Article

An Incentive-Based Optimization Approach for Load Scheduling Problem in Smart Building Communities

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Abstract: The impact of load growth on electricity peak demand is becoming a vital concern for utilities. To prevent the need to build new power plants or upgrade transmission lines, power companies are trying to design new demand response programs. These programs can reduce the peak demand and be beneficial for both energy consumers and suppliers. One of the most popular demand response programs is the building load scheduling for energy-saving and peak-shaving. This paper presents an autonomous incentive-based multi-objective nonlinear optimization approach for load scheduling problems (LSP) in smart building communities. This model's objectives are three-fold: minimizing total electricity costs, maximizing assigned incentives for each customer, and minimizing inconvenience level. In this model, two groups of assets are considered: time-shiftable assets, including electronic appliances and plug-in electric vehicle (PEV) charging facilities, and thermal assets such as heating, ventilation, and air conditioning (HVAC) systems and electric water heaters. For each group, specific energy consumption and inconvenience level models were developed. The designed model assigned the incentives to the participants based on their willingness to reschedule their assets. The LSP is a discrete–continuous problem and is formulated based on a mixed-integer nonlinear programming approach. Zoutendijk's method is used to solve the nonlinear optimization model. This formulation helps capture the building collaboration to achieve the objectives. Illustrative case studies are demonstrated to assess the proposed model's effect on building communities consisting of residential and commercial buildings. The results show the efficiency of the proposed model in reducing the total energy cost as well as increasing the participants' satisfaction. The findings also reveal that we can shave the peak demand by 53% and have a smooth aggregate load profile in a large-scale building community containing 500 residential and commercial buildings.

Keywords: demand response; load scheduling; incentives; inconvenience; mixed-integer nonlinear programming; smart building community; multi-objective optimization

1. Introduction

Energy consumption has grown significantly in the last decade, and the immediate need to develop and implement intelligent models to efficiently use energy resources is required. The building sector, including both residential and commercial buildings, presents around 43% of the United States total energy consumption, 54% of natural gas consumption, and 71% of national electricity consumption. The need to manage energy consumption in the building sector is a critical activity to reduce peak demand. Appropriate building asset rescheduling is a practical solution for load shifting during peak hours that helps the grid, especially when facing the growing penetration of plug-in electric vehicles (PEV).

The main problem with these programs is that although they can be financially beneficial to energy consumers, they may affect their thermal comfort and hinder their ability to

operate assets more conveniently. To deal with the former problem, researchers have been developing new models to minimize the deviation between the original operation time and the optimal operation time for buildings' assets while maximizing human comfort. To address the latter, end-users can be incentivized for participation by registering their assets for a demand response (DR) program and letting the utility company manage the operation time of those assets, considering the submitted start and end time by the users, in order to get incentives in the form of utility bill reduction. This results in a beneficial trade-off for both energy consumers and suppliers, making building asset rescheduling a practical solution to reduce the peak demand.

The emergence of smart homes (SH) is a major step towards reaching the energy efficiency and demand response goals discussed above. Implementing DR programs in SHs by adjusting the on–off status of electrical assets and smart control of thermal assets affects the electricity peak reduction. Gelazanskas and Gamage categorized DR programs into two different groups: price-based and incentive-based [1]. Price-based programs aim to motivate participating customers to alter their consumption patterns in response to time-varying electricity prices. In contrast, incentive-based programs aim to reduce customers' energy consumption by providing fixed or time-varying incentives considering power system stress periods [2]. DR programs can also be categorized based on their modeling approach. Farzan et al. illustrated that a top-down approach is often employed for long-term models, while short-term models usually use a bottom-up approach [3]. Individual assets of each building are not identified as a contributor in a top-down demand response approach. On the other hand, the contribution of individual assets of each building is considered in the aggregate load profile in a bottom-up approach. So, asset rescheduling problems should be categorized into short-term models that employ a bottom-up approach.

1.1. Literature Review

Many researchers have focused on developing different models to improve demand response programs and building energy management to deal with the dramatic expansion of electricity demand in the building sector. Missaoui et al. [4] presented a global model-based anticipative building energy management system to control household energy. This model was able to optimize a compromise between user comfort and energy cost that took into account occupant expectations and physical constraints like energy price and power limitations. In [5], the authors developed hybrid data-driven approaches to capture building heat transfer nonlinearities to predict building zone-level average temperature response. In another article, Ghofrani et al. proposed a methodology to assign heating, ventilation, and air conditioning (HVAC) operation planning schemes for connected buildings with the objective of energy-saving and load leveling [6]. In [7], a brief overview of the architecture and functional modules of a smart home energy management system (HEMS) was presented and the advanced HEMS infrastructures and home appliances in smart houses were thoroughly analyzed. A systematic review of the scientific literature on electric load management was done in [8] in order to describe and summarize the most relevant terminologies in this field. In [9], a building energy and comfort management model was presented, through occupant behavior pattern detection, based on a large-scale environmental sensor network.

One of the most popular methods to manage building energy is the rescheduling of asset usage. Recently, many researchers have drawn their attention to reschedule building assets in order to reduce their total energy costs, proposing frameworks for appliance scheduling based on cost minimization. In their model, each user in the system would find an optimal start time and operating mode for the appliances in response to the varying electricity prices [10–12]. Meanwhile, other researchers have taken consumer convenience and satisfaction into account while minimizing the total energy cost. Sou et al. [13] presented a mixed-integer linear programming model to minimize the electricity cost of smart home appliances' scheduling problems while satisfying technical operation constraints and consumer preferences. Setlhaolo and Xia [14] introduced a nonlinear integer optimization

model for the optimal scheduling of appliances. They introduced inconvenience to appliance scheduling's general problem to measure the baseline and optimal schedule disparity. In [15], a home power management system was proposed to minimize electricity costs and reduce high peak demand while maintaining user comfort. Rasheed et al. proposed an optimization algorithm for electricity bill minimization of the residential user in the time-of-use pricing models and peak shaving of the demand curve. For this purpose, they considered three types of smart appliances: without delay, a delay of one hour, and a delay of five hours. Özkan [16] proposed a real-time appliance-based home power management system to reduce electricity costs, improve energy efficiency, and consider user comfort. In [17], the authors proposed a cluster-based approach for building assets rescheduling in a smart building community to minimize the total energy cost as well as the peak-to-average ratio.

A limited number of papers have considered the convenience level as an objective of their problem. In most of the studies related to asset rescheduling problems, consumer satisfaction is included in the constraints. Setlhaolo et al. [18] presented a mixed-integer nonlinear optimization model based on the time-of-use electricity tariff. Besides total energy cost, they also included the inconvenience that comes from the new schedule to the objective function. In [19], the authors extended their previous work and added carbon emission minimization to their model and solved it for multiple households. In [20], the authors developed an occupancy-based nonlinear optimization algorithm for building cooling systems control to reduce energy consumption and costs considering human thermal comfort efficiently. Yahia and Pradhan [21] proposed a binary integer linear programming model to solve the residential load scheduling problem while considering consumer's preferences. Muhsen et al. [22] presented a multi-objective optimization differential evolution algorithm to obtain a set of optimal solutions by minimizing the cost and peak of a load simultaneously. Then, a multi-criteria decision-making method was used for sorting the optimal solutions' set, from the best to worst, to enable the customer to choose the appropriate operating time. In [23], the authors proposed a multi-objective mixed-integer linear programming (MILP) model to minimize electricity costs, consumer inconvenience, and electrical peak load. In [24], a preference-based demand response model was presented based on real-time electricity price to solve the problem of optimal residential load management. The purpose of their model is to minimize the costs associated with energy consumption, the inconvenience caused to consumers, and environmental pollution.

Thermal assets of buildings are considered in a few studies in the building appliance scheduling problem literature. In addition, some of the studies that take into account thermal assets have found an optimal on-off status for these assets. However, assigning on-off status to a thermal asset does not usually maintain the building's occupants' thermal comfort for bigger time-steps. It can neither be applicable for large residential or commercial buildings to turn on or off a thermal asset for a couple of hours. So, there is a need to consider these assets as power shiftable assets, in which their power can be changed based on the electricity price and consumer preference while satisfying user comfort. Caprino et al. [25] described an approach for the peak shaving problem that leverages the real-time scheduling discipline to coordinate the activation/deactivation of a set of loads. Their study focused on some specific appliances such as HVAC systems, washing machines, dishwashers, and electric ovens. In [26], the authors demonstrated an optimal household appliance scheduling problem with a battery as an energy storage system under the time-of-use electricity tariff. They found the optimal on–off status for all electronic and thermal assets and achieved cost-saving, peak shaving, and valley filling through load shifting. Shirazi and Jadid [27] proposed an automatic and optimal residential energy consumption scheduling technique to minimize the energy costs and the inconvenience for the operation of both electrical and thermal assets in a smart home environment. Zhu et al. [28] presented an improved cooperative optimization algorithm for household appliance scheduling. They considered both thermal and electronic assets in their model and formulated the problem as a discrete–continuous nonlinear problem.

