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Study of the Nonuniform Consolidation Characteristics of Soft Soils Using a Novel Model

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Abstract: The degree of soil nonuniformity consolidation varies continuously with the passage of consolidation time and is accompanied by nonlinear alterations in soil parameters. Existing theoretical models often assume a constant relationship between the parameters of the two soil layers, failing to consider the effect of nonuniform consolidation. This assumption does not align with real-world conditions and can lead to significant errors in calculation results. Hence, this study aims to investigate the dynamic changes in soil undergoing nonuniform consolidation and develop a mathematical model that accounts for this phenomenon. Based on the large-strain and double-layer models, an improved consolidation model was proposed, which considers nonuniform variations in consolidation with a vertical drain and corrections to calculations under the influence of the nonlinear relationships of soil parameters. The proposed improved model was validated by comparison with field test data, and the results were compared with those of the classical model. Finally, the effects of different consolidation parameters on consolidation behavior were investigated. The research is a reliable calculation method that incorporates the dynamic nonuniform changes in consolidated soil, enabling more accurate predictions of consolidation of foundations treated by vertical drains.

Keywords: nonuniform consolidation; vertical drains; foundation treatment; novel model; nonlinearity

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1. Introduction

Vertical drains, such as PVD, stone columns, and sand drains, have been widely used to accelerate the consolidation of soft soil foundations in recent years [1–7]. The purpose of installing vertical drains in soft soils is to reduce the seepage path and induce the water in the soil to flow radially into drains. This way, the consolidation of the foundation is completed, and the strength of the foundation is improved.

Numerous studies have found that the process of vertical drain installation forms a smear layer by squeezing the soil around the vertical drain [8–10]. According to the characteristics of the process of vertical drain installation, the layered expression centered on the vertical drain was the basic idea for building the vertical drain consolidation model [3]. Based on the single-layer ideal model, Hansbo et al. proposed a double-layer model that included the smear effect and completed the derivation [3]. Currently, theoretical models using two-layer expressions have been widely recognized and developed [11–14]. According to this series of double-layer theoretical models, the permeability coefficient (k_s) of the smear zone is much smaller than that of the undisturbed zone (k_h), and the consolidation rate in the theoretical calculations is greatly influenced by the ratio k_h/k_s [14–16]. Most current theories assume that the state relationship between the two soil layers remains constant (e.g., k_h/k_s is assumed to be a constant) and often uses a value of k_h/k_s from 3 to 8 for the calculation [3,10,14,17]. In fact, this only reflects the initial state when the smear layer is formed and does not consider the influence of changes during the consolidation process. Several studies have shown that the degree of soil nonuniformity in undisturbed and disturbed zones constantly changes during consolidation; ignoring the dynamics of nonuniform consolidation can severely affect the results of consolidation calculations [7,18–20]. To consider this dynamic development process, Zhou et al. proposed the concept of 'equivalent smear' for consolidation calculations [18]. However, this method ignores nonlinear variations in soil parameters with consolidation and assumes a constant value that is divided into multiple segments. Considering the clogging effect in the vacuum preloading treatment of dredged slurry and assuming that the clogging zone forms rapidly and constantly, Zhou et al. proposed using the ratio of the initial permeability coefficient to the permeability coefficient of the clogging zone to reflect the initial nonuniformity [7]. The introduction of the initial permeability coefficient and its expression in combination with the nonlinear relationships of soil parameters does not misrepresent the nonuniform properties of the consolidation process. However, the model is limited by the specificity of the vacuum preloading process, assuming that clogging is instantaneously generated and constant. In summary, no theoretical model considering the radial nonuniform dynamic changes in the soil in the vertical drain has been reported.

