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Structural Performance and Reasonable Cross-Ratio of Cross-Cable Multi-Tower Cable-Stayed Bridges

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Abstract: The Queensferry Crossing in the UK is the first multi-tower cable-stayed bridge in the world to use mid-span cross-stayed cables to improve structural rigidity. To study the structural performance and economy of cross-cable multi-tower cable-stayed bridges, a total of 11 finite element models were established using two cross-cable setting methods. By changing the number of crossed cables in the mid-span, the variation laws of structural deformation and internal force are obtained. The cross-cable efficiency based on structural stiffness and the cross-cable economy based on the consumption of cables used in the entire bridge are quantitatively analyzed, and it is considered that there is a reasonable cross-ratio of cross-cables. Combined with the current design specification and the empirical data of the actual bridge, the limits of the double indicators were determined, and a scheme comparison chart was formed. The results show that under the action of unbalanced load, the cross cable can greatly reduce deformation and balance the internal force of the structure. The optimal solution is to form a mid-span cable crossing by adjusting the cable spacing, and the reasonable range of cross-ratio is 15%~35%. If the structural stiffness is improved by adding additional cross cables, the cross-ratio should be controlled within 16% to ensure structural economy. This provides a reference for the design and research of cross-cable multi-tower cable-stayed bridges in the future.

Keywords: multi-tower cable-stayed bridges; cross cable; double indicators; reasonable cross-ratio

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1. Introduction

The multi-tower cable-stayed bridge adopts a multi-tower and multi-span arrangement, which can achieve a large spanning capacity. However, due to the lack of end anchor cables and auxiliary piers in the middle tower, structural rigidity is insufficient and the deformation is too large under the unbalanced live load [1,2]. Due to the excessive structural deformation, the stress of the components of the multi-tower cable-stayed bridge under the action of live load changes greatly. This results in components prone to stress fatigue. The bridge designers have proposed a variety of conceptual design schemes to increase the stiffness of the middle tower [3–7]. Structural engineers have also conducted relevant research on the deformation of tall towers and components and the stiffness of cables [8–11]. The research object of this paper is cross-cable multi-tower cable-stayed bridges that improve the rigidity of the mid-tower by setting cross-cables in the mid-span.

The Queensferry Crossing, Scotland, UK, is a three-tower cable-stayed bridge with a main span of 650 m [12–15]. It is the world's first multi-tower cable-stayed bridge with crossed cables. This special design makes it possible to become a 21st century Landmark project [12,13]. The Bianyuzhou Yangtze River Bridge was completed by the end of December 2021. It is the first cross-cable multi-tower cable-stayed bridge in China [16]. The design and construction of the bridge provided a reliable basis for the study of cross cables. Currently, most research on cross cables publicly released at home and abroad is in the conceptual design stage. There are few in-depth studies and calculations for such structures. Only by conducting a comparative analysis was it concluded that cross cables have advantages over other constraint forms. This paper will analyze and calculate a three-tower cable-stayed bridge with crossed cables. In this manner, the stiffness response of this type of structure is obtained. The cross-ratio is one of the most important design parameters for this type of structure. We will calculate a reasonable cross-ratio based on structural performance and economics. The results of this paper will provide a reference for the design and analysis of cross-cable multi-tower cable-stayed bridges in the future.

2. Literature Review

Before carrying out the analysis and calculation of cross-cable multi-tower cable-stayed bridges, it is necessary to review the various schemes proposed by the predecessors. The most direct way to improve the structural stiffness of multi-tower cable-stayed bridges is to increase the stiffness of the towers. Danish researchers believe that adequate longitudinal stiffness can be obtained with A-shaped towers, but this is not the only method [3,4]. The rigid middle tower has poor economic performance. Rationally configuring the stiffness of beams, towers, and pier columns to bear internal forces and limit deformation is the best solution [17]. Some bridge designers proposed measures to increase the stiffness of multi-tower cable-stayed bridges, including the use of horizontal stability cables, cross-stayed cables, and external stability cables [18]. A foreign academician of the Chinese Academy of Engineering studied the effects of different cable-stay arrangements on the stiffness of multi-tower cable-stayed bridges [19]. He made calculations for bridges using three systems of horizontal stabilizing cables, inclined stabilizing cables, and mid-span crossing cables. By analyzing the bending moment and deflection of the structure, he concluded that it is feasible to arrange the cross-stayed cables in the mid-span to improve the overall stiffness of the structure. However, there are some problems with the cross cable, such as the amount of stay cables increases, complex anchorage of cables in crossover areas, and possible continuous damage [20]. It should be selected according to the actual situation [21]. Bridge researchers in China took a four-tower cable-stayed bridge with a main span of 408 m as the research object and analyzed the structural response under various stabilizing cable schemes [22]. The calculation results show that under the tower-beam joined together system, the vertical deflections of the main beams of various stiffening methods are not much different. If the stress performance of the main beam is considered, the cross cable-stay and the A-shaped tower are superior to other stiffening methods. By conducting analyses, they believe that there is an optimal amount of cables in the cross-cable system. The amount of cables used will increase sharply with too many crossings. A young scholar studied the effect of cross cable arrangement on the mechanical properties of multi-tower cable-stayed bridges [23]. He also provided a simplified calculation method for the deformation of the cross-cable multi-tower cable-stayed bridges [24]. The calculation results show that the spacing arrangement and asymmetric arrangement of the cross cables can effectively improve the restraint on the center tower and increase structural rigidity. The above literature research will provide ideas and guidelines for the modeling and research of this paper.

3. Modeling Methods

3.1. Structural Parameters

The design parameters of the Queensferry Crossing were determined through complex analysis and research. This paper uses bridge structure parameters similar to the Queensferry Crossing in the modeling process [12,15]. In this manner, it can ensure that erroneous conclusions will not be generated due to improper parameter selection.

The calculation structure is a three-tower cable-stayed bridge with two main spans of 650 m. The bridge tower adopts a single column type and the cross-section is a rectangular box. The side tower is 203.3 m high, and the middle tower is 210.7 m high. The main beam is a steel box girder with a width of 37.6 m and a height of 4.465 m. There are 23 pairs of stay cables on one side of each tower. Each cable has the same cross-section. The entire bridge is set up with double parallel cable planes. The side tower cable plane spacing is 2.9 m, and the middle tower cable plane spacing is 1.9 m (Figure 1).

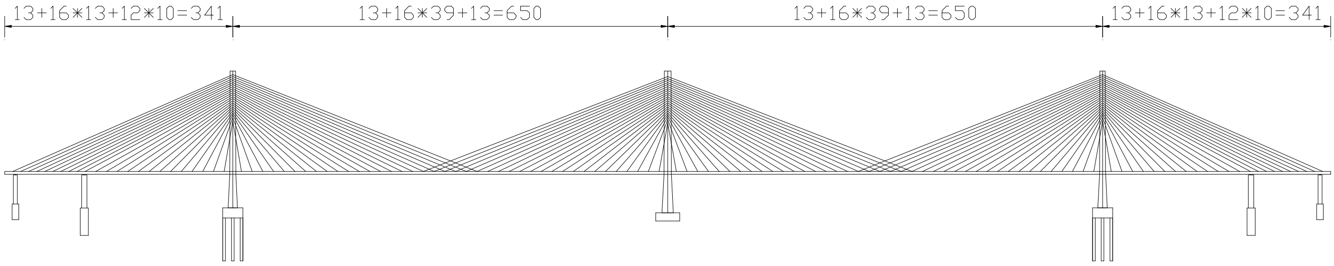


Figure 1. Elevation of structure (m).

