Tire Models

The Finite element method (FEM) is one of the most useful and powerful tools used for designing and analyzing tires. The first application of the finite element method in tire industries returned to 1970 decay by the introduction of tire science technology journals. The first effort to model two-dimensional tires referred to the 1970 decay and the first footprint modeling refers to 1980.

In 1969, Dugoff et al. [4] presented a theoretical study of the effect of tire-mechanics properties on vehicle performance. This study determined the combined longitudinal and lateral forces of the tire during various automobile maneuvers. A tire model was simulated to determine the vehicle's response under skidding conditions. The simulation included the corresponding analytical functions representing the relationships between the longitudinal and lateral tire shear force components. The analytical functions were described as a function of various parameters including tire normal load, sideslip and inclination angles, and longitudinal slip. Finally, the tire shear force equations were assessed through the comparison with experimental results and showed a good agreement with the physical test data.

In 2005, Cho et al. [5] performed an explicit finite element analysis to predict the transient dynamic response of a rolling tire traveling with a small cleat. The three-dimensional tire model was generated with detailed tread blocks to realistically simulate the tire-cleat procedure. The frictional dynamic contact algorithm was defined with the total Lagrangian scheme and the penalty method. Time-history and frequency responses of the dynamic forces were determined with the numerical simulations. Additionally, the influence of the tire rolling speed and inflation pressure on the transient dynamic response was studied. Finally, the FE results were verified by the experimental results.

In 2006, Chae [6] performed a nonlinear analysis of a truck tire 295/75R22.5 by implementing membrane elements for tire cords, and hyperelastic solid elements were implemented for the rubbery parts. The tire ring was modeled with a rigid ring model. The finite element analysis model was validated using experimental measurements. For this purpose, the vertical stiffness of the tire, forces, and moments were compared with the physical test results. Finally, the FEA showed a good agreement with the experimental data.

In 2008, Hublau [7] attempted to develop a mathematical model for the definition of rolling resistance by presenting a mechanical justification of the ISO 18164 norm with the help of the first law of thermodynamics. It must be pointed out, that an interesting finding of this research was the comparison of the equations used in ISO 18164 standards and the physical equation that showed the norm neglects the differences between the bearing friction of the tire and the drum in the skim test. Finally, the order of the bearing friction was investigated as 0.1 kg/t at 80 km/h speed.

In 2012, Behroozi [8] discussed the effect of the complexity of the finite element models on the accuracy of the predicted tire behavior for the aircraft tire. Three FE models were analyzed to investigate the accuracy of the results. All the output results were compared to measured test data to define the rubber behavior, and the Yeoh hyperelastic material model was defined. All the simulations were carried out using Abaqus/CAE for generating a two-dimensional tire model and an Abaqus command line for three-dimensional analysis such as inflation pressure. All the simulation results were compared with the physical tests and the FE results and showed a good agreement with real-size measurements of tire profiles. Moreover, tire burst analysis was performed with various tire inflation pressures. Consequently, it was determined that maximum stress in the rubber structures of the tire is directly proportional to the mesh size variation; however, the mesh size has a negligible effect on the maximum stress in reinforcement cords and maximum deformation of the tire.

In 2016, Lardner [9] presented the validation and modeling of a wide base truck tire size 445/50R22.5. A finite element analysis is employed for determining the first vertical and longitudinal mode of frequencies. For this purpose, a frequency analysis was performed on the truck tire using the Fast Fourier Transform (FFT). In addition, the effect of tire speed and inflation pressure on the sidewall damping coefficient and modes of vibration were predicted at various operating conditions including inflation pressure varied from 50 psi to 165 psi, and speed range changed between 20 km/h and 100 km/h, and vertical load for the tire center from 1500 lbs. to 9000 lbs. The results were compared and validated in good agreement with the previous experimental studies.

In 2017, Ludvigsen et al. [1] provided a study on the wet grip for winter tires by the weight of the car and the frictional forces in tire-terrain interactions. The finite element analysis was conducted using the ANSYS Workbench 18.0. This study investigated the comparison between the effect of air-filled and non-air-filled tires on grip improvement. To predict the friction, the friction coefficient was defined as a function of the contact pressure. The various pressure trends for the tire FE models were simulated and the results were analyzed for the loading conditions. Finally, it was observed that airless tires had a significant effect on increasing the grip compared with conventional tires.

In 2018, Tigdemir [10] presented a numerical study for wheel–snow interaction using finite element techniques. Firstly, the tire structural model was generated by SOLIDWORKS and ANSYS (version 2016) design modeler software. The analysis of the tire was carried out using ANSYS Explicit Dynamics. To model the rubber behavior, Mooney–Rivlin hyperelastic material was used. The frictional force between tire and snow particles was predicted considering the linear behavior of the snow erosion. It must be noted that six different mesh sizes were adopted for this study, and the effect of mesh size was studied on the accuracy of the results. Finally, the simulation results were validated, and it was found that the model with 0.025 m and 0.02 m mesh sizes provides more precise results than other models.

In 2019, Rafei et al. [11] developed the full material sensitivity analysis for rolling tires with Abaqus (2016) finite element software. The tire cross section is shown in Figure 2. The results were compared to the experimental data, and they showed good agreement. Finally, it was proved that the nonlinear viscoelastic material could predict the accurate rolling resistance of tires due to the capability to simulate the exact behavior of material losses and rubber behavior.

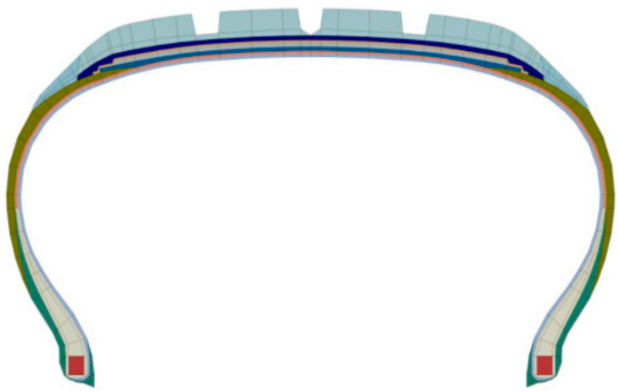


Figure 2. Two-dimensional passenger car finite element model [11].

In 2020, Xu et al. [12] focused on developing an intelligent tire system that used machine learning techniques for tire force estimation. It presented a novel approach by integrating a tri-axial accelerometer installed in the tire with neural network algorithms to predict longitudinal, lateral, and vertical forces acting on the tire in real time. Accelerometer data, collected from various driving conditions such as cornering and free-rolling, were processed through machine learning models—Neural Network, Random Forest, and Recurrent Neural Network (RNN)—to estimate tire forces accurately. The neural network with the Rprop algorithm was identified as the most effective due to its balance of accuracy and simplicity for real-time applications. The study demonstrated the potential of intelligent tires to improve vehicle safety and control by providing more accurate, real-time tire data, particularly valuable for autonomous driving and advanced vehicle systems. However, the Random Forest model struggled with large slip angles and extreme driving conditions, and the system required substantial training data and computational power, which could challenge real-time deployment in less-equipped systems. Additionally, the controlled testing conditions indicated the need for further validation in more unpredictable real-world environments.

In 2020, Ali [13] presented a model based on finite element methods to estimate rolling resistance for a passenger car radial (PCR) tire. For this purpose, the deflection of the several tires under certain loading conditions was measured and calculated. To achieve a clearer perception of this phenomenon, the value of the deformation force in tires for each component during deformation was computed. In this research, a total force that occurred due to deformation was represented as energy loss or hysteresis dissipation caused by rolling resistance. In addition, the experimental test was performed using three various tire designs: two grooves, three grooves, and four grooves. It was found that the four-groove tire cluster showed the lowest rolling resistance coefficient (RRC). Consequently, all the finite element simulations were compared with the experimental data, and it was observed that the lowest rolling resistance was the result of the largest crown radius.

In 2021, Aalto et al. [14] investigated the effect of the pressure distribution under a tire on the rolling resistance. For this purpose, a two-dimensional tire model was generated using the finite element method. The tire model was built with two layers, one for the sidewall and one for the belt in the ANSYS (version 2021 R1) software. To control the boundary conditions of the problem easier, the authors used MATLAB. A hypothesis was composed, and it was stated that the pressure distribution was represented to be offset in the rolling direction. Then, the mechanism for generating rolling resistance was explained, and the FE models were made to extract this behavior. Finally, an offset for the pressure distribution beneath the tire was observed in the finite element analysis.

In 2022, Maria et al. [15] presented a novel framework for tire analysis. For this reason, closed unclamped B-splines were implemented for a fully continuous explanation of geometry and field variables. The advantage of the proposed model was that the tire analysis using this approach was less sensitive to the applied discretization compared with standard FEA models. In addition, an Arbitrary Lagrangian–Eulerian technique was deployed to describe the rolling phenomena. For this purpose, a direct computation of second-order gradients caused by the use of higher-order basis functions was used. The results of numerical simulations were validated by the experimental tests for a passenger car tire and the analysis results showed a good agreement with the physical measurements for the actual tire.

In 2023, Fathi et al. [16] presented a finite element model to investigate tire cornering characteristics in tire–road interaction for a Regional Haul Steer II, RHS 315/80 R22.5 truck tire rolling in a dry rigid surface. The simulations were carried out using Pam-Crash (version ESI 10.0) software. To define rubber behavior in tire compounds the Mooney–Rivlin model was implemented. Tire cornering force was calculated in various operating conditions, including three different speeds concerning various positive slip angles. The local and global frame coordinates were deployed to compute the cornering force. Moreover, tire moments including self-aligning moment was measured from the tire section for all operating conditions. Additionally, by comparison between the FE results, it was observed that the tire lateral force was directly affected by the variation in the slip angles. Furthermore, it was noticed that tire inflation pressure was the main indicator for tire–road interaction characteristics.

In 2023, Liang et al. [17] studied the effect of the belt pressure distribution ratio on the carcass contour based on the new non-natural equilibrium contour theory. For this reason, a 385/65R22.5 all-steel radial tire was modeled using finite element techniques. By performing finite element analysis, the belt pressure share ratio was calculated. Finally, it was found that by increasing the belt pressure share ratio, the clamping effect of the belt layer on the carcass will be strengthened. Moreover, the curvature of the tire crown in carcass contour was increased and the crown curve was reshaped to be more rounded. Additionally, according to the results of the finite element simulation, it was found that the increase in the belt pressure share ratio resulted in increasing tire wear and lowering rolling resistance.