The degree of nonuniformity of vertical drain consolidation varies continuously with consolidation time and is accompanied by nonlinear changes in soil parameters. Although the theoretical calculation is thought to be more reasonable, it also significantly increases the difficulty of application in practical engineering. This is because it is frequently difficult to solve under the impact of nonlinear parameters and complex models. In previous studies, to obtain closed-form solutions for the vertical strain of soft soils, parameters such as permeability and compression coefficients were generally assumed to be constant, and stress–strain curves conformed to linear behavior. In fact, these assumptions of constant values were incompatible with nonuniform variations in the model and nonlinear variations in the soil parameters. At present, considering the effect of nonlinear variations in soil parameters has attracted the attention of many scholars [21–23]. Considering that nonlinear variations in soil parameters in consolidation calculations often render a very complicated solution, scholars have tried to perform simplified calculations [17,24]. Indraratna et al. completed the theoretical derivation of a two-layer model based on classical void ratio–effective stress and void ratio–permeability relationships and gave a solution by mathematical treatment [17]. However, the proposed theory ignored the effect of nonuniform consolidation (e.g., k_h/k_s was still assumed to be constant). The solution method proposed in the paper, which was compared by taking different values of c_c/c_k, often led to errors. To better reflect real changes in the soil parameters, a large-strain model of vertical drain consolidation was introduced [20,22]. Although vertical drain consolidation theories were derived based on large deformation coordinates, the influence of smears was not considered, and the presence of nonuniform consolidation was neglected. The complexity of nonlinear parameters makes analytical solutions more challenging, but scientifically equivalent solutions can truly benefit the application and promotion of engineering [7,24]. Considering the nonlinear variation in soil parameters and calculating with scientifically equivalent models are important tasks in the field of vertical drain consolidation.

Therefore, in this study, an improved consolidation model based on the large-strain and double-layer model was proposed. This model considers nonuniform variations in vertical drain consolidation and modified calculations of the nonlinear relationships of soil parameters. It was validated by field test data, and the results of the classical solution were compared with the proposed solution. Finally, the effects of different consolidation parameters on consolidation behavior were investigated.

2. The Related Developments of Vertical Drain Theory

Based on Barron's classical consolidation theory, a radial consolidation model considering smears under the assumption of equal strain was derived [3]. The average degree of consolidation was almost the same under equal strain and free strain, but the solution was simpler under the assumption of equal strain. Thus, it has been common to use equal strain in most radial drainage consolidation analyses. Figure 1 shows an axisymmetric model with the smear effect, which includes the drain well, the smear zone, and the undisturbed zone of radii r_{w}, r_{s}, and r_{e}, respectively. Assuming that the soft soil parameters are constant, the equation was established as follows:

$$-\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{k_{\mathrm{s}}}{\gamma_{\mathrm{w}}}r\frac{\partial u}{\partial r}\right) = \frac{\partial \varepsilon_{\mathrm{v}}}{\partial t}\ r_{\mathrm{w}} < r \leq r_{\mathrm{s}} \tag{1}$$

$$-\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{k_{\mathrm{h}}}{\gamma_{\mathrm{w}}}r\frac{\partial u}{\partial r}\right) = \frac{\partial \varepsilon_{\mathrm{v}}}{\partial t}\ r_{\mathrm{s}} < r \leq r_{\mathrm{e}} \tag{2}$$

where k_{h} is the horizontal permeability of soft soils, k_{s} is the permeability of the smear zone, γ_{w} is the unit weight of water, u is the excess pore pressure, and ε_{v} is the vertical strain.

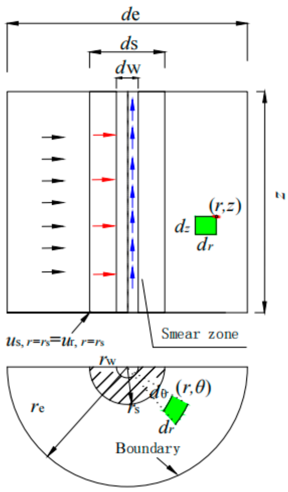

Figure 1. Consolidation model for vertical drains including smears.