3.2. Model Description

According to the design parameters, a finite element 3D model is established by *Midas civil* software. This model has a total of 1074 nodes and 917 units, and they constitute the framework of the entire model. Among them, the bridge tower and main beam adopt general beam elements. Because this paper analyzes the state of the bridge after completion, the nonlinear effect of the stay cable is ignored. First, the stay cables are simulated with truss elements. After static linear analysis, the "unknown load coefficient" function in the software is used to obtain the initial cable force according to the influence matrix. Then, the stay cables use tension-only cable elements. When performing static linear analysis, the software automatically calculates the stay cables as equivalent truss elements taking into account the correction of Ernst's formula.

At the middle tower, the tower and the main beam are joined together, and at the side towers, the tower and the main beam are separated. With this boundary constraint, the overall stiffness of the structure is better, and the temperature internal force in the main beam is small. The overall diagram of the model and the detailed diagram of the crossed cables are shown in Figure 2.

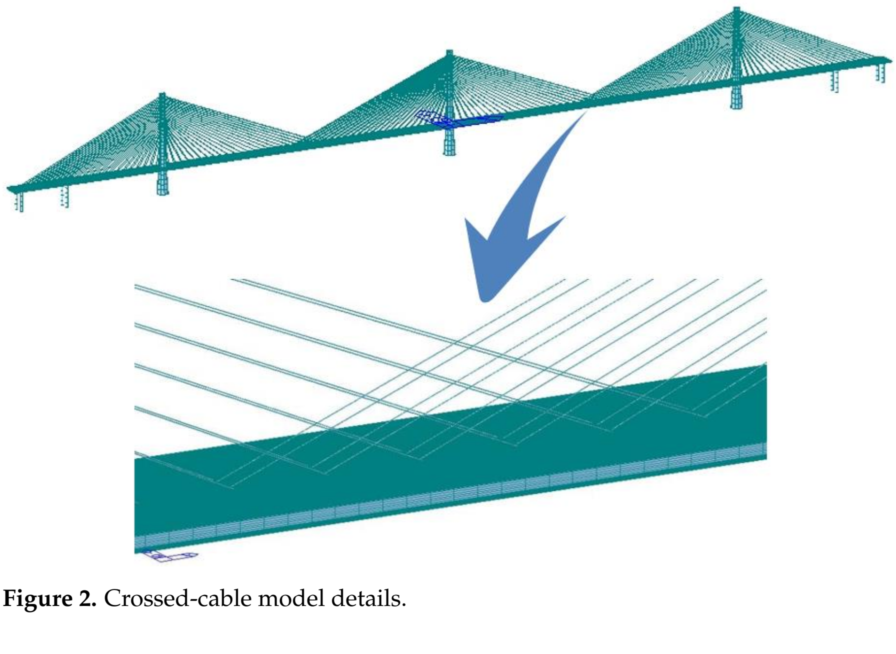


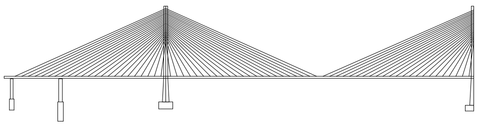
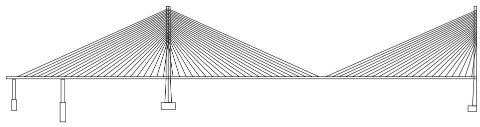
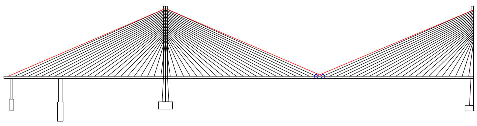
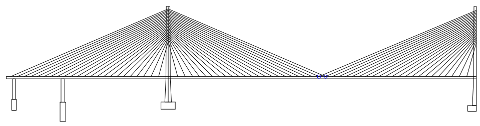
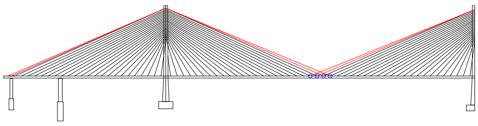
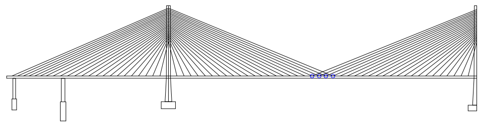
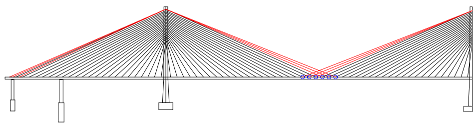
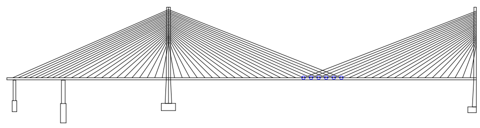
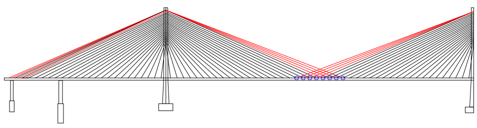
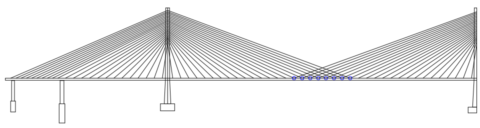
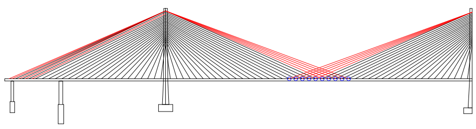
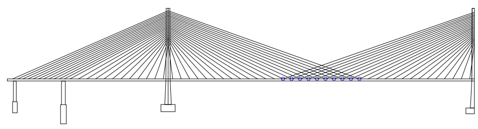
Figure 2. Crossed-cable model details.

3.3. Cross Cable Setup Scheme

In this paper, there are two methods to set the cross-cable. One method is to increase the number of stay cables based on the original design to form mid-span crossed stay cables; the other method is to keep the original number of stay cables unchanged and adjust the cable distance to form mid-span crossed stay cables.

According to the above ideas, two sets of comparison schemes are set up. See Table 1 for details.

Table 1. Parameters of contrast scheme with crossed cables.

			Method 1				**Method 2**
I	**II**	**III**	**Half Elevation View**	**I**	**II**	**III**	**Half Elevation View**
I-the scheme number; **II**-the number of pairs of stay cables on one side of the tower; **III**-the number of pairs of crossed cables in mid-span.							
a	23	0		A	23	0	
b	24	2		B	23	2	
c	25	4		C	23	4	
d	26	6		D	23	6	
e	27	8		E	23	8	
f	28	10		F	23	10	

It can be seen from Table 1 that plan a and plan A are the same scheme. Therefore, a total of 11 three-tower cable-stayed bridges were established based on the parameters in the table above. The boundary conditions of the structures are the same. The most unfavorable load case of a three-tower cable-stayed bridge is the full span live load acting on one side of the main span [25–28]. See Figure 3 for details. The unbalanced load q is increased from 10 kN/m to 50 kN/m to obtain the variation of structural stiffness.