In 2024, Fathi et al. [18] modeled and validated a 235/55R19 all-season tire from continental manufacture. The tire-road model was simulated using the finite element method in the Pam-Crash (version ESI 10.0) software. The road was modeled as a rigid dry hard surface. The Mooney–Rivlin material model was defined for modeling tire rubber compounds. The passenger car tire was calibrated based on the tire weight, and bead diameter. Moreover, the numerical validation was performed based on the static stiffness, rolling resistance coefficient, and cornering stiffness coefficient. Tire dynamics was numerically validated based on the drum-cleat test. Tire–road interaction characteristics including tire rolling resistance were predicted based on the ISO58580. The gap in this study was the lack of experimental validation for tire rolling resistance and tire cornering force.

3.3. Experimental Testing

Experimental testing of tires involves a range of controlled tests to measure tire performance across variables like load, slip angle, speed, and temperature. These tests are crucial for validating tire models, refining tire design, and ensuring consistent safety and performance under diverse driving conditions. Various types of testing are commonly used. Static testing measures the structural characteristics of tires, such as stiffness and deformation under load, without rotation. These tests, including vertical, lateral, and radial stiffness tests, provide foundational insights into how tires handle static loads. Dynamic testing is conducted with the tire in motion, assessing performance under different speeds, slip angles, and road conditions. These tests, covering cornering, braking, and acceleration, measure forces, moments, and frictional behaviors, which are essential for understanding handling, grip, and stability. Environmental testing subjects tires to varying temperatures, humidity levels, and surface conditions to see how factors like wet or icy roads impact durability, traction, and braking. High-speed and wear testing assesses tire longevity and wear patterns under high-speed and prolonged use conditions.

In 1995, Allen R.W. et al. [19] addressed the need for tire force measurements on wet pavement to develop a low-coefficient friction tire model, as existing data primarily focused on dry conditions. It detailed the process of enhancing tire models for the National Advanced Driving Simulator (NADS) by conducting tests with tires shaved to 4/32 inches to simulate reduced friction levels. Although shaving tires did not replicate the effects of normal wear, it was sufficient for the study's purpose of reducing the coefficient of friction. Tire forces were measured on a water-coated surface to mimic wet pavement conditions across various tests, including cornering and braking, at five loading conditions. The results indicated a decrease in effective lateral stiffness and longitudinal force with increasing tire speed. These data were then used to create a tire model for dynamic simulations, with the paper concluding by presenting the model's performance in relation to loading conditions and tire slip angles.

In 2000, Kabe et al. [20] presented both implicit and explicit simulation using finite element techniques to model a passenger car tire. Moreover, the cornering simulation was performed to extract the lateral force of the tire under the state stage. The results were validated using the experimental data from an MTS Flat tire test. The predicted cornering results in two different methods including implicit FEM and explicit FEM were compared to the experimental ones and they showed a good correlation between the simulation and the real-time cornering data from the MTS Flat-Test Tire Test.

Research into temperature and wear effects has shown how these environmental factors affect tire friction and tread elasticity, impacting handling and braking efficiency. In 2013, Unice et al. [21] highlighted the need for tire materials with improved thermal stability, spurring advancements in materials that enhance tire performance across a wider range of temperatures.

In 2013, Virkar et al. [22] presented a comprehensive review of tire performance parameters and experimental setups used to evaluate them. The study discussed critical factors affecting tire performance, such as rolling resistance, tire wear, temperature, cornering properties, and noise, and details testing setups like drum-type, flat-track, and flat-rotating disc machines. While the study effectively synthesized previous research, it lacks original experimental data and detailed statistical analysis to validate its claims on optimal testing methods. The absence of testing resulted in limited practical applicability, as the theoretical approach may not account for all variables encountered under dynamic road conditions.

Recently [23], data from the Formula SAE Tire Testing Consortium (TTC) collected at the Calspan tire testing facility were processed, analyzed, and modeled. The raw data were formatted for convenient processing in Matlab, where it was organized according to testing parameters. Graphical techniques were employed to represent the data, ensuring that the selection criteria effectively captured the relevant data segments and that the data were well-structured. Various modeling techniques were compared, and the limitations of modeling based on the available data were examined. A modified version of the non-dimensional tire theory was researched and discussed, and Matlab scripts were developed to fit the non-dimensional model using regression fits to populate response surfaces for the model coefficients. The results of the modeling were presented, along with suggestions for future modifications and improvements.

However, experimental tire testing faces significant challenges due to the complexity of replicating real-world conditions and the inherent variability in tire behavior. Accurately simulating diverse conditions, such as different road surfaces, weather, and loads, is resource-intensive and often difficult to generalize to real-world applications. Ensuring data accuracy and consistency can also be problematic, as minor variations in temperature, humidity, or surface texture can impact results, making it hard to compare tests conducted at different facilities. Additionally, testing is costly and time-consuming, requiring specialized equipment and facilities like high-speed dynamometers, climatic chambers, and test tracks to assess durability, wear, and performance comprehensively. Capturing high-frequency dynamics, such as vibrations and noise characteristics (NVH), poses further challenges, as these require advanced sensors and data systems to record complex, fast-transient responses. Analyzing the multi-dimensional data generated, covering forces, moments, slip, wear, and deformation, adds another layer of complexity, demanding both specialized expertise and sophisticated data analysis to interpret accurately.

3.4. Summary of Tire Modeling and Analysis

In this section, a review of the tire axis terminology and tire mechanics was covered to provide a perspective of tire forces and moments in a static and dynamic rolling maneuver.

Then, a review of tire modeling techniques, including numerical, empirical, semi-empirical, analytical, and experimental modeling was provided.

Among the various modeling approaches and validation for tires, implementing the finite element method was found to be a powerful technique to predict tire forces and moments estimation and has been known as a time-saving process and cost-effective method for tire modeling since the 1980s.

Additionally, tire validation based on experimental data has consistently shown that the finite element method provides reliable predictions for tire performance, particularly at high speed, including rolling resistance, and cornering characteristics compared to other methods such as implementing a neural network that limits the tire force range and speeds.

Moreover, instead of performing expensive experimental testing cyclic testing, and physical measurements such as coast down test or tire rolling resistance test using the RR machine, performing trailer test, and numerical analysis highlights the beneficial aspect of tire modeling using the finite element method.

It was introduced that among the recent research on tires, there is a major drawback to all-season tires, particularly studying the effect of material definition on tire performance especially in countries like the United States, Canada, etc. Therefore, this literature review could improve the aspects of material effect and ambient temperature on tire performance to reduce fuel consumption. Table 1 summarizes the most significant research papers provided in the literature in chronological order.

Table 1. Summary and critical observations of tire modeling research.

Year, Author	Topic	Comments/Critical Analysis
1969, Dugoff et al. [4]	Tire forces, FEA	Limited by its reliance on a simulation model that, despite showing qualitative agreement with experimental data, might not fully capture the complexities of real-world tire performance.
2000, Kabe et al. [20]	Tire forces, FEA	Overlooked the complexities and variabilities of tire behavior, as the reliance on idealized models and specific testing conditions could limit the generalizability and practical applicability of its findings.
2005, Cho et al. [5]	Tire transient forces	Used a simplified model of the tire-cleat interaction and artificial damping to simulate dynamic viscosity which may compromise the accuracy of the results.
2006, Chae [6]	Tire forces, experimental	Limited experimental testing and complex tire model that is computationally expensive.
2007, Qi [24]	Tire forces, FEA	Undermine its own contributions by acknowledging that solution accuracy may degrade significantly in certain cases.
2008, Hublau [7]	Tire forces, Derivation	Formal derivation of what is already known, no significant contribution.
2012, Behroozi [8]	Tire forces, FEA	Fail to address the broader implications of mesh size sensitivity in rubber components and its potential impact on the reliability of predictions.
2016, Lardner [9]	Vibration analysis, FEA	Overlook the potential interactions between varying parameters, which might not accurately reflect the complex, dynamic behaviors observed in real-world tire performance.
2017, Ludvigsen et al. [1]	Wet grip, friction coefficient	Underestimate the complexities associated with the overall performance of such tires, particularly regarding factors like durability, and heat dissipation with icy surfaces that could impact the feasibility and effectiveness of the proposed solution.
2018, Lee et al. [25]	Tire forces, Model approach	Rely on model-based methods without sufficiently addressing the inherent limitations of sensor data accuracy
2019, Rafei et al. [11]	Tire forces	Not thoroughly addressing the broader implications of model selection on other tire performance aspects particularly given that the relationship between rolling resistance and factors such as temperature, wear, and road conditions remains complex.
2020, Pelc, Józef [26]	Lateral stiffness	Overly focused on the technical advancements without adequately addressing the potential limitations of the model in terms of scalability, or validation against diverse scenarios, which could affect its overall utility in tire design and analysis.
2020, Zitelli et al. [27]	Dissipation of the viscosity energy	Overlooking the nonlinear behavior and other critical parameters that can significantly influence energy dissipation and overall tire performance in diverse driving conditions.
2020, Nazari et al. [28]	Tire cornering force, hydroplaning speed	Underestimate the challenges associated with computational complexity and the potential limitations of the validation process, as the reliance on existing literature for model validation could compromise the generalizability of the results to varied real-world conditions and tire designs.

Table 1. *Cont.*

Year, Author	Topic	Comments/Critical Analysis
2020, Ali [13]	Tire deflection Hysteresis dissipation Rolling resistance coefficient	Lack of a thorough exploration of the broader implications of design choices on other performance metrics, such as traction, handling, and durability, which could limit the applicability of the findings in a holistic tire design context.
2021, Aalto et al. [14]	Rolling resistance	Lacking robust conclusions due to the ambiguous results regarding the direction of the pressure offset and fails to address the influence of vertical load variation, which could significantly enhance the understanding of the underlying mechanics of rolling resistance in tires.
2022, Maria et al. [15]	Tire rolling	Lack of detailed comparisons with existing methods and the dependence on experimental validation raises questions about its practical applicability and effectiveness.
2023, Fathi et al. [16]	Cornering force Self-aligning moment	Lacks a thorough examination of the validation of the numerical model against experimental data, which is crucial for ensuring the reliability of the simulation results.
2023, Liang et al. [17]	Wear	Lacks detailed explanations of the experimental validation process for the finite element analysis results.
2023, Ge et al. [29]	Tire lateral force, sideslip angle, camber angle, and rolling resistance	Could be enhanced by providing a more thorough discussion on the limitations of the numerical model and its generalizability to other tire designs or conditions.
2024, Fathi et al. [18]	Rolling resistance, cornering force	It could benefit from a more in-depth discussion on the implications of material model selection and the potential influence of varying operational conditions on the tire's performance beyond the tested parameters.

4. Contact-Friction Modeling in Tire–Road Interaction

To model the contact between tire and road, various models have been investigated and some of the models are presented in the following sections.