The average excess pore water pressure for vertical drains can be expressed as follows:

$$\overline{u} = u_0 \exp\left(\frac{-8T_{\mathrm{h}}}{\mu}\right) \tag{3}$$

where $\mu = \frac{n^2}{n^2-1}\left(\ln\frac{n}{s} + \frac{k_{\mathrm{h}}}{k_{\mathrm{s}}}\ln s - \frac{3}{4}\right) + \frac{s^2}{n^2-1}\left(1 - \frac{s^2}{4n^2}\right) + \frac{k_{\mathrm{h}}}{k_{\mathrm{s}}}\frac{1}{n^2-1}\left(\frac{s^4-1}{4n^2} - s^2 + 1\right)$, $s = \frac{r_{\mathrm{s}}}{r_{\mathrm{w}}}$, and $n = \frac{r_{\mathrm{e}}}{r_{\mathrm{w}}}$.

Nonlinear variations in permeability and compressibility with respect to the void ratio should be considered for the constitutive behavior of clay, which undergoes large-strain deformations. The void ratio–effective stress and void ratio–permeability relationships can be expressed as:

$$e = e_0 - c_{\mathrm{c}} \log\left(\frac{\sigma'_{\mathrm{v}}}{\sigma'_0}\right) \tag{4}$$

$$e = e_0 + c_{\mathrm{k}} \log\left(\frac{k_{\mathrm{h}}}{k_{\mathrm{h0}}}\right) \tag{5}$$

where e_0 is the initial void ratio, e is the current void ratio, σ'_0 is the initial effective stress of soft soils, σ'_{v} is the effective stress, k_{h0} is the initial horizontal permeability of soft soils, c_{c} is the compression index, and c_{k} is the permeability index.

Based on Hansbo's model and combining Equations (4) and (5), Indraratna et al. [17] expressed the excess pore pressure under equal strain as:

$$\bar{u} = u_0 \exp\left(-P_{\mathrm{av}} \frac{8T_{\mathrm{h0}}}{\mu}\right) \tag{6}$$

Equation (6) is very similar to Equation (3). The main difference between them is that the average value between the starting and ending values of the coefficient of consolidation is represented by the parameter P_{av} in Equation (6). Although the log-linear void ratio–stress relationship (i.e., Equations (4) and (5)) was used for calculations in the research of Indraratna et al. [17], it does not reflect the nonuniform consolidation over time. As a result, the solving method may lead to significant errors.

3. Nonuniform and Nonlinear Characteristics

3.1. Variation in Nonuniform Consolidation

In terms of consolidation time, an initial nonuniform effect (i.e., t = 0) is produced after a vertical drain is installed. As consolidation proceeds, the nonuniformity dynamically changes, and when the time is sufficiently long, the soil nonuniformity tends to be uniform (i.e., $t = t_\infty$). On the basis of the effective stress principle, the excess pore water pressure of soft soil at any location continues to dissipate under ideal conditions and infinite consolidation time [3,9,14]. Therefore, as shown in Figure 2, the variation in permeability coefficients in the disturbed and undisturbed zones throughout consolidation can be simply divided into three states. The permeability coefficient in the vertical drain is assumed to always be much greater than that of soft soils. When $t = t_0$ (i.e., when the vertical drain is installed), the soft soils around the vertical drain are compacted by mechanical action to form a layer of soft soil with low permeability, while the permeability coefficient of the soft soils in the undisturbed area remains unchanged. At $t = t_\infty$ (i.e., when the consolidation time is sufficiently long), the excess pore water pressure of the whole soft soil layer decreases to 0, and the permeability coefficient of the undisturbed area is consistent with that of the smear area. Parameter λ is introduced to express the relationship between k_{h} and k_{s} at any moment, as follows:

$$\lambda = \frac{k_{\mathrm{h}}}{k_{\mathrm{s}}} \tag{7}$$

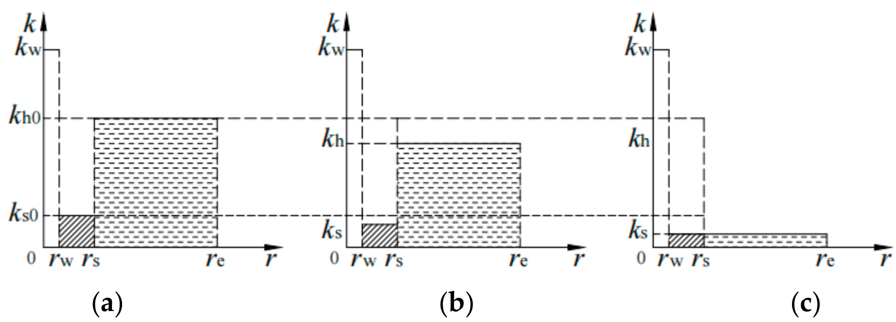

Figure 2. Radial distribution of the permeability coefficient. (**a**) $t = t_0$. (**b**) $t = t$. (**c**) $t = t_\infty$.

Considering that the soil parameters are nonlinear and combining Equations (4) and (5) yields a modification of Equation (7), as follows:

$$\lambda = \frac{k_{\mathrm{h0}}}{k_{\mathrm{s0}}} \left(\frac{\sigma'_{\mathrm{v0}} + \Delta\overline{\sigma'}_{\mathrm{v}}}{\sigma'_{\mathrm{v0}}} \frac{\sigma'_{\mathrm{vs}}}{\sigma'_{\mathrm{vs}} + \Delta\overline{\sigma'}_{\mathrm{v}}} \right)^{-c_{\mathrm{c}}/c_{\mathrm{k}}} \tag{8}$$

According to Equation (3), λ is very important for the consolidation calculation, and it is one of the most important parameters affecting the consolidation rate. According to the analysis of Figure 2, when $t = t_0$, the maximum value $\lambda_{\max} = k_{\mathrm{h0}}/k_{\mathrm{s0}}$ is obtained, and λ is 1 at $t = t_\infty$. Then, during the whole consolidation process, λ is in the range from 1 to $\lambda_{\max}$. In the proposed model, the influence of λ is considered in Equation (21).

3.2. Compressibility and Permeability Nonlinearity

According to Equations (4) and (5), the constitutive laws relating the void ratio e to the permeability k_h and effective stress σ'_v are obtained as follows (Figure 3). Differentiating Equation (4) with respect to the effective stress, and then introducing parameters J, L, and Q, yields:

$$J = 1 + \left(\frac{\Delta\sigma}{\sigma'_0}\right) - \left(\frac{u_t}{\sigma'_0}\right) \tag{9}$$

$$L = \left(\frac{\overline{\sigma'}_v}{\sigma'_0}\right)^{-c_c/c_k} = \left[1 + \left(\frac{\Delta\sigma}{\sigma'_0}\right) - \left(\frac{u_t}{\sigma'_0}\right)\right]^{-(c_c/c_k)} \tag{10}$$

$$L = \left(\frac{\overline{\sigma'}_v}{\sigma'_0}\right)^{-c_c/c_k} = \left[1 + \left(\frac{\Delta\sigma}{\sigma'_0}\right) - \left(\frac{u_t}{\sigma'_0}\right)\right]^{-(c_c/c_k)} \tag{11}$$

where $\Delta\sigma$ is the additional load of soft soils and u_t is the excess pore pressure.

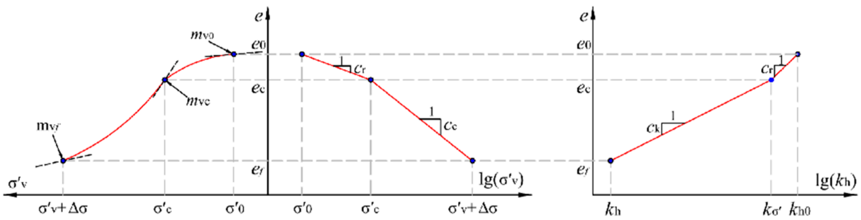

Figure 3. The relationship of e-lg k and e-lg σ_v.

