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Review

An Objective Classification Scheme for Solar-System Bodies Based on Surface Gravity

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Abstract: We introduce succinct and objective definitions of the various classes of objects in the solar system. Unlike the formal definitions adopted by the International Astronomical Union in 2006, group separation is obtained from measured physical properties of the objects. Thus, this classification scheme does not rely on orbital/environmental factors that are subject to debate—the physical parameters are intrinsic properties of the objects themselves. Surface gravity g is the property that single-handedly differentiates (a) planets from all other objects (and it leaves no room for questioning the demotion of Pluto), and (b) the six largest ($g > 1$ m s^{-2}) of the large satellites from dwarf planets. Large satellites are separated from small satellites by their sizes and masses/densities, which may serve as higher-order qualifiers for class membership. Size considerations are also sufficient for the classification of (i) main-belt asteroids (except possibly Ceres) as small solar-system bodies similar in physical properties to the small satellites; and (ii) a group of large Kuiper-belt objects as dwarf planets similar in physical properties to the large (but not the largest) satellites in our solar system. The selection criteria are simple and clear and reinforce the argument that body shape and environmental factors need not be considered in stipulating class membership of solar as well as extrasolar bodies.

Keywords: asteroids; gravitation; minor planets; planets; satellites; solar system

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1. Introduction

We present some intriguing results of wide interest to the community concerning a meta-analysis of the physical properties of classes of objects observed in our present-day solar system—a strictly scientific endeavor as opposed to subjective panel debates and voting resolutions. We begin with a brief historical overview of the ancient definition of planets, including references to the geocentric and heliocentric models, and the modern definitions of planets and dwarf planets adopted by the International Astronomical Union (IAU) in 2006. This overview provides the context that we need to furnish in order to expose and rectify a contemporary technical lapse on the subject of defining planets that has heretofore gone unnoticed.

Several other researchers have worked on this topic in the past 20 years (e.g., Refs. [1–9]), but their results are subject to debate, and they have seemingly given rise to some confrontations and disagreements. Our results do not belong to the same category; the divisions of solar system objects according to their physical properties that are presented below are not questionable as they do not use body shapes or orbital/environmental characteristics; the various separatrices between classes of objects are absolutely clear-cut. In essence, our methodology uses the same taxonomical principles put forth recently by McIntyre et al. [8] and Dick [9], who stressed the importance of analyzing and understanding in depth the intrinsic physical properties of systems to be assigned to classes, types, subtypes, etc.

1.1. Historical Overview

In antiquity, people watched some lights wandering in the night sky against a background of other lights that appeared to be standing still. They agreed to describe the wandering lights by the Greek term "planet" (meaning "drifter"). Ancient scientists knew nothing about the origin or the properties of planets, except for a *single orbital characteristic* —their occasional apparent retrograde motion [10,11]—which they failed to explain despite intense efforts spanning several centuries. This failure culminated in the Roman era with the Ptolemaic model of the then-known universe, a "*puzzling collection of superficial analogies and unfounded assertions*" [12], described by Ptolemy in his then influential work *Almagest*.[1]

Ptolemy was a firm believer in the geocentric theory (which had been previously supported by such great minds as Aristotle and Plato), and he had the power to enforce its adoption by the largest scientific community of his time, the Alexandrian School. More than three centuries before Ptolemy, a faint glimmer of hope for rapid progress, the heliocentric model of Aristarchos of Samos, had been expeditiously discarded by all scientists of the Alexandrian School [13–15]. Only the great Archimedes of Syracuse, who was about 20 years younger than Aristarchos, found the heliocentric model plausible, and he actually used it to estimate the size of the Universe in *The Sand Reckoner*[2] [16].

The point to keep in mind is that the great minds of antiquity did not know anything about the physical properties of planets (with the notable exception of Aristarchos of Samos whose estimates were not accurate), and Ptolemy's rise to prominence in Roman times made things worse. Lacking additional information, the first scientists had no choice but to use the observed orbital properties of planets in their definitions and descriptions. On the other hand, we, the children of the twentieth century, have no excuse for doing the same since we do not face such a categorical dearth of information about the various objects residing in our solar system.

At later times, the definition was improved, based on the realization that the few known large planets do not really wander, but they revolve around the sun in nearly coplanar and nearly circular orbits. This superficial description based again on a *single orbital characteristic* was finally questioned by the IAU, when the organization recognized the need for a new definition of planets that would reflect much of the currently available scientific knowledge.[3] Related resolutions 5A and 6A were passed by the 2006 General Assembly[4] after considerable debates, alternative proposals, and quite a few substantial rewrites.[5]

In retrospect, no one in the community seemed to realize an insidious technical lapse that could potentially undermine the effort and diminish the value of the achievement: the ancient scientists had absolutely no information about the *physical properties of objects* in the solar system, so they had to work with the only tangible fact known at each time, the orbits of the planets. In stark contrast, we presently have "tons" of accurate information about these objects, their physical characteristics and their orbits,[6] so we would be remiss if we kept clinging solely to orbital characteristics before we had at least examined and compared the known physical properties of these objects.

Sadly, this is exactly what transpired. Two of the three legs in the new IAU definition rely on observations of orbits, and only one leg refers to an unassuming physical property, the nearly spherical shape of planets (Note 4). A spherical or spheroidal shape results from hydrostatic (or mechanical stress) equilibrium in the presence of sufficiently strong self-gravity, such as that of planets and stars; on the other hand, there also exist quite a few small moons and small Kuiper-belt objects (KBOs) that appear (at modest resolutions) to be fairly round, or at least not too irregular, presumably because their self-gravitational forces are strong enough for their small sizes. Some examples are Saturn's pumice-moon Hyperion[7,8] with mean diameter $D = 270$ km; the KBO binary components Logos and Zoe[9] with diameters $D = 77$ and 66 km, respectively; and Jupiter's small moon Adrastea[10,11] with mean diameter $D = 16.5$ km.

In general, many astrophysical systems, especially those that rotate rapidly, reach equilibrium states that are spheroidal or triaxial in shape [19]. On the other hand, a purely physical property, such as mean surface gravity g, can be determined on average for all objects irrespective of shape or internal structure and it is therefore a superior diagnostic tool with which to assess the strength of self-gravity [8]. This elucidation should be contrasted with the strategy followed in Refs. [2,20,21], where a round shape was used as a primary qualifier of "sufficiently strongly" self-gravitating objects.

1.2. A Serendipitous Discovery

In our work, we were meta-analyzing solar-system data[12] with no regard to the IAU definitions of planets and dwarf planets. We were mostly interested in comparing planetary and satellite surface densities $\sigma \equiv M/(4\pi R^2)$, where M is the mass and R is the mean radius of a body. That was part of a multi-year effort to investigate the surface densities of objects and systems at various astrophysical scales [22–24]. In the solar system, we normalized the σ-values to the corresponding Earth value of $\sigma_\oplus = 1.169 \times 10^{10}$ kg m^{-2}.

The resulting dimensionless ratios $\sigma/\sigma_\oplus$ are equal to the ratios of surface gravities $g/g_\oplus$ since $g \propto \sigma$ according to Newton's gravitational law; the proportionality constant is the Gaussian factor $4\pi G$, where G is the Newtonian gravitational constant, assumed to be the same everywhere in the solar system. In what follows, we will use the better-known physical quantity

$$g = \frac{GM}{R^2}\,, \tag{1}$$

which we recast in the convenient normalized form

$$g = g_\oplus \left(\frac{M/M_\oplus}{A/A_\oplus} \right), \tag{2}$$

because mass M and surface area A are typically tabulated quantities for most objects in the solar system. Here, quantities with the subscript $\oplus$ denote the corresponding values for the Earth. In the few cases in which A-values were not tabulated, we obtained their mean values from the equation $A = 4\pi R^2$, using the estimated mean radii R.