4.1. LuGre Friction Model

In 1993, De Wit et al. [30] proposed a new dynamic friction model that was capable of capturing all the friction phenomena in the actual experiments. The proposed model represented the arbitrary steady-state friction characteristics. The hysteretic behavior due to frictional lag and spring-like behavior in stiction were completely observed with the model. Moreover, the proposed model gave a variation on the break-away force based on the rate of differences in the applied force. Finally, all the mentioned criteria were simplified into a first-order nonlinear differential equation.

In 1999, Canudas de Wit, C. et al. [31] presented a friction model to capture real-time changes in the road profile in the tire–road interaction. As shown in Figure 3, several dynamical friction models were introduced to estimate the real contact friction in rolling conditions. Moreover, in the presence of these models, the road surface could be described with one single parameter. It was shown that the distributed parameter version of these models could estimate stationary shape profiles between normalized friction and slip rate which follows closely the experimental data of the Magic formula.

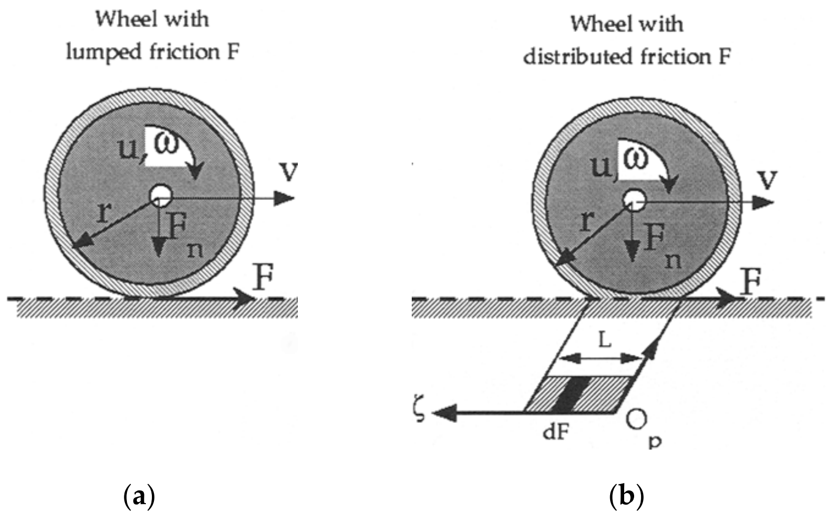


Figure 3. One-wheel system with (**a**) lumped friction, and (**b**) distributed friction [31].

4.2. Early Tire Friction Model

In 2003, Canudas-de-Wit et al. [32] proposed a new dynamic friction model for tires in the longitudinal tire–terrain interactions. For this purpose, a dynamic friction model for contact-point friction problems, known as the LuGre model was implemented. A partial differential equation for the distribution of the friction force in the tire contact zone was developed according to a contact patch between the tire and the road. For the friction force, an ordinary differential equation (the lumped model) was derived considering the patch boundary conditions and the normal force distribution in the tire footprint. This lumped model was calculated to be approximately close to the distributed friction model. This friction model was able to measure the exact transient behavior of the frictional force generated in the tire contact patch in the tire–terrain interactions in the barking and acceleration conditions. A velocity-dependent, steady-state description of the friction force versus the slip coefficient is also derived that facilitates easy tuning of the model parameters by comparing them with the steady-state experimental data. Experimental results verified the accuracy of the new tire friction model in the investigation of the friction force during transient vehicle motion.

In 2004, Wang et al. [33] presented the real-time tire–road friction measurement of the friction forces of tires. A setup was adopted with a different GPS which specified the non-linear longitudinal force of the tire. Tire–road friction coefficient was measured on various surfaces, particularly during the winter season under various vehicle operating conditions. All the simulation results were compared with the winter maintenance vehicle called the 'SAFEPLOW' and a good agreement was found between the results and the measured data. It was observed that compared with the physical data, the friction estimation for tire–road interaction rolling over various road textures provided reliable results.

In 2006 Kuwajima et al. [34], presented an analytical model to define the effect of tire friction on the surface roughness of terrain. Considering several experimental procedures, the rolling/sliding friction of tire tread rubber was determined against abrasive papers (representing the surface texture) under low slip velocities. To validate the measured data for the interaction between rubber and the abrasive papers, a two-dimensional explicit finite element analysis was performed. The results showed that the total contact patch area and local partial slip were increased for the finer surfaces under the same normal and tangential forces. Moreover, it was found that the velocity-dependent friction resulted in a higher local slip. It was investigated that local frictional behavior at microslip regions at asperity contacts has a significant effect on rolling/sliding friction at low slip ratio.

Figure 4 shows the tire model with the combination of the radial springs and radial dampers as a part of real time simulations of the tire-road interactions.

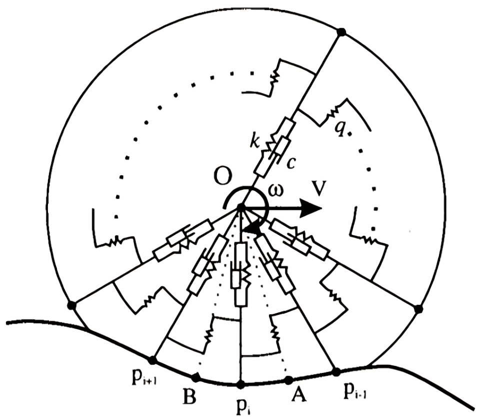


Figure 4. A physical-based Radial- Inter radial model of the tire in the Spring-Damper forms [35].

4.3. Recent Tire Friction Model

In 2013, Lee [36], developed a model for friction estimation for tires rolling in snowy terrain. For this purpose, Gaussian process-based stochastic metamodels were implemented to verify test data in drawbar pull and traction for tires defined as a function of slip. Then, the parameters were transformed from a deterministic physically based tire–snow interaction into a stochastic one. The mechanical properties of the tire and the frictional coefficient between the tire and terrain (natural snow) were validated by interval-based local and global validation metrics between simulation and physical data. Consequently, a reliable agreement was reported by the authors between the model and test data.

In 2014 Conte [37], expanded a new tire model, based on a simple Brush tire model as shown in Figure 5 to analyze tire forces and dissipated energy under various dynamic conditions. All the characteristic equations of the rubber material were imported to the 3D multi-line brush tire model. Finally, rolling resistance and dissipated energy were estimated considering tire geometry.

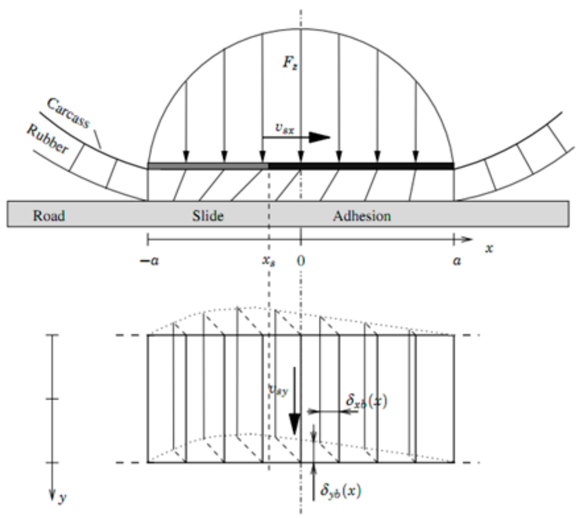


Figure 5. Brush tire model, stretching of the small volumes of rubber in the tire contact patch [37].

Figure 6 demonstrates a rigid ring model with a wheel plane (left) and out of plane (right) on a hard surface.

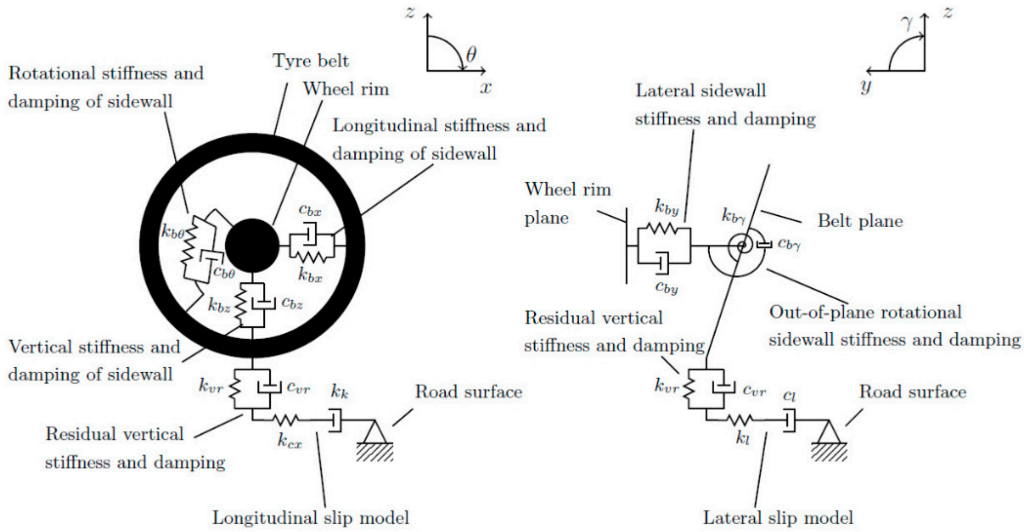
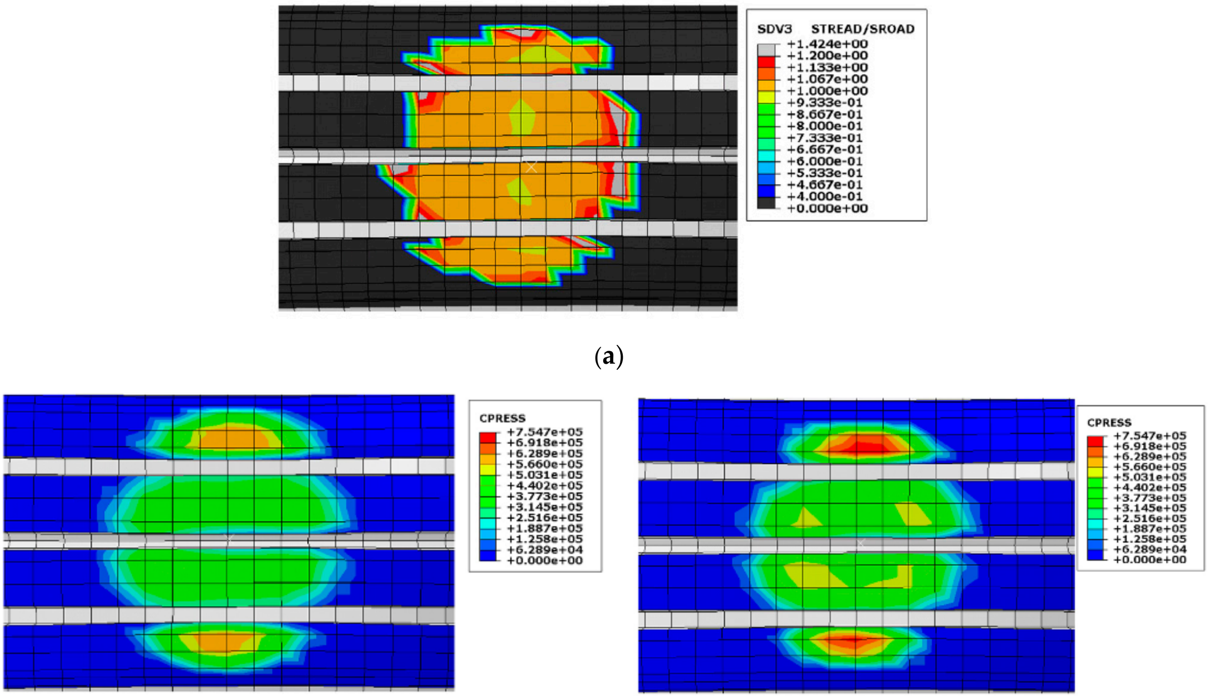


Figure 6. In-plane and out-of-plane rigid ring tire models on hard surface [38].