There are limited studies that consider scheduling problems for commercial buildings. In [29], a multi-objective demand-side management (DSM) solution based on an integer genetic algorithm was presented to benefit both utilities and consumers. The authors proposed a load shifting technique to schedule controllable appliances of commercial and industrial consumers at various hours of the day. Yalcintas et al. [30] presented a load shifting and scheduling model to provide several measures to shift electricity usage to off-peak times for commercial and industrial customers when electricity prices are lower. Vaziri et al. [31] presented a bi-objective formulation to minimize energy costs and dissatisfaction by scheduling the activities that happen in a hospital by considering the hospital's specific constraints and limitations.

One of the reasonable concerns about building asset scheduling formulations is considering consumers' willingness to participate in these programs. For this purpose, assigning incentives to the customers is a viable solution that can tempt them to register for this DR program. There are limited models that take into account incentives and their objective functions. The authors in [21] presented a model that considered the incentives offered to users for participating in a demand response program. Paudyal and Ni [32] proposed an incentive-based demand management scheme for schedulable appliances in a residential community. This compensation scheme was adopted for the shifting of task-based appliances based on the level of inconvenience.

1.2. Study Contributions

Based on the above-mentioned literature review, most building asset rescheduling models only consider time-shiftable assets and ignore the impact of thermal assets on energy-saving and peak-shaving. They also employ models for single residential buildings, single commercial buildings, or multiple residential buildings, and none of them consider a community consisting of multiple residential and commercial buildings together.

In this paper, an autonomous model is presented to find the optimal schedule of time-shiftable assets and manage power-shiftable assets' energy consumption (e.g., thermal assets). This model is an incentive-based model in which the users get incentivized for participating in this DR program. The users should register their desired assets along with their characteristics and their preferred start and end time for the operation. They will get incentives from the utility company in the form of utility bill reductions.

The other aspect of the proposed model is to consider human comfort. Deviation from the preferred operation time of an asset and the optimal proposed time may inconvenience the user. In addition, the difference between the buildings' desired indoor temperature and the actual indoor temperature is another inconvenience source for the DR program participants. The proposed model takes into account these inconveniences and tries to minimize them.

This problem is formulated as a discrete–continuous nonlinear problem and solved for a connected community consisting of some connected residential and commercial buildings. This mixed-integer nonlinear programming (MINLP) model is developed to consider all inconvenience sources (e.g., appliance rescheduling, thermal discomfort) and assigned incentives from the utility companies. For solving the problem, the method of feasible directions by Zoutendijk is used to deal with the nonlinearities. The problem is solved for both small-scale and large-scale building communities with three configurations: (i) buildings operate as usual, (ii) buildings minimize their objectives separately without collaboration, and (iii) buildings collaborate to minimize the whole community's objectives. Based on what was mentioned, the contributions of this work are listed as follows:

- The impact of thermal assets on the building energy consumption and human comfort are considered in the formulation.
- The user inconvenience levels for time-shiftable and thermal assets are considered in the formulation.
- The DR program participants' incentives are included in the optimization model that helps them get incentivized based on the amount of energy they are willing to shift.

- For solving the nonlinear optimization problem, the method of feasible direction is discussed and employed.
- A complete data set for residential and office buildings' appliances is prepared that can be used by other researchers in the field.

2. Problem Statement and Preliminaries

In most demand response programs, the focus is on interactions between the utility company and each customer. For example, in the direct load control (DLR), the utility or an aggregator (which is managed by the utility) can remotely control the operations and energy consumption of specific appliances in a household [33]. The most important concern with these programs is the users' privacy, which can be a barrier to run programs like this. Instead of concentrating on single users, it is assumed that the building owners in a community participate in a DR program in which they are enabled to interact with the other users as well as the utility company. Figure 1 shows how the buildings are connected in a smart community.
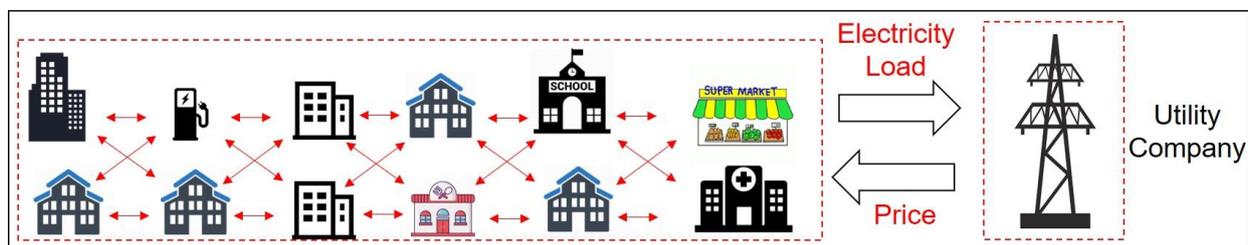

Figure 1. The interaction between a building and the other end-users as well as the utility company in a smart connected community.

In this problem, we will propose a DR program in which peak demand will be reduced by asset rescheduling in different buildings in a smart connected community. The load shifting becomes more critical as PEVs are being used more and more, and they are gradually changing the shape of the aggregate electricity load profile. Unbalanced conditions resulting from an increasing number of PEVs could result in degradation of power quality, increased harmonics and voltage problems, and increasing line losses, and they also could potentially damage utility and customer equipment [34]. The asset rescheduling removes the distributors' pressure in the peak time and shifts the energy to off-peak hours. It also helps the customers to save money on peak demand charges.

This connected community has two types of buildings, namely, residential apartments and office buildings. Considering the fact that each building has a different set of assets, they are categorized into two general groups: shiftable and non-shiftable assets. Non-shiftable assets are the ones that cannot participate in a DR program because the users need them at different times of the day at their convenience. On the other hand, shiftable assets can be operated at different times, considering their preferences. These assets include flexible time-shiftable assets, including electronic appliances and PEV charging facilities, and power-shiftable assets, such as thermal assets. Electronic appliances are assets, such as dishwashers, cloth washing machines, and cloth dryers, which can be operated during a given range, not strictly at a specific time. PEV charging facilities in residential or office buildings have a couple of charging outlets that can be scheduled at different times, considering the specific constraints for PEV charging. Thermal assets, including HVAC systems and electrical domestic water heaters, can also be managed to provide human thermal comfort. They should also be programmed to consume their required energy based on the aggregate electricity load profile to reduce peak demand.

There are significant differences between these two groups (flexible time-shiftable assets and power-shiftable assets). The optimal on-off status of flexible time-shiftable assets can be decided by using binary decision variables. For thermal assets, their power consumption will be calculated based on the thermodynamics equation considering the

human comfort level. Hence, the power of thermal assets is a continuous decision variable. Therefore, the problem is discrete–continuous and formulated as a mixed-integer nonlinear model. The proposed model aims to maximize the incentives a customer will get, minimize their inconvenience level, and minimize total energy cost. The flowchart of the proposed methodology is shown in Figure 2.
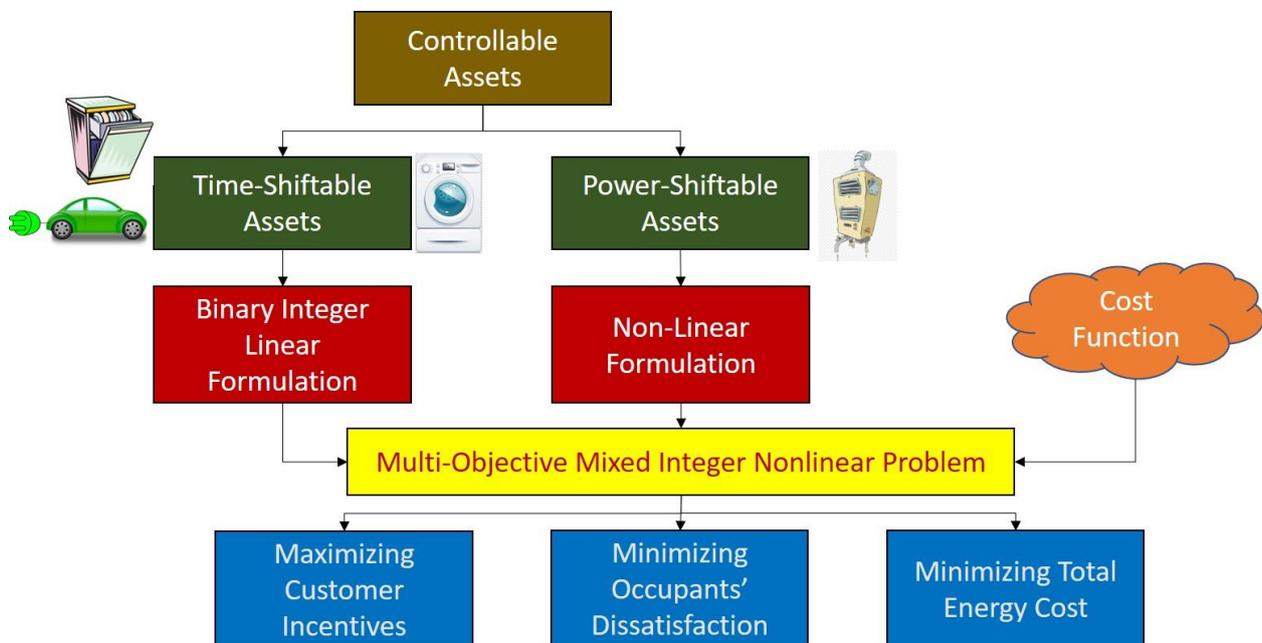

Figure 2. The flowchart of the proposed methodology.