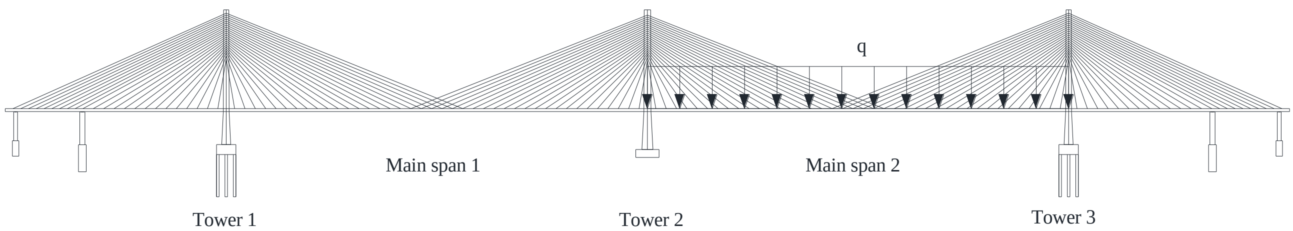


Figure 3. The sketch of the unbalanced load.

4. Results and Discussion

4.1. Structural Performance

In this paper, finite element software is used for modeling and calculation analysis [29–31]. The deformation and internal force of the structure under the action of the cross cable are calculated, and the variation law of the structural performance is obtained by comparison. This reflects the mechanical properties of cross-cables.

4.1.1. Horizontal Displacement of the Top of the Mid-Tower (Tower 2)

The comparison of the horizontal displacement of the top of the middle tower is shown in Figure 4. The calculation results show that the following: (1) The horizontal displacement value of the top of tower 2 increases linearly with the increase in unbalanced load q. (2) Under the load, with the increase in the number of cross-cables, the horizontal displacement of the tower gradually decreases, and structural stiffness gradually increases. It is worth noting that the efficiency of increasing stiffness becomes slower. (3) With the increase in unbalanced load q, the effect of the ability of the cross-cable to increase the stiffness of the middle tower becomes more and more obvious.

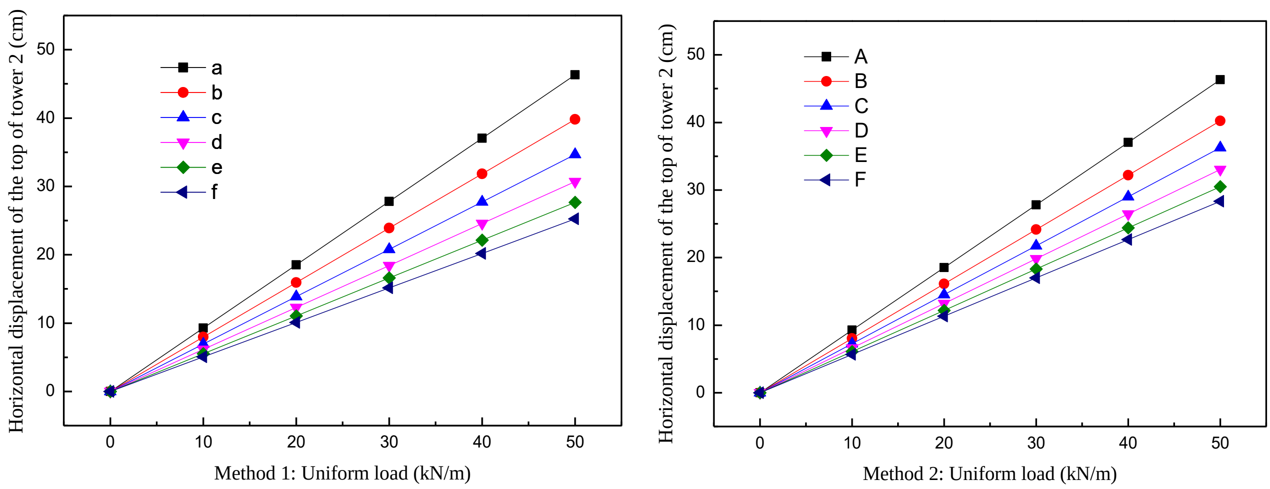


Figure 4. Comparison of horizontal displacement of mid-tower.

In the same set of six pairs of cross cables, when the unbalanced load q = 50 kN/m, the tower displacement in scheme d is reduced by 33.70%, and the tower displacement in scheme D is reduced by 28.63%. Method 1 is slightly better than method 2 for increasing the stiffness of the mid-tower.

4.1.2. Deflection of Main Span (Main Span 2)

The comparison of the deflection of the main span is shown in Figure 5. The calculation results show the following: (1) The deflection value of the main span 2 increases linearly with the increase in unbalanced load q. (2) Under load, with the increase in the number of cross-cables, the deflection of the main span gradually decreases, and structural stiffness gradually increases. It is worth noting that the efficiency of increasing stiffness becomes slower. (3) With the increase in unbalanced load q, the effect of the cross-cable to reduce the deflection of the main span becomes more and more obvious.

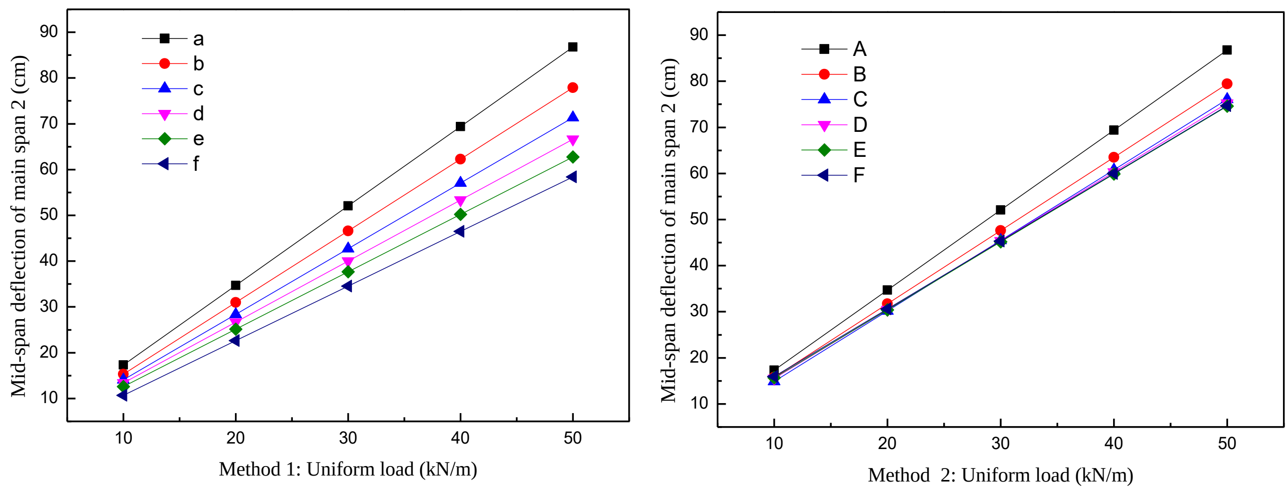


Figure 5. Comparison of deflection of main span.

In the same set of six pairs of cross cables, when the unbalanced load is q = 50 kN/m, the deflection of the main span in scheme d is reduced by 23.17%, and scheme D is reduced by 13.26%. Method 1 is obviously better than method 2 for increasing the stiffness of the main span. After four pairs of cross cables are set by method 2, increasing the cross cables has little effect on reducing the deflection of the main span.