In 2018, El-Sayegh et al. [39] developed a finite element model for a wide base truck tire 445/50R22.5 in a virtual software package Pam-Crash. The cornering behavior of the tire was investigated over dry and wet surfaces. Moreover, the truck tire is validated using finite element techniques. To define the Water model, the smoothed particle hydrodynamics method was implemented. The contact algorithm between tire-water interaction was defined using the node-symmetric node-to-segment contact with edge treatment. Several simulations were carried out with various tire speeds and different inflation pressures to predict tire behavior. Furthermore, Tire cornering characteristics were predicted such as the lateral force, rolling resistance, cornering stiffness, and self-aligning moment. Additionally, all the FE results were verified according to the previously published experimental data.

In 2018, Rafei et al. [40], presented an advanced thermomechanical finite element model for a rolling tire that captures the friction coefficient in tire–road interaction based on implementing the various material models. For this purpose, friction was modeled using a thermo-mechanical procedure by developing two Abaqus subroutines. In addition, different friction models for the tire–road interaction were implemented to model rolling tires under cornering operating conditions. The experimental validations were performed on the FEM simulation results using output data from a Flat-Trac Machine Test. The FEM results revealed that the complexity of the friction models does have a negligible effect on the lateral force compared with aligning moment and footprint pressure. Moreover, as a pointer-related effect with the material definition, it was investigated that the complexity of the material models does not have a significant effect on the lateral force value compared with aligning moment and pressure distribution on the footprint as seen in Figure 7.



(**a**)

(**b**) (**c**)

Figure 7. Distribution of the friction coefficient in tire contact patch: (**a**) Deployed of Savkoor's model and Effect of temperature-dependent material properties for tire compound on tire footprint; (**b**) Modeling without thermal coupling; (**c**) Modeling with thermo-mechanical coupling [40].

In 2019, Ozerem et al. [41] presented a simple brush-type tire model for the Formula SAE tires with a novel method for the contact length formulation. Moreover, sets of ordinary differential equations (ODE) were deployed to investigate the thermal effect of the tire model. The output results of the proposed model were verified against experimental tire test measurements which revealed good agreement and proved that the model can provide a precise prediction of the tire forces and thermal effect for the thermal tire model. In addition, the temperature-dependent tire model was employed in a two-track model of Oxford Brookes Racing's Formula SAE vehicle to investigate the abilities of the model under transient handling simulation. The results showed that the proposed model can capture tire behavior in approximately close to real operating conditions. An interesting finding of this research was that tire temperature has a significant effect on the vehicle dynamic behavior particularly during on-limit scenarios.

In 2020, Yang et al. [42] proposed an improved contact model that is capable of predicting the influence of grain shape and tread rubber deformation for tire-terrain interactions. In the present contact model, the interfaces between the sand grains capture the form of the contact patch of the tire (footprint), compared with conventional point contact models. The contact calculation in conventional point contact captures four interactions, i.e., normal force, tangential force, rolling resistance, and twisting resistance. On the other hand, the contact between tread rubber and sand grains is defined as the surface contact in the tread–sand contact model. This surface contact contains the rolling resistance and twisting resistance on the grains due to rubber deformation in tire–sand interactions. Then, a sandpile simulation was carried out and a comparison of coarse particles was performed to verify the sand–sand contact model. Consequently, the novel contact model was implemented for the rubber tire–sand terrain simulations, and the results of numerical simulations were compared with the single-wheel experimental tests. In addition, a good correlation between FE and experiments was observed.

In 2021, Gupta et al. [43] modeled the static behavior of a radial mini truck tire for the road cum rail automobile in interactions with rails. For this purpose, the finite element method was implemented using the ANSYS (version R19.2) software program to achieve the rubber tire behaviors in contact with the rail underneath inflation stress values. The overall performance of tires on the rail was investigated using computational methods via finite element analysis. Finally, the effect of inflation pressure was studied in the equivalent strain in rubber tires and rims, deformation, and contact patch of rubber tire.

In 2021, Sufian et al. [44] presented a simulation to investigate tire–road contact behavior according to the tire tread pattern. The simulation was conducted using finite element methods. Moreover, several basic designs of tire treads were presented and were modeled using finite element methods. It was observed that the tread pattern design significantly affected the stress distribution of the internal layers of the tire structure. As a result, it should be noted that the effective contact increased with increasing the intensity of contact stress. Moreover, large distributions showed good stability corresponding to the uniform distributed force inside the tire structure.

In 2022, Ge et al. [45] introduced the contact dynamics method to predict the interactions between tire and road. For this purpose, a coupling of the finite element method (FEM) and the discrete element method (DEM) was used. To model tire FEM was implemented and it computed the contact stresses on the pavement surface, while DEM was deployed to simulate the heterogeneous structure of an asphalt mixture. The tire contact model was calibrated using contact stress distributions for free-rolling and full-braking conditions. It was found that, according to the particles of the pavement, mixtures tended to flow along the longitudinal direction and undergo a high tangential contact force under full braking according to the particle displacement and force distributions.

In 2022, Nakajima [46] developed a new theoretical tire model to predict tire forces and moments by implementing a two-dimensional contact patch of a tire with a rib pattern. For this reason, a finite element analysis was performed to predict tire forces and moments analytically. In addition, tire side force (Lateral force) was calculated using the theoretical tire model. Furthermore, the predicted self-aligning torque with the theoretical tire model follows close to the FEM computation. Using the proposed model, the shear force distribution in a two-dimensional footprint was also calculated under slip angle and the results showed a good correlation with the FEM computation. Additionally, the theoretical tire model provided sufficient accuracy compared with the FEM computation for calculating the distribution of the adhesion region and sliding region in a two-dimensional contact patch.

4.4. Summary of Contract-Friction Models

This section reviewed significant advancements in tire–road friction modeling, highlighting the evolution and specialized focus of each approach. The LuGre friction model, introduced and laid a foundation by capturing steady-state friction and frictional lag behaviors; further enhanced in 1999 to account for real-time road profile changes. Canudas-de-Wit et al. (2003) [32] expanded upon this with a dynamic model to simulate longitudinal tire–terrain interactions, validated through experimental braking and acceleration scenarios. Recent developments have introduced stochastic and thermomechanical models, such as snow-specific friction estimation and temperature-dependent models for transient behavior. The most recent work integrates finite element methods and DEM/FEM coupling, offering realistic simulations of tire interactions with diverse road textures. Together, these models enhance predictive accuracy and applicability under varied environmental and operational conditions, providing a comprehensive framework for tire performance analysis. Table 2 summarizes the most significant papers provided in the literature in chronological order.

Table 2. Summary and critical observations of contact-friction modeling.

Year, Author	Topic	Comments/Critical Analysis
1993, De Wit et al. [30]	Dynamic friction	While the proposed dynamic friction model effectively captures several key friction behaviors and supports control design, it lacks empirical validation across diverse operating conditions, which limits its practical robustness and applicability.
1999, Canudas de Wit, C. et al. [31]	Road friction	While the proposed method offers an innovative approach to estimating road condition changes online, it relies on idealized conditions (such as a non-vanishing slip rate) that may limit its variable driving dynamics.
2003, Canudas-de-Wit et al. [32]	Frictional force	Limitations in accurately predicting friction behavior across diverse conditions due to its reliance on idealized assumptions about the contact patch and normal force distribution.
2004, Wang et al. [33]	Contact friction	While the real-time friction measurement system demonstrates reliable performance across diverse conditions and applications, its dependence on differential GPS may limit its effectiveness in environments where GPS signals are weak or obstructed, such as urban or densely wooded areas.
2006, Kuwajima et al. [34]	Contact friction	Although this study provides valuable insights into the relationship between tire friction and pavement roughness, its reliance on abrasive paper as a proxy for pavement surfaces may oversimplify the complex road textures.
2013, Lee [36]	Frictional coefficient	While the approach employs a robust statistical framework and Gaussian process models to address the validation of vehicle interactions with snowy terrain, the description lacks clarity on how the findings translate into practical applications or improvements in model accuracy and usability.
2014, Conte [37]	Dissipated energy	Fails to provide concrete evidence or examples demonstrating how the autonomous corner module (ACM) tangibly improves vehicle performance or reduces fuel consumption compared to traditional systems.
2018, El-Sayegh et al. [39]	Wet friction	Although the paper offers a comprehensive investigation into the cornering characteristics of a wide base truck tire, it lacks a critical discussion on the limitations of the simulation and the potential discrepancies between the modeled and real tire behavior under varying conditions.
2018, Rafei et al. [40]	Friction coefficient	Fails to adequately address the implications of its findings for real-world applications or provide a clear framework for how these insights can enhance tire design or performance.
2019, Ozerem et al. [41]	Brush model	Presented a simple brush-type tire model for the Formula SAE tires with a novel method for the contact length formulation; however, lacks experimental validation of the tire model.
2021, Sufian et al. [44]	Contact stress	While the study attempts to explore the impact of tire tread design on tire-road contact behavior through finite element analysis, it lacks a thorough examination of how these findings translate into practical improvements in tire performance or safety for different driving conditions.
2021, Gupta et al. [43]	Contact patch	Although the paper focuses on modeling and simulating the behavior of a radial truck tire in contact with rails, it lacks a clear articulation of how the findings will inform practical improvements in the design or maintenance of road cum rail vehicles.
2022, Ge et al. [45]	Dynamic friction	While the study introduces a novel approach by integrating contact dynamics with FEM and DEM, it would benefit from a more detailed discussion on the practical implications of these findings for improving pavement design.
2022, Nakajima [46]	Tire side force	Lack of quantitative analysis on the extent of discrepancies between the model and FEM results diminishes its applicability in practical tire design or performance optimization.

