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Real-Time Modeling for Design and Control of Material Additive Manufacturing Processes

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Abstract: The use of digital twin and shadow concepts for industrial material processes has introduced new approaches to bridge the gap between physical and cyber manufacturing processes. Consequently, many multidisciplinary areas, such as advanced sensor technologies, material science, data analytics, and machine learning algorithms, are employed to create these hybrid systems. Meanwhile, new additive manufacturing (AM) processes for metals and polymers, based on emerging technologies, have shown promise for the manufacturing of sophisticated parts with complex geometries. These processes are undergoing a major transformation with the advent of digital technology, hybrid physical-data-driven modeling, and fast-reduced models. This study presents a fresh perspective on hybrid physical-data-driven and reduced order modeling (ROM) techniques for the digitalization of AM processes within a digital twin concept. The main contribution of this study is to demonstrate the benefits of ROM and machine learning (ML) technologies for process data handling, optimization/control, and their integration into the real-time assessment of AM processes. Therefore, a novel combination of efficient data-solver technology and an architecturally designed neural network (NN) module is developed for transient manufacturing processes with high heating/cooling rates. Furthermore, a real-world case study is presented, showcasing the use of hybrid modeling with ROM and ML schemes for an industrial wire arc AM (WAAM) process.

Keywords: additive manufacturing; real time modeling; machine learning; hybrid physical-data-driven modeling

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1. Introduction

The role of real-time ROM technology, along with hybrid and smart ML schemes, has already revolutionized many industrial processes. The development of digital twin and digital shadow concepts has optimized process control and design. The introduction of the process-data paradigm and the use of ML technologies have significantly modernized process control concepts, enabling real-time prediction and correction for better control of industrial processes. Live sensor and remote sensing data, offline experimental and literature data, and numerical simulation predictions are all part of this processing paradigm, where data acquisition, handling, and training can greatly influence the manufacturing process. Furthermore, hybrid and smart data-handling technologies can help assess the performance of final parts regarding their service life. However, the introduction of these new technologies has not rendered conventional analytical, experimental, and numerical modeling redundant; rather, it has made their use smarter by integrating their results into evolving databases. Thus, dynamic and evolving databases can be accumulated for model training using data generated from analytical, experimental, and further offline detailed numerical simulation studies [1–3].

The systematic integration of these new technologies into AM processes can help establish faster and more accurate predictive models, avoiding long and expensive additional experimental and numerical efforts. Therefore, appropriate ROM techniques have been developed to enhance the digital twin and digital shadow concepts [4,5]. In this study, a combination of hybrid physical-data-driven and ROM techniques, along with ML modules, is employed for the WAAM deposition process. Additionally, a numerical simulation framework is set up to provide sufficient data for the ROM and ML schemes. Furthermore, a snapshot matrix is generated for the deposition process using various input variables (e.g., initial temperature, deposition speed, and torch power) to examine their effects on the final component. To ensure the ROM predictive model works effectively, some of the data are used to derive the reduced model (including its training), while the rest of the data are used to validate the performance of the ROM.

2. Simulation of AM Processes—Methodology

Different modeling techniques have been employed for the simulation of AM processes, considering many inherent thermal and mechanical characteristics. Although various AM processes have been developed for metallic and non-metallic components, many of the process functionalities are similar in terms of numerical modeling approaches [6–8]. The finite element (FE) method has been extensively used for simulating processes like AM, where different approaches have been developed to address the dynamic nature of these processes [9–12]. As AM parts are deposited and built up layer by layer, conventional FE methods, with their fixed domain size, can only simulate the final geometry without considering the progressive development of the layered structure. Although many alternative FE techniques have been developed to alleviate these shortcomings through dynamic mesh generation, deactivation/activation, and hybrid techniques, accurately modeling deposition processes remains a challenging task [13–16].

2.1. Dynamic FE Techniques

The application of alternative FE methods for dynamic manufacturing processes like casting and AM has recently been promoted to simulate progressively generated parts using both fixed-size and variable-size numerical domains. Since these processes inherently involve multi-physical thermal, fluid, and solid features and may also include multi-scale microstructural evolution phenomena, the proposed dynamic FE methods need to handle the different physical and dimensional aspects of these processes. This poses challenges for conventional FE methods, where geometries of domains are prepared and discretized at the start of the simulation, and all system matrices are initially assembled to calculate the full system responses. To alleviate these numerical complications, different simulation strategies, including deactivation and activation methods and novel dynamic mesh-generating frameworks, have been proposed [9,17].

For the so-called Deactivation and Activation Technique (DAT) for AM processes, the common practice is to deactivate almost all of the mesh related to the finished part at the start of the simulation (except initial base elements) and re-activate it (layer by layer or block-wise) as the simulation progresses [16,18,19]. Although this helps simulate the dynamic nature of the process using conventional FE methods, the deactivation process does not remove elements from assembled matrices; rather, it reduces their numerical impact to very small values (e.g., using very small multipliers). Additionally, any changes in material properties and system boundaries during the process cannot be simulated, as the system matrices are assembled at the start of the simulation. This means the solver needs to solve full system matrices from the start, which is computationally inefficient. Hence, dynamic mesh techniques (DMTs) have recently been proposed to overcome obstacles related to DAT by generating or splitting mesh during the simulation, where new mesh blocks or layers can be adapted into mainstream FE techniques for AM applications [4,12]. These techniques treat the progressive generation of AM parts during the simulation by generating and appending mesh blocks in a predefined or calculated manner, which are then attached to the main numerical domain.