4.1.3. Internal Force of Main Beam

The comparison of the maximum positive bending moment of the main beam is shown in Figure 6. The calculation results show the following: (1) With the increase in unbalanced load q, the maximum positive bending moment of the main beam keeps increasing. (2) Under the dead load, the setting of the cross cable leads to the increase in the internal force of the main beam. When the unbalanced load is q = 50 kN/m, the setting of the cross cable leads to a reduction in the internal force of the main beam. (3) In general, the growth of the number of cross cables will increase the internal force of the main girder under the dead load. However, as the unbalanced live load increases, the negative effect of the cross cable gradually disappears, which reduces the internal force of the main beam. The internal force of the beam tends to be balanced and stable.

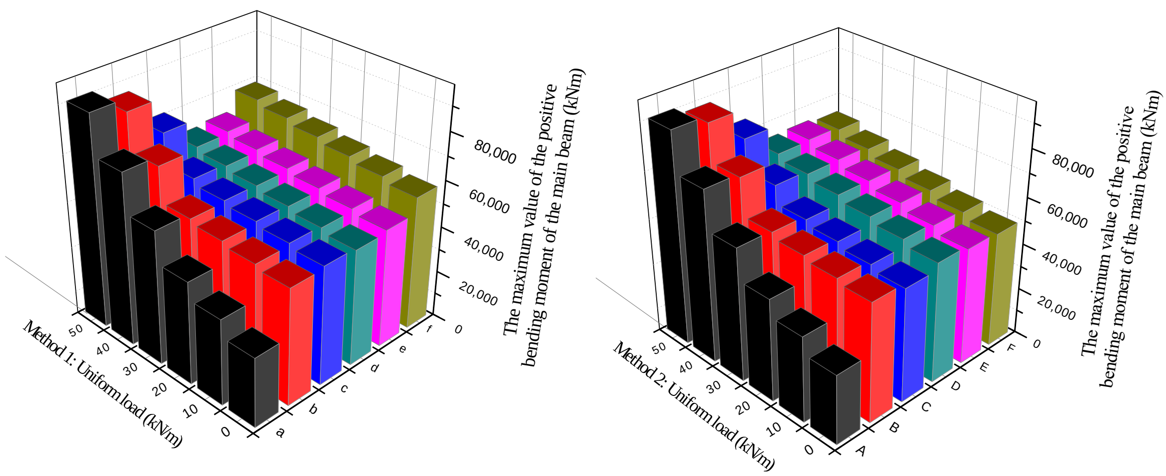


Figure 6. Comparison of maximum positive bending moment of the main beam.

In the same set of six pairs of cross cables, when the unbalanced load is q = 50 kN/m, the maximum positive bending moment of the main beam in scheme d is reduced by 34.65%, and the scheme D is reduced by 29.80%. Method 1 is slightly better than method 2 for reducing the internal force of the main beam.

4.1.4. Cable Force

In this paper, 10 pairs of stay cables in the mid-span of main span 1 (the cross cables and their adjacent stay cables) are selected as the analysis objects. The cable numbers are shown in Figure 7. To avoid the redundancy of the comparison diagram caused by a large amount of data, scheme a, d, f, scheme A, D, and F are selected for analysis. Calculate the cable force under the dead load and the combined action of the dead load and 50 kN/m unbalanced load, and the difference between the two is the change value of the cable force caused by the unbalanced load.

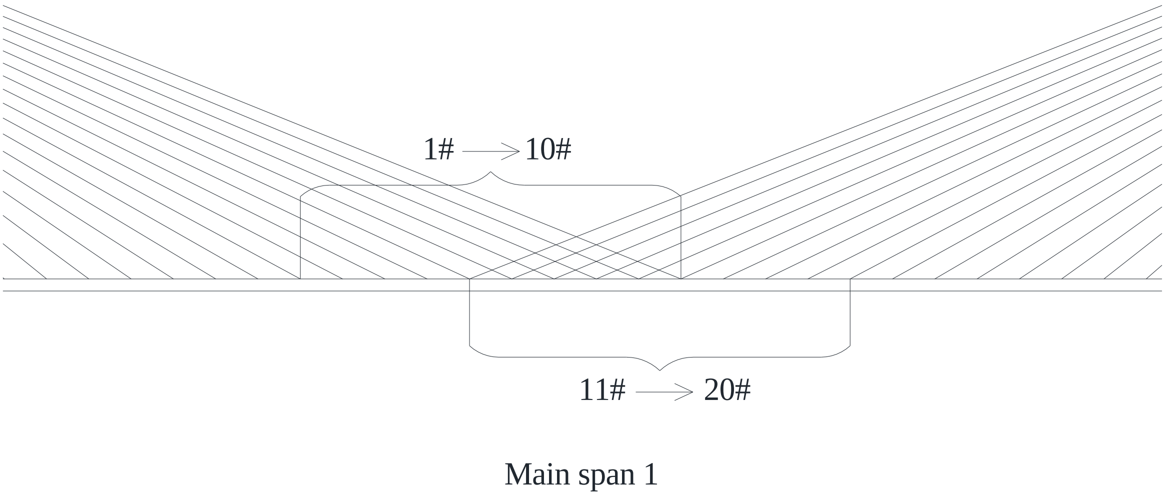


Figure 7. Numbering diagram of stay cables.

The comparison of the change value of the cable force is shown in Figure 8. The calculation results show that the following: (1) Due to the unbalanced load, the cable force on one side of the main span 1 increases, and the cable force on the other side decreases. (2) With the growth of the number of cross cables, the change value of the cable force tends to be balanced. (3) Under the action of an unbalanced load, the crossed cable makes more cables share the external force and weakens the peak internal force of the main force-bearing components.

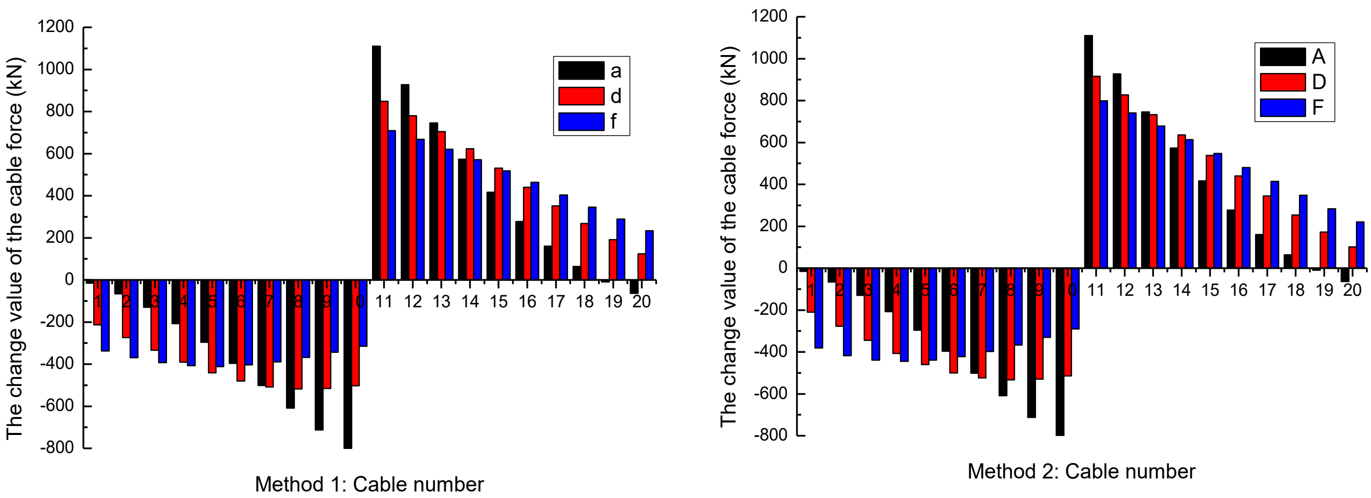


Figure 8. Comparison of the change value of the cable force.