5. Thermal Analysis for the Rolling Tire

Pneumatic tires exhibit thermomechanical behavior which could be described as a highly sophisticated transient phenomenon that, overall, a dynamic nonlinear coupled thermos-viscoelasticity problem must be solved with heat sources generated from internal dissipation and contact and friction [47].

5.1. Tire Internal Temperature and Hysteresis Analysis

In 1976, Clark [48] presented an idealized model to predict the heat generated by a pneumatic tire. A solution was investigated for the temperature rise in the tire section. For this purpose, the thermal properties of the rubber were defined as known coefficients. Finally, the required time to achieve the equilibrium in the thermal state was calculated. It should be noted that the authors reported the total heat generation from the carcass viscoelastic losses which leads to bulk heat generation but also the contact patch heating effect.

In 1978, Wong J.Y. [2] proposed a model based on experimental measurements that captured the effect of internal tire temperature on the rolling resistance coefficient for pneumatic tires. Figure 8 reveals the variation of the tire rolling resistance based on the tire internal temperature. Furthermore, based on Figure 9, the rolling resistance was calculated as a function of the temperature in the tire shoulder area.

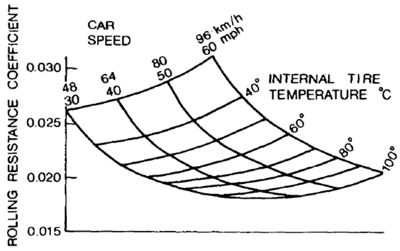


Figure 8. Effect of internal temperature on the rolling resistance coefficient [2].

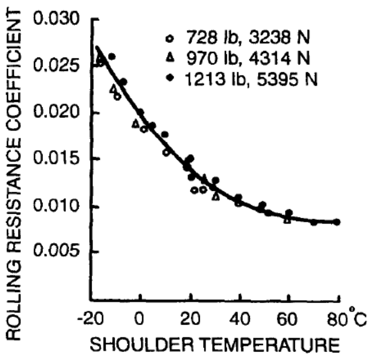


Figure 9. Variation in the rolling resistance coefficient with shoulder temperature for a car tire [2].

In 1996, MC Allen et al. [49] presented a numerical mode to predict the temperature rise in a free-rolling aircraft tire. For this purpose, the main indicator for the heat generation source was considered as dissipated energy from cyclic inelastic deformation in the tire. It must be noted that the analysis to define deformation was captured while the tire rolling in a steady-state frame. In this way, all the concurrent cycles are described as the same as the first cycle. To determine the inelastic energy, a phase lag was imposed between the strain and the stress fields. It should be considered that to keep the realistic validation between the simulation and the experimental data, a frequency-independent stage was defined for the phase lag.

In 2010, Li et al. [50] presented a three-dimensional axisymmetric finite element model of the 205/75R15 PCR tire. The FE tire model was generated in the Adina finite element analysis software. The steady temperature field for a rolling tire was modeled. Moreover, the thermal distribution contours of the rolling tire were extracted from the simulations. The steady-state temperature field was specified for each section of the tire, and it was analyzed to optimize the tire structure (piles) and to provide a better understanding of the rubber compound.

In 2012, Yokota et al. [51] estimated the temperature distribution for a running tire. Running tires were in thermal equilibrium between the generation of internal heat due to the viscoelasticity of the rubber material and the effects of their thermal environment. Since viscoelasticity showed temperature dependency, there was a strong correlation between this thermal equilibrium and rolling resistance. However, there are few methods available for estimating the rolling resistance of an actual running vehicle using this correlation. In this paper, the authors aimed to estimate the temperature distribution in the running tire using the quasi-static model of FEM, considering the thermal environment of real road driving, and introduced an approach for estimating the rolling resistance of an actual running vehicle based on the correlation between the tire's temperature distribution and rolling resistance.

In 2022, Cong NT et al. [52] provided a finite element approach to predict the temperature distributions of a steady-state rolling tire. Firstly, the Mooney–Rivlin model was implemented to describe the nonlinear mechanical behavior of the rubber in tire structure. The behavior of reinforcement parts of the tire including body-ply, wire, and rim were modeled with the linear elastic material model. The influence of the inflation pressure and vehicle loading were predicted using the coupled calculation method. In addition, hysteresis energy loss was implemented as the link to relate the strain energy density to the heat source in rolling tires. The temperature distribution of rolling tires was extracted via the steady-state thermal analysis. Moreover, an efficient computational method was introduced to reduce computation time for coupled 3D dynamic rolling simulation of the tire. Finally, the results of the numerical finite element simulation showed that loading has a significant effect on the prediction of the temperature field, and it was recognized as the main indicator for the temperature distributions of a steady-state rolling tire.

5.2. Tire Ambient Temperature Analysis

In 2004, Lin et al. [53] developed a numerical model to predict temperature distribution for a smooth tread bias tire of a light truck under various operating conditions including different speeds inflation pressures, and different applied loads. The main source of temperature rise in tires corresponded to the dissipated energy from the cyclic deformation of the tire. To validate the results, two different experimental tests were performed including dynamic mechanical testing and material testing to obtain hysteresis and total strain energy, respectively. Hysteresis energy loss was found to be a link between the strain energy density and the source of heat generation in rolling tires. Therefore, temperature distribution files could be predicted by the steady-state thermal analysis. This study was developed to facilitate tire temperature distribution in rolling tires.

In 2006, Narasimha Rao et al. [54] provided a technique for modeling non-axisymmetric tires with tread patterns to predict tire temperature. An algorithm was developed to investigate the temperature distribution of a tire using a simple decoupled procedure. An iterative process was implemented to predict the temperature as loss modulus. Both tire temperature and loss modulus properties were described as the function of strain and temperature. Moreover, tire operating conditions were measured experimentally. Finally, the generated algorithm for a smooth-treaded tire was compared with the finite element results, and a good agreement was found.

In 2012, Li et al. [55] developed the analytical model for a radial tire using the finite element method, using Algor (version 2002) software to study the effect of temperature. A virtual FEM model of the actual construction of the Bridgestone 24.00R35 tire was generated. The effect of ambient temperature on stress and deformation of the tire was investigated. Tire rubber material was modeled using the Mooney–Rivlin constitutive model. The steady-state heat transfer analysis was carried out and the results were validated with the real data under the ambient temperature from −40 to 40 °C at the Syncrude mine in Canada.

In 2022, Premarathna et al. [56] presented a numerical analysis for determining the effect of operational factors on the behavior of the solid tire under various temperature conditions. This method was a combination with the design of experiments (DoE). For this purpose, a three-dimensional static finite element analysis was performed. Then, the thermal models were generated for the solid resilient tires considering the relaxation characteristics of rubber, and temperature-dependent properties of hyperelastic material models defined for tire compounds. To predict the temperature-dependent mechanical properties of tire rubber compounds a dynamic mechanical analysis (DMA) and the William Landel Ferry (WLF) tests were carried out. Moreover, tire performance including tire blasting region was predicted using the proposed FE thermal model. Then, the FE results were validated using experimental data. At the second stage of the analysis, a two-level fractional factorial DoE analysis was performed using response readings of maximum stresses, deformations, and contact pressure extracted from the validated FE thermal model. The number of rubber layers, applied load, rolling speed, ramp angle, and aperture level were deployed as factors of DoE analysis. Finally, the results revealed the potential for considerable stress reduction with a minimum variation in the tire deformation and footprint. This method is very cost-effective and induced the capability of the combined FEM and statistical design procedure for decreasing the amount of performed physical tests.

5.3. Rolling Resistance, Hysteresis Loss, and Wear Analysis

In 1998, Song et al. [57] presented a finite element model to calculate the tire deformation due to the inflation and vertical loads. Moreover, the FE model investigated the heat generation due to the deformation. Both analyses were carried out using ABAQUS commercial software. To determine the temperature distribution field, a heat generation equation, and an effective strain amplitude equation were developed. It should be noted that the temperature dependency of the loss factor of rubber was taken into account for this analysis. The rubber properties were interpolated for the computed temperatures. Moreover, the temperature estimation was repeated as a cyclic procedure. To measure the internal temperature of the tire, the thermocouples were located at the shoulder and bead filler. In addition, the inner tire air temperature was captured using an implanted thermocouple in the rim valve hole. Finally, rolling resistance was calculated by the SAE J1269 (4 steps) method and a good correlation was found between FE results and experimental data.

In 2012, Li et al. [58] established a three-dimensional axial symmetry FEA (finite element analysis) model for a 165/70R13 radial tire. The model was constructed via ANSYS Workbench FEA software. For this purpose, the UI (user interface) of ANSYS Workbench was implemented and the thermal analysis function of this software was used. Moreover, a numerical simulation of the temperature field was carried out. The output of the simulation was the temperature distribution at each part of the tire intuitively. The finding was used for the better tire design. Finally, it was shown that the computed results of temperature distribution followed closely to real data.

In 2013, Cho et al. [59], developed a three-dimensional full-patterned tire model to predict the rolling resistance (RR) and the temperature distribution. For this purpose, a 3-D periodic patterned tire model was generated using finite element techniques. This analysis aimed to determine the rolling resistance numerically by implementing the hysteretic loss of viscoelastic rubber compounds. It must be noted that a 3-D periodic patterned tire model was constructed by repeating the one-sector mesh of the tire section in the circumferential direction. The strain cycles during one revolution were estimated using the 3-D static tire contact analysis. In addition, the hysteretic loss of the tire rubber compound during one revolution was determined according to the loss modulus of the rubber compound and the maximum principal value of the half-amplitudes of six strain components. Finally, the numerical simulation results were validated with the experimental data and the rolling resistance and temperature distribution were compared with the simply grooved tire. The final contour of the temperature distribution for the tire cross section for a patterned tire is presented in Figure 10 and the tire shoulders showed the highest sensitivity in this analysis.

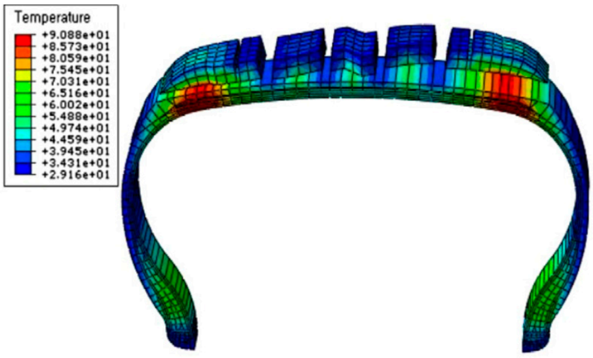


Figure 10. Temperature distributions of a patterned tire model at 140 km/h [59].