We realized that there exist large prominent gaps in the distributions of several physical properties of various objects in the solar system. These gaps separate and segregate various known classes of objects, in which case there can be no subjective opinion concerning membership in each group. So, we have no need to create new groups or new names or introduce exceptions in this physically motivated classification scheme; we can simply use the obvious gaps in physical quantities to define succinctly the existing groups. During this procedure, solar-system objects do not change their current group memberships—except in an alternative scenario in which asteroid Ceres and KBO dwarf candidates Orcus, Salacia, and 2002 XV$_{93}$ are placed in the class of small solar-system bodies (see Section 4.3 below).

In the end, we find no reason to hold onto the vacillating definitions of the past (except for satellites; see Section 2) because the two most important physical properties (surface gravities and mean diameters) that we employ as qualifiers of classes are capable of defining all of the known groups of solar-system objects physically and, to a very large extent, objectively. In this way, clarity [9] and simplicity in the classification scheme are restored and, by Occam's razor [25], the older definitions are disfavored without prejudice.

1.3. Outline

The remaining sections are organized as follows. In Section 2, we list 60 solar-system objects that we consider in our study, and we show that their volume densities, despite being fundamental in nature, are inadequate for classifying any and all groups. In Sections 3–5, we present a combined analysis of the distributions of surface gravities, mean diameters, and masses, respectively. The simple physical criteria/definitions that we have alluded to above are all formulated in these sections and they are subjected to a preliminary "extrasolar" test by analyzing 19 exoplanets with extreme and/or desirable properties (Sections 3.2 and 5). In Section 6, we discuss the lessons learned from our meta-analysis, we briefly report on another sample of 103 exoplanets, and we summarize our conclusions.

2. List of Objects, Inadequacy of Volume Densities, and Definition of Satellites

2.1. Volume Densities

A fundamental physical quantity of all objects considered in this work—the mean volume density ρ—turns out to be inadequate as a primary or secondary qualifier of class membership. Figure 1 shows that all types of solar-system objects commingle with one another, and no segregation can be deduced for any of the known groups.

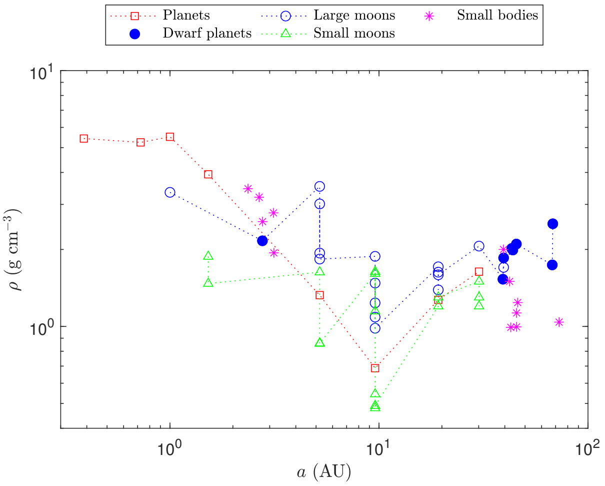

Figure 1. Volume densities ρ versus semimajor axes a of the orbits around the sun. Adopted units are traditionally used in solar-system astronomy. The color scheme is specified in the legend. Objects plotted at the same mean distance from the sun are arranged downward in order of decreasing surface gravity. The dotted line segments have no physical meaning; they aid with the identification of patterns, if any, for each class. For the distinction between small and large moons, see Section 4.2 below.

If there is a trend in Figure 1, it is the V-shape of the radial distributions $\rho(a)$ of planets and satellites with vertices at the location of Saturn, the only planet with $\rho < 1$ g cm^{-3}. Large dwarf planets are filling the right side of the V-shape of the major planets. It is known that lower densities reveal the presence of significant amounts of icy material and/or porosity in the interiors of such objects. Perhaps, large dense dwarfs can be differentiated from small light dwarfs by volume density [26]. This issue will arise again below, where low density may be used as a tertiary qualifier of smallness in the effort to separate objectively KBOs into dwarf planets and small solar-system bodies.

The Saturnian system of low-density satellites (several of them below the density of water) is also discernible in this plot at the semimajor axis $a = 9.58$ AU.

2.2. List of Solar-System Objects

We have obtained solar-system data from the NASA archives and related websites (Notes 6 and 12). In Figure 1 and all subsequent figures, we plot object properties in order of increasing mean distance from the sun (semimajor axis a), and we arrange those objects lying at the same a-value downward in order of decreasing surface gravity (see also Figure 2 below). All sizable objects were included, and some of the very small ones were also added for comparison purposes. The list of the 60 highlighted objects, in the order shown in Figure 2, is as follows:

(a) eight planets (red squares): Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune.
(b) eight dwarf planets (filled blue circles): Ceres, Orcus, Pluto, Haumea, Quaoar, Makemake, Gonggong, Eris.
(c) 16 large satellites (open blue circles): Earth's Moon; Jupiter's Io, Ganymede, Europa, Callisto (the four Galilean satellites); Saturn's Titan, Rhea, Dione, Iapetus, Tethys; Uranus' Titania, Oberon, Umbriel, Ariel; Neptune's Triton; Pluto's Charon.

(d) 16 small satellites (green triangles): MARS’ Phobos, Deimos; JUPITER’S Himalia, Amalthea, Adrastea;[13] SATURN’S Enceladus, Mimas, Phoebe, Hyperion, Prometheus, Pandora; URANUS’ Miranda, Puck; NEPTUNE’S Nereid, Proteus, Larissa.

(e) 12 small solar-system bodies (magenta asterisks): ASTEROIDS Vesta, Juno, Pallas, Themis, Hygiea;[14] KUIPER BELT OBJECTS 2002 XV_{93},[15] Salacia, Varuna,[16] Logos, Zoe, Varda,[17] 2007 UK_{126}.[18]

2.3. Satellites

A distinction between satellites and all other objects is made according to the existing definition, which we also adopt:

Definition 1 (*Satellites*)**.** *Satellites (a.k.a. moons) orbit around larger nearby bodies other than the sun.*

In doing so, we do not subscribe to the proposal that the location of the center of mass ought to be taken into account (as this was communicated to the public in Ref. [32]). We give two reasons: (a) We do not see the need to add yet another orbital characteristic to any of the definitions. (b) The center of mass may change with time as a binary system continues to evolve via external tidal interactions.

A distinction between large and small satellites is made on physical grounds in Section 4, where we also determine a subclass containing the (six) largest of the large satellites with $g > 1$ m s^{-2} as a secondary qualifier. For obvious reasons (we consider only physical properties), we make no distinction between regular and irregular satellites and we have included both types in the list of solar-system objects.

3. The Surface Gravities of Planets, Dwarf Planets, and Satellites

3.1. Planets and Satellites in Our Solar System

Figure 2 shows a surprising result. When surface gravities g from Equation (2) are plotted versus semimajor axes a from the sun, a large gap is evident between $g = 1.8$ m s^{-2} and 3.6 m s^{-2} (top/bottom = 2.0) that separates the eight major planets from all other objects. The lower boundary is specified by the Jovian moon Io ($a = 5.20$ AU), not by a dwarf planet. The upper boundary is specified by planets Mercury ($a = 0.387$ AU) and Mars ($a = 1.524$ AU).