In the same set of six pairs of cross cables, when the unbalanced load is q = 50 kN/m, the cable force of the 11# cable in scheme d is reduced by 23.56%, and the scheme D is reduced by 17.55%. The cable force of the 10# cable in scheme d is reduced by 49.25%, and scheme D is reduced by 46.34%. Method 1 is slightly better than method 2 for reducing the cable force.

This section discusses the structural performance of cross-cable multi-tower cable-stayed bridges from four aspects: tower deformation, main beam deformation and internal force, and cable internal force. The results show that, under the action of unbalanced load, setting the mid-span cross cable can greatly reduce the structural deformation, balance the internal force of the components and ensure structural rigidity. The cross-cable acts as a connection, which increases the force of the components that were unstressed or less stressed before, and weakens the peak internal force of the main force-bearing components. As a result, the force of the entire bridge is more balanced, and the load capacity of each component can be fully utilized. This means that, with the same component materials, the cross-cable multi-tower cable-stayed bridge can achieve a larger span than the traditional multi-tower cable-stayed bridge. Bridge designers can improve the overall stiffness of multi-tower cable-stayed bridges by adjusting the spacing of stay cables to form crossings at the design stage. Additional cross-stayed cables can also be added to solve the problem of insufficient stiffness of existing multi-tower cable-stayed bridges. However, from all calculation results, it can be found that with the increase in the number of cross-cables, the degree of improvement of the structural stiffness decreases gradually. Bridge designers should pay attention to this to ensure the rationality and economy of the structure.

4.2. Economic Analysis of Cross-Cable

The more cross-cables are set, the more rigid the structure will be [32,33]. It can be observed from the calculation results that, regardless of method 1 or method 2, with the increase in the number of cross-cables, the efficiency of its action gradually decreases. At the same time, the consumption of the cables continues to increase. In this paper, the economy of stay cables is studied from the two aspects of efficiency and consumption [33,34].

4.2.1. Efficiency of the Cross Cable

In this paper, the efficiency of the cable action is reflected by calculating the stiffness of the middle tower. A 20,000 kN horizontal concentrated force F is added on the top of the middle tower. Calculate the displacement value Δ of the middle tower. Then, the stiffness K of the middle tower is equal to the following.

$$K = \frac{F}{\Delta} \tag{1}$$

To more directly represent the contribution of each additional two pairs of cross-cables relative to the structural stiffness, the cross-cable efficiency index γ is described as follows.

$$\gamma = \frac{\delta K}{K_0} \tag{2}$$

In the above formula, we have the following:

δK—the difference in the stiffness of the tower after adding two pairs of cross cables;
K_0—the stiffness of the tower without adding two pairs of cross cables.

It indicates the ratio of the increase in stiffness of the middle tower after adding two pairs of crossed cables to the stiffness without adding these two pairs of crossed cables.

The calculation results are shown in Figure 9.

The calculation results show that the following: (1) With the increase in the number of cross-cables, its effect on the middle tower gradually decreases. (2) The cross-cable efficiency of method 2 is slightly lower than method 1.

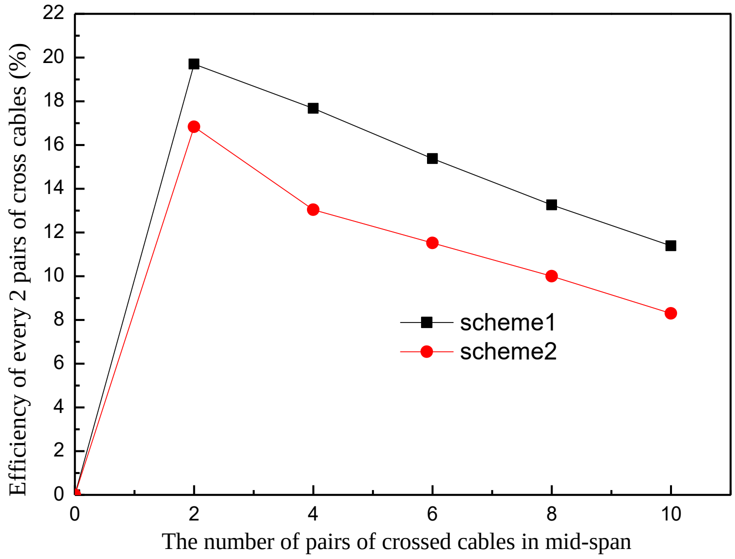


Figure 9. Efficiency of every 2 pairs of cross cables.

4.2.2. Consumption of the Cables

With the increase in the number of cross-cables, the consumption of the cables continues to increase. Cable consumption is an economic indicator of structure. Reasonable cable consumption is the basis for designing and constructing a multi-tower cable-stayed bridge with reasonable structure, excellent performance, and economical application. Cable consumption for the entire bridge calculated in this paper is shown in Figure 10.

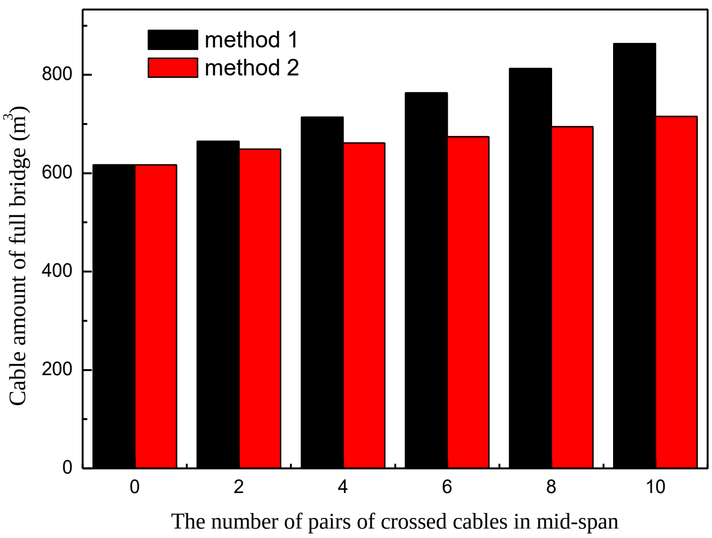


Figure 10. Comparison of the consumption of the cables.

The calculation results show that the following: (1) For setting the same number of mid-span cross cables, the consumption of cables used by method 1 is significantly larger than method 2. (2) With the increase in the number of cross-cables, the consumption of cables by method 2 increases gently, and the consumption of cables by method 1 increases sharply.