3. Problem Formulation

In this section, we present the mathematical model for building assets rescheduling as a discrete–continuous problem. We have three objectives in this problem: total energy cost minimization, customers' incentives maximization, and inconvenience level minimization. Because of the quadratic cost function and a nonlinear constraint for the thermal inconvenience, our problem is nonlinear. Therefore, this problem is formulated as a multi-objective mixed-integer nonlinear problem. In the following, first, the mathematical models for thermal assets and power-shiftable assets are presented. Then, the model for flexible time-shiftable assets is presented. The objective functions are then discussed, and finally, the solution method is shown.

3.1. Time-Shiftable Assets

Equation (1) illustrates the disparity between the optimal on–off status for a time-shiftable appliance ($x_{i,j}(t)$) and the consumer's preferred on–off status of that asset ($u_{i,j}(t)$). For this purpose, the binary variable $z_{i,j}(t)$ is defined for inconvenience level, which equals one if there is a mismatch between the preferred schedule and the optimal schedule for deferrable asset i of building j at time t.

$$z_{i,j}(t) = \left| x_{i,j}(t) - u_{i,j}(t) \right| = \begin{cases} 1, & \text{if } x_{i,j}(t) \neq u_{i,j}(t) \\ 0, & \text{if } x_{i,j}(t) = u_{i,j}(t) \end{cases} \tag{1}$$

The second constraint for the time-shiftable appliance makes sure that the scheduled-ON time slots for deferrable asset i of building j are within the operating time window [$\alpha_{i,j}$,

$\beta_{i,j}$] and are equal to the required number of time slots to execute the time-shiftable asset operation i of building j ($\omega_{i,j}$).

$$\sum_{\alpha_{i,j}}^{\beta_{i,j}} x_{i,j}(t) \geq \omega_{i,j} \tag{2}$$

Equation (3) guarantees the total energy associated with the optimal asset schedule does not exceed the amount that the consumer j is willing to participate in one day (H_j).

$$\sum_{t=t_s}^{\tau} \sum_{i=1}^{a} x_{i,j}(t) S_{i,j} \leq H_j \tag{3}$$

Equation (4) allows consumers to earn an incentive only when they switch off their electronic assets during peak times. In this equation, $w_{i,j}(t)$ is a binary number for incentives, which equals one if consumers earn incentives because they switched off asset i of building j at time t, against their preference, or is zero otherwise.

$$x_{i,j}(t) - u_{i,j}(t) \geq w_{i,j}(t) \tag{4}$$

Equations (5)–(8) ensure continuous, uninterrupted operation of the time-shiftable assets and that the assigned time slots for each appliance are successive. For assets that may be operated more than once per day (i.e., oven operation for lunch and dinner), the appliance can be treated as two separate appliances. A new auxiliary binary decision variable ($y_{i,j}(t)$) is used to state that the operation of asset i of building *j*. If $y_{i,j}(t) = 0$, the operation of this asset is just completed during time slot t. Hence, the corresponding $x_{i,j}(t)$ must be zero.

$$x_{i,j}(t) \leq y_{i,j}(t), \forall t \tag{5}$$

$$x_{i,j}(t) - x_{i,j}(t-1) \geq y_{i,j}(t) - 1, \forall t \geq 2 \tag{6}$$

$$y_{i,j}(t-1) \geq y_{i,j}(t), \forall t \geq 2 \tag{7}$$

$$1 - x_{i,j}(t) \geq y_{i,j}(t), \forall t \tag{8}$$

3.2. Thermal Assets

3.2.1. HVAC Systems

In this paper, we model HVAC systems to maintain the building's human comfort level using hygrothermal equations. In this regard, the dwelling temperature $T_{in}(t+1)$ at each time step t+1 is calculated using the dynamic energy balance equation. In this model, we relate the indoor ($T_{in}(t)$) and the outdoor temperature ($T_{amb}(t)$) between two successive time steps using Equation (9) [35].

$$T_{in}(t+1) = \varepsilon T_{in}(t) + (1-\varepsilon)\left(T_{amb}(t) \pm \frac{\varphi P_{HVAC,j}(t)}{K}\right) \tag{9}$$

In Equation (9), the plus (+) and minus (−) symbols are used for heating and cooling air-conditioning, respectively. Also, ε is the factor of inertia, φ is the coefficient of performance, and K is the thermal conductivity of the system (kW/°C). For using this equation, we need to make some assumptions as follows: (i) the air mass and the shell and other contents of the HVAC system have a single total thermal mass, (ii) the control of humidity is neglected, (iii) internal heat sources are neglected, (iv) the cycling effect of the thermostat is neglected [35]. This equation was used in [36,37]. In Equation (9), ε can be found as follows:

$$\varepsilon = \exp\left(\frac{-K \Delta t}{mc}\right)$$

where Δt is the control period of the problem, and mc is the total thermal mass (kWh/°C). Equation (16) is a modified version of a more general model that can be found in [38].

To be able to use this equation for modeling HVAC systems, some assumptions should be made, including (i) the shell, the air mass, and the other contents of the space have a total thermal mass; (ii) no independent thermal storage is coupled to the main heating or cooling equipment; (iii) the control of humidity is neglected; and (iv) internal heat sources are neglected. Furthermore, it should be considered that the power consumption of HVAC systems is in a predefined range, as is shown in Equation (10).

$$P_{HVAC_{min}} \leq P_{HVAC,j}(t) \leq P_{HVAC_{max}} \tag{10}$$

3.2.2. Electric Water Heater Systems

In this section, a single-element electric water heater (EWH) system for use in residential and small office buildings was formulated. This model is based on the energy flow analysis as follows [39]:

$$T_{EWH}(t+1) = T_{EWH}(t) \exp\left(-\frac{\Delta t}{\gamma\,C}\right) + \gamma\left(G\,T_{amb}(t) + F\,T_{iw} + P_{EWH,j}(t)\right)\left(1 - \exp\left(-\frac{\Delta t}{\gamma\,C}\right)\right) \tag{11}$$

In Equation (11), $T_{EWH}(t)$ is water temperature inside EWH at time t, T_{iw} is the incoming water temperature, $T_{amb}(t)$ is the ambient temperature, $P_{EWH,j}(t)$ is the heating element power for the electric water heater of building j at time t, and R is the tank insulation thermal resistance. The coefficients C, G, F, and γ can be calculated by:

$$C = V_{tank}\,\rho_{water}\,Cp_{water}\quad G = \frac{A_{tank}}{R}\quad F = \rho_{water}\,\dot{m}_{water}\,Cp_{water}\quad \gamma = \frac{1}{F+G}$$

where V_{tank} is the capacity of the tank, ρ_{water} is the density of water, Cp_{water} is the specific heat of water, A_{tank} is the surface area of the tank, and $\dot{m}_{water}$ is the flow rate of water. Equation (12) illustrates that the power consumption of EWH systems is in a predefined range:

$$P_{EWH_{min}} \leq P_{EWH,j}(t) \leq P_{EWH_{max}} \tag{12}$$

3.2.3. The Inconvenience Level for Thermal Assets

Equation (13) illustrates the inconvenience level for thermal assets, which is defined as a function of the difference between the desired temperature and actual temperature.

$$v_{i,j}(t) = \begin{cases} \dfrac{\left(\Delta T_{i,j}(t)\right)^2}{\psi_{i,j}}, & \text{if } \Delta T_{i,j}(t) \neq 0 \\ 0, & \text{if } \Delta T_{i,j}(t) = 0 \end{cases} \quad ;\ (i \in \{HVAC, EWH\}) \tag{13}$$

In Equation (13), $\psi_{i,j}$ is the maximum allowable difference between the desired temperature and actual temperature. $v_{i,j}(t)$ is crucial in this problem to make sure human comfort constraint is respected. $\psi_{i,j}$ is assumed 25 for hot water temperature and 4 for building indoor temperature. Based on this assumption and Equation (13), $v_{i,j}(t)$ would be a number between 0 to 1. Equation (13) demonstrates that if the difference between the actual temperature and the desired one is zero, the inconvenience level would be zero. If this difference becomes closer to the maximum allowable difference, $v_{i,j}(t)$ will converge to 1. The value of this difference can be found by Equation (14) for the HVAC systems and Equation (15) for EWH systems:

$$\Delta T_{HVAC,j}(t) = |T_{des}(t) - T_{in}(t)| \tag{14}$$

$$\Delta T_{EWH,j}(t) = |T_{w,des}(t) - T_{EWH}(t)| \tag{15}$$

In these equations, $T_{des}(t)$ is the desired temperature of the indoor of the building and $T_{w,des}(t)$ is the desired temperature for the hot water. Based on Equations (13)–(15), the values of $\Delta T_{HVAC,j}(t)$ and $\Delta T_{EWH,j}(t)$ will be between 0 to 2 and 0 to 5, respectively. This will meet the occupants' thermal comfort level in terms of building indoor temperature and hot water temperature.