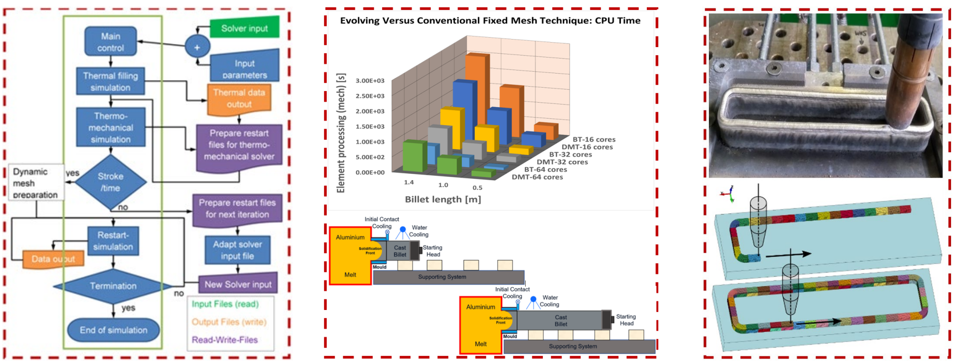
2.2. Evolving Domain Technique

One of the recent DMT frameworks proposed for AM processes is the evolving domain technique. This technique considers the transient nature of the numerical domain and its continuous expansion throughout the processing time. The new technique, along with its accompanying block-mesh scheme, has been developed to address the numerical issues associated with domain expansion and the dynamic discretization of evolving domains. The expansion of the numerical domain is managed using a dynamic zone, where new layers of mesh (or mesh blocks) can be appended to the existing domain in a tempo-spatial manner. The spatial directions for the expansion of the numerical domain can be defined using a predefined path (e.g., material deposition direction) or by calculating the required growth during the transient simulation. This technique eliminates the need for generating a steady-state mesh with birth and death features and also removes the necessity for splitting element layers at the deposition front.

For AM process simulations, continuous deposition processes in various spatial directions can be simulated using a mesh evolution technique at the deposition front. In this research, an in-house code was developed to control and implement dynamic mesh generation and insertion (in Python) as follows:

- ➢ First, the initial geometry of necessary components, including the baseplate, is generated and meshed.
- ➢ In the second step, the initial thermal and mechanical boundaries are considered, and the initial system matrices are assembled.
- ➢ The first time-steps/iterations are subsequently solved using a thermal–mechanical solver, and the deposition front coordinates are updated.
- ➢ The domain geometry and mesh are later adapted by inserting a mesh block based on the deposition direction and speed.
- ➢ In the next step, the domain matrices are updated with new mesh entries, and the extra input energy is disseminated amongst the domain using the mapping technique.
- ➢ In the final step of the loop, after achieving thermal energy balance, the previous converged solution is used as a first step for a newly updated domain, and the simulation scheme continues with the new geometry/mesh till the next evolution step is triggered.

The computational performance of the technique for industrial casting applications has already been examined, and extensive numerical investigations have revealed that higher speed and accuracy can be achieved using the evolving domain technique [16]. Figure 1a,b show the flowchart for the evolving algorithm and a comparison study of the numerical performance of this technique versus conventional methods (for horizontal casting cases). Figure 1c illustrates the mesh-insertion technique for simulating the WAAM deposition process, where new meshes are appended to the existing domain to match the material deposition process [12]. More comprehensive discussions about the evolving domain technique can be found in [20].



(a) (b) (c)

Figure 1. (**a**) Flowchart for evolving algorithm, (**b**) CPU time comparison for DAT and DMT for horizontal casting case using different numbers of computational cores. Reprinted from Ref. [12], and (**c**) dynamic WAAM deposition and its numerical mesh-insertion technique for DMT. Reprinted with permission from refs. [16]. 2020, ELSEVIER.

3. Methods for AM Reduced Models

Cyber–physical modeling, along with the new concepts of digital twins and shadows, has already found its way into industrial processes, revolutionizing optimized design and control through digitalization schemes. Cyber manufacturing processes, often described as the combination of physical and cyber tools/digital replicas and twinning systems, are well known. However, in this research, only the aspects of digital replicas and twinning systems were considered, with physical testing and design of experiments (DOEs) conducted solely for model calibration. Sensing technologies, data gathering, data filtering, and data-processing schemes, along with reduced-order models (ROMs) for fast predictive–corrective analyses, form a crucial part of the digitalization framework for a process-data paradigm. ROM technologies have also been rapidly evolving to achieve fast and reliable real-time predictions using new analytical and mathematical concepts. Although conventional reduced modeling techniques have been employed over the last fifty years to limit computational time and resources for detailed process simulations, they are unable to provide real-time predictions during processes. Consequently, recent attention has focused on the new generation of ROMs, which enable fast predictions and corrections in real-time during AM processes.

3.1. ROM Techniques for AM Processes

The increasing demand for real-time simulations of complex manufacturing processes has spurred numerous research activities aimed at developing fast ROMs using novel mathematical and analytical concepts [21,22]. Despite the exploitation of various techniques to create suitable ROMs for both transient and steady-state processes, challenges related to accuracy, speed, and solution stability remain unresolved for many multi-physical aspects of industrial processes. Consequently, numerous mathematical decomposition, projection, and dimensional reduction techniques have been developed to mitigate the extensive computational efforts required for modeling the multi-physical nature of these processes. In AM processes, detailed thermal–mechanical and/or fluid–thermal–mechanical simulations are routinely performed to optimize power, material delivery, deposition speed, heating, and cooling. However, the reduced versions of these simulations are cumbersome due to the transient nature of AM processes and their continuous domain expansion, which are difficult to predict using any simplification scheme.