To simplify the calculation, mathematical processing was performed [17,24]. In Indraratna et al.'s study [17], the value of P was given by means of linear equivalence (i.e., Q in Equation (11)), as in Equation (12).

$$P = P_{av} = 0.5\left[1 + \left(1 + \frac{\Delta p}{\sigma'_i}\right)^{1-(c_c/c_k)}\right] \tag{12}$$

where Δp is the additional load of soft soils and σ'_i is the initial effective stress.

Furthermore, the solution method causes large errors when some values are taken. Therefore, this paper gives a new calculation method for Q. The calculation method for L is also provided. Hence, by the method of average integration, the average values of Equations (10) and (11) are given by:

$$\overline{L} = \frac{\sigma'_0\left[\left(1 + \left(\frac{\Delta\sigma}{\sigma'_0}\right)\right)^{1-(c_c/c_k)} - 1\right]}{\Delta\sigma(1 - (c_c/c_k))} \tag{13}$$

$$\overline{Q} = \frac{\sigma'_0\left[\left(1 + \left(\frac{\Delta\sigma}{\sigma'_0}\right)\right)^{2-(c_c/c_k)} - 1\right]}{\Delta\sigma(2 - (c_c/c_k))} \tag{14}$$

By assuming σ'_0 = 2 kPa and $\Delta\sigma$ = 80 kPa, the proposed method is compared with linear equivalence. As shown in Figure 4, when c_c/c_k = 0.5, the results of the two methods differ by 15%; when c_c/c_k = 1.5, the difference is nearly twice. Combined with the actual form of Q, this shows that the method proposed in this paper can better reflect the result.

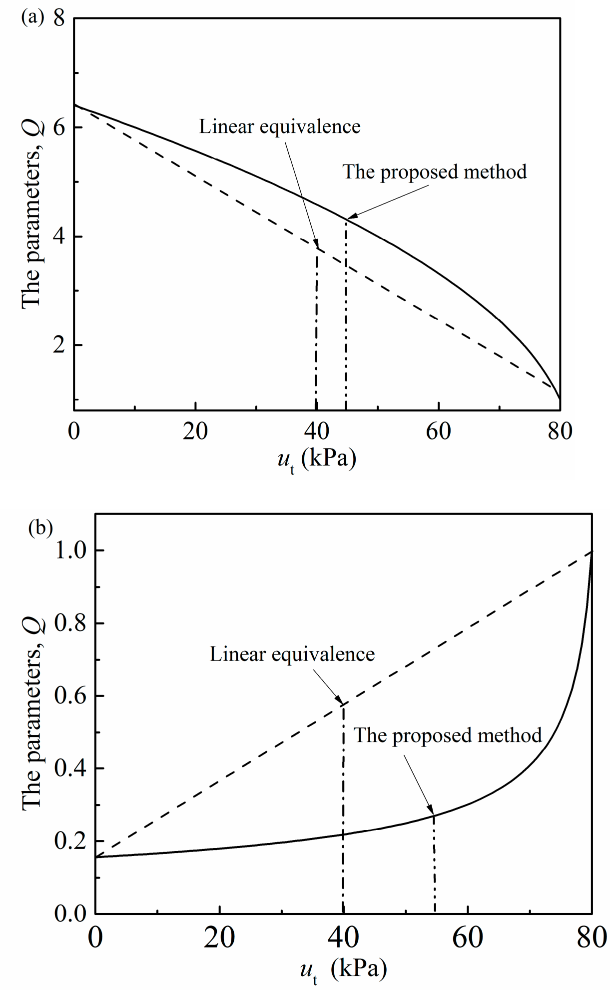

Figure 4. Comparative analysis of Q: (**a**) c_c/c_k = 0.5 and (**b**) c_c/c_k = 1.5.