In 2015, Behnke et al. [60] presented a thermo-mechanical analysis of a three-dimensional tire structure. The analysis was carried out in a steady state rolling. The analysis was a combination of sequentially coupled mechanical and thermal simulation modules. To perform the mechanical analysis the Arbitrary Lagrangian–Eulerian (ALE) framework was implemented with a 3D finite element tire model to reflect the temperature-dependent behavior of tire rubber compound (obtained by the viscoelastic behavior of rubber) in a steady state rolling motion. Heat generation sources including energy dissipation from the tire compound and friction in the tire–terrain contact patch were estimated through experimental tests. The data obtained from the physical testing were imported as an input parameter for the thermal module. To perform the thermal simulation, a tire cross-section was deployed and the temperature distribution from total heat sources was computed in a transient thermal analysis. In addition, numerical simulations were carried out for three types of tire performance in free-rolling conditions. Finally, the FE results were validated physically using the test rig setup.

In 2019, Wu et al. [61] analyzed the effect of temperature on tire performance and wear. The study was performed for the tread rubber of the aircraft tire since aircraft tires are directly affected by wear. The wear tests were conducted under various temperatures under different slip angles for the tire tread compounds. Wear surface morphology and wear mechanism were explained. The wear analysis was carried out based on the finite element techniques of the rubber wheel. Finally, it was found that the slip angle had a significant effect on the wear rate for temperatures beyond 45 °C. Moreover, it was determined that higher temperatures (greater than 65 °C) reduced the roughness of the wear surface. Consequently, frictional power density and wear rate revealed a good correlation with each other.

In 2020, Farroni et al. [62] presented a real-time thermal model for motorcycle tires. This research provided a physical model for motorcycle tires, according to the application of the Fourier thermodynamic equations. A three-dimensional domain was considered for all heating sources in tires, including friction power at the tire-road interface and the periodic heat generation at each revolution for a rolling tire over asphalt. The heat fluxes in the system were completely modeled. Finally, the simulation results were compared with the experimental findings.

In 2021, Usmanova et al. [63] presented a decoupled numerical analysis of automotive tires to investigate hysteretic loss and temperature distribution. It must be noted that temperature-dependent material characteristics were utilized in heat generation analysis. For this purpose, the cyclic change in strain energy values was calculated from the three-dimensional deformation analysis. Later, these values were exported as an input parameter for thermal analysis to determine the temperature distribution and thermal heat generation that occurred due to hysteretic loss. Finally, deformation analysis results were validated using the experimental data available and a good agreement was found among the rolling resistance and peak temperature values in the one-way-coupled techniques. Additionally, the results were found to follow closely to the actual tire behavior.

In 2022, Park et al. [64] proposed a new simulation methodology to investigate the temperature distribution and the rolling resistance of a passenger car tire. Moreover, the thermo-mechanical characteristics of the cap ply were considered in this analysis. To obtain the stress-strain–temperature curves, a combination of the tensile test data and the shrinkage test data of Nylon 6.6 cap ply and Hybrid cap ply, were implemented. Then, the obtained curves were imported into an ABAQUS user subroutine. Two different analyses including a deformation analysis and a thermal analysis were iteratively carried out until the stable distribution in the temperature field. Finally, the hysteretic loss, the rolling resistance, and the rolling resistance coefficient were computed. Additionally, all the finite element simulation results were compared to the experimental data, and a reasonable agreement was found between the test data and FE results. In addition, it was found that Nylon 6.6 cap ply and Hybrid cap ply are two main indicators that significantly affect the rolling resistance coefficient of the tire.

In 2023, Ma S. et al. [65] presented a novel approach to estimate the hysteresis loss of rubber compounds from ultra-large off-the-road tires. Based on the cyclic tensile tests that are performed on the tire tread compounds, the hysteresis curves can be derived. Furthermore, the other parameters such as peak stress, residual strain, and hysteresis loss at two different levels of strain (six and eight strain components) were extracted and repeated for 14 rubber samples. Finally, according to the experimental data, a hysteresis loss model considering strain levels, strain rates, and rubber temperatures has been proposed. The model was developed based on the modified and novel strain energy function from the traditional Mooney–Rivlin model.

In the same year, Hyttinen et al. [66] presented the rolling resistance prediction and temperature distribution of a truck tire in a climate wind tunnel. Tire temperatures at the shoulder and tread were calculated in the climate wind tunnel tests, experimentally. The tire temperature distribution was simulated using a thermal tire model with speed-variable thermal inertia. Finally, the tire temperature results from simulations were compared with the measured inner-liner and shoulder temperatures. Overall, a good agreement was shown with the test data. Moreover, rolling resistance was modeled by the superposition of time-temperature parameters. Furthermore, a master curve for determining the rolling resistance and a curve for tire temperature shift was created. Additionally, considering these curves, the final values of rolling resistance and tire temperature were verified using the experimental data. It was found that tire shoulder temperature is a significant indicator for predicting the rolling resistance rather than infrared measurements. As shown in Figure 11, rolling resistance is plotted over tire shoulder temperature at various vehicle speeds (solid line). The speed-stabilized rolling resistance coefficients can be extracted from the measurements (dotted line).

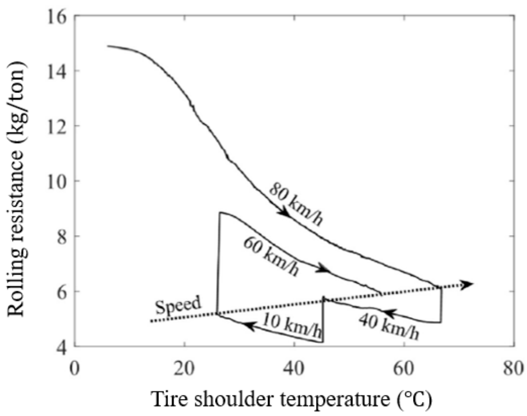


Figure 11. Rolling resistance versus tire shoulder temperature at various speeds [66].

In 2023, Chen et al. [67] presented the theories of dissipated energy and heat transfer in a rolling tire. For this purpose, a thermo-mechanical–abrasive (TMA) coupling analysis technique was implemented. In addition, temperature distribution and abrasion under various operating conditions including different slip angles were demonstrated experimentally. Finally, it was found that sideslip conditions resulted in increasing the friction energy dissipation and wear rate. Moreover, sideslip conditions led to an increase in the peak value of the asymmetric distribution of the temperature field. Additionally, it can be noted that abrasion had a significant effect on the accuracy of the estimated thermomechanical properties. Consequently, a good agreement was found between the predicted temperature and abrasion profile compared with the experimental data.

5.4. Summary Thermal Models

Pneumatic tires exhibit complex thermomechanical behavior that can be modeled as a dynamic nonlinear coupled thermos-viscoelasticity problem, necessitating the consideration of heat generated from internal dissipation and friction. Over the years, various studies have focused on predicting tire temperature and hysteresis losses under different operating conditions. Notable advancements include the establishment of the relationship between heat generation and tire temperature rise, a model linking internal tire temperature to rolling resistance, and subsequent numerical models from researchers that utilized finite element methods (FEM) to analyze temperature distribution and heat generation due to cyclic deformation. Recent developments involve using Mooney–Rivlin models for material behavior and incorporating operational factors affecting tire performance. Studies in 2022 and 2023 further explored the hysteresis loss in rubber compounds, rolling resistance prediction, and the influence of tire design on thermal characteristics, demonstrating improved accuracy and correlations with experimental data. Overall, the evolution of these models and methodologies highlights the intricate interplay between thermal dynamics and mechanical performance in pneumatic tires, emphasizing the importance of temperature in assessing tire efficiency and safety. Some of the limitations of such models are summarized in Table 3 in chronological order.

Table 3. Summary and critical observations of thermal analysis for tires.

Year, Author	Topics	Comments/Critical Analysis
1976, Clark [48]	Tire generated heat	The proposed model for tire heating, while providing an estimation of thermal equilibrium time, lacks validation against experimental data and does not account for real-world complexities like variable road conditions and dynamic heat generation during tire operation.
1978, Wong J.Y., [2]	Internal temperature	A model on experimental measurements that captured the effect of internal tire temperature on the rolling resistance coefficient for pneumatic tires.
1996, MC Allen et al. [49]	Internal temperature	Oversimplifies the deformation process by assuming identical cycles and a frequency-independent phase lag, which may limit the model's accuracy under variable operational conditions or differing material responses.
2010, Li et al. [50]	Thermal distribution	The study lacks a quantitative analysis linking thermal distribution insights to specific recommendations for tire structure or rubber compound enhancements.
2012, Yokota et al. [51]	Internal temperature	While the paper aims to leverage the correlation between temperature distribution and rolling resistance to estimate rolling resistance in real-world conditions, it lacks specific validation data or comparative results to demonstrate the model's accuracy and practical applicability for real driving scenarios.
2022, Cong NT et al. [52]	Hysteresis energy loss	Oversimplifies by assuming linear elasticity for non-rubber components and lacks experimental validation, which may limit the model's accuracy in capturing real-world temperature behavior under diverse conditions.
2004, Lin et al. [53]	Hysteresis energy loss	Although the study effectively utilizes hysteresis energy loss to model heat generation and temperature distribution in a rolling tire, it lacks a detailed comparison of its simulation results with experimental data, limiting confidence in its predictive accuracy under diverse operating conditions.