4.2.3. Economic Indicator of Crossed Cables

In this paper, the economic indicator Q of the crossed cable is solved according to the maximum vertical deflection ω of the main beam under the live load and the consumption β of the cables for the full bridge. Economic indicator Q is the consumption of cables to reduce the deflection of the main beam by 1 cm using the cross cable. The deflection of the main beam under live load and the consumption of cables in scheme a(A) is (ω_0, β_0). After setting the cross cable, the reduction value of the vertical deflection of the main beam and the increased value of cable consumption is $(\omega_0 - \omega_i, \beta_i - \beta_0)$.

Then, the economic indicator Q is described as follows.

$$Q = \frac{\beta_i - \beta_0}{\omega_0 - \omega_i} \tag{3}$$

The calculation results of the crossed cable economic indicator Q of each scheme are shown in Figure 11.

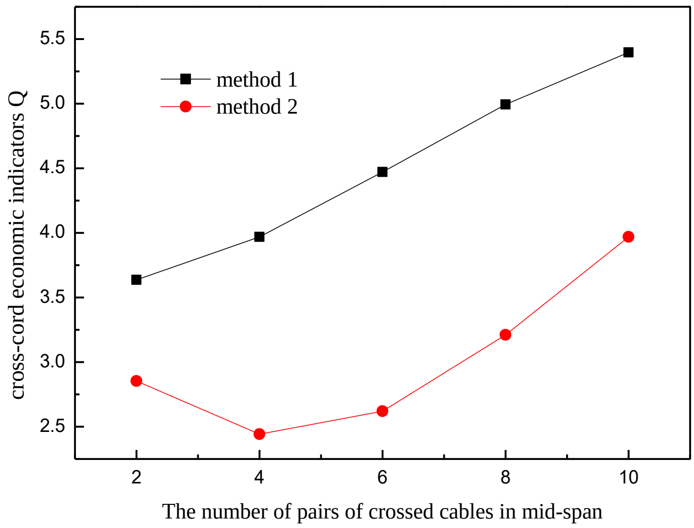


Figure 11. Comparison of economic indicator of crossed cables.

The calculation results show that the following: (1) The economic indicator Q value of method 1 ranges from 3.636 to 5.697, and the economic indicator Q value of method 2 ranges from 2.853 to 3.969. That is to say that the economy of the crossed cable in method 1 is worse than method 2. (2) With the increase in the number of cross-cables, the economics of the cross cable in method 1 becomes worse and worse. The economic indicator of method 2 decreases first and then increases. Method 2 shows better economy when setting four and six pairs of mid-span cross cables.

4.3. Reasonable Cross-Ratio of Crossed Cables

The cross-ratio of stay cables is an extremely important design parameter for cross-cable multi-tower cable-stayed bridges. The two aspects of structural stiffness and cable consumption were studied in depth in the previous section. The results show that too many cable crossings will lead to the problem of excessive cable consumption. There is an optimal solution for the cross-ratio. Under the requirement of structural rigidity, the scheme with less cable consumption should be selected as much as possible.

The rigidity requirements of cable-stayed bridges in China's Specifications for Design of Highway Cable-stayed Bridge (JTG/T 3365-01-2020) are mainly for conventional highway cable-stayed bridges [35]. There is no clear regulation specifically for the rigidity limits of multi-tower cable-stayed bridges [36]. Li Z.S. proposed in the literature to use the deflection-span ratio to reflect the stiffness of the multi-tower cable-stayed bridge [37]. By conducting an analysis of the multi-tower cable-stayed bridges that have been built and operated well in China, they obtained their deflection-span ratio is roughly 1/534~1/770. Combined with the relevant bridge specifications in Japan and the United States, the range of the deflection-span ratio of the multi-tower cable-stayed bridge under live load is limited to 1/550~1/900. According to the calculated value of the cross-cable economic indicator Q of 11 schemes, take the interval median $Q = 4$ as the boundary of the cross-cable economy.

Take the maximum vertical deflection of the main beam of the multi-tower cable-stayed bridge under live load as the abscissa and the cable consumption of the entire bridge as the ordinate. The calculation results of 11 schemes are obtained, as shown in Figure 12. According to the reasonable range of deflection-span ratio, the allowable deformation range of the maximum vertical deflection under live load is obtained (the red dotted line in the figure). It is believed that the cross-cable multi-tower cable-stayed bridge in this range meets the structural rigidity requirements. Draw the cable economic indicator boundary in Figure 12 (the black dotted line in the figure). The schemes located in the upper part of the boundary are economically inferior and costly. The schemes located at the lower part of the boundary are economical.

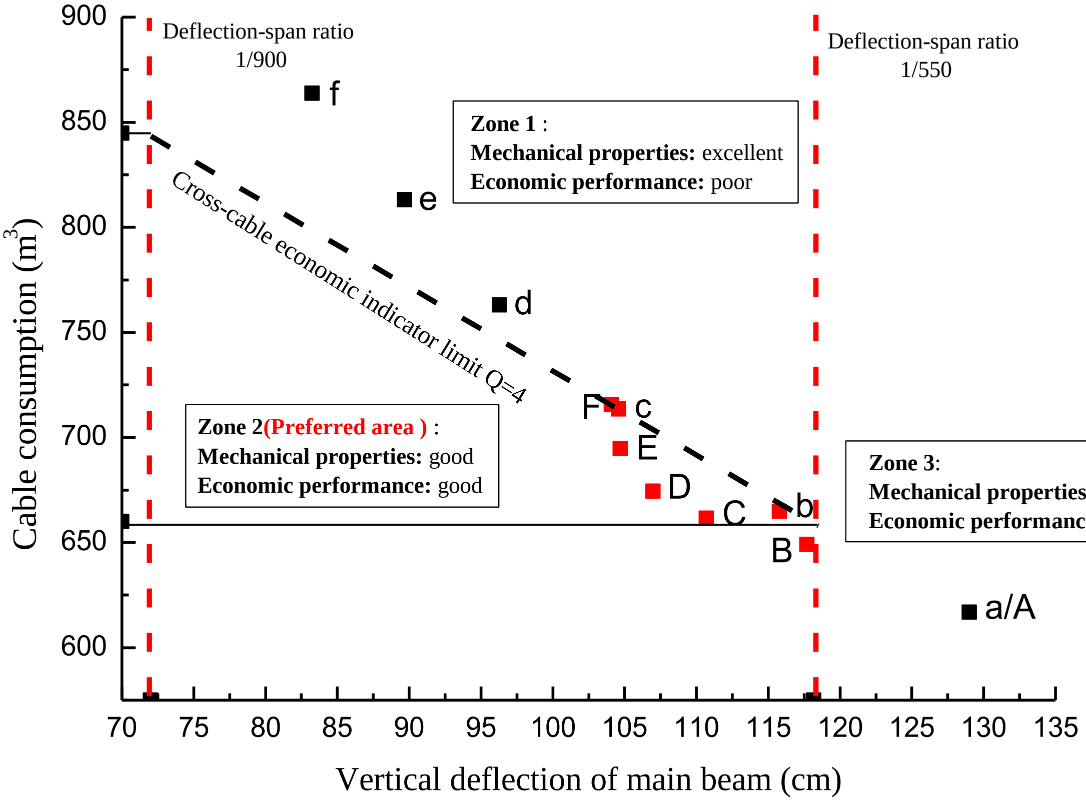


Figure 12. Scheme comparison chart.