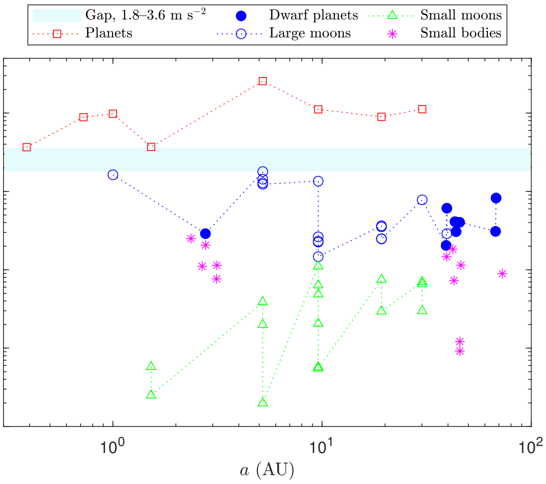

Figure 2. As in Figure 1, but for surface gravities g versus semimajor axes a. A large gap between $g = 1.8$ m s^{-2} and 3.6 m s^{-2} clearly separates the 8 major planets from all other objects. The lower boundary of the gap is set by Jupiter’s moon Io, not by a dwarf planet. Dwarf planets commingle with large satellites in g-space.

Large satellites have comparable surface gravities to dwarf planets, and the six largest satellites (our Moon, Jupiter's Galilean satellites, and Saturn's Titan) have significantly higher g-values than all of the dwarf planets (Section 4.2). The gap between planets and dwarf planets in Figure 2 is even larger ($g = 0.82$–3.6 m s^{-2}, top/bottom = 4.4). In this case, the lower boundary is set by dwarf planet Eris ($a = 67.9$ AU). We are not going to use this larger gap in the definition of planets to avoid making a subjective distinction between planets and dwarf planets.

Assuming that Orcus will be accepted as a dwarf planet by the IAU (which is widely expected), then Orcus will set a lower limit of $g = 0.20$ m s^{-2} for the known dwarf planets. This limit creates an issue: asteroids Vesta and Pallas have larger surface gravities ($g = 0.25$ and 0.21 m s^{-2}, respectively) but are not considered dwarf planets, probably because they are much smaller than Ceres. That, of course, was a subjective choice since the proposed "dominance of an object in its neighborhood" [3,6,7] was not included in any of the IAU definitions. In our case, this issue is resolved conclusively in Sections 4 and 5, where, based on physical properties, we show that no asteroid (besides possibly Ceres) belongs to the dwarf class. The same holds true for many mid-sized KBOs (small solar-system bodies), which the current definitions are unable to classify judiciously.

The separation seen in Figure 2 between small and large satellites is documented in Section 4. Here, it suffices to say that small satellites have weak surface gravities ($g < 0.13$ m s^{-2}) and do not create any additional problems for the classification scheme. Furthermore, the six largest satellites ($g > 1$ m s^{-2}) are also obviously distinct from the other large satellites whose surface gravities are very much comparable to those of the dwarf planets; it is uncanny, but both lists extend from $g = 0.2$ m s^{-2} (Orcus, Umbriel) to $g = 0.8$ m s^{-2} (Eris, Triton).

With the information presented so far, we can split the Io–Mercury gap in the middle to obtain a mean critical g-value, viz.

$$g_{\rm crit} = 2.7 \text{ m s}^{-2}\,, \tag{3}$$

and use it to formulate a succinct definition of planets:

Definition 2 (*Planets*)**.** *Planets are objects in our solar system, other than the sun, with mean surface gravities* $g > g_{\rm crit}$.

Obviously, there is no need to include here any orbital characteristics or any additional physical properties of the objects involved. In our solar system, there are no marginal objects to within ±0.9 m s^{-2} of the critical value, and we doubt that any new objects will ever challenge the threshold given by Equation (3). For instance, we find that a hypothetical Planet Nine [33] would have a minimum surface gravity of $g = 3$ m s^{-2} for a minimum mass of $5M_\oplus$ and a maximum radius of $4R_\oplus$ and a maximum g-value comparable to that of Jupiter at the opposite extreme of $10M_\oplus$ and $2R_\oplus$.

3.2. Brief Digression to Exoplanets

We hope that Definition 2 will prove useful in the classification of exoplanets and rogue planets as well. A preliminary investigation of 19 exoplanets exhibiting extreme and/or desirable properties (Table 1) is encouraging, to say the least. The results reinforce our argument that there is no need to consider binarity, size, volume density, semimajor axis, or any environmental[19] characteristics in classifying planets.

The gemstone of this table is, of course, the miniature compact planetary system of TRAPPIST-1 with seven rocky objects comparable to or smaller than the Earth, orbiting within the inner 0.062 AU from the star in a pristine resonant ($\frac{1}{8}$:$\frac{1}{5}$:$\frac{1}{3}$:$\frac{1}{2}$:$\frac{3}{4}$:1:$\frac{3}{2}$) configuration. All seven objects qualify as planets, and none of them land in the g-gap shown in Figure 2 for our solar system. In fact, their surface gravities are all larger than those of Mercury and Mars by factors of $\sim$1.3–2.5.

Table 1. Extreme and/or Desirable (E/D) Properties and Classification of Selected Exoplanets.

Object Name	E/D Properties	Ref.	a (AU)	ρ (g cm^{-3})	M (kg)	D (km)	g (m s^{-2})	Classification (This Paper)
TRAPPIST-1 b	(a)	(1)	0.0115	3.99	6.09×10^{24}	1.43×10^{4}	7.97	P
TRAPPIST-1 c	(a)	(1)	0.0158	4.79	6.93×10^{24}	1.40×10^{4}	9.39	P
TRAPPIST-1 d	(a)	(1)	0.0223	3.36	1.77×10^{24}	1.00×10^{4}	4.71	P
TRAPPIST-1 e	(a)	(1)	0.0293	5.63	4.61×10^{24}	1.16×10^{4}	9.13	P
TRAPPIST-1 f	(a)	(1)	0.0370	4.56	5.58×10^{24}	1.33×10^{4}	8.46	P
TRAPPIST-1 g	(a)	(1)	0.0469	4.16	6.87×10^{24}	1.47×10^{4}	8.52	P
TRAPPIST-1 h	(a)	(1)	0.0619	3.95	1.98×10^{24}	9.86×10^{3}	5.44	P
LHS 1140 b	(a)	(2)	0.0875	12.5	3.97×10^{25}	1.82×10^{4}	31.9	P
Kepler-51 b	(b)	(3)	0.247	0.064	2.20×10^{25}	8.79×10^{4}	0.76	DP
Kepler-51 c	(b)	(3)	0.377	0.034	2.65×10^{25}	1.15×10^{5}	0.54	DP
Kepler-51 d	(b)	(3)	0.500	0.048	3.40×10^{25}	1.21×10^{5}	0.62	DP
Kepler-47 b	(c)	(4)	0.296	1.67	5.03×10^{25}	3.86×10^{4}	9.01	P
Kepler-176 b	(d)	(5)	0.05723	21.3	5.49×10^{25}	1.70×10^{4}	50.7	P
Kepler-176 c	(d)	(5)	0.1011	2.03	2.68×10^{25}	2.93×10^{4}	8.32	P
Kepler-176 d	(d)	(5)	0.1615	1.77	2.77×10^{25}	3.10×10^{4}	7.68	P
Kepler-176 e	(d)	(5)	0.2552	0.444	1.22×10^{24}	1.74×10^{4}	1.08	DP
WD 1145 + 017 b	(e)	(6)	0.005	1.09	3.99×10^{21}	1.91×10^{3}	0.29	DP
GQ Lup b	(f)	(7)	103	4.89	4.08×10^{28}	2.52×10^{5}	172	P (or BD)
Kepler-1649 c	(g)	(8)	0.0827	5.79	7.47×10^{24}	1.35×10^{4}	10.9	P