3.3. Objective Functions

This problem is formulated as a multi-objective problem to maximize customers' incentives and minimize their dissatisfaction and utility bills. In this problem, we have three objectives, namely, maximization of user incentives, minimization of total energy cost, and minimization of human discomfort.

Equation (16) illustrates the objective that aims at maximizing the total incentives the customers can get by rescheduling their assets from their preferred time to another time.

$$\text{Max} \left\{ \sum_{t=t_s}^{\tau} \sum_{j=1}^{N} \sum_{i=1}^{A_j} \left(w_{i,j}(t) \, d(t) \right) \Delta t \right\} \tag{16}$$

Additionally, the inconvenience level is minimized in Equation (17) by considering two types of dissatisfactions: (i) the deviation from the desired building indoor temperature, and (ii) the inconvenience for the customers that occurs by operating their flexible assets and charging their PEVs outside of their preferred times.

$$\text{Min} \left\{ \sum_{t=t_s}^{\tau} \sum_{j=1}^{N} \sum_{i=1}^{A_j} \left(v_{i,j}(t) + z_{i,j}(t) \right) \right\} \tag{17}$$

Equation (18) shows the third objective function that minimizes the energy costs on a specific time horizon.

$$\text{Min} \left\{ \sum_{t=t_s}^{\tau} \sum_{j=1}^{N} f(L(t)) \right\} \tag{18}$$

In Equation (18), $f(L(t))$ is the cost function that indicates the cost of electricity generation and distribution at each hour. In this study, a quadratic cost function is used to control the load demand better. Equation (19) describes the cost function of this model.

$$f(L(t)) = \mu_1 \left(L_j(t) \right)^2 + \mu_2 \, L_j(t) + \mu_3 \tag{19}$$

In Equation (19), coefficients μ_1, μ_2 and μ_3 are non-negative values to help calculate the energy cost where $\mu_1 > 0$ and $\mu_2, \mu_3 \geq 0$. The total load for building *j* is obtained as:

$$L_j^t = \sum_{i=1}^{A_j} \left[x_{i,j}(t) \, S_{i,j} + P_{i,j}(t) \right] \tag{20}$$

This equation helps calculate the total load for each building by adding up the energy consumption of time-shiftable and thermal assets.

3.4. The Optimization Model

We have three objective functions in this problem, namely, electricity costs, customer incentives, and the inconvenience level. To solve this problem, we can write objective functions to show all of them in a single function. As mentioned before, one of the main contributions of this work is to consider the collaboration between buildings. For this purpose, three different cases are defined in this study. The first one is the baseline case, such that buildings are working without implementing any smart models. In the second case, each building tries to minimize its own objectives without considering other buildings in the community. Equation (21) shows the final objective function for this case.

$$\sum_{j=1}^{N} \text{Min} \left\{ \left(\sum_{t=t_s}^{\tau} f(L(t)) - \sum_{t=t_s}^{\tau} \sum_{i=1}^{A_j} \left(w_{i,j}(t) \, d(t) \right) \Delta t \right) + \lambda \sum_{t=t_s}^{\tau} \sum_{i=1}^{A_j} \left(v_{i,j}(t) + z_{i,j}(t) \right) \right\} \tag{21}$$

In contrast, Equation (22) describes the third case that illustrates the cost and inconvenience functions when buildings collaborate to minimize the community's energy costs and inconvenience level.

$$\text{Min}\left\{\sum_{t=t_s}^{\tau}\sum_{j=1}^{N}\left(f(L(t)) - \sum_{i=1}^{A_j}\left(w_{i,j}(t)d(t)\right)\Delta t\right) + \lambda\sum_{t=t_s}^{\tau}\sum_{j=1}^{N}\sum_{i=1}^{A_j}\left(v_{i,j}(t) + z_{i,j}(t)\right)\right\} \quad (22)$$

In these two equations, λ is a non-negative penalty coefficient for the inconvenience objective function. This value determines the importance of total energy cost and incentives versus the occupant's inconvenience level. So, objective Functions (21) and (22) can be written as: $\text{Min}\left\{f_{\cos t} + \lambda f_{\text{inconvenience}}\right\}$.

3.5. The Solution Approach

This scheduling problem is a nonlinear optimization problem due to Equations (13) and (19). There are some methods that we can use to solve this problem. One of these methods is the method of feasible directions (we call it Zoutendijk's method). This method generates an improving feasible direction at each iteration and then optimizes along that direction. A complete explanation and proof of this method can be found in [40]. In order to use Zoutendijk's method, we should have a nonlinear objective function and linear constraints. So, the objective functions for each case should be written as:

$$\sum_{j=1}^{N}\text{Min}\left\{\left(\sum_{t=t_s}^{\tau}f(L(t)) - \sum_{t=t_s}^{\tau}\sum_{i=1}^{A_j}\left(w_{i,j}(t)\,d(t)\right)\Delta t\right) + \lambda\sum_{t=t_s}^{\tau}\sum_{i=1}^{A_j}\left(\frac{\left(\Delta T_{i,j}\,(t)\right)^2}{\psi_{i,j}} + z_{i,j}(t)\right)\right\} \quad (23)$$

$$\text{Min}\left\{\sum_{t=t_s}^{\tau}\sum_{j=1}^{N}\left(f\left(L(t)\right) - \sum_{i=1}^{A_j}\left(w_{i,j}(t)\,d(t)\right)\Delta t\right) + \lambda\sum_{t=t_s}^{\tau}\sum_{j=1}^{N}\sum_{i=1}^{A_j}\left(\frac{\left(\Delta T_{i,j}\,(t)\right)^2}{\psi_{i,j}} + z_{i,j}(t)\right)\right\} \quad (24)$$

The difference between the above equations and Equations (21) and (22) is that we replaced $v_{i,j}(t)$ with $\frac{\left(\Delta T_{i,j}(t)\right)^2}{\psi_{i,j}}$ to have all nonlinearities of the problem in the objective function. Now, we have a nonlinear objective function with linear constraints, and we can use Zoutendijk's method to solve it. Equations (23) and (24) can be solved as an MINLP problem subject to Equations (1)–(12), (14) and (15).

4. Case Study Design

The proposed model is validated with illustrative case studies. In these cases, a combination of residential and commercial buildings were investigated. These examples analyzed the MINLP model for connected buildings in a community. The time-step (Δt) for the asset rescheduling problem was arbitrarily selected to be 10 min, meaning that one day, which was the time horizon for solving the scheduling problem in this work, was divided into 144 time-slots.

4.1. Case Scenarios

This paper considers two building communities to evaluate our proposed model's effectiveness on energy consumption and peak demand. The two communities differed in terms of the number of buildings. The first one is a small-scale community with 12 buildings, including ten residential and two commercial buildings. The second community is a large-scale one with 500 buildings, consisting of 480 residential units and 20 commercial units. In each community, three configurations were investigated to illustrate the proposed model's accuracy to solve the electrical and thermal asset rescheduling problem. The first configuration was the baseline model in which buildings consumed energy as usual without any control algorithms. In the second one, the proposed model was applied to each building and they tried to minimize their own objectives and did not collaborate. The third configuration depicted the case that all buildings were connected and collaborated to minimize energy costs and inconveniences.

4.2. Building Functionalities

Based on the literature review, only a few works have focused on commercial buildings to solve the load scheduling problem. In this work, we included a typical office building and some residential buildings in our design to assess the effects of the asset rescheduling model on the energy consumptions for both types of buildings and the whole community. Thus, a set of schedules for time-shiftable appliances for residential and office buildings are provided that can be used for future reference as a benchmarking framework. Tables 1 and 2 show the appliance data for a residential building and a small office building, respectively.

Table 1. Time-shiftable assets for a residential building.

No.	Asset	Rated Power (kW)	User Preferred Time	The Operation Range	Duration (Minutes)
1	Clothes washing machine	3.5	17:20–18:00	15–21	40
2	Clothes dryer	3.2	18:10–19:20	15–22:30	70
3	Dishwasher	2.8	20:10–22:10	18–23:30	120
4	Microwave	0.9	18:30–18:40	18:20–19:10	10
5	Electric kettle	1.8	7:30–7:40 / 19–19:10	7:10–7:50 / 18:50–19:30	10
6	Electric stove	5.2	7–7:40 / 18–18:40	6:30–7:50 / 17:30–19:20	40
7	Blender	0.8	17:40–17:50	17:20–18:40	10
8	Hair dryer	1.5	6:40–6:50	6:30–8	10
9	Steam iron	1.4	20–20:20	19:30–23:30	20
10	Vacuum cleaner	1.35	19:30–20	14–21	30
11	Coffee maker	1.1	7:10–7:20	6:40–8	10
12	Phone charger	0.01	21:30–23:30	18–1 (next day)	120

Table 2. Time-shiftable assets for an office building.