4. Analytical Solution of the Governing Equation

After Gibson et al. [25], the relationship between the Lagrangian coordinate a and the convective coordinate ξ is obtained:

$$\frac{\partial \xi}{\partial a} = \frac{1+e}{1+e_0} \tag{15}$$

Based on the large-strain radial consolidation analysis of Geng et al. [22], Nguyen et al. [13] and Zhou et al. [7] established a large-strain governing equation featuring radial flow, as shown in Equation (16):

$$2\pi r k_h \frac{1}{\gamma_w}\frac{\partial u}{\partial r}\frac{\partial \xi}{\partial a}\mathrm{d}t\mathrm{d}a = -\frac{1}{1+e_0}\frac{\partial e}{\partial t}\pi\left(r_e^2 - r^2\right)\mathrm{d}t\mathrm{d}a \tag{16}$$

Then, Equation (16) can be written as:

$$\frac{\partial u}{\partial r} = -\frac{\gamma_w}{2rk_h}\left(r_e^2 - r^2\right)\frac{1}{1+e}\frac{\partial e}{\partial t} \tag{17}$$

Using the boundary condition u = 0 at r = r_w, the excess pore pressure in the smear zone can be derived:

$$u_s = -\frac{\gamma_w}{2k_s}\left(r_e^2 \ln\frac{r}{r_w} - \frac{r^2 - {r_w}^2}{2}\right)\frac{1}{1+e}\frac{\partial e}{\partial t},\ r_w < r \le r_s \tag{18}$$

where u_s is the excess pore pressure in the smear zone.

The excess pore pressure at the outer boundary of the smear zone ($u_{s,r=rs}$) is equal to the inner boundary of the undisturbed zone ($u_{r,r=rs}$); then:

$$u_r = -\frac{\gamma_w}{2k_h}\left(r_e^2 \ln\frac{r}{r_s} - \frac{r^2 - {r_s}^2}{2}\right)\frac{1}{1+e}\frac{\partial e}{\partial t} - \frac{\gamma_w}{2k_s}\left(r_e^2 \ln\frac{r_s}{r_w} - \frac{{r_s}^2 - {r_w}^2}{2}\right)\frac{1}{1+e}\frac{\partial e}{\partial t},\ r_s < r \le r_e \tag{19}$$

where u_r is the excess pore pressure in the undisturbed zone.

The average excess pore pressure $\overline{u}_t$ at depth for a given time is:

$$\overline{u}_t = \frac{\int_0^l \int_{r_w}^{r_s} 2\pi r \cdot u_s \mathrm{d}r\mathrm{d}z + \int_0^l \int_{r_s}^{r_e} 2\pi r \cdot u_r \mathrm{d}r\mathrm{d}z}{\pi(r_e^2 - r_w^2)l} \tag{20}$$

where l is the depth of the influential area.

By substituting Equations (18) and (19) into Equation (20) and integrating:

$$\overline{u}_t = -\frac{\gamma_w r_e^2}{2k_h}\frac{1}{1+e}\frac{\partial e}{\partial t} \cdot \mu \tag{21}$$

where $\mu = \frac{n^2}{n^2-s^2}\left(\ln\frac{n}{s} - \frac{3}{4} + \frac{s^2}{n^2} - \frac{s^4}{4n^4}\right) + \lambda\left(\frac{n^2-s^2}{n^2 s}\right)(s-1)$.

Nonlinear variation in the permeability and compressibility with respect to the void ratio should be considered for the constitutive behavior of clay, which undergoes large-strain deformation. The void ratio–effective stress and void ratio–permeability relationships can be expressed as Equations (4) and (5), respectively. Differentiating Equation (4) with respect to effective stress gives:

$$\frac{m_{v0}}{m_v} = J \tag{22}$$

Under the assumption of constant strain, consolidation is obtained:

$$-\frac{1}{1+e}\frac{\partial e}{\partial t} = m_v \frac{\partial \sigma'_v}{\partial t} \tag{23}$$

where m_v is the coefficient of volume change, $m_v = m_{v0}\frac{\sigma'_{v0}}{\sigma'_v}$ [7,17].