Table 3. *Cont.*

Year, Author	Topics	Comments/Critical Analysis
2006, Narasimha Rao et al. [54]	Temperature prediction	While the study presents a novel finite element algorithm to extend temperature prediction to non-axisymmetric tires, it lacks comprehensive validation against experimental data, which limits the reliability of its applicability to complex treaded tire geometries.
2012, Li et al. [55]	Ambient temperature	While the study provides a detailed analysis of ambient temperature effects on tire stress, it could strengthen its practical relevance by validating simulation outcomes against experimental field data.
2022, Premarathna et al. [56]	Temperature distribution	While the study effectively combines numerical modeling and design of experiments (DoE) to optimize solid tire performance, it could be improved by including a more thorough sensitivity analysis of the design and operational factors.
1998, Song et al. [57]	Tire internal temperature	The study would benefit from a deeper discussion of the limitations of its assumptions, such as the interpolation of material properties and the focus on specific temperature measurement points.
2012, Li et al. [58]	Temperature distribution	Lacks a comprehensive validation against experimental data, which is essential to establish the reliability of the simulation results for informing practical improvements in tire design and performance.
2013, Cho et al. [59]	Rolling resistance	Lacks a detailed explanation of the assumptions made in neglecting the tire rolling effect and how these assumptions may impact the accuracy of the results.
2015, Behnke et al. [60]	Tire heat generation	While the numerical framework presented offers a comprehensive approach for thermo-mechanical analysis of 3D tire structures, its effectiveness could be further demonstrated discussing potential limitations in the generalizability of the model results beyond the tested conditions.
2019, Wu et al. [61]	Wear and slip angle	The study would benefit from a more comprehensive analysis of the long-term effects of varying operating conditions on wear mechanisms.
2020, Farroni et al. [62]	Tire heating source	Enhance its practical applicability by incorporating testing data to validate the model's predictions and explore how variations in environmental conditions might influence tire performance.
2021, Usmanova et al. [63]	Hysteretic loss,	The study falls short by not fully integrating temperature-dependent material properties throughout the entire modeling process, which could lead to significant discrepancies.
2022, Park et al. [64]	Rolling resistance	Although the proposed simulation methodology demonstrates an innovative approach to predicting temperature distribution and rolling resistance in passenger car tires, its reliance on specific material data for Nylon 6.6 and Hybrid cap plies may limit its applicability to other tire designs or materials.
2023, Ma S. et al. [65]	Hysteresis loss of rubber	While the HLSRT model shows promise in accurately predicting hysteresis loss for rubber compounds in ultra-large OTR tires, the dependence on a narrow range of strain levels, strain rates, and temperatures may limit its applicability to more diverse operating conditions.
2023, Hyttinen et al. [66]	Rolling resistance	While the study effectively correlates tire shoulder temperature with rolling resistance, it may overlook the potential influence of other factors such as road surface variability, tire wear, and environmental conditions on rolling resistance.
2023, Chen et al. [67]	Dissipated energy	The proposed thermo-mechanical–abrasive (TMA) coupling analysis method significantly enhances the accuracy of temperature and abrasion predictions under high-speed sideslip landing conditions; however, the study may benefit from a more comprehensive exploration of the long-term implications of these heating and wear patterns on tire lifespan and performance under varying operational conditions.

6. Tire Material and Constitutive Modeling

In this section, the tire material modeling along with the constitutive modeling of the rubber compounded are described and discussed.

6.1. Tire Tread Material Review

In 1987, Futamura [68] discussed the influence of the modulus of tire cords in stabilizers and body plies on radial automobile tire behavior. For this purpose, polyester, rayon, and aramid materials were used to vary the cord modulus systematically. This did not affect high speed, endurance, or plunger energy. It was found that in the body ply, the aramid cord generated higher rolling resistance than the polyester cord, but no significant effect of cord material in stabilizer plies was observed. However, increasing the cord modulus in the stabilizer ply resulted in a significant increase in the cornering coefficient. Additionally, wear resistance also increased particularly under high severity conditions.

In 2004, Karabi et al. [69] investigated the relaxation time (λ k) and the viscosity index (η k) of a tire tread compound. The rheological characteristics of the sample were identified by a parallel plate rheometer and the rheological material functions, such as complex shear viscosity (η k), elastic shear modulus (G'), and viscous shear modulus (G'') were calculated in the frequency sweep test (FST). A generalized Maxwell model (GMM) over a limited range of frequency and temperature was adopted to calculate λ k and η k. A steady state region for determining the rheological material functions was extracted from time sweep test results at different temperatures and frequencies. A stress sweep test was performed to investigate the linear viscoelastic zone of the compound at a given temperature and frequency. Finally, output parameters of frequency sweep tests were measured including the complex shear viscosity (η^*), elastic shear modulus (G'), and viscous shear modulus (G''), and then the relaxation time was calculated. The results revealed a good correlation between the rheological behavior of the compound and the GMM. Moreover, it was investigated that relaxation time could be described as a decreasing function of both temperature and frequency.

In 2007, Ghosh et al. [70] presented a nonlinear finite element analysis method for a passenger car radial tire varying in constructional details. Tire performance was determined concerning several parameters including belt angle, tread material, apex height, and ply turn-up height. Moreover, the overall constructional behavior of the tire. To study the effect of individual parameters and their interactions, single and composite design matrix was considered, and the simulation results were verified with them. Finally, a good correlation was found between theory and the experimental test data of tire performance. Additionally, it must be noted that the results of tire stiffness and handling parameters closely followed the predicted trend.

In 2010, Yang et al. [71] proposed a simplified and effective approach to generate tire materials data adopted in tire finite element analysis (FEA). For this purpose, all tire samples were extracted from an actual tire product based on special dimensions for the test specimen and test standards and test criteria for the reinforcement cords in the tire structure. The hyperelastic and viscoelastic material properties of the tire rubber compound were calculated using a simple uni-axial tension test. A Dynamic mechanical analysis (DMA) test was performed in the low-frequency domain to determine the reinforcement elastic modulus. The nonlinear material characteristics were simulated using Abaqus/CAE software. Furthermore, an appropriate rubber strain energy model that evaluates the experimental data correctly is selected for the measured data. It must be noted that to design tire geometry and structure, an image processing technique was implemented. Finally, the tire vertical stiffness and footprint area from the FE simulations were compared with the experimental data, and a good agreement was found between the obtained FE results and the experiment. Hence, various FEAs of tire performance can be carried out for the future application of tire materials testing, also preliminary use of the proposed model was introduced in the tire noise, vibration, and harshness (NVH) application.

In 2013, Ghoreishy et al. [72] presented an experimental and analytical analysis of the tire tread compound reinforced by silica and carbon black to investigate the material properties of the tire rubber compound. For this purpose, a tensile load was applied to a dumbbell-shaped specimen and three rubber-strip specimens with different widths. Heat buildup tests were also carried out, and the temperature distribution was calculated.

In addition, traditional hyperelastic models (Marlow and Yeoh) were implemented to define the time-independent behavior of the rubber. Moreover, the viscosity of rubber was described with two linear (Prony series) and nonlinear [Bergstrom–Boyce (BB)] models. It was shown that ignoring the nonlinear viscosity of the rubber compound resulted in a noticeable error in the determination of mechanical deformation. It was found that the linear viscoelastic model could not be capable of investigating the accurate behavior under large strain, significantly for wider samples. In contrast, the combination of the Yeoh hyperelastic model and the BB equation illustrates mechanical behavior at low to medium strains, as well as large strains. It was found that both linear and nonlinear hyper viscoelastic models precisely predicted rubber hysteresis, and this could be used to determine tire rolling resistance.

In 2014, Gosh et al. [73] performed a finite element analysis of three different tires 10.00R20, 295/80R22.5, and 315/80R22.5 with a nanocomposite tread base compound. Rolling resistance values were measured for these three tires under an experimental test carried out by a pulley wheel machine. Moreover, a temperature equation was developed for determining tire temperature distribution. The dissipated energy was evaluated by implementing the product of elastic strain energy and loss tangent of materials through post-processing using rolling resistance. Moreover, the loss tangent of materials versus strain was examined at two various temperatures using experimental tests. A good correlation between the predicted values of the rolling resistance and the experimental result was found (90%) in all cases. It was proved that according to finite element simulation, nanocomposite-based tread compounds have a significant effect on lowering rolling resistance compared with general carbon black tread compounds or silica tread compounds.

In 2016, Maghami, S. [74] presented various dynamic mechanical measuring techniques. The study aimed to achieve a reliable viscoelastic master curve and describe the interpretations of the outputs. As silica technology in passenger car tire tread applications was introduced, the filler–polymer interactions were considered a key factor. The goal of this thesis was to decrease the hydrophobic properties of the hydrocarbon polymers to increase compatibility with hydrophilic silica. The dynamic mechanical measurements on the tread compound were performed and it was found that adding modified SBRs had a significant effect on the lowering rolling resistance of the tire tread.

In 2018, Aldhufairi [75] reported that the rolling resistance can be responsible for 20–30% of the total vehicle fuel consumption. It was found that the lower rolling resistance resulted in less fuel consumption (i.e., CO_2, NO_x, and hydrocarbon emissions). It was determined that the main source of rolling resistance corresponds to the tire deformation and structural deflection in the tire which is mainly described as hysteresis damping. The hysteresis damping can be accounted for 80–95% of the total rolling resistance. The research was particularly focused on the initial source for the rolling resistance named mechanical hysteresis damping. The reviews were based on three points of view including the structural lay-up, the dimensional features, and the material compounds of the tire.

In 2019, Korunović et al. [76] presented a material modeling simulation for tire reinforcements. Furthermore, various material definitions including linear, Yeoh, and Marlow, were analyzed for tire rubber compounds. A real tire model was defined as a general system and the corresponding results were discussed in terms of accuracy, computational efficiency, and sophisticated characteristics identification. Consequently, the major benefits of the use of nonlinear material models, particularly of the Marlow model, were found. Finally, the finite element results especially for the cords model were compared with an actual tire model.

In 2020, Mousavi, H. et al. [77] presented a model to investigate the effect of various tread compounds on the tire-ice interaction. Several experimental analyses were carried out on three identical tires from the same company. The only difference between the tires was the definition of rubber material. Two approaches were performed in this study. For the first approach, several tests were carried out for the selected tires at the Terramechanics, Multibody, and Vehicle Systems (TMVS) laboratory at Virginia Tech University to compare their behavior experimentally. For the second approach, a tire-ice model was implemented to calculate the height of the water film generated at the tire footprint. Finally, it was found that an increase in the height of the water film resulted in a decrease in the friction coefficient, which is one of the most vital parameters for determining tire performance. The performance of the three tires was observed as well. A good correlation was shown by simulation and the experimental data. The results of this study identified the sensitivity of the magnitude of the tractive force in terms of parameters such as tire temperature, normal load, etc. Furthermore, the results showed that traction is directly affected by Young's modulus with the highest traction corresponding to the tire with the lowest Young's modulus.

In 2022, Wang et al. [78] established a non-contact test rig to determine the rolling mechanical properties of the rubber tread block. Furthermore, the distribution characteristics of the rubber tread block were extracted by the improved DIC techniques for a rolling tire. To present the nonlinear viscoelasticity of rubber materials, a Fourier series model was introduced. Finally, the energy dissipation, rolling resistance, and rolling resistance coefficient of the rubber tread block were computed.

In 2022, Shenvi et al. [79] simulated the tire–ice interaction using sixteen tires which were completely the same in all aspects of design and structure except for the tread rubber compound. The simulations were carried out using the in-house developed ATIIM, ATIIM 2.0, and modified versions of three simplified classical models. A comparison was performed through the simulations to achieve the benefits and drawbacks of the proposed approach.