E/D Properties: (a) All objects are orbiting within 0.1 AU of their host stars, and LHS 1140 b as well as TRAPPIST-1 d–g are orbiting inside habitable zones. (b) Extremely low volume densities, all constrained to be below 0.07 g cm^{-3}. (c) Smallest orbit around a binary star. (d) Planets c–d–e are locked in a $\frac{1}{2}$:1:2 Laplace resonance [34,35] in which the most massive planet b does not participate. (e) Least massive exoplanet, orbiting very close to a white dwarf. (f) Very large mass (thus, it could be a brown dwarf of mass $M = 21.5\,M_{\rm Jup}$) and shrinking toward Jupiter's size. (g) An Earth-sized planet orbiting in the habitable zone around an M-dwarf type star. **References**: (1) Refs. [35,36]. (2) Ref. [37]. (3) Refs. [38,39]. (4) Ref. [40]. (5) Refs. [35,41]. (6) en.wikipedia.org/wiki/WD_1145%2B017_b, accessed on 20 August 2024. (7) exoplanet.eu/catalog/gq_lup_b, accessed on 20 August 2024. (8) Vanderburg et al. [42], who kindly sent us an estimate of the mass ($1.25 M_\oplus$) based on the Weiss–Marcy relation [43]. The object would still be classified as a planet (with $g = 2.9$ m s^{-2}) if its mass were as low as $M_\oplus/3 = 2 \times 10^{24}$ kg. **Notes**: (i) D denotes mean diameter. (ii) Classification: P for planets and DP for dwarf planets (according to Definitions 2 and 3, respectively). (iii) BD for brown dwarf, a substellar object that fuses deuterium or lithium and emits light primarily at infrared wavelengths.

3.3. *Dwarf Planets in Our Solar System, a Partial Definition*

We can now begin formulating a partial definition of dwarf planets as objects in our solar system, other than satellites, with mean surface gravities $g < g_{\rm crit}$. This definition is incomplete because, in the absence of a lower limit, many small solar-system bodies (asteroids and KBOs, shown by asterisks in Figure 2) would literally litter the dwarf class. The IAU are also considering this issue but have not resolved it so far.

Brown [21] proposed that a cutoff in size ought to be adopted for small objects (a diameter of $D = 400$ km, obtained from Mimas, the smallest clearly round moon in the solar system, which however, is not in hydrostatic equilibrium since $g \neq \Omega^2 R$, where Ω is its spin angular velocity). As reasonable as this specification of "smallness" may be, it is subjective and not supported by the satellite data (see Section 4.3 below). Furthermore, it would promote the asteroids Vesta, Pallas, and Hygiea to the dwarf class, along with another $\sim$120 KBOs. In any case, we believe that we can resolve the lower cutoff issue objectively (Section 4), but not by adopting the above-mentioned critical value of $g_{\rm low} = 0.13$ m s^{-2} obtained from satellites because that would be just another subjective choice.

4. Mean Diameters of Dwarf Planets and Satellites

4.1. *Dwarf Planets Revisited*

Figure 3 shows another surprising result. When mean diameters $D = 2R$ are plotted versus semimajor axes a from the sun, a large gap is evident between $D = 525$ and 910 km (top/bottom = 1.7) that separates large satellites and dwarf planets from small satellites and small solar-system bodies. The upper boundary is set by Orcus ($a = 39.2$ AU), and the lower boundary is set by Vesta ($a = 2.36$ AU). Only some mid-sized KBOs are found with diameters in this range, and they have received considerable attention recently because of their exceptionally small densities in an area dominated by rocky objects [26]. By and large, this physical property differentiates these objects from larger KBOs lying above the gap's upper boundary (filled blue circles in Figure 3).

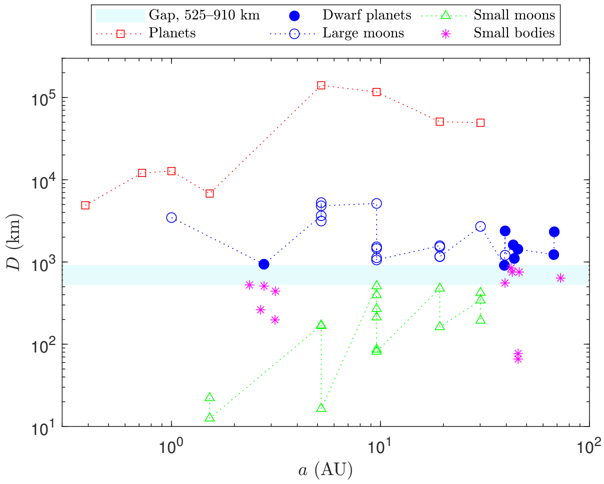

Figure 3. As in Figure 1, but for mean diameters D versus semimajor axes a. A large gap between $D = 525$ and 910 km separates the large satellites commingling with the dwarf planets from the small satellites commingling with small solar-system bodies.

In our case, the upper boundary of this gap provides an additional threshold,[20] viz.

$$D_{\text{crit}} = 900 \text{ km}, \tag{4}$$

that we need in order to complete the definitions of dwarf planets and small solar-system bodies:

Definition 3 (*Dwarf Planets*)**.** *Dwarf planets are objects in our solar system, other than satellites, with mean surface gravities* $g < g_{\text{crit}}$ *and mean diameters* $D > D_{\text{crit}}$.

Definition 4 (*Small Solar-System Bodies*)**.** *Small solar-system bodies are objects other than the sun, the planets, the dwarf planets, and the satellites.*

Asteroid Ceres ($g = 0.28$ m s^{-2}, $D = 939$ km) barely qualifies for dwarf-planet status according to Definition 3, but all other asteroids are excluded because their diameters $D \ll 900$ km. In this case, we did not split the gap in the middle in order to leave out of the dwarf class many smaller, porous, weakly self-gravitating KBOs, such as 2002 MS$_4$, Salacia, Varuna, 2002 AW$_{197}$, 2003 AZ$_{84}$, 2002 XV$_{93}$, and many more.[21] That is not a subjective choice: by examining physical properties such as size and volume density (to us, secondary and tertiary qualifiers, respectively),[22] Grundy et al. [26] argued convincingly in 2019 that these KBOs do not belong to the dwarf class. Definition 3 is generally consistent with their suggestion, although the D_{crit}-value of Equation (4) does not quite reach their high watermark of $D = 1000$ km (but see also Section 4.3 below, where we explore the consequences of adopting the high watermark from Ref. [26] for D_{crit}).

Recent radiometric measurements [28] show that 2002 MS$_4$ ($D = 726$ km or 796 km [44]) and Salacia ($D = 866$ km [45]) may have somewhat larger sizes ($D_{\text{new}} = 960$ km and 921 km, respectively); thus, they may qualify for dwarf-planet status (in addition to Quaoar, Orcus, and Sedna which are near certainty). That would make a total of 11 dwarf planets. In this list, Orcus may appear with a diameter of 983 km [28] (corresponding to $g = 0.175$ m s^{-2}), larger than that of Ceres. So, it seems that Ceres (or Salacia) may finally end up setting the upper boundary of the size gap, which would be marginally higher (by <3%) than that shown in Figure 3 (or maybe not; see Section 4.3 below for a tweaked classification scheme for dwarf planets and small solar-system bodies, which we think may be more objective and which dispenses with the tangled web just described).