No.	Asset	Rated Power (kW)	User Preferred Time	The Operation Range	Duration (Minutes)
1	Microwave	0.9	12:10–12:20 / 12:20–12:30 / 12:30–12:40	11:40–13:00	10
2	Electric kettle	1.8	8:30–8:40 / 14–14:10	8:00–9:20 / 13:30–14:30	10
3	Bottleless water cooler and heater	5.1	9–9:30 / 14–14:30	8:30–10 / 12:30–15	30
4	Paper shredder	0.15	15–15:20	14–17	20
5	Coffee maker	1.1	8:20–8:30 / 11–11:10 / 14–14:10	8–8:40 / 10:30–11:30 / 13:30–15	10

Table 1 shows that some assets, such as the electric kettle and electric stove, were used twice a day; hence, assigning an optimal schedule to each asset was necessary. in addition, some assets needed to respect the logical sequence between their operations. For example, a washing machine's operation cycle should be completed before a clothes dryer's operation cycle will be processed. Regarding the time-shiftable assets in the office building, the microwave is preferred to operate in three consecutive cycles; however, it can be rescheduled in three separate 10-min cycles between 11:40 to 13 also.

PEV charging schedules for a residential building are different from an office building. Table 3 describes the schedules for each type of buildings along with the characteristics of PEV batteries. In order to have a case study similar to real-world cases, it was assumed that the preferred time of use for PEVs was different for each residential building. For this problem, we assumed that there were three patterns for charging the vehicles in the residential units. The car owners in the first two types of residential buildings preferred to charge their vehicles when they arrived home. In contrast, the car owner in the third type of residential buildings charged their car in the afternoon without any flexibility in the scheduling, then preferred to charge it again in the evening until the following day—that is why their car's total charging duration was one hour more than the other cars in the residential building charging facilities. In the office building, it was assumed that the installed PEV charging outlets could charge the vehicles faster with a higher power rate, therefore, the duration of charging for one car was less than the charging duration in a residential building charging facility.

Table 3. The schedules of PEV charging assets.

Building Type	Number of PEVs	User Preferred Time	The Operation Range	Charging Duration for One Car (mins)
Residential unit, Type 1	1	17–22	17–6 (next day)	300
Residential unit, Type 2	2	19–24	18:30–6 (next day)	300
Residential unit, Type 3	1	17–18 21–2 (next day)	17–18 21–6 (next day)	360
Office building	10	9–12	8:30–16:30	180

5. Results and Discussion

The proposed MINLP model solved LSP for each case with a code written in MATLAB using the commercial solver Gurobi. All tests are run on an Intel Core i7 (2.9 GHz) with 16 GB of RAM running under Windows X. The problem was solved for four case studies. It was assumed that the residential building occupants were not at home between 8 a.m. to 4 p.m. The rest of the time, they were considered at home.

The case scenarios are based on building communities in Newark, NJ, USA. We wanted to solve the problem for a representative day in the cooling season. Figure 3 shows the ambient temperature of the experiment day, which was a typical day in July. The buildings' cooling systems were considered in the formulation based on Equation (9). The parameters of such a system, as well as the electric water heater parameters, are shown in Table 4.
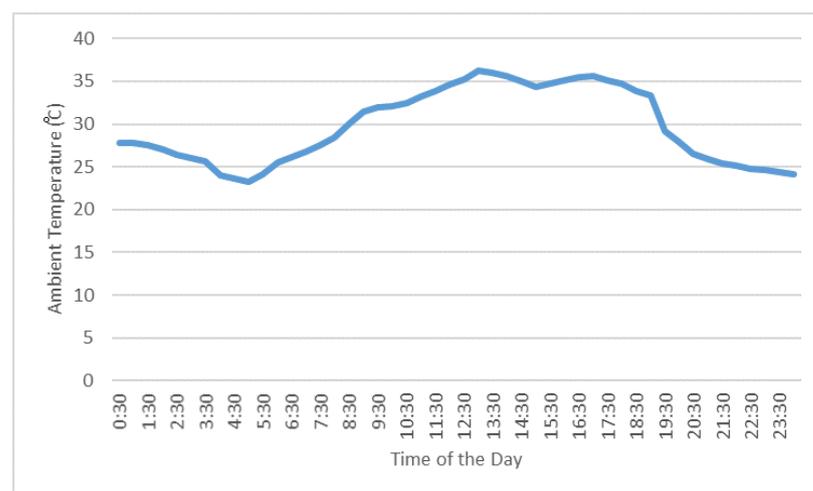
Figure 3. The ambient temperature for a typical day in June in Newark, NJ, USA.

Table 4. The parameters of the case study.

Parameters	Value
Thermal conductivity	$K = 0.45$ (kW/°C)
The total thermal mass of the fluid of the cooling system	$mc = 6.3$ (kWh/°C)
The coefficient of performance of the cooling system	$\varphi = 3.2$
The temperature of incoming water to the EWH system	$T_{iw} = 25$ (°C)
The volume of the storage tank of the EWH system	$V_{tank} = 150$ (l)
The surface area of the storage tank of the EWH system	$A_{tank} = 0.04$ (m²)
The density of water	$\rho_{water} = 998$ (kg/m³)
The specific heat of water	$Cp_{water} = 4186.8$ (J/(kg·°C))
The thermal resistance of the water storage tank	$R = 1.309$ (m²°C/W)
The flow rate of the hot water	$\dot{m}_{water} = 40$ (1/h)
Power range of the EWH system	$P_{hvac,j}^{t} \in [0, 4.5]$kW
Power range of the cooling system	$P_{ewh,j}^{t} \in [0, 2.8]$kW

The energy cost function is quadratic, as in [33]. It is assumed that coefficients μ_2 and μ_3 are zero for all hours. The values for coefficient μ_1 is found based on the following equation:

$$\mu_1(t) = \begin{cases} 0.3 \text{ cents/kWh Between midnight to 7 a.m., between 10 a.m. to 6 p.m., and between 9 p.m. to midnight} \\ 0.4 \text{ cents/kWh Between 8 a.m. to 10 a.m.} \\ 0.5 \text{ cents/kWh Between 6 p.m. to 9 p.m.} \end{cases}$$

The assigned incentives for shifting each kilowatt of electricity follows a rule-based model as follows:

$$d(t) = \begin{cases} 0.05 \text{ cents/kWh Between midnight to 7 a.m., between 10 a.m. to 6 p.m., and between 9 p.m. to midnight} \\ 0.10 \text{ cents/kWh Between 8 a.m. to 10 a.m.} \\ 0.15 \text{ cents/kWh Between 6 p.m. to 9 p.m.} \end{cases}$$

In this study, parameter λ is a fixed value of 0.5, representing a reasonable balance between energy costs and inconvenience functions. In order to solve the MINLP problem by Zoutendijk's method, we have to define the initial points. We assume the initial state for each device ($x_{1,i}^t$) equals to the preference given by the user (u_i^t). For building indoor temperature and hot water temperature, the initial values are assumed 22.5 °C and 40 °C, respectively.

5.1. The Small-Scale Community

This section investigates the impact of applying the proposed model on a small-scale building community consisting of 12 buildings. In order to evaluate the model, three different configurations were considered: the baseline case, the case without building collaboration, and the case with building cooperation. Then, we compared these three configurations' results to see how much this model improved energy efficiency and decreases energy consumption in this community.

5.1.1. The Baseline Case

This case showed the buildings' energy consumption in a small-scale community without implementing the smart controls. The buildings did not collaborate and did not use a DSM algorithm. It was assumed the occupants used their appliances when they needed them. There were no smart scheduling controls that avoided using electrical and thermal appliances during peak hours, thus, people turned on the building assets at their preferred time to maximize their comfort in terms of scheduling and thermal comfort; this increased total energy costs since the peak hours have the highest energy tariffs, and it also escalated the peak demand that puts pressure on the main grid.

5.1.2. The Case without Building Collaboration

In this case, each building tried to optimize its own objectives by rescheduling its time-shiftable and power-shiftable assets. Controllable building assets were employed to minimize the user inconvenience level. They were shifted to off-peak hours while considering the building owner's constraints. Figures 4 and 5 describe the appliances' operating schedules for a residential building and an office building in this community.