In 2023, Yoon et al. [80] provided a finite element analysis to investigate the effect of tread composite material models on the numerical prediction of rolling tire noises. For this purpose, a coupled acoustic-structural finite element analysis was performed. A viscoelastic master curve was presented for the frequency-dependent damping in the vibrations that occurred in the tire structure. It was investigated that the tread composites with 230.4% and 1428% larger storage (E′) and loss moduli (E″), respectively, provided 1.16–9.79% louder structure-borne noises. In addition, the elastic modulus (E) of the tread composites correlated with tire noise, indicating that lower viscoelastic and elastic moduli were advantageous for low-noise tires. In the context of autonomous electric vehicles and urban air mobility, this study could help understand how composite material properties, particularly viscoelasticity, can be used to control structural noise and vibration.

6.2. Material Constitutive Modeling

In 1988, Kern et al. [81] presented the development of tread rubber considering the significance of macrostructure and the glass transition temperature of the polymer for providing the best balance of the characteristics. Basic differences in structure and properties between emulsion and solution SBR were demonstrated. Additionally, the influence of micro and macrostructure of solution rubbers on the performance characteristics of tread materials was investigated based on the tire performance data.

Recently in 2024, Fathi et al. [82] explored how different rubber material models impact the performance of truck tires, particularly focusing on rolling resistance and cornering forces. Using a finite element analysis (FEA) model of a truck tire, the authors analyzed four material models: Mooney–Rivlin, visco-Mooney–Rivlin, linear viscoelastic, and nonlinear viscoelastic. The authors examined how these materials behave under various operating conditions, including different speeds and slip angles, and assessed their effects on tire–road interaction characteristics. The study found that the material model used can significantly affect the tire's lateral grip and rolling resistance, with the nonlinear viscoelastic model offering the lowest rolling resistance but reduced lateral grip. Overall, the results provide insights into optimizing tire tread materials for improved performance and efficiency. A weakness of this research is its reliance on simulations without incorporating real-world testing data for validation under all scenarios, particularly for the nonlinear viscoelastic model. This limits the study's applicability to practical situations where factors like temperature and road conditions may play a significant role in tire behavior.

In the same year, Fathi et al. [83] investigated the interaction between a passenger car tire and a rigid road surface using finite element analysis (FEA) in the Abaqus software. The study focused on various complex material models, including linear visco-hyperelastic, parallel rheological framework, and Mullins effect, to predict rolling resistance and wheel force. The analysis compared both steady-state rolling and transient dynamic simulations to highlight the effects of material selection on performance metrics such as tire footprint, contact pressure distribution, and rolling resistance. The study revealed that incorporating complex models like the Mullins effect and nonlinear viscoelasticity enhanced the accuracy of predictions regarding rolling resistance, though it complicated the numerical process. The findings showed that more advanced material models offered better energy dissipation tracking and improved tire performance simulations. The limited amount of experimental testing restricted the generalizability of the findings, particularly when assessing the applicability of the advanced models under varied temperatures or real-world road conditions.

In the same year, Ly et al. [84] compared the performance of a mixed service drive truck tire using two different rubber material models: the legacy Mooney–Rivlin (MR) and the more advanced Ogden hyperelastic material model. The study conducted finite element analysis (FEA) in a virtual environment to assess the rolling resistance coefficient (RRC) of the tire under varying inflation pressures and vertical loads. The simulations showed that the differences in RRC between the two models were minimal, with the largest observed difference being 7.335%. The research concluded that the Ogden model, designed for improved thermal and wear simulations, was comparable to the MR model for typical tire–road interaction scenarios, validating its use in future thermal and wear applications. A key weakness of the paper was its reliance on simulation results without experimental validation for the newer Ogden material model. Additionally, while the study highlighted differences between the material models, it did not explore other key factors like temperature or wear performance, limiting its applicability to broader real-world conditions.

6.3. Summary of Tire Material Modeling

This section summarized the development of tire material modeling and the constitutive modeling of rubber compounds, detailing key research advancements from 1987 to 2024. It began with the examination of the impact of tire cord modulus on performance, which revealed findings on rolling resistance and cornering coefficients. Subsequent studies explored rheological characteristics, nonlinear finite element analysis, and hyperelastic properties while investigating various tire construction parameters and their effects on performance metrics such as stiffness and rolling resistance. Later research focused on the significance of material composition, such as the comparison between silica and carbon black in rubber, and the influence of viscoelastic properties on energy dissipation and noise generation. More recent studies, utilized advanced finite element analysis to assess the effects of complex material models on tire performance, revealing significant impacts on rolling resistance and grip under varying conditions, though many studies lacked real-world validation. Overall, these investigations contributed to optimizing tire materials for enhanced efficiency and performance. The summary and critical review of the most significant papers are presented in Table 4 in chronological order.

Table 4. Summary and critical observations of tire material modeling.

Year, Author	Topic	Comments/Critical Analysis
1987, Futamura [68]	Tread Material	The discussion lacks depth in exploring the underlying mechanisms that contribute to the observed performance differences, particularly regarding the absence of effects on high speed, and plunger energy.
2004, Karabi et al. [69]	Relaxation time	The study fails to provide a comprehensive analysis of the potential limitations of the generalized Maxwell model in capturing the full complexity of the tire tread compound's rheological behavior.
2007, Ghosh et al. [70]	Tire stiffness	The study inadequately addresses the inherent trade-offs and complexities involved in tire design optimization, particularly how conflicting property requirements might influence the overall performance.
2010, Yang et al. [71]	Rubber strain energy	The study fails to sufficiently acknowledge the potential inaccuracies and limitations of using simplified test procedures for generating tire material properties.
2013, Ghoreishy et al. [72]	Rubber material	Investigation of the material properties of the tire rubber compound with Bergstrom–Boyce (BB) (BB) models
2014, Gosh et al. [73]	Rubber material	The study oversimplifies the complex interactions between tire materials and environmental factors, potentially limiting the generalizability of its findings across different tire designs.
2016, Maghami, S., [74]	Tire material	While the thesis addresses the critical balance between rolling resistance and wet skid performance in tire design, it may oversimplify the complex interplay of material properties and environmental factors by focusing primarily on viscoelastic properties without adequately considering the broader implications of tire performance under varied real-world conditions.
2018, Aldhufairi [75]	Hysteresis damping	Lack of a comprehensive examination of the trade-offs involved in reducing rolling resistance, such as potential compromises in safety and performance, which are crucial for holistic tire development.
2019, Korunović et al. [76]	Tire cords	Although the paper provides a thorough analysis of various material models for tire reinforcements, it fails to address the potential limitations and challenges of implementing these models in real-world scenarios.
2020, Mousavi, H. et al. [77]	Rubber material	While the study effectively demonstrates the impact of rubber compound properties on tire performance in icy conditions through both experimental and simulation approaches, it lacks a comprehensive exploration of the broader implications of these findings on tire design and selection for varying environmental conditions, which limits its practical applicability.
2022, Wang et al. [78]	Rubber material	While the study introduces an innovative non-contact method for calculating the rolling resistance of rubber materials, it inadequately addresses potential variations in tire performance across different environmental conditions and tire designs.
2022, Shenvi et al. [79]	Rubber material	Although the study highlights the necessity of accurate simulation methods for tire performance on ice, it lacks a comprehensive evaluation of how the different tread rubber compounds specifically influence traction and handling under various ice conditions.
2023, Yoon et al. [80]	Composite material	While the study effectively illustrates the correlation between tread composite material properties and rolling tire noise, it fails to adequately address potential trade-offs between noise reduction and other critical tire performance characteristics, such as durability and traction.

Table 4. *Cont.*

Year, Author	Topic	Comments/Critical Analysis
1988, Kern et al. [81]	Rubber temperature	Basic differences in structure and properties between emulsion and solution SBR were demonstrated. The influence of the micro and macrostructure of solution rubbers on the performance characteristics of tread materials was investigated based on the tire performance data.
2024, Fathi et al. [82]	Material analysis	While the research effectively utilizes finite element analysis to explore the sensitivity of truck tire rubber compounds on tire–road interaction, it fails to provide specific numerical results or comparisons that would illustrate the practical implications of the different material models on performance metrics.
2024, Fathi et al. [83]	Rubber material	While the research demonstrates a comprehensive approach to studying tire–road interaction using advanced finite element modeling and a variety of material models, it lacks clarity regarding the practical implications of the results, particularly in how the different modeling approaches directly influence the rolling resistance and footprint characteristics of the tire under various loading conditions.
2024, Ly et al. [84]	Rubber material	Although the paper provides a thorough investigation into the tire–road interaction using both Mooney–Rivlin and Ogden hyperelastic material definitions, it lacks a detailed discussion on the implications of the minimal differences in rolling resistance results, which raises questions about the practical significance of adopting the more complex Ogden model for wear and thermal applications in tire design.

7. Conclusions

The paper provides an extensive overview of various modeling approaches and experimental methods for analyzing tire–road interactions, highlighting several commonalities, differences, and recurring themes. A shared focus across studies is the reliance on finite element analysis (FEA) and semi-empirical models as primary tools for predicting tire performance metrics such as rolling resistance, cornering stiffness, and wear. These models share the common goal of enhancing accuracy in simulated tire dynamics, with a particular emphasis on capturing tire deformation, stress distribution, and friction under diverse conditions. However, notable differences arise in the scope and computational complexity of each approach, while FEA is widely applied for its high precision in complex interactions, it is often resource-intensive, whereas semi-empirical models are favored for their efficiency and ease of application but may lack the depth required for intricate scenarios. Another recurring theme is the impact of environmental factors, such as temperature, load, and surface conditions, on tire behavior, which is consistently addressed across studies yet with varying degrees of emphasis. FEA studies often incorporate environmental variables in detailed simulations, while empirical models typically approximate these factors due to computational limitations.

Furthermore, many tire models have been presented to define an approximate estimation of actual friction for contact-friction algorithms of tire–terrain interactions. In addition, regardless of tire finite element modeling, several tire models including the rigid ring model and brush model were introduced by many researchers to generate and represent the tire structure for describing some of the friction problems.

Additionally, the effect of temperature and rolling resistance was studied as a coupled method such as thermomechanical analysis of rolling tires. In addition, the temperature distribution field of tires was studied according to the thermal energy dissipation of different tire structures including reinforcement belts, carcasses, and particularly tread patterns.

It must be noted that several material models have been introduced for rubber to describe tire tread compounds. Many hyper-viscoelastic material models were implemented for tire tread compounds to generate an actual behavior of tire performance under various conditions and to describe the effect of hysteresis loss and energy dissipations of the viscoelasticity of tire compounds.

It was found that among the recent research on tires, there is a major drawback on winter tires particularly the study of the effect of material definition on winter tire performance. The performance analysis of winter tires under extreme weather conditions is especially vital in countries such as the United States and Canada. Therefore, future research work will focus on the aspects of material effect on tire performance in severe climate conditions and will potentially help to improve tire performance including traction and braking.

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