4.2. Large Versus Small Satellites

The gap seen in Figure 3 strictly between small and large satellites, $D = 510$–1058 km, is a little wider (top/bottom = 2.1) than the 525–910 km gap discussed above. The upper boundary is set by Tethys, and the lower boundary is set by Enceladus (both at $a = 9.58$ AU). We doubt that we will ever find a large satellite in the solar system that will land in this gap (we are confident that all moons with $D > 500$ km have already been discovered[23]). So, the KBO critical value of $D_{\rm crit} = 900$ km appears to be applicable in the satellite group as well. We use it to formulate a brand-new definition of two distinct groups of satellites:

Definition 5 (*Large versus Small Satellites*)**.** *Large satellites have mean diameters* $D > D_{\rm crit}$*, whereas small satellites have* $D < D_{\rm crit}$*.*

Furthermore, we make an important distinction within the class of large satellites: There exists a sizable g-gap (top/bottom = 1.6, width $\Delta g = 0.46$ m s^{-2}) between $g = 1.24$ m s^{-2} (Callisto) and $g = 0.78$ m s^{-2} (Triton); thus, we define the subclass of the (six) largest of the large satellites:

Definition 6 (*The Largest of the Large Satellites*)**.** *The largest of the large satellites have surface gravities* $g > 1$ m s^{-2}.

From the viewpoint of sizes, it is evident from Figure 3 that large satellites (but not the largest ones captured by Definition 6) commingle with dwarf planets and that small satellites commingle with small solar-system bodies (asteroids and KBOs). The same holds true for masses (Section 5 below) and, for the most part, in Figure 2 above (surface gravities, excluding again the largest of the large satellites).

We also note that planets and the largest of the large satellites cannot be differentiated by size. We see in Figure 3 that the two largest moons, Ganymede and Titan ($D = 5268$ and 5150 km, respectively), are larger than Mercury ($D = 4880$ km); Callisto ($D = 4821$ km) is barely smaller by 1% in size than Mercury.

4.3. A Resolute Alternative View

We chose above the critical diameter shown in Equation (4) out of respect for the (subjective) opinion of the majority in the community that KBO Orcus deserves to be promoted to the dwarf class, especially since it may be larger than Ceres [28]. On the other hand, the highest critical D-value suggested by the wide gap in the distribution of satellite diameters is closer to 1050 than 900 km. This is seen in Figure 4, where we plot the distribution of satellites with diameters between 25 km and 1600 km. This figure reveals nature's segregation of small from large satellites (there are no significant gaps for $D < 500$ km). It also implies that the physical processes that create these types of objects are inefficient in making large satellites: only 11% of the moons with $D > 0.3$ km have $D > 500$ km, and a mere 4.3% have $D > 1600$ km.

The occasional presence of large satellites indicates that nature does try to run a continuous formation process at larger sizes too, although the physical processes mostly do not succeed, likely because the favorable initial conditions do not materialize often in the accretion disks around planets.[24] That could constitute evidence against the gravitational-instability formation scenario for moons and KBOs and in favor of the core accretion hypothesis [48]: Gravitational instability is expected to form preferentially large objects, whereas core accretion will not form large objects if there is not enough material at the formation site. This is a subject for future work.

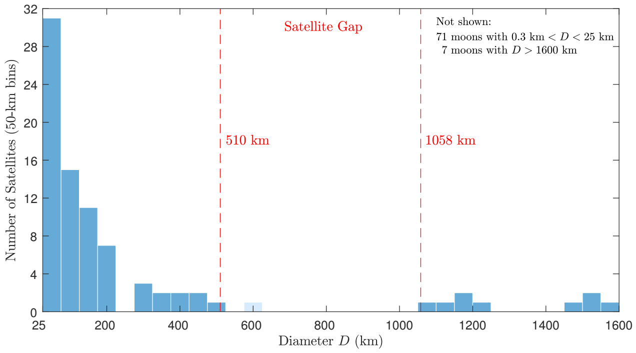

Figure 4. Histogram of satellites in our solar system with $D > 0.3$ km. We zoom in a little closer to the area of the 510–1058 km gap, leaving out 71 tiny moons with $D < 25$ km and 7 sparsely distributed large moons with $D > 1600$ km (in order of increasing size: Triton, Europa, Moon, Io, Callisto, Titan, Ganymede). The lightly colored bin represents Dysnomia (Note 23).

Based on Figure 4, a rounded critical D-value of 1000 km is a firm separatrix of large from small satellites—why, then, not also of all large from small objects in our solar system and in extrasolar systems? If nature uses the same processes for building such objects in the scales and the environments considered, then the extended separation created by the gap in Figure 4 cannot possibly be applicable only to satellites.[25] This hypothesis will be tested in exoplanetary systems as we continue to discover more of them. At present, it gives us a more objective classification scheme that separates large from small objects in our solar system.

There already exists physical support for such a size separation in the case of KBOs [26]: KBOs with diameters $400\text{ km} < D < 1000\text{ km}$, densities $\rho \lesssim 1\text{ g cm}^{-3}$, and albedos $\alpha \lesssim 0.2$ are effectively small in size, and they should not be included in the dwarf class because they are different in internal structure than the larger and much denser KBOs. Grundy et al. [26] call such objects "transitional" between the large and small KBOs; we may classify them as small, especially if their surface gravities are $g < g_{\text{low}} = 0.13\text{ m s}^{-2}$ (Section 3), which may be used as a higher-order indicator of smallness in the Kuiper belt.[26]

In this alternative classification scheme, the critical diameter between small and large objects in the solar system is set to

$$D_{\text{crit}} = 1000\text{ km}, \tag{5}$$

in which case both Orcus and Ceres are too small to be dwarf planets. Salacia and 2002 MS$_4$, with recently measured larger diameters but still under 1000 km [28], would also be excluded, leaving thus only 7 dwarfs: in order of increasing distance from the sun, these are Pluto, Haumea, Quaoar, Makemake, Gonggong, Eris, and Sedna; all in the Kuiper belt and all with $D > 1000$ km. Among the seven dwarfs, Sedna has the smallest diameter ($D = 1041$ km; Refs. [28,49]).

It is rather surprising to observe such salient transitions of objects between classes when D_{crit} is changed from 900 km to 1000 km, and it is mostly KBOs that blur the separatrix. Such transitions between classes show how sensitive the scheme is to subjective choices. If the corresponding satellite gap is not taken into account, then one can move the bar from Equation (5) down to 900 km (Equation (4)), effectively asking for several more KBOs and Ceres to be promoted to the dwarf class; or even down to 525 km asking effectively for asteroid Vesta and many more KBOs (listed in Ref. [28]) to also be promoted; or down to Mimas' 400 km [28] because one might think that some of the remaining asteroids and KBOs also deserve to be promoted in the off-chance that they are round, compact, and thus "sufficiently" self-gravitating.

This vicious spiral has no end, and this is the price to be paid for not considering the intrinsic physical properties. In our classification, some degree of subjectivity remains in choosing D_{crit} for Definitions 3 and 5. We generally feel more comfortable with Equation (5) because it takes into account the volume densities (a tertiary qualifier) of transitional KBOs [26] implicitly. In any case, the "best" or "final" classification scheme is not for us to decide, and we gladly leave this task to the community and the IAU—although we would like to see a revised classification scheme being managed by measured physical properties of these objects, as opposed to their orbital/environmental characteristics.

5. Masses of Planets and Satellites

Consideration of masses was not needed above in order to differentiate between the various groups of objects in the solar system. With the definitions established in the previous sections, we examine here the distributions of masses among various groups in order to find out whether object masses (a higher-order qualifier) are consistent with the results obtained so far. Mass gaps between groups are striking features and, as usual, they reveal valuable intrinsic physical information.