Figure 12 is the scheme comparison chart on the double indicators. The area in the figure is divided into three areas by the deflection-span ratio boundary and the cable economic indicator boundary.

Zone 1 (Scheme d, e, f): The cross-cable scheme in this area exhibits excellent mechanical properties, with small vertical deformation of the main beam under live load and high structural rigidity. However, the cable consumption of the entire bridge has increased sharply, resulting in a poor economy. This area schemes are suitable for the design plan that has very strict requirements on structural rigidity, and the economy is only used as an auxiliary reference.

Zone 2 (Scheme B, b, C, c, D, E, F): The cross-cable scheme in this area shows good mechanical performance, the vertical deformation of the main girder under live load is moderate, and it meets the requirements of the structural rigidity limit. The cable consumption of the entire bridge is also appropriate, and the economy is good. The scheme of this area takes into account both the structural stiffness and the consumption of cables and achieves a more reasonable design goal. It should be used as the optimal design scheme for the cross-cable multi-tower cable-stayed bridge.

It is worth noting that schemes B, b, c, and F are all close to the structural rigidity boundary or the cable economic boundary. Therefore, schemes C, D, and E should be preferred.

Zone 3 (Scheme A/a): The cross-cable scheme in this area shows poor mechanical properties. Although the cable consumption of the full-bridge is small, it cannot meet the limit requirements of structural rigidity. It is an unsafe structure and should not be considered.

To apply the analysis conclusions to multi-tower cable-stayed bridges with cross-cables, this paper uses a dimensionless cross-cable ratio to indicate the number of cross-cables. The cross-cable ratio k is as follows.

$$k = \frac{m}{M} \tag{4}$$

In the above formula, we have the following:

m—number of cross cables;
M—the number of stay cables on one side of the tower; see Figure 13 for details.

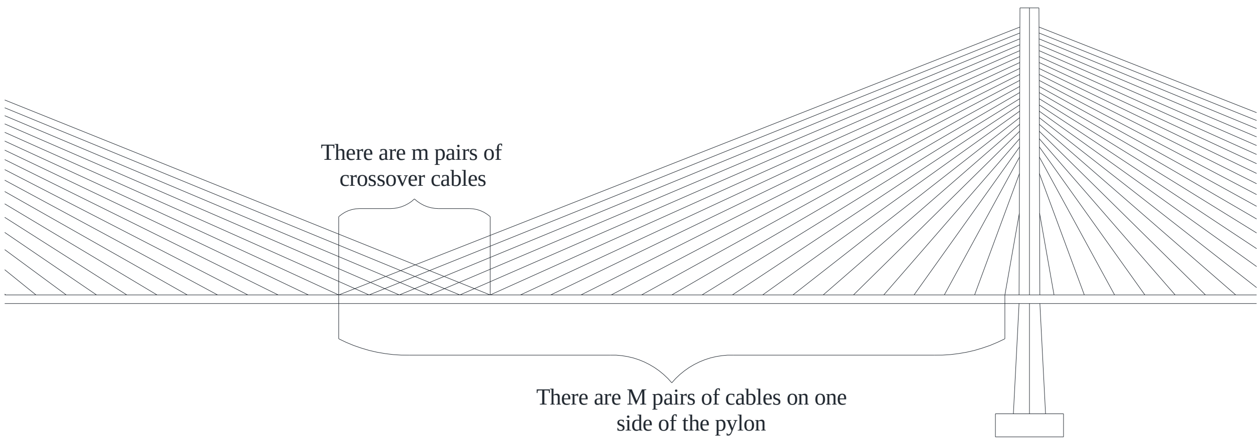


Figure 13. Cross-cable ratio calculation diagram.

According to the scheme comparison chart, the optimal scheme are schemes C, D, and E, and their cross-cable ratios are 17.39%, 26.09%, and 34.78%. The results show that the most reasonable method is to form a mid-span cable crossing by adjusting the cable spacing, and the reasonable range of cross-ratio is 15%~35%. If structural stiffness is improved by adding additional cross cables, the cross-ratio should be controlled within 16%. Otherwise, the structure will show the problem of an excessive consumption of cables for the entire bridge.

The design of the Queensferry Crossing is exactly like this. The setup method of the cross cable is to form a mid-span cross cable by adjusting the cable spacing, and the cross-cable ratio is 25%. This further validates the conclusion of this paper.

5. Conclusions and Recommendations

With the special design of Queensferry Crossing as the research background, this paper conducts in-depth research on cross-cable multi-tower cable-stayed bridges. From the aspects of structural stiffness and economy, the influence of cross-cables on structural performance is clarified, and a reasonable cross-ratio is proposed.

This paper proposes two methods to set up cross cables in the mid-span and carries out a series of finite element analyses of 11 contrasting schemes. Comparing scheme A (no cross-cable) and scheme D (six pairs of mid-span crossed cables are formed by adjusting the spacing of stay cables). When the unbalanced load is 50 kN/m, the horizontal displacement of the middle tower of scheme D is reduced by 28.63%, the deflection of the main beam is reduced by 13.26%, the maximum positive bending moment of the main beam is reduced by 29.80%, and the change value of the mid-span stay cable force is reduced by 17.55% and 46.34%. The result shows that the cross cable acts as a connection to balance the internal force of the component and can greatly improve structural performance. Under the unbalanced load, the cross cables can effectively solve the problems of insufficient stiffness and excessive deformation of the middle tower. Cross-cable multi-tower cable-stayed bridges have better spanning capacity than conventional multi-tower cable-stayed bridges using the same materials.

The cross-ratio is an important parameter of cross-cable multi-tower cable-stayed bridges. With the increase in the number of cross-cables, structural stiffness increases. However, cable efficiency decreases, and cable consumption increases. This paper proposes an optimal design scheme for the multi-tower cable-stayed bridge with crossed cables based on the double indicators of structural stiffness and cable consumption. That is to form mid-span cables crossing by adjusting the spacing of the cables, and the reasonable range of the cross-ratio is 15%~35%. If the structural stiffness is improved by adding additional cross cables, the cross-ratio should be controlled within 16% to ensure structural economy.

Cross-cable multi-tower cable-stayed bridges are more complex than traditional multi-tower cable-stayed bridges. It is very meaningful to analyze the local stress of its main beam and the anchorage area of cross cables. Sensitivity analysis of cross-cable multi-tower cable-stayed bridges with different spans, materials, and design parameters is worthy of further studies in the future.