The concepts of characteristic equations, eigenvalue analyses, and matrix factorization have been employed in this research to create a fast ROM framework for transient AM processes. This framework enables real-time predictive and corrective measures using previously measured or calculated data. To begin, let us consider a conventional second-order differential equation as follows:

$$[K][u] + [M][\ddot{u}] = 0 \tag{1}$$

where $[K]$, $[M]$, and $[u]$ are the stiffness, mass, and displacement matrices of the system. The simple solution, along with the eigenvalue model representation for the system, can be written as

$$u(t) = Ae^{i\omega t} \rightarrow \{[K] - \omega^2[M]\}[u] = 0 \tag{2}$$

where ω and A are eigenvalues (natural frequencies) and a constant multiplier, respectively. If the tempo-spatial characteristic of the system is decomposed, the characteristic equation of the system along with its solution for a point in space can then be written as

$$\left(\omega^2\right)^N + c_1\left(\omega^2\right)^{N-1} + c_2\left(\omega^2\right)^{N-2} + \ldots\ldots c_N = 0 \quad u(x,t) = \sum_n \hat{u}_n(x,\omega_n)e^{i\omega_n t} \tag{3}$$

where ω_n and $\hat{u}_n$ are the system's natural frequencies and Fourier coefficients, respectively. The characteristic equation of the system is calculated using properties such as geometry, material properties, and boundaries. However, solving the system equations using conventional time and frequency domain solvers requires substantial computational time and effort. This makes the calculation of system characteristics for engineering problems and processes lengthy and costly, rendering it unsuitable for real-time digital twinning. Consequently, much research has been conducted to reduce computational time and effort by developing specialized types of reduced-order models (ROMs) for industrial processes [3,23,24]. One interesting alternative technique, also used in the current research, is to estimate the characteristic equation of systems and processes using smart data-driven techniques instead of calculating them directly.

This ROM technology is initially based on the same mathematical concepts of eigenvalues and orthogonal eigenvectors to find the characteristic equation of the process. However, instead of using system properties to formulate the system matrices, it relies on calculated or measured responses of the system across a range of variables. If the responses of the system under varying parameters are known, a matrix of system responses under changing variables can be constructed, which holds the basic characteristic information about the system. The estimation of the system's characteristic equation is then performed using mathematical decomposition techniques traditionally used for eigenvalue analyses. To understand the basis of the mode decomposition technique for estimating the characteristic equation, let us consider the governing equation for a vibrational problem as follows:

$$[K][u(x,t)] + [M]\left[u(\ddot{x},t)\right] = 0 \rightarrow \quad \left([K] - \omega^2[M]\right).[u(x,t)] = 0 \rightarrow \quad [K_d]^{-1}.[u(x,t)] = 0 \tag{4}$$

where $[K_d]^{-1} = [K] - \omega^2[M]$ is called the dynamic stiffness matrix. Now, we can consider the alternative governing equation for the response of the system as

$$[Y(x,t)] = [C].\ [X(x,t)] \tag{5}$$

where $[Y(x,t)]$ can be interpreted as the desired response at specific values for variables, $[C]$ is the estimated system characteristics (using data techniques), and $[X(x,t)]$ is the input values. In this research work, Singular Value Decomposition (SVD) is used to decompose the system response matrix, which gives the optimal expansion of the matrix as

$$[Y(x,t)] = U \sum V^T \tag{6}$$

where U represents the spatial eigenvector decomposition, $\sum$ is the singular value diagonal matrix (e.g., eigenvalues), and V^T represent temporal decomposition for transient system responses. The SVD technique is a popular reduction method based on linear algebra that can reduce the size of predictive models in terms of dimensions and time. A more detailed mathematical description of the SVD method can be found in [25,26]. Using these reduced models, the transient responses of the system can be estimated in real-time using a database of previous responses.

3.2. Hybrid ROM-ML Techniques

The advancement of ROM techniques for industrial process simulations has encountered challenges related to the accuracy of data extrapolation and rate dependency (e.g., for high cooling/heating rates). In many of these processes, the limited amount of available data, combined with the large number of influential parameters, has promoted the use of integrated ROM-ML hybrid schemes, where further data training can be performed. To establish a foundation for such a hybrid framework, appropriate ML and ROM techniques must be employed to enhance the predictive power of these models. This combined ROM-ML technique can facilitate the development of agile and efficient models for transient thermal–mechanical and/or fluid–thermal–mechanical WAAM process modeling.

For data training in this research, a neural network (NN) ML scheme was implemented using in-house code. This NN scheme can employ either Multi-Layer Perceptron (MLP) or Back-Propagation Neural Networks (BPNN) for training system response data. For data interpolation and extrapolation, data extracted from experimental and detailed simulations were stored in a file. During each exemplar epoch, a certain number of data points were selected randomly without repeating any learning vector. Since batching concurrent inputs is computationally more efficient than sequential inputs, epochs and total computation of errors were used as follows:

$$E_{epoch} = \frac{1}{n_{epoch}} \sum_{n=1}^{n_{epoch}} \sum_{u=1}^{U} \left(t_u - y_u\right)^2 \tag{7}$$

where t_u and y_u are desired and model outputs, and the network error is computed based on the RMS standard formulation, where it is propagated into the network until convergence is reached. In the back-propagating networks, the modification of weights is proportional to error differential ∂E with respect to weight ∂w as

$$\Delta w_{ij} \propto -\frac{\partial E_{epoch}}{\partial w_{ij}} \tag{8}$$

After data training, the resulting ROM-ML models can further update themselves using the NN scheme, enhancing thermal and mechanical predictions for AM processes. Additionally, these hybrid models can be used for modeling sub-processes such as heating and cooling during AM processes [4].

3.3. Case Study: Reduced Models for WAAM Process

Digital twin and cyber–physical modeling, along with real-time prediction and correction concepts, have already been employed in some AM industrial processes. The optimization and active control of these processes are among the main goals of twinning technology, where sensing technology, data generation, and data-processing schemes can be combined for greener production. In this case study, attention is focused on the WAAM process, a type of directed energy deposition (DED) scheme. In this technique, single or multiple plasma or electric arc torches are used to weld layers of wire feedstock onto a predefined geometry. The heat generated by the moving torch melts the wire, allowing layers of material to be built up through an inter-layer fusion process.