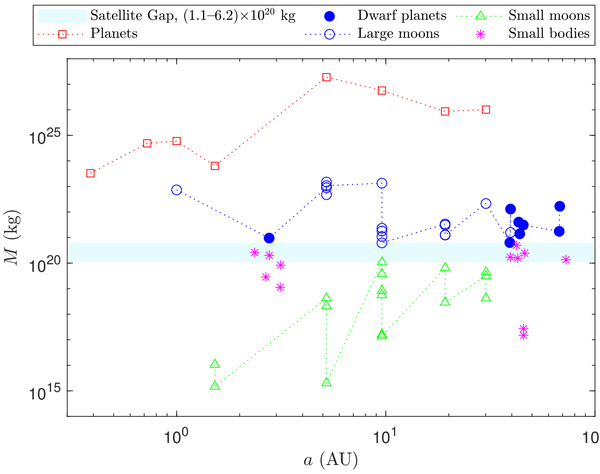

Figure 5. As in Figure 1, but for masses M versus semimajor axes a. As seen in Figure 3, a large gap between $M = 1.1 \times 10^{20}$ and 6.2×10^{20} kg separates small from large moons in the solar system. A narrow gap (not highlighted in the plot) between $M = 1.5 \times 10^{23}$ kg and 3.3×10^{23} kg separates planets from the largest satellites (By now, it should be clear that there is not a single dwarf planet that can compare to the 6 largest of the satellites by mass, size, or surface gravity).

Figure 5 shows that when masses M are plotted versus semimajor axes a from the sun, a huge gap, $M = (1.1–6.2) \times 10^{20}$ kg (top/bottom = 5.6) is found between the previously identified small and large satellite groups. The upper boundary is set by Tethys, and the lower boundary is set by Enceladus (both at $a = 9.58$ AU), just as for satellite sizes in Figure 3. So, the separatrix deduced from these two moons is solid no matter whether one considers mass or size, but this is not obvious for surface gravity: $g = 0.146$ m s^{-2} and $g = 0.113$ m s^{-2}, respectively (top/bottom = 1.3). The two moons have relaxed to roughly the same gravitational equilibrium state despite being very different internally ($\rho = 0.984$ and 1.61 g cm^{-3}, respectively, in addition to their widely differing sizes and masses). From the average of the g-values of the two moons, we obtain the threshold of

$$g_{\rm low} = 0.13 \ \text{m s}^{-2}\,, \tag{6}$$

mentioned earlier in the paper (a tertiary qualifier).

Some of the larger asteroids and the mid-sized KBOs marked by asterisks in Figure 5 land inside the mass gap or place slightly higher (Ceres and Orcus), washing out the distinction between large and small bodies. For this reason, size (Section 4) appears to be a better indicator of smallness than mass. On the other hand, even if one were to choose for critical mass that of Enceladus (the most massive of the small moons), then Vesta and Pallas would land only marginally into the gap. Thus, we conclude that there is no physical property that would justify the promotion of these two asteroids to the dwarf class.

The satellite mass gap described above extends approximately between $M_{\rm S} = 1 \times 10^{20}$ and 6×10^{20} kg. We can run a stringent consistency check by using the critical values from Equations (4)–(6) as follows: By substituting $g_{\rm low}$ and either one of the $R_{\rm crit} = D_{\rm crit}/2$ values into Equation (1), we derive critical masses ($M_{\rm S,crit}$) for separating large from small satellites, as predicted from the definitions given in Section 4. We find that

$$M_{\rm S,crit} \simeq \begin{cases} 4 \times 10^{20}\ {\rm kg}, & \text{for } D_{\rm crit} = 900\ {\rm km} \\ 5 \times 10^{20}\ {\rm kg}, & \text{for } D_{\rm crit} = 1000\ {\rm km} \end{cases} . \tag{7}$$

These critical values land just above the average value of the satellite mass gap $M_{\rm S,gap} = 3.65 \times 10^{20}$ kg. This result is important: It shows that no matter which value is finally adopted for the critical size that separates large from small objects, the remarkable division actually observed among solar-system satellites is reproduced by Equation (1) for $g = g_{\rm low}$. Unfortunately, the observed satellite mass gap is too wide, and we cannot use Equation (7) to distinguish objectively between the above two $D_{\rm crit}$-values; so, this remains a partly subjective choice in our classification scheme; although one may argue that the results of Grundy et al. [26], who considered additionally the mean densities and the albedos of KBOs with $D > 400$ km, may be tipping the scale in favor of the alternative scheme of Section 4.3 and $D_{\rm crit} = 1000$ km.

There is another significant gap (top/bottom = 2.2) in Figure 5 (not highlighted) between the least massive major planet, Mercury ($M = 3.3 \times 10^{23}$ kg), and the most massive satellite, Ganymede ($M = 1.5 \times 10^{23}$ kg, which is 9 times more massive than the most massive dwarf planet Eris). The midpoint of this gap lies at

$$M_{\rm P,crit} = 2.4 \times 10^{23}\ {\rm kg}\,, \tag{8}$$

with a half-width of $\pm 9 \times 10^{22}$ kg. This critical mass may serve as a secondary qualifier for the planet class,[27] but only together with the primary qualifier $g_{\rm crit}$ (Equation (3)). We did not use $M_{\rm P,crit}$ in our Definition 2 because it does not take into account object sizes; it is the self-gravity ($\propto g$), not the mass, which shapes and controls the dynamics of all large objects as well as their total internal pressures ($\propto g^2$) in hydrostatic equilibrium. Furthermore, surface gravity (Equation (1)), contains aggregate (thus intertwined) physical information, as opposed to mass or size alone.

Although the gaps situated below the major planets are not defined by the same exact objects, the gap in masses is the main source of the corresponding gap in surface gravities (Figure 2), since there is no separation in diameters between planets and the largest of the large satellites in the data (Figure 3). Nevertheless, we can determine a mean critical radius $R_{\rm P,crit}$ implied for major planets by substituting the critical values $g_{\rm crit}$ and $M_{\rm P,crit}$ into Equation (1). We find that

$$R_{\rm P,crit} = 2436\ {\rm km}\,, \tag{9}$$

with a half-width of ± 611 km assuming uncorrelated half-widths in $M_{\rm P,crit}$ and $g_{\rm crit}$. This mean value is barely below Mercury's mean radius ($R_{\rm Me} = 2440$ km; Ref. [50]). Since Ganymede and Titan are larger than Mercury (Section 4.2), $R_{\rm P,crit}$ cannot distinguish planets from large satellites, but it can certainly distinguish planets from dwarfs: Pluto ($R = 1188$ km) is marginally larger in size than Eris [28,51,52], but Pluto's radius is only 49% of $R_{\rm P,crit}$ and 65% of the lower limit of the "error bar" $R_{\rm P,min} = R_{\rm P,crit} - 611\ {\rm km} = 1825$ km (which coincidentally is also the mean radius of Jupiter's moon Io; Ref. [53]).

Finally, we note that the only extrasolar body small enough in Table 1, with a size below $R_{\rm P,crit}$ (and below $R_{\rm P,min}$), is WD 1145 + 017 b, an object that happens to be orbiting very close to a white dwarf. Its low surface gravity and low mass also indicate that the object is not a planet. Since its diameter $D > 1000$ km and its volume density $\rho > 1\ {\rm g\ cm^{-3}}$, it was classified unambiguously as a dwarf planet in Table 1.

6. Summary and Concluding Remarks

6.1. Summary

We have investigated the physical properties of various groups of bodies in the solar system. We have discovered large gaps in the distributions of certain fundamental physical properties that separate the various known groups of objects in distinct classes defined by one, two, or three measured quantities (in order of decreasing strength, these are surface gravity, mean diameter, and mean volume density, respectively). These gaps can be used to define the various classes succinctly and, to a large degree, objectively. A summary of our findings is as follows:

(1) Planets are differentiated from all other objects by their large surface gravities ($g > 2.7$ m s^{-2}). There is no need to refer to any orbital characteristic or any additional physical property. This is a surprising result: Definition 2 in Section 3.1 can identify a planet without any reference to the object's orbital state or environment (and it seems to work for extreme exoplanets, too; see Section 3.2). This is also an affirmation of a general principle that we have espoused long ago—gravity, and only gravity, has the final word in the formation of satellites, planets, and other objects grown to solar-system scales; internal pressure or mechanical stresses only resist until the bitter end.