These two figures illustrate that in this case, buildings tried to defer their assets from the original schedule to as low as possible while attempting to minimize total energy cost. In residential buildings, one of the most considerable deferrals was the PEV charging schedules. Since their required power is significant and the original schedule shows the user's preference to charge it during the evening, the model shifted these controllable assets to midnight to reduce the evening peak hours' energy consumption. In the office buildings, the amount of rescheduling was not as much as the residential buildings since the model realized that the difference between energy cost units is not that significant compared to the inconveniences these deferrals could cause.
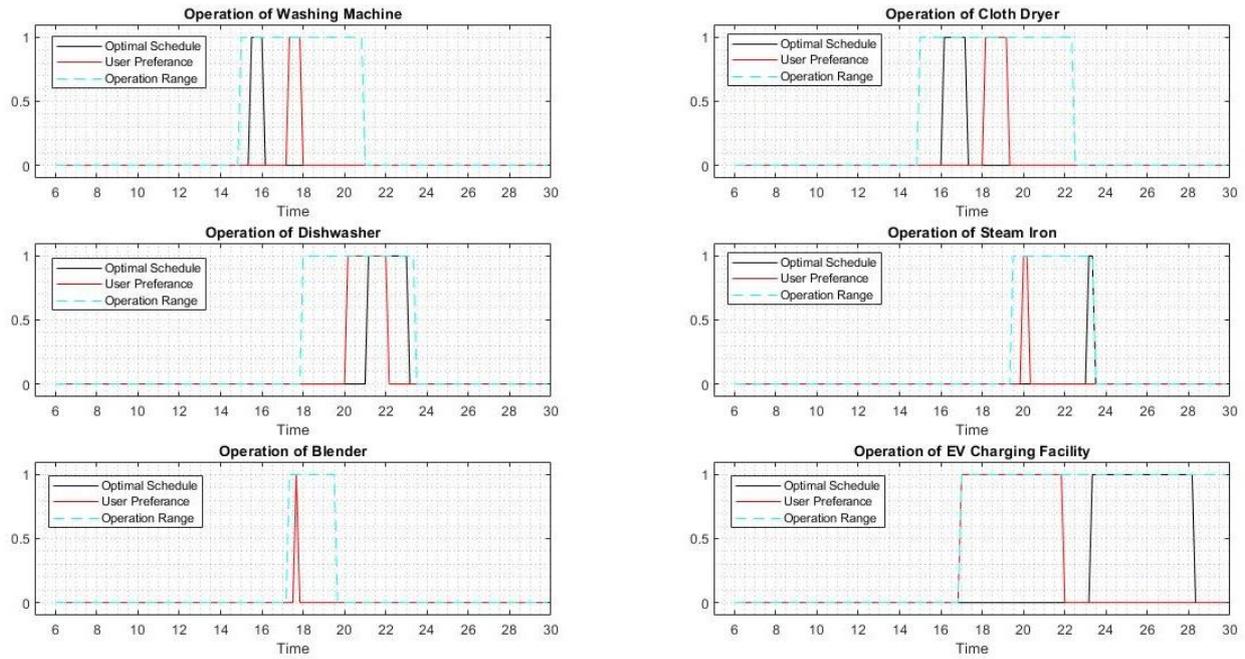
Figure 4. The operation of some assets in one of the residential buildings in the case where they did not collaborate.
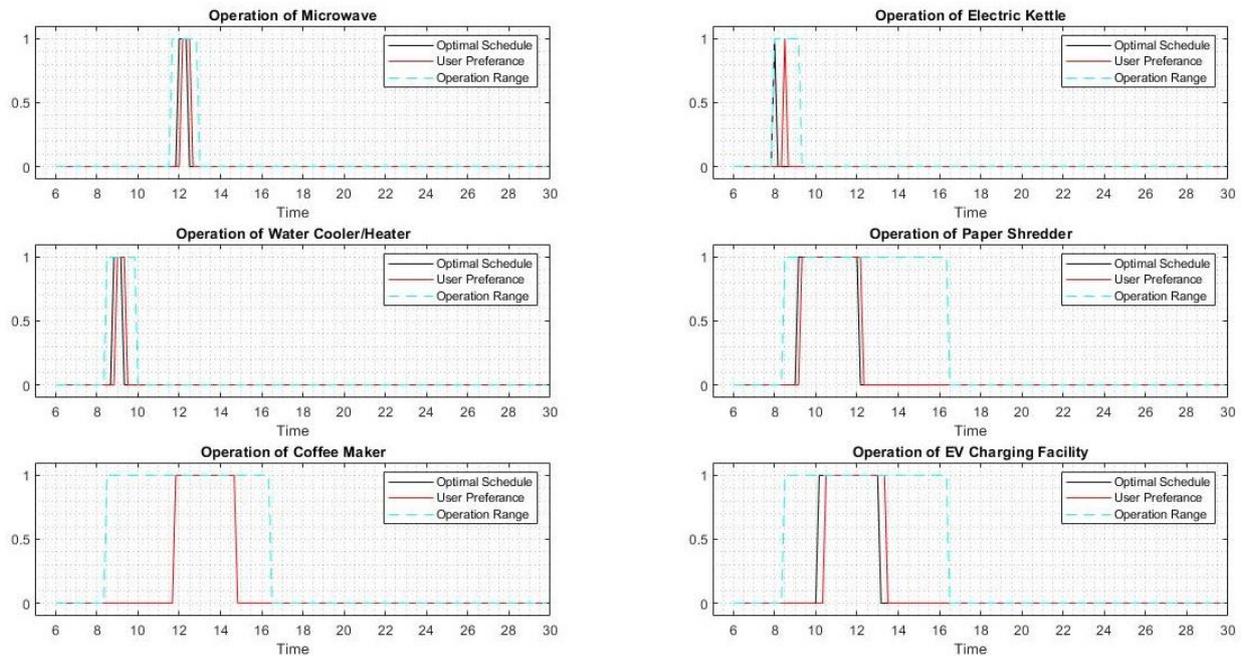

Figure 5. The operation of some assets in one of the office buildings in the case where they did not collaborate.

5.1.3. The Case with Building Collaboration

The third case described how all residential and commercial buildings worked together in a smart platform to minimize the energy costs and inconvenience levels and maximize the utility companies' assigned incentives. Figures 6 and 7 show some assets' operation in one of the residential and office buildings, respectively.
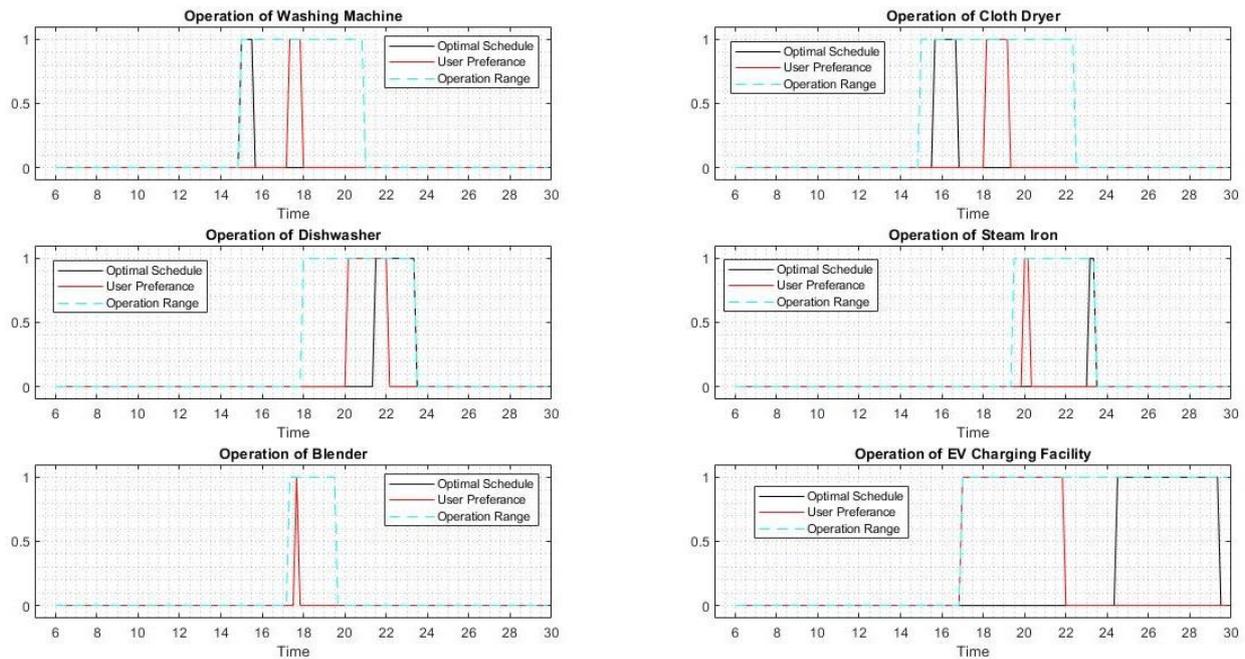
Figure 6. The operation of some assets in one of the residential buildings in the case they collaborate.
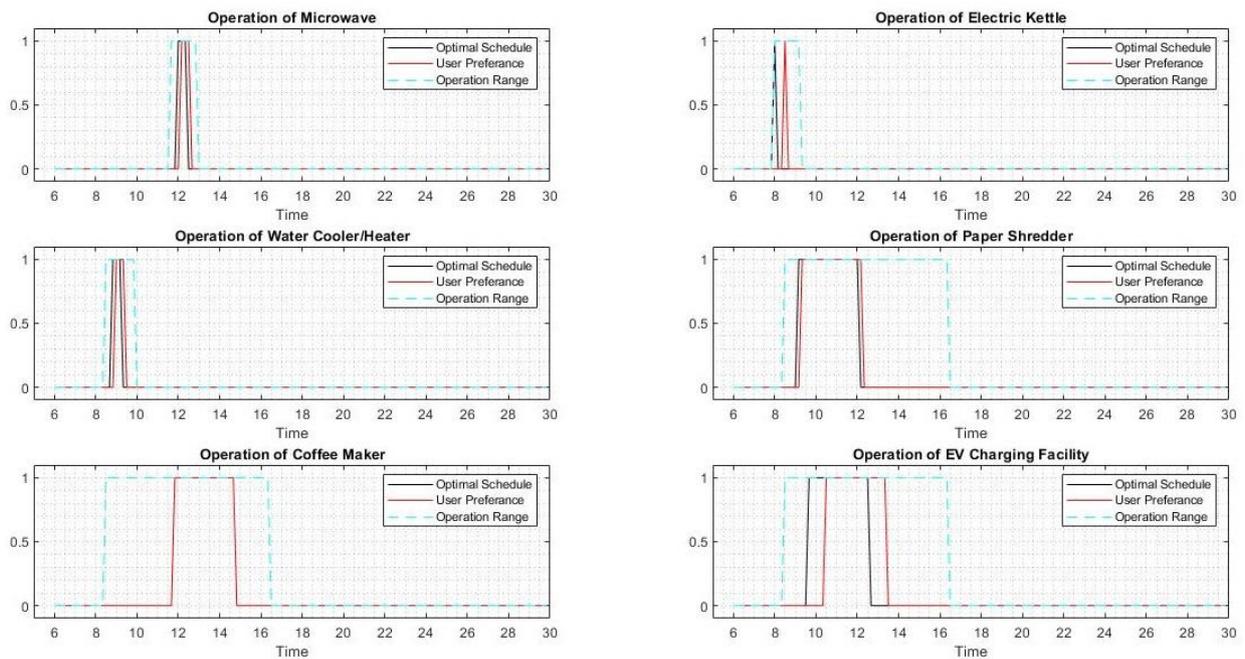