Although various sophisticated transient and steady-state numerical simulations of WAAM processes have already been developed, their use in digital twinning is limited due to the substantial computational time and resources required. Therefore, for real-time predictions and corrections, the predictive power of ROM and ML can be employed. We can consider Equation (6) to predict temperature (and stress) responses during a WAAM process as follows:

$$[Y(x,t)] = U\sum V^T \;\rightarrow\; T_k(x,y,z,t) = U_k \sum\nolimits_k V_k^T \qquad \sigma_k(x,y,z,t) = U_{k\sigma} \sum\nolimits_{k\sigma} V_{k\sigma}^T \tag{9}$$

where T_k σ_k are temperatures and stresses at process time t, and the snapshot results can be interpolated using the radial basis function (RBF) or adaptive redial basis function (ARBF) as follows [27,28]:

$$f(T) = \sum_{i=1}^{k} a_i \varphi(||T - T_i||) \tag{10}$$

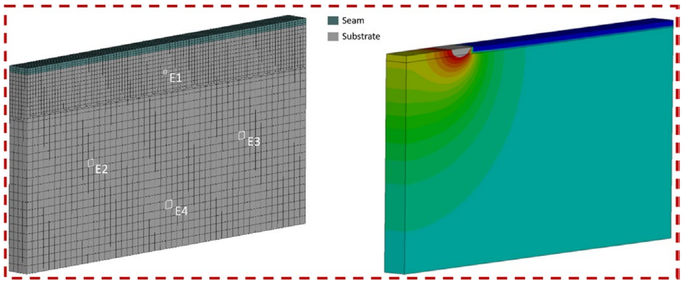
where a_i and φ are the weighting coefficient for the RBF. Here, the temperature at the any location can be predicted during the WAAM process using a sum of k radial basis functions, which are weighted by appropriate coefficients. For the RBF definition of WAAM processes with their high heating rate (passing of a torch over a measuring point), the artificial network architecture developed earlier can be employed, where an input, hidden, and output layer can be defined as

$$\varphi_i = e^{\left(-\frac{||\overline{T}-\overline{T}_i||^2}{2\mu_i^2}\right)} \tag{11}$$

where $\overline{T}$, $\overline{T}_i$, μ, and φ_i are the temperature input vector, ith neuron sample vector, ith neuron bandwidth, and ith neuron output, respectively. The innovative combination of SVD and the neural network RBF for implementing a reduced model for WAAM can produce fast and reliable predictions, even in zones with high-gradient data (e.g., high heating rates). To investigate the accuracy and reliability of this reducing technique, a WAAM case study was conducted to simulate the welding of a single layer on top of a thermally pre-conditioned wall. The wall, with dimensions of 100 mm × 50 mm × 6 mm, was welded with aluminum 6061 alloy using a plasma technique. For the initial verification study, experimental work was conducted using thermocouples to measure the temperature at selected locations. The results of these studies were then compared to thermal–mechanical FE simulations for verification and calibration. After completing the initial verification study, a snapshot matrix was built using varying process parameters, such as torch power, deposition speed, and initial temperatures [5]. A series of FE simulations were then performed using these scenarios to generate a database for building the reduced model. Nine scenarios were initially used to build the model, while three DOE cases were later carried out for the validation of the reduced models. Figure 2 shows the snapshot matrix along with the FE mesh and its typical temperature contour results during the torch passage.

Scenario No.	ROM ID	Initial Temp [°]	Power [W]	Deposition Vel. [m/s]
1	B1	25	2200	0.007
2	B2	75	2000	0.0075
3	B3	115	1800	0.006
4	B4	160	1600	0.008
5	B5	200	1500	0.007
6	B6	230	1400	0.0075
7	B7	250	1300	0.007
8	B8	300	1150	0.0065
9	B9	350	1100	0.008
10	V1	90	1950	0.006
11	V2	180	1530	0.007
12	V3	320	1120	0.0065

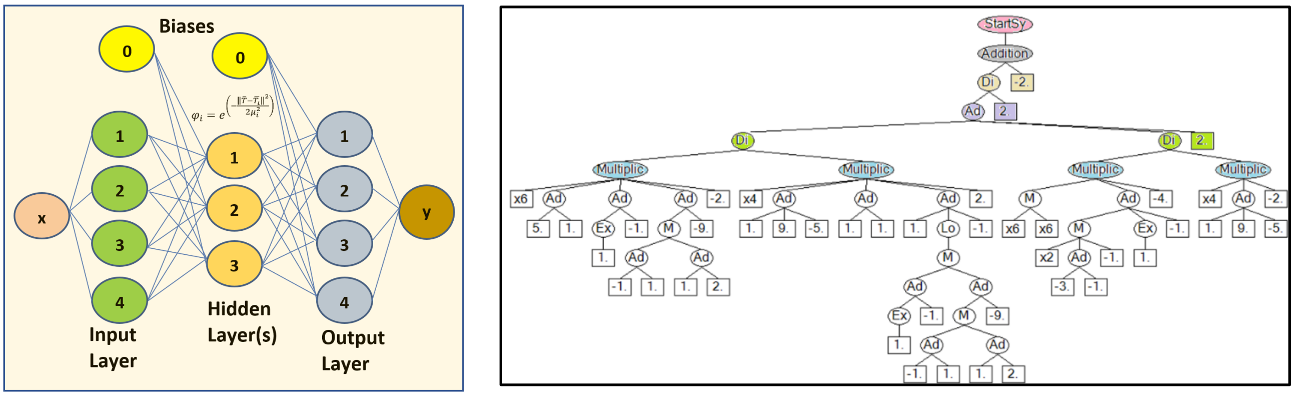
(**a**)



(**b**)