(2) Large satellites are differentiated from small satellites by their sizes and masses but not by their surface gravities or volume densities. This result is sensible since gravity is no longer dominant at smaller scales. The size and mass gaps are so wide (Figures 3 and 5) that the results suggest strongly a division between all large and small objects in the solar system at a critical diameter of $D_{\rm crit} \simeq$ 900–1000 km and a critical mass of $M_{\rm crit} \simeq (1\text{–}6) \times 10^{20}$ kg (Sections 4 and 5, respectively).

(3) Dwarf planets certainly have $g < 2.7$ m s^{-2}, but we also need to make use of additional, higher-order qualifiers such as thresholds in size and/or density. A size threshold is suggested by the strongly segregated satellites (Figure 4): $D_{\rm crit} = 900$ km or 1000 km—a subjective choice (see Section 4). A density threshold is provided for KBOs by the work of Grundy et al. [26], who relied on sizes, volume densities, and albedos to suggest on physical grounds that $D_{\rm crit} \approx 1000$ km and $\rho_{\rm crit} \approx 1$ g cm^{-3} are likely objective thresholds. On the other hand, the definitions adopted by the IAU in 2006 are matched only if a critical diameter of 900 km is adopted—otherwise (for $D_{\rm crit} = 1000$ km), Ceres, Orcus, and some other nascent dwarfs will have to be demoted to the class of small solar-system bodies.

(4) Main-belt asteroids are all excluded from the dwarf class for a choice of $D_{\rm crit} = 1000$ km (Section 4.3). Otherwise (for $D_{\rm crit} = 900$ km), Ceres is the only asteroid that belongs to the dwarf class (Section 4.1).

(5) KBOs are almost continuously distributed in size and can only be separated into small and large objects by the critical diameter determined by satellites (the two groups commingle in the figures). There is, however, an apparent gap in diameters between 756 and 1041 km in which only three mid-sized KBOs are found[28] (Orcus, 2002 MS$_4$, and Salacia). Depending on the choice of critical diameter, the dwarf class may end up with seven or eleven members. If $D_{\rm crit} = 1000$ km is adopted and volume density [26] is used as a tertiary qualifier (Section 4.3), then the class will contain only seven dwarfs, all in the Kuiper belt: in order of increasing distance from the sun, these are Pluto, Haumea, Quaoar, Makemake, Gonggong, Eris, and Sedna.

(6) Surface gravity g turned out to be the most important physical quantity for groups containing large objects (Section 3.1) in the classification scheme that we described. Looking at the g-data altogether, we can offer a general partition of classes of large objects in the solar system that will never have to be revised (we believe that all large objects have already been discovered) and it is also aesthetically pleasing: in metric units (m s^{-2}), planets have $g > 3$; satellites have $g < 2$; and dwarf planets have $g < 1$ (The precise thresholds are $g_{\rm crit} = 2.7$ (g-gap average), 1.8 (Io), and 0.82 (Eris), respectively).

(7) We have also determined a threshold for surface gravities of small objects ($g_{\text{low}} = 0.13$ g cm^{-3}) from the clear division of sizes and masses between small and large satellites (Section 5). But the condition $g < g_{\text{low}}$ could only be a tertiary indicator of smallness after size and mass. In the case of KBOs, this higher-order threshold and the critical density of $\rho_{\text{crit}} \approx 1$ g cm^{-3} [26] allow for three mid-sized objects (Orcus, Salacia, and 2002 XV$_{93}$) to be placed in the dwarf class. Between them, 2002 XV$_{93}$ has the lowest surface gravity $g = 0.15$ m s^{-2} and Salacia has the lowest volume density $\rho = 1.26$–1.5 g cm^{-3} [45,55].

(8) Calculations based on the large surface-gravity gap and the large mass gap just below the major planets (i.e., just below Mercury) produce theoretical thresholds (secondary indicators) of mass and size above which an object should likely qualify as a planet (Equations (8) and (9) in Section 5). The critical mass is 73% of Mercury's mass but the critical radius is only 4 km smaller than Mercury's mean radius of $R = 2440$ km [50]. So, this threshold does not leave any room for planets smaller than Mercury in our solar system and perhaps beyond (Table 1 and Section 3.2). Such a lower limit in size for major planets has been discussed in the past (by all standards, Mercury is a singularly small planet), but this viewpoint has not been met with strong quantitative support until now (Section 5).

6.2. Concluding Remarks

6.2.1. Physical Properties (as Opposed to Orbital Properties)

Based on the observed wide gaps in physical properties, we formulated new objective definitions for the various classes of objects observed in our solar system (Sections 2–4). We took the opposite view of the IAU standard and we chose to pay firm attention to inherent physical properties of solar-system bodies rather than their conspicuous orbital characteristics (Section 1). In fact, we eschewed all orbital properties in Definitions 2–6 given above.

Only the old transparent Definition 1 of Satellites (traced back to Galileo; Ref. [56]) uses orbits in order to distinguish moons from all other bodies in the solar system. But Definition 1 was necessary in order to separate ab initio satellites from planets; the two classes overlap when sizes are considered (Section 4.2).

6.2.2. Exoplanets

A preliminary test of our planet-related Definitions 2–4 that uses 19 exoplanets with extreme and/or desirable properties (Table 1) shows that our set of definitions may actually work for exoplanetary systems as well. Our definitions can distinguish between such diverse objects as the seven planets in the miniature resonant system TRAPPIST-1 and the three extremely light dwarf planets in the Kepler-51 system (Section 3.2).

In a second test, we surveyed another sample of 25 multiplanet systems[29] containing a total of 103 exoplanets for which mass and radius information is available from observations. We found only 5 bodies that qualify for the class of dwarf planets ($g < 2.7$ m s^{-2}). These are listed in Table 2 for future reference. Their mean radii and masses spread over one and two orders of magnitude, respectively, whereas their surface gravities vary by less than a factor of 2. It is interesting that all five g-values are larger than the upper dwarf value (item (6) in Section 6.1) of $g = 1$ m s^{-2} in the solar system (which seems to also work for the 'extreme' extrasolar sample listed in Table 1); but this may be due to a selection effect—large exoplanets are easier to detect.

Table 2. Extrasolar dwarf planets ($g < 2.7$ m s^{-2}).

Object Name	a **(AU)**	M **($M_\oplus$)**	R **($R_\oplus$)**	g **(m s^{-2})**	**Refs.**
HIP 41378 f	1.37	12	9.2	1.39	[57,58]
K2-138 f	0.1045	1.63	2.9	1.89	[59,60]
Kepler-223 e	0.1486	4.8	4.6	2.22	[61,62]
Kepler-444 d	0.0578	0.048	0.453	2.31	[63,64]
Kepler-444 e	0.0671	0.045	0.475	1.94	[63,64]

6.2.3. Our Moon and Io

Returning to our solar system, we show in Table 3 that our Moon (although slightly smaller) is markedly similar in physical properties to Jupiter's moon Io. Our natural satellite is 18% less massive, and this mild difference may be the main reason that their physical similarity is not widely appreciated. Furthermore, two orbital properties (inclination to the ecliptic $i_{\rm ecl}$ and eccentricity e) are very different and make the comparison muddier. On the other hand, their mean distances from their respective central bodies (local semimajor axes $a_{\rm cb}$) are quite similar.