Figure 7. The operation of some assets in one of the office buildings where they collaborated.

The proposed model helped the community to defer the assets such that they could reduce the energy costs. Figures 6 and 7 describe how the assets were deferred from the original time to another time in an attempt to reduce total power and total inconvenience. It suggests that this schedule may not be the most optimal schedule for a single residential or commercial building, but it definitely is the best schedule when considering the total energy cost and inconvenience levels of the whole community.

5.1.4. Comparison between Case Studies

The intelligent control models make the building energy consumptions spread during the day instead of increasing the peak loads in the peak hours. The advantages of these methods are two-fold: the energy costs will be reduced for the building owners by reducing the consumption in the peak hours, and the peak loads will be relieved, which helps the utility companies since it mitigates the pressure on the power grid.

Furthermore, the implementation of this method will not bother the occupants in terms of thermal comfort. Figure 8 shows the impact of the proposed method on the residential building's indoor temperature. In the baseline case, the building occupants can rapidly reach their desired temperature when they are at home. There is no smart control model that is concerned about energy consumption, especially during the evening peak hours. Without building coordination, the actual temperature resulting from smart control is somehow similar to the baseline case until the afternoon. After this time, since the model is trying to minimize the energy costs, the temperature is slightly more than the desired temperature, which helps the building save energy while maintaining human comfort and respecting the 2 °C temperature deviation constraint. In the last case study, the proposed model tried to minimize the energy costs the whole day since it was concerned with the other residential and commercial buildings' energy consumption in the community. The model realized that reducing the energy consumption of thermal assets in the residential buildings helped reduce the community's total energy consumption since the office buildings used the majority of their required energy during the morning peak hours. Note that the model still maintains the thermal comfort of occupants and respects the 2 °C temperature deviation constraint.
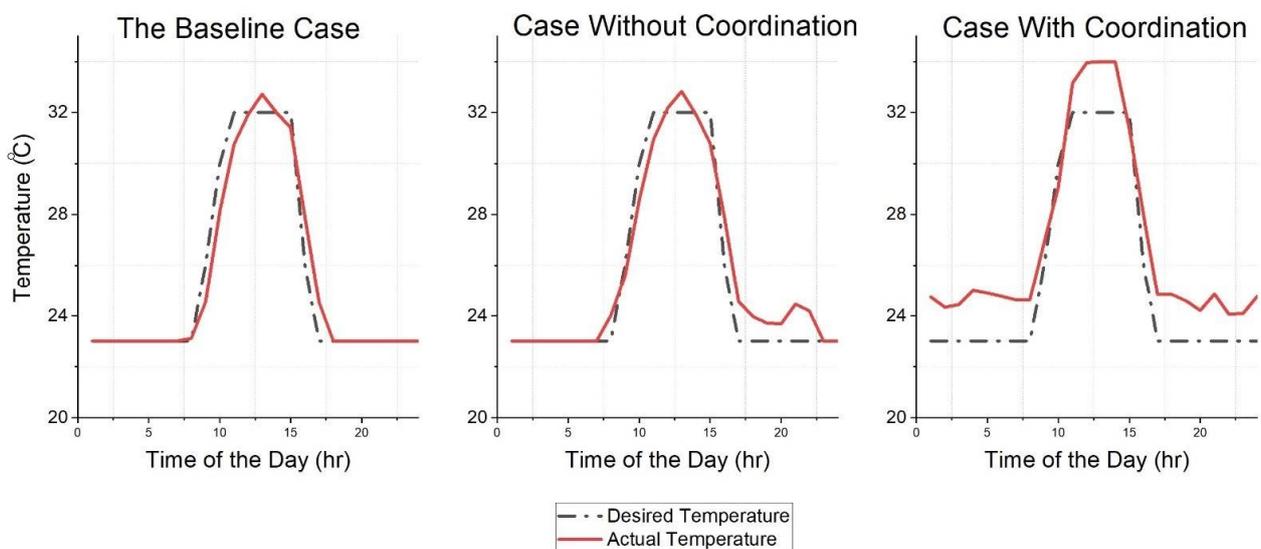

Figure 8. The comparison of the mean indoor temperature for a residential building.

Figure 9 describes how the model changes the aggregate load profile's shape by shifting controllable assets to off-peak hours. The green graph shows that the baseline case's peak loads were in peak hours, undeniably increasing the energy costs. The red chart belongs to the case that buildings were trying to optimize their objectives separately. The building shifted its assets to the low-price periods considering the user-defined constraints. It can be understood all PEVs were charged during midnight, and the evening peak loads were shaved, which helped the building save money but created another peak at midnight that can pressure the grid. The third plot illustrates the results of implementing the MINLP method in a case with building collaboration. It shows the peak loads were shaved and distributed during the day almost equally. There were not significant morning or evening peak loads. It helped remove the pressure from the main grid while reducing

the building owners' energy costs. This method can shave peak demands in communities with residential and commercial buildings that collaborate via an intelligent platform.
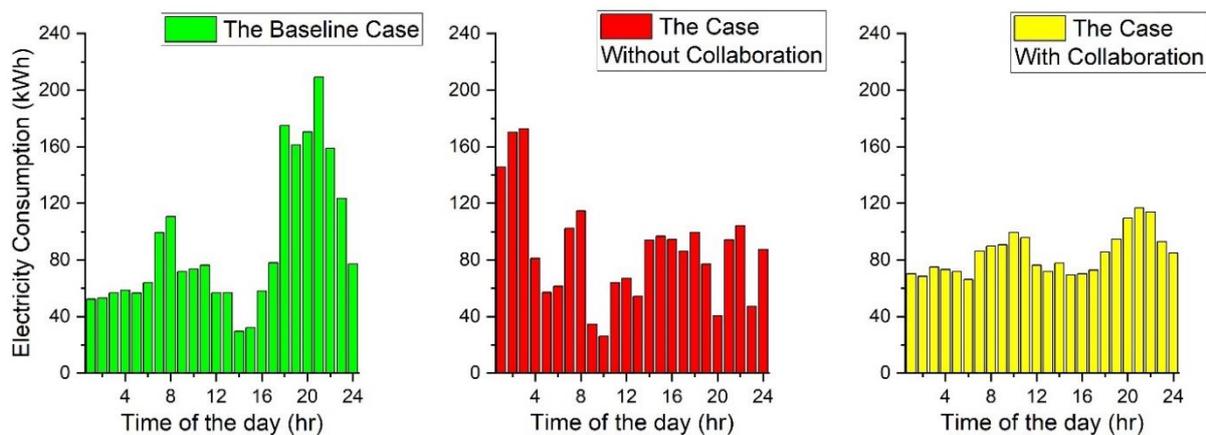

Figure 9. The comparison of total electricity demand for a small-scale building community.