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References

1. Yu, B.C.; Wang, J.S.; Ai, J.; Sun, R.Y. The structural form and development of cable-stayed bridge. In Proceedings of the 2017 2nd International Conference on Advances in Materials, Mechatronics and Civil Engineering (ICAMMCE 2017), Guangzhou, China, 19–20 January 2017; Volume 121, pp. 61–64.
2. Wang, P.H.; Yang, C.G. Parametric studies on cable-stayed bridges. *Comput. Struct.* **1996**, *60*, 243–260. [CrossRef]
3. Gimsing, N.J. *Cable Supported Bridges: Concept & Design*; John Wiley & Sons, Ltd. Publication: Hoboken, NJ, USA, 1997.
4. Gimsing, N.J. The modern cable-stayed bridge—50 years of development from 1955 to 2005. In Proceedings of the International Symposium on Innovation & Sustainability of Structures in Civil Engineering, Nanjing, China, 1 January 2005; Volume 1, pp. 47–64.
5. Cid, C.; Baldomir, A.; Hernandez, S. Optimum crossing cable system in multi-span cable-stayed bridges. *Eng. Struct.* **2018**, *160*, 342–355. [CrossRef]
6. Shao, X.D.; Hu, J.; Deng, L.; Cao, J.H. Conceptual design of superspan partial ground-anchored cable-stayed bridge with crossing stay cables. *J. Bridge Eng.* **2014**, *19*, 06013001. [CrossRef]
7. Ruiz-Teran, A.M.; Aparicio, A.C. Two new types of bridges: Under-deck cable-stayed bridges and combined cable-stayed bridges—The state of the art. *Can. J. Civ. Eng.* **2007**, *34*, 1003–1015. [CrossRef]
8. Akhlaq, H.; Butt, F.; Alwetaishi, M.; Riaz, M.; Benjeddou, O.; Hussein, E.E. Structural identification of a 90 m high minaret of a landmark structure under ambient vibrations. *Buildings* **2022**, *12*, 252. [CrossRef]
9. Xia, G.P.; Zhang, Z. Cable deflection and gravity stiffness of cable-stayed suspension bridge. In *Advanced Materials Research*; Trans Tech Publications Ltd.: Freienbach, Switzerland, 2011; Volume 255, pp. 1039–1042.
10. Polak, M.; Micka, T.; Klier, T.; Plachy, T.; Simler, M. An experimental analysis of large vibrations of selected cable-stays on a cable-stayed bridge. *Appl. Mech. Mater.* **2016**, *837*, 85–88. [CrossRef]
11. Wang, D.W.; Shang, M.F.; Sun, P.X. Deformation performance analysis of a truss structure based on the deformation decomposition method. *Buildings* **2022**, *12*, 258. [CrossRef]
12. Kite, S.E.; Hornby, R.; Minto, B.; Carter, M.T.; Hussain, N.M. Queensferry Crossing, Scotland-scheme, specimen and definition designs. *J. Bridge Eng.* **2019**, *172*, 92–112.

13. Carter, M.T.; Kite, S.E.; Hussainm, N.M.; Minto, B. Design of the Forth replacement crossing, Scotland. *Proc. Inst. Civ. Eng.* **2010**, *163*, 91–99. [CrossRef]
14. Kite, S.E.; Hussain, N.M.; Carter, M.T. Forth replacement crossing-Scotland, UK. *Procedia Eng.* **2011**, *14*, 1480–1484. [CrossRef]
15. Yang, X.Y. Summer. The design of FRC in Scotland. *Bridge* **2011**, *2*, 5–7.
16. Ning, B.W. Overall design of Bianyuzhou Yangtze river bridge on newly-built Anjiu high-speed railway. *Bridge Constr.* **2020**, *50*, 86–91.
17. Virlogeux, M. Bridges with multiple cable-stayed spans. *Struct. Eng. Int.* **2001**, *11*, 61–82. [CrossRef]
18. Rodado, J.; Manterola, J. Multi-span cable stayed bridges. In Proceedings of the IABSE Conference: Cable-Supported Bridges—Challenging Technical Limits, Seoul, South Korea, 12–14 June 2001; pp. 25–33. [CrossRef]
19. Tang, M.C. Cable-stayed bridges. In *Bridge Engineering Handbook*; Chen, W.-F., Duan, L., Eds.; CRC Press: Boca Raton, FL, USA, 2000.
20. Yan, G.M. Re-discussion on some cable-stayed bridges, and also on multi-tower cable-stayed bridges. In Proceedings of the 13th National Bridge Academic Conference, Shanghai, China, 16–19 November 1998; pp. 178–182.
21. Zhou, N.X. Prospect of design conception of cable-stayed bridge in the 21st century. In Proceedings of the 13th National Bridge Academic Conference, Shanghai, China, 16–19 November 1998; pp. 161–170.
22. Chen, A.R.; Xiang, H.F. Conceptual design of multi-tower cable-stayed bridge. In Proceedings of the 13th National Bridge Academic Conference, Shanghai, China, 16–19 November 1998; pp. 171–177.
23. Chai, S.B.; Zhang, R.L.; Wang, X.L. Influence of crossed-cables arrangement on mechanical performance of multi-tower cable stayed bridge. *Sci. Technol. Eng.* **2021**, *21*, 13131–13138.
24. Chai, S.B.; Wang, X.L. Simplified calculation method for deformation of multi-tower cable-stayed bridges with crossed cables. *Eng. Struct.* **2019**, *181*, 354–361. [CrossRef]
25. Cao, S.S.; Lei, J.Q.; Li, Z.S.; Lin, D.J.; Wang, R.G. Stiffness analysis of multi-tower cable-stayed bridge. *J. World Bridge* **2012**, *40*, 55–59.
26. Yu, M.; Liao, H.L.; Li, Q.; Ma, C.M.; Li, M.S. Research on the structural system of multi-tower cable-stayed bridges. *J. Railw. Build.* **2015**, *3*, 12–15.
27. Miao, J.W. *Research on Design Theory of Super-Span Cable-Stayed Bridge*; Tongji University: Shanghai, China, 2006.
28. Lin, D.J.; Li, Z.S.; Wang, R.G. Study on mechanical properties of multi-tower cable-stayed bridges. *Highway* **2013**, *7*, 317–321.
29. Mao, Y.F. Mathematical modeling of cable—Stayed bridge. In Proceedings of the 7th International Conference on Education, Management, Information and Mechanical Engineering, Shenyang, China, 28–30 April 2017; Volume 76, pp. 1816–1820.
30. Yutaka, O.; Shunichi, N. Static and seismic studies on steel/concrete hybrid towers for multi-span cable-stayed bridges. *J. Constr. Steel Res.* **2011**, *67*, 203–210.
31. Alsayed, M.; Lin, L.; Hassan, J. Static behavior of partially earth-anchored cable-stayed bridge of different side-to-main span ratios: Super-long span system with crossing cables. *Structures* **2021**, *33*, 3736–3745. [CrossRef]
32. Xiong, F.; Chen, C.C. A comparative study on the economic characteristics of the extradosed cable-stayed bridge and cable stayed bridge. *Appl. Mech. Mater.* **2015**, *721*, 744–746. [CrossRef]
33. Arellano, H.; Tolentino, D.; Gomez, R. Optimum criss crossing cables in multi-span cable-stayed bridges using genetic algorithms. *J. Civ. Eng.* **2019**, *23*, 719–728. [CrossRef]
34. Juszczyk, M. On the search of models for early cost estimates of bridges: An SVM-based approach. *Buildings* **2020**, *10*, 2. [CrossRef]
35. *JTG/T 3365-01-2020*; Specifications for Design of Highway Cable-stayed Bridge. Ministry of Transport of the People's Republic of China: Beijing, China, 2020.
36. Chai, S.B.; Xiao, R.C.; Wang, X.L. Longitudinal restraint stiffness of cross-cables in multi-tower cable-stayed bridges. *J. Harbin Inst. Technol.* **2016**, *9*, 354–361.
37. Li, Z.S. *Based on the Static and Dynamic Characteristics of Long Span Multi Tower Cable-Stayed Bridge Structure Stiffness Research*; Beijing Jiaotong University: Beijing, China, 2014.