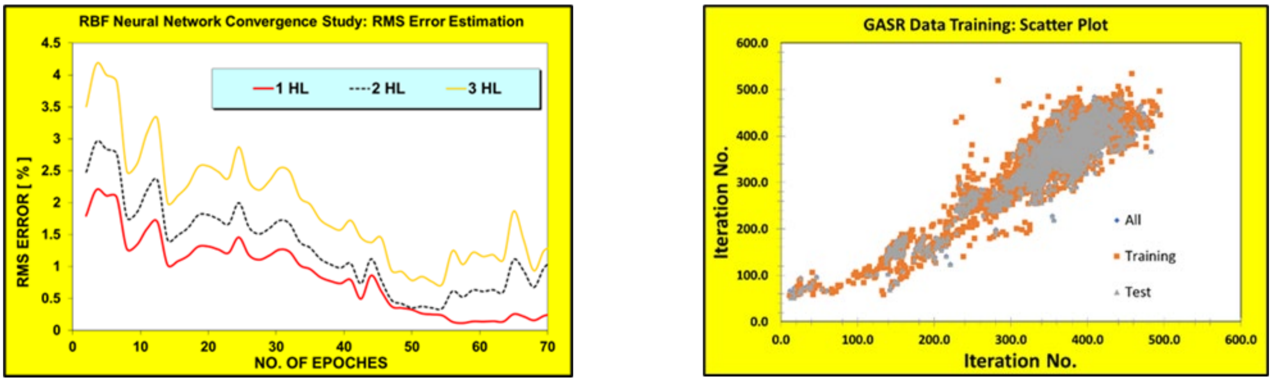
Figure 2. (**a**) Snapshot matrix for WAAM scenarios with varying process parameters; (**b**) FE mesh with selected measuring points and representative temperature contour during passage of torch. Reprinted with permission from ref. [5].

For data interpolation and training of the reduced models, two different techniques are used: genetic algorithm symbolic regression (GASR) and neural networks. The GASR technique is capable of data processing, handling, and fitting, which can be performed using available computer codes. Specifically, the academic code HeuristicLab, described in [29], was used in this research for data handling and training. GASR is a type of regression analysis that searches multi-dimensional space for mathematical expressions and operators using a genetic algorithm to find the most suitable model. For the RBF neural network, convergence is achieved using dynamic node creation [30] and an overlapping scheme for the test and training data. These techniques enhance the network's ability near the search space boundaries and even beyond (i.e., extrapolation). Since the RBF technique proved to be more practical and yielded more accurate results for this study, the main reduced model was built using a combination of SVD and RBF techniques. Figure 3 shows the neural network architecture and the GASR tree-like diagram, while Figure 4 illustrates their convergence and data training plots.



(**a**) (**b**)

Figure 3. (**a**) NN architecture for RBF interpolation; (**b**) tree-like GASR diagram for genetic algorithm fitting.



(**a**) (**b**)

Figure 4. (**a**) NN convergence of networks with different number of hidden layers, and; (**b**) scatter plot for training process of GASR.

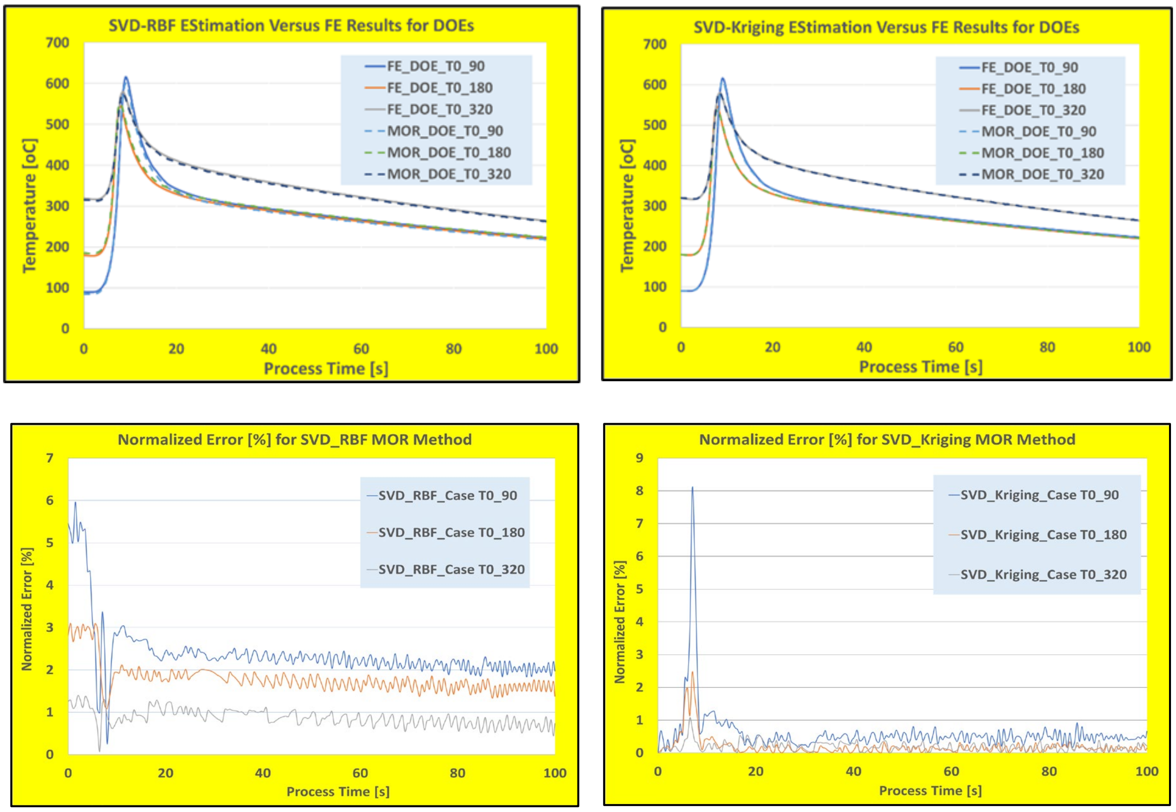
In the final part of the study, the performance of the SVD-RBF reduced model was evaluated by comparing its results with other popular model-building techniques. Five different techniques—kriging, regression, support vector machine (SVM), clustering, and inverse distance (InvD)—were used to create alternative WAAM reduced models using the available data [31–33]. The performances of these reduced models were then compared to the trained SVD-RBF model using the results from three additional DOEs. Temperature time histories for the deposition process were collected for specific nodes along the wall, and the history results were compiled for the entire simulation time. The performances of these reduced models were then compared with the SVD-RBF model by calculating the normalized error histories.