5.2. *The Large-Scale Building Community*

This case evaluated the model in a 500-building community to observe the optimization model's impact on energy consumption and peak demand. This case is a good representative of a real-world building community. For this case, we compared a baseline case in which buildings are operating as usual without any smart control systems with a completely connected community that was controlled by a central control system that applies the proposed optimization model. Figure 10 illustrates the impact of the model on the energy consumption profile of the building community.
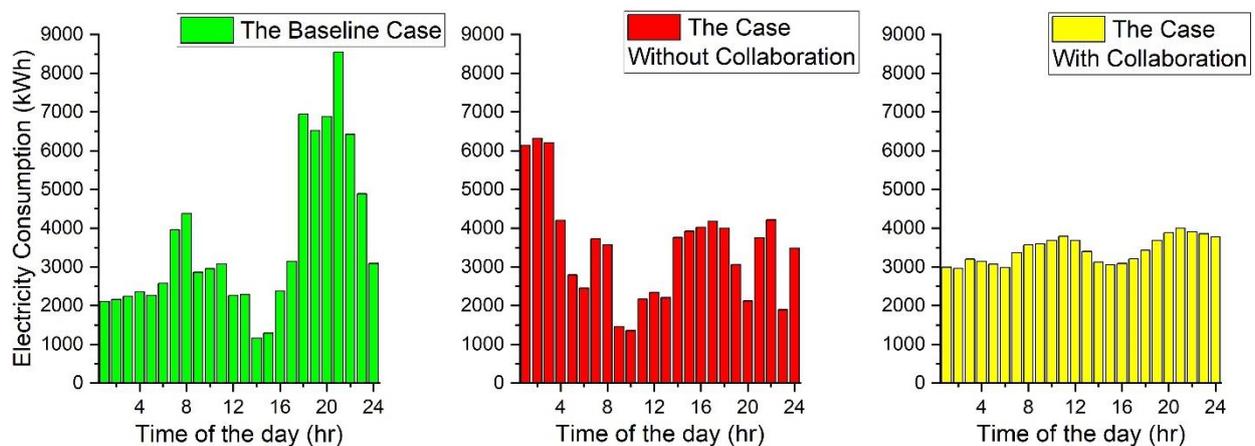

Figure 10. The comparison of total electricity demand for a large-scale building community.

This figure investigates the impact of the proposed model on the baseline case. The green graph shows the peak demand becomes even more significant for a larger community, making the utility costs higher. The second case shows the scenario that buildings minimize their own objectives and shift their assets' operations to off-peak hours. The third graph belongs to the case that uses the proposed model to reduce their energy costs and occupants' discomfort. This graph shows even more significant peak shaving in comparison with the small-scale community. The reason for that is by increasing the building community's size, the opportunities to shift the assets and smoothen out the aggregate load profile become more and more, resulting in a smoother aggregate load profile.

5.3. Discussion

Table 5 describes the results of the MINLP method for energy consumption and electricity bill savings. The table illustrates the efficiency of the model for the whole building community. The findings show that there are better opportunities for energy consumption reduction, peak demand decrease, and also energy cost saving in larger building communities. The results also describe that the percentage of energy consumption reduction for both communities is very similar, while the peak demand is significantly reduced after implementing the new model. One of the main reasons for that is the decrease in energy use happens when thermal assets are rescheduled; however, most of the time, the energy-saving by changing the setpoint temperature was not that significant, and the optimization model found the human discomfort minimization more critical. On the other hand, peak demand reduction is remarkable due to the electrical asset rescheduling from peak hours to off-peak hours. It shows that the model is able to perfectly shave the peak demand and reduce the pressure on the power grid.

Table 5. The comparison of cost-saving percentages for different types of buildings.

Case		Energy Consumption Reduction	Peak Demand Reduction	Energy-Cost Saving
Small-scale building community	Without collaboration	3.98%	17.35%	8.76%
	With collaboration	5.31%	44.15%	10.47%
Large-scale building community	Without collaboration	4.05%	26.13%	9.91%
	With collaboration	5.43%	53.15%	13.02%

The most critical electrical assets for peak shaving are PEVs. These assets consume a considerable amount of energy in each time period and are more flexible in terms of operation time. Consequently, shifting their operation schedules to midnight adds more flexibility to the systems and helps the model reschedule their operation time to save more energy costs. Based on these explanations and knowing that we have a quadratic cost function, the mentioned peak demand reduction helps save notable energy costs for the community. It makes even greater energy-cost savings for larger-scale building communities since the aggregate load profile would be smoother.

Table 5 shows a slightly better cost-saving when the buildings are connected and collaborate. The fact that the case without collaboration adds a new peak at midnight to the system is worth considering. It is definitely not desirable for the utility companies since it puts pressure on the main grid again. Hence, the collaboration between buildings in a community not only helps the owners save energy and money, but also helps the utility companies increase the grid reliability. It should be noted that using the proposed model for the large-scale building community would increase the computational time up to 45 min. So, this method would better be used once each day for day-ahead planning and control of the assets. However, for real-time scheduling, some faster algorithms such as reinforcement learning might work better.

6. Conclusions

This paper presented an autonomous nonlinear optimization model for load scheduling problems in a smart building community. The proposed bottom-up approach is able to schedule all existing assets of both residential and commercial buildings. This model's objectives were three-fold: minimizing the total electricity costs, maximizing the assigned incentives for each customer, and minimizing the inconvenience level. The assets were categorized into two groups, namely, time-shiftable assets and power-shiftable assets such as HVAC systems and electric water heaters. For each group, specific energy consumption and inconvenience level models were developed. The designed model has assigned the incentives to the participants based on their willingness to reschedule their assets. The problem is a discrete–continuous problem with many decision variables. A mixed-integer nonlinear programming model was developed to find the optimal schedule for all assets in a building community. This formulation takes building collaborations in a community

into account. The model was implemented in two building communities with three different configurations consisting of residential and office buildings to reveal the proposed model's effect.

The MINLP model described in this work can be used for operational planning and designing novel incentive-based demand response programs. The model can be used in smart cities with more residential and commercial buildings with various functionalities. The proposed model can be extended in the future by considering the stochasticity of the load scheduling problem. Also, the extension of the described model on a smart community can be seen in the presence of different distributed energy resources (DER), such as photovoltaic panels, small wind turbines, and energy storage systems.

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Nomenclature

$x_{i,j}(t)$	a binary number to represent the optimal on or off status of a flexible time-shiftable asset i of building j at time t, which equals 1 if asset i is to be turned on at time t and zero otherwise
$y_{i,j}(t)$	an auxiliary binary decision variable to state the status operation of asset i of building j at time t. If it is zero, the operation of this asset is just completed during time slot t and the corresponding $x_{i,j}(t)$ must be zero
$z_{i,j}(t)$	a binary number for the inconvenience, which equals 1 if there is a miss-match between the preferred schedule and the optimal schedule for deferrable asset i of building j at time t
$w_{i,j}(t)$	a binary number for incentives, which equals 1 if consumers earn incentives since they switched off asset i of building j at time t, against their preference, and zero otherwise
$u_{i,j}(t)$	the consumer's preferred on-off status of a flexible time-shiftable asset i of building j at time t
$v_{i,j}(t)$	the inconvenience level for thermal asset i of building j at time t
$P_{HVAC,j}(t)$	power consumption of the HVAC system in building j at time t (kW)
$P_{EWH,j}(t)$	power consumption of the EWH system in building j at time t (kW)
$T_{in}(t)$	building indoor temperature at time t (°C)
$T_{EWH}(t)$	hot water temperature at time t (°C)
$\alpha_{i,j}$	the start time of operation range for asset i of building j
$\beta_{i,j}$	the end time of operation range for asset i of building j
$\omega_{i,j}$	the required number of time slots to operate the time-shiftable asset i of building j
$d(t)$	the incentive offered at time t
Δt	the time step length (h)

$S_{i,j}$	rated power of time-shiftable asset i of building j
t_s	start time
τ	planning horizon (h)
ε	factor of inertia
$\Delta T_{i,j}(t)$	difference between the actual temperature and the desired one (°C)
K	thermal conductivity (kW/°C)
φ	coefficient of performance
mc	total thermal mass (kWh/°C)
T_{iw}	the temperature of incoming water to the electric water heater system (°C)
T_{amb}	ambient temperature (°C)
V_{tank}	the capacity of the tank (l)
A_{tank}	the surface area of the tank (m^2)
ρ_{water}	the density of water (kg/m^3)
Cp_{water}	the specific heat of water (J/(kg·°C))
$\dot{m}_{water}$	the flow rate of water (l/h)
$L_j(t)$	total hourly load for building j
R	the thermal resistance of the water storage tank (m^2·°C/W)
$\psi_{i,j}$	maximum allowable difference between the desired temperature and actual temperature (($°C)^2$)
μ_1, μ_2, μ_3	non-negative constants for the quadratic cost function

Indices

i	index of building assets
j	index of buildings
t	index of time

Sets

A_j	set of assets in building j
N	set of buildings in the community

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