Table 3. Io–moon comparison of properties.

Property	**Io** ^{a,b}	**Moon** ^{c}	**Moon/Io Ratio**
g (m s^{-2})	1.80	1.62	0.90
D (km)	3643	3475	0.95
M (10^{22} kg)	8.93	7.35	0.82
ρ (g cm^{-3})	3.528	3.344	0.95
$a_{\rm cb}$ (km)	421,800	384,400	0.91
$i_{\rm ecl}$ (deg)	2.213	5.145	2.32
e	0.0041	0.055	13.4

[a] solarsystem.nasa.gov/moons/jupiter-moons/io, accessed on 20 August 2024. [b] en.wikipedia.org/wiki/Io_(moon), accessed on 20 August 2024. [c] solarsystem.nasa.gov/moons/earths-moon, accessed on 20 August 2024.

The g-values of these moons are the two highest ones among all satellites in the solar system. The third highest g-value belongs to Ganymede ($g = 1.43$ m s^{-2}) and it is 12% smaller than that of the Moon. The volume densities of Io and the Moon are also the two highest among satellites (Table 3 and Figure 1), which means that gravity has pushed as hard as it could have in collapsing these objects. So, it seems that nature comes up against a pressure barrier when making strongly self-gravitating satellites ($g_{\rm S,max} \approx 2$ m s^{-2}). It tried to make such objects in two different settings within the solar system, but it could not get past this barrier.[30] This strengthens our belief that satellites with $g_{\rm S} > 2$ m s^{-2} will not be found in extrasolar systems either.

There is a physical explanation for having a pressure barrier resisting successfully at about that $g_{\rm S,max}$ threshold, and we alluded to it in Section 5: In the final stages of contraction with the mass collected in an object being locked in and unchanging, gravity takes over and pushes the object to smaller radii. The force of gravity ($\propto g$) finds resistance from internal pressure forces that, in principle, do not allow for unlimited shrinking. Internal pressure scales as g^2, so when $g > 1$, the resisting force grows faster than g, it competes more efficiently against gravity and eventually halts the contraction. Apparently, the contraction of satellites is halted before g reaches the $g_{\rm S,max} \approx 2$ m s^{-2} threshold. For Io, $g^2 = 1.80^2 = 3.24$, so internal pressure more than tripled compared to an earlier, more diffuse state in which $g^2 = 1$ and its size was $1.34D = 4883$ km.

Such estimates cannot be carried out for KBOs because their g-values are smaller than 1 m s^{-2}. Thus, it is not surprising that most of these objects are not in hydrostatic equilibrium. At such low g-values (also low hydrostatic pressures) and small sizes, ordinary matter must find other ways to oppose gravity, such as purely mechanical stresses supported by high porosity, which is inferred from the unusually low mean densities of such objects (see Ref. [26] and references therein on related laboratory work).

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Abbreviations

The following abbreviations are used in this manuscript:

BD	Brown Dwarf (Table 1)
DP	Dwarf Planet (Table 1)
IAU	International Astronomical Union
LSST	Large Synoptic Survey Telescope
KBO	Kuiper-Belt Object
P	Planet (Table 1)

Notes

[1] Ptolemy's Almagest, 1998, Translated and annotated by G. J. Toomer, Princeton Univ. Press, Princeton.

[2] en.wikipedia.org/wiki/The_Sand_Reckoner, accessed on 20 August 2024.

[3] www.iau.org/news/pressreleases/detail/iau0602, accessed on 20 August 2024.

[4] www.iau.org/news/pressreleases/detail/iau0603, accessed on 20 August 2024.

[5] en.wikipedia.org/wiki/IAU_definition_of_planet, accessed on 20 August 2024.

[6] ssd.jpl.nasa.gov, accessed on 20 August 2024; solarsystem.nasa.gov/planets, accessed on 20 August 2024.

[7] en.wikipedia.org/wiki/Hyperion_(moon), accessed on 20 August 2024.

[8] A triaxial ellipsoid with $D^3 = (360.2 \times 266.0 \times 205.4)$ km^3 [17] and degree of triaxiality $D/(360.2\text{ km}) = 0.750$.

[9] en.wikipedia.org/wiki/58534_Logos, accessed on 20 August 2024.

[10] en.wikipedia.org/wiki/Adrastea_(moon), accessed on 20 August 2024.

[11] Adrastea's shape has been called "irregular" [18], although we actually see a triaxial ellipsoid measuring $D^3 = (20 \times 16 \times 14)$ km^3 with a mild degree of triaxiality $D/(20\text{ km}) = 0.825$.

[12] Note 6 and en.wikipedia.org/wiki/Solar_System, accessed on 20 August 2024.

[13] simple.wikipedia.org/wiki/Adrastea_(moon), accessed on 20 August 2024.

[14] en.wikipedia.org/wiki/Asteroid, accessed on 20 August 2024.

[15] Ref. [27].

[16] Refs. [28,29]

[17] Ref. [30].

[18] Refs. [26,31].

[19] The concept of "dynamical dominance," an extrinsic property, has recently gained traction [3,6,7]. From our point of view, such classification criteria are not necessary since the dominance of a self-gravitating body is manifested, above all, by the strength of its surface gravity; it diminishes as we move out and into the body's neighborhood.

[20] This is a subjective choice that we make in this work, bowing to the majority that wants Ceres and Orcus in the dwarf class. We discuss an alternative view in Section 4.3 below.

[21] en.wikipedia.org/wiki/List_of_Solar_System_objects, accessed on 20 August 2024; and Ref. [28].

[22] We note that 2002 XV$_{93}$ is excluded by size ($D = 564$ km), not by density ($\rho = 2$ g cm^{-3}).

[23] Eris' moon Dysnomia, with $D = (700 \pm 115)$ km and a very low albedo of $\alpha = 0.04^{+0.02}_{-0.01}$ [46], is not included in our meta-analysis because it is possible that the reported estimates could be inaccurate; see also recent work in Ref. [47], where $D = 615^{+60}_{-50}$ km and $\alpha = 0.05 \pm 0.01$. At any rate, Dysnomia (the lightly colored bin in Figure 4 below) may shrink the satellite size gap of $D = 510$–1058 km, but it does not affect the choice of D_{crit}.

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[24] For instance, there is a large gap of width ≈1200 km between the diameters of Jupiter's Io and Callisto, viz. 3643 km $< D <$ 4821 km, without any moons present. Thus, it seems that there was not enough material anywhere in the protosatellite accretion disks for a moon of that size to be formed.

[25] There is an analogous gap in the distribution of KBOs between 756 and 1041 km that contains only three objects [28]. This is hardly surprising since 2247 objects with $D >$ 100 km have been observed so far in the Kuiper belt [28], and only 68 of them (3%) have $D >$ 500 km. As in Figure 4 for large moons, nature was prevented from assembling but a few large KBOs.

[26] We are aware of three KBOs with $D <$ 1000 km that have $g > g_{\rm low}$: Orcus, Salacia, and 2002 XV_{93}.

[27] The planets of the Kepler-51 system (Table 1) show that object mass alone fails as a primary qualifier, but surface gravity succeeds in a difficult case involving extrasolar bodies with extreme properties.

[28] The Euclid and LSST surveys [54] may potentially detect more KBOs, not necessarily orbiting near the ecliptic.

[29] NASA Exoplanet Archive, exoplanetarchive.ipac.caltech.edu, accessed on 20 August 2024.

[30] Even if the Moon had been made from a larger mass such as that of Io, and if gravity had managed to push the contraction down to the present size of the Moon (which we seriously doubt), then g would still be slightly lower than 2 m s^{-2}.

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