4. Discussion

The development of accurate and real-time reduced models for metallic and polymeric AM processes, with their multi-physical aspects, is challenging and requires careful consideration of data training and testing. In this study, the eigen-based SVD technique was enhanced with the interpolation and training power of RBF and neural networks to establish an efficient and accurate reduced model. The verification of the proposed reduced model showed that it can reliably produce real-time predictions of temperature and stress/strain time histories. However, due to space constraints, only the temperature results are presented here, excluding mechanical deformations, stress, and strain results. At first glance, these models appear capable of predicting responses even with small and limited-size databases and a large range of process parameter variations. However, several challenges in generating and training these types of reduced models need to be addressed before they can be solidly employed within digital twin concepts, namely:

- ➢ For WAAM processes with thermal–mechanical and multi-physical aspects, reduced models need to cope with rapidly changing data, especially for processes with a high cooling and heating rate.
- ➢ The size and variation of data within the snapshot matrix can significantly affect the prediction power of these models. Different sampling techniques should be employed to cover the entire multi-dimensional search space (e.g., Sobol and Latin Hypercube).
- ➢ To carefully verify the performance of these models, a rigorous validation criterion is required, examining performance maps at internal, near-boundary, and extreme conditions (extrapolation) of the search space.
- ➢ Although the use of neural network and GASR techniques can greatly increase the predictive power of reduced models, customized training schemes are necessary for proper data interpolation and fitting.

Figure 5 shows the estimated time history temperature for a computational node along the deposited wall for the FE simulation, trained SVD-RBF, and SVD-Kriging reduced models.



(**a**) (**b**)

Figure 5. Comparison of temperature time histories for FE versus ROMs along with their normalized errors for (**a**) SVB-RBF trained, and (**b**) SVD-Kriging ROM techniques.

The computational time for the FE simulation is approximately 2880 s per scenario (i.e., wall clock time), whereas the reduced models take only about 0.72 s to estimate the time history responses. Figure 5a compares the temperature predictions and normalized error graphs (relative to verified FE results) for the proposed SVD-RBF method across the three verification DOEs. Figure 5b presents these results for the popular SVD-Kriging reduced model. The Kriging method is one of the best interpolation techniques, using a limited set of sampled data points to evaluate spatio-temporal variables over a continuous search space.

At first glance, the calculated errors show that during periods of high heating rates (when the torch is passing over the measuring point), the proposed SVD-RBF-trained model exhibits lower error margins, indicating almost no data rate dependency. In contrast, the popular SVD-Kriging model shows higher error margins during these times but maintains a consistently lower error margin throughout the rest of the time history results. Further investigation of the data rate dependency for these two techniques is shown in Figure 6a,b, where the correlation of normalized error time histories with the rate of temperature changes (heating rate) is plotted for both techniques. Figure 6c,d illustrate the accumulated normalized error over the entire time history and the Pearson correlation index for both techniques.

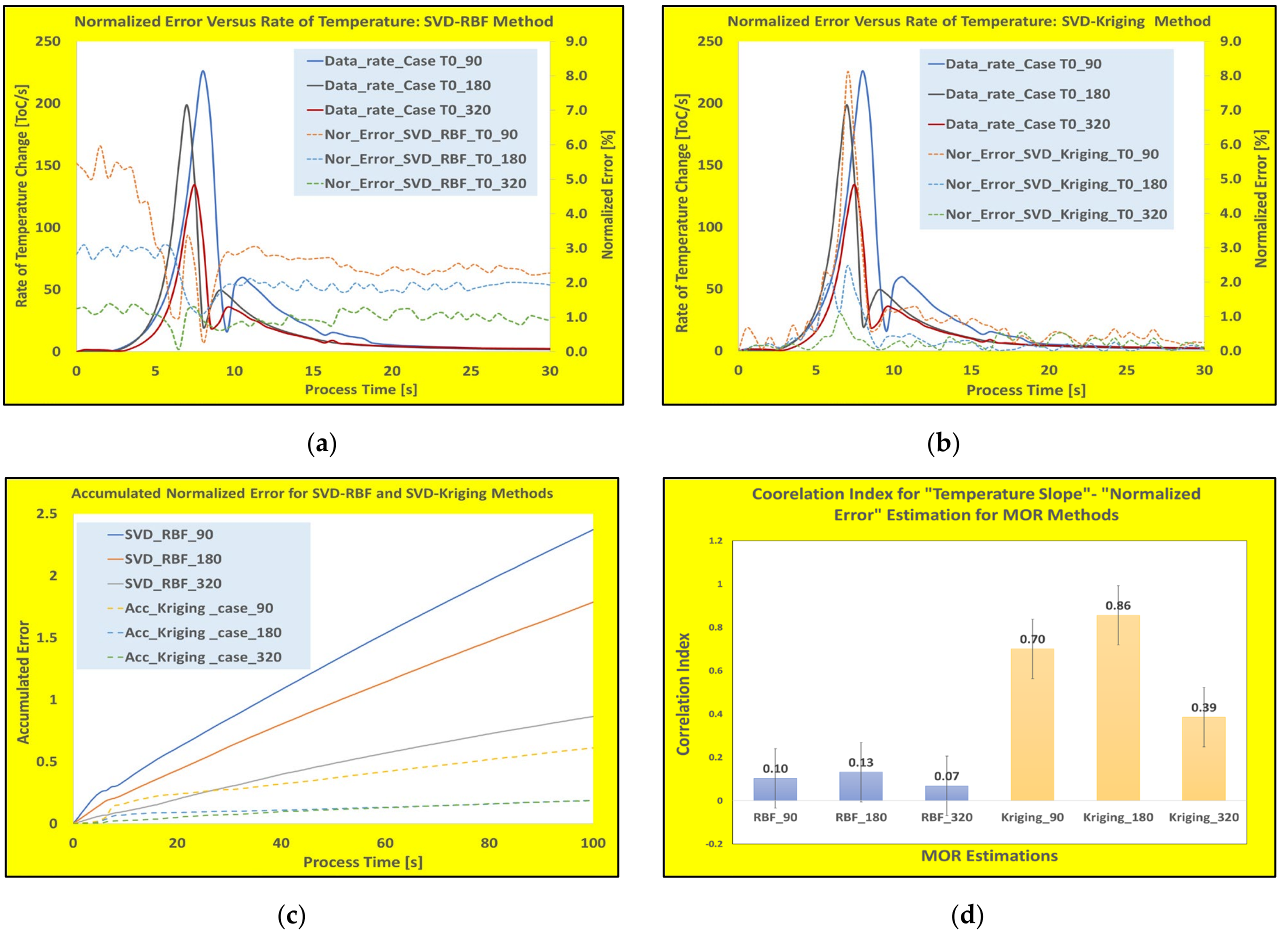
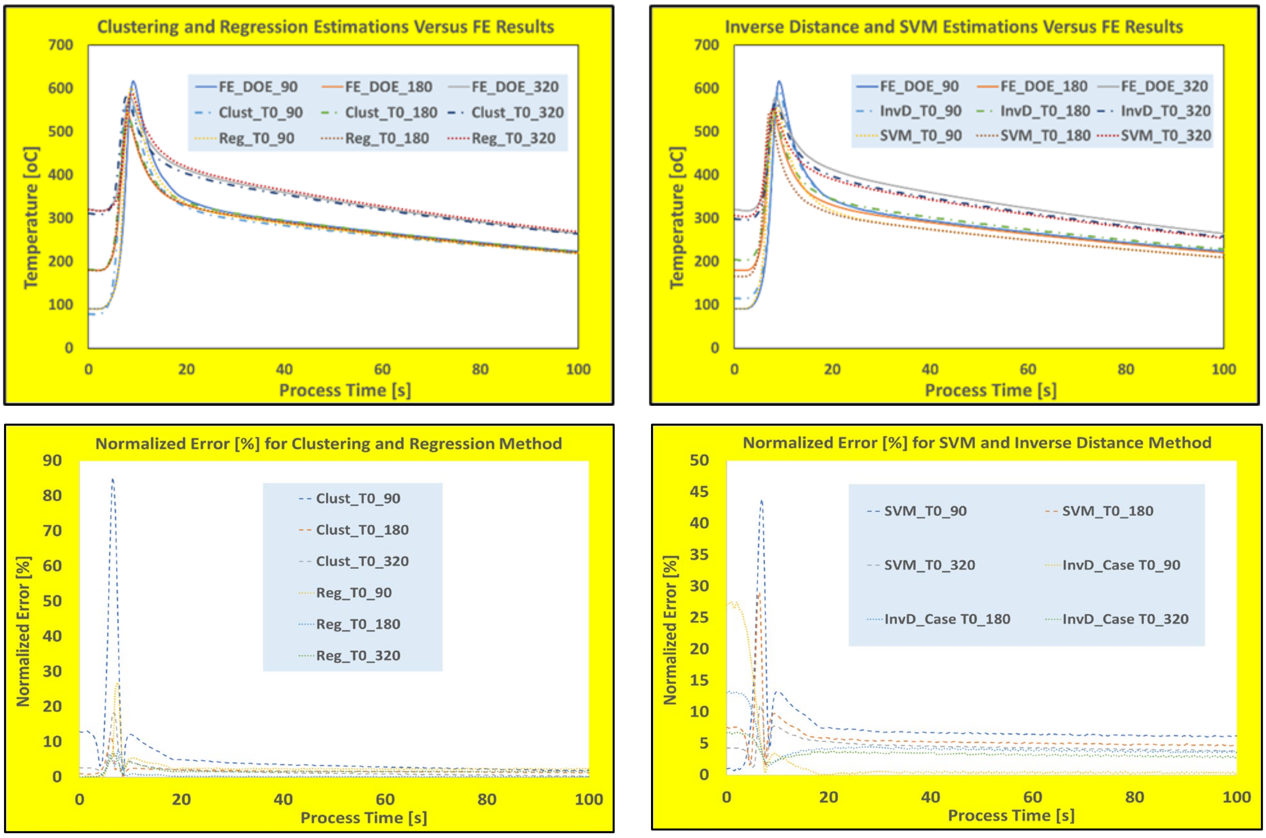


Figure 6. Rate of temperature changes (heating rate) versus normalized errors for (**a**) SVD-RBF trained, (**b**) SVD-Kriging, (**c**) accumulated normalized errors for SVD-RBF and SVD-Kriging, and (**d**) Pearson correlation index for heating rate with respect to normalized error for both techniques.

As these results indicate, while the SVD-RBF model predictions are more accurate in high-heating-rate zones (i.e., almost rate-independent), the SVD-Kriging normalized error results show strong heating rate dependency. However, the SVD-Kriging technique produces about 1% lower error margin after the torch passes over the measuring point, indicating better data fitting at low temperature changes. Therefore, due to their effective sampling and interpolation schemes, both SVD-RBF and SVD-Kriging techniques are suitable for WAAM processes with moderate heating/cooling rates. For high-heating/cooling-rate processes, it is recommended to use the trained SVD-RBF technique for model-building procedures. To demonstrate the superiority of these techniques for real-time modeling, the final part of the study is dedicated to showing the performance of other popular reduced model techniques for WAAM processes. Figure 7 presents the time history temperature predictions along with normalized error plots for clustering, regression, InvD, and SVM model-building techniques.

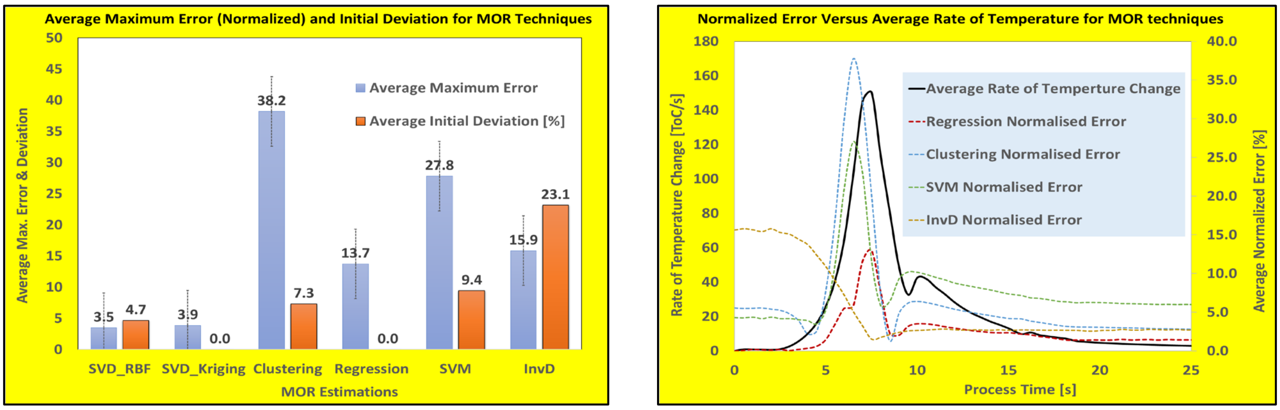


(**a**) (**b**)

Figure 7. Comparison of calculated temperature time histories for FE versus ROMs along with their normalized errors for (**a**) clustering and regression and (**b**) InvD and SVM techniques.

As clearly demonstrated in Figure 7, most of these popular reduced model techniques exhibit either heating rate dependency or an inability to fit the predefined initial conditions of the process (initial base temperature). The sampling rate for the thermal calculations of all these WAAM scenarios is half a second (2 Hz), with heating rates of up to 200 K/s, which may increase the error margin for real-time predictions. Furthermore, the heating rate dependency and deviation of initial temperatures are shown in Figure 8a, where the average maximum errors (over three DOEs) and the deviation of initial conditions (in percentages) are plotted for all six methods (including trained SVD-RBF and SVD-Kriging methods). Figure 8b shows the time history of the average normalized errors (over three DOEs) with the calculated heating rates for the four popular model-building techniques (for the first 25 s of the WAAM process).

As shown in these figures, the predictions based on these techniques exhibit either some degree of heating rate dependency or significant deviation in predicting the initial thermal conditions. Both trained SVD-RBF and SVD-Kriging models demonstrate superior overall performance, with low maximum errors and limited or zero deviation in the initial thermal conditions. Besides these two techniques, the best performance is achieved by the regression method, which shows relatively low rate dependency and no deviation in the initial conditions.



(**a**) (**b**)

Figure 8. (**a**) Average maximum errors (over three DOEs) and deviation of initial conditions (in percentages) for six ROM techniques; (**b**) correlation of heating rates with normalized errors for clustering, regression, InvD, and SVM techniques.

5. Conclusions

The reduced models and their fast and real-time prediction potentials can introduce opportunities for further developments toward the digitalization of additive manufacturing processes. In this study, different reduced model-building techniques were examined to evaluate their performance for optimizing and controlling WAAM processes. Initially, a brief description of the numerical simulation techniques for these dynamic processes was presented, along with some technical aspects of the new evolving-domain and dynamic-mesh technique. Furthermore, a brief mathematical description of popular reduced model-building techniques was provided, and the impacts of hybrid ML-ROM trends on the accuracy and reliability of the reduced models were discussed. In the following sections, the ROM description of a practical case study for the WAAM process was elaborated, examining its critical aspects of accuracy, heat rate dependency, and thermal initial boundary conditions.

There are challenges related to the accuracy of ROM techniques for industrial process modeling, including data extrapolation, initial boundary fitting, and rate dependency (e.g., for high cooling/heating rates). Additionally, for many of these processes with a large number of input parameters, the quality and size of the database can greatly influence the accuracy of predictions. The results presented here indicate that for the two developed SVD-RBF and SVD-Kriging methods, both the issues of heating/cooling rate dependency and modeling of initial boundary conditions can be partially resolved. However, with further ML training (e.g., NN data training), even better results can be achieved for high-gradient heating/cooling processes, where almost no rate dependency is observed in the predictions. Detailed investigations into the best performance “solver-interpolator” combinations have shown that some well-known solvers (e.g., SVD, POD) can produce more accurate results when used with the right interpolating schemes. Furthermore, there is potential to use ML for training and improving data representations within the hybrid ROM-ML framework to achieve superior accuracy and agility. This will promote the advancement of more adapted MOR-ML technologies for AM processes, where the features of both ML and MOR can be combined for more accurate reduced models.

As a final statement, this study aims to encourage the use of ROM-ML schemes for the real-time modeling of dynamic material processes like WAAM. Although these hybrid reduced models are not intended to replace other experimental or detailed numerical process simulations, they can be valuable assets within the digitalization framework for control and optimization. The opportunity to apply the hybrid modeling scheme to other multi-physical aspects of these processes, such as mechanical stress/strain and deformation/warping, will be explored in future publications.

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