By combining Equations (22) and (23), Equation (21) can be rewritten in the following terms:

$$\overline{u}_t = \frac{\gamma_w m_{v0}}{2k_{h0}}\frac{\partial \sigma'_v}{\partial t} r_e^2 \cdot \mu' \cdot \left(\frac{\sigma'_v}{\sigma'_0}\right)^{c_c/c_k - 1} \tag{24}$$

where $\mu' = \frac{n^2}{n^2-s^2}\left(\ln\frac{n}{s} - \frac{3}{4} + \frac{s^2}{n^2} - \frac{s^4}{4n^4}\right) + \lambda_{max}\left(\frac{\sigma'_{v0}+\Delta\overline{\sigma'}_v}{\sigma'_{v0}}\frac{\sigma'_{vs}}{\sigma'_{vs}+\Delta\overline{\sigma'}_v}\right)^{-c_c/c_k}\left(\frac{n^2-s^2}{n^2 s}\right)(s-1)$, $\lambda_{max} = \frac{k_{h0}}{k_{s0}}$, and $m_{v0} = \frac{c_c}{\mathrm{In}10\ (1+\overline{e}_0)\ \sigma'_0}$.

Based on the principle of effective stress, the average excess pore pressure can be expressed as:

$$\overline{u}_t = \sigma'_0 + \Delta\sigma - \sigma'_v \tag{25}$$

Substituting Equations (10), (11) and (25) into (24) yields:

$$\overline{u}_t = \frac{\gamma_w m_{v0}}{2k_{h0}}\frac{\partial \overline{u}_r}{\partial t} r_e^2 \cdot \mu \cdot \frac{1}{Q} \tag{26}$$

where $\mu' = \frac{n^2}{n^2-s^2}\left(\ln\frac{n}{s} - \frac{3}{4} + \frac{s^2}{n^2} - \frac{s^4}{4n^4}\right) + \lambda_{max}\frac{L_{h0}}{L_s}\left(\frac{n^2-s^2}{n^2 s}\right)(s-1)$.

The governing equation of large-strain radial nonuniform consolidation based on compression and permeability nonlinearity is shown in Equation (26), which does not have a general solution. By incorporating the above assumption, Equation (26) can be written as:

$$\overline{u}_{\mathrm{t}}=\frac{\gamma_{\mathrm{w}}m_{\mathrm{v0}}}{2k_{\mathrm{h0}}}\frac{\partial\overline{u}_{\mathrm{r}}}{\partial t}r_{\mathrm{e}}^{2}\cdot\overline{\mu}\cdot\frac{1}{Q} \tag{27}$$

where $\overline{\mu}=\frac{n^2}{n^2-s^2}\left(\ln\frac{n}{s}-\frac{3}{4}+\frac{s^2}{n^2}-\frac{s^4}{4n^4}\right)+\lambda_{\max}\frac{L_{\mathrm{h0}}}{L_{\mathrm{s}}}\left(\frac{n^2-s^2}{n^2 s}\right)(s-1)$.

The average degree of consolidation (U_{r} in %) can be calculated conveniently by the following equation:

$$U_{\mathrm{h}}=\frac{u_0-\overline{u}_{\mathrm{t}}}{u_0-u_\infty}\times 100\% \tag{28}$$

5. Verification of the Proposed Model

This section describes the comparison of the proposed model with the results given by Hansbo [3] and Indraratna et al. [17]. According to Berry et al. [26], the values of $c_{\mathrm{c}}/c_{\mathrm{k}}$ for soft soils in the range of 0.5–2.0 were used in the analysis. Table 1 lists the relevant parameters used for comparison with different models. Some constants of Hansbo's model can be calculated by using the parameters in Table 1, e.g., the coefficient of volume compressibility, $m_{\mathrm{v}} = c_{\mathrm{c}}/[\ln 10\ (1 + e)\ \sigma_{\mathrm{v}})]$.

Table 1. Parameters used for comparison with different models.

Parameter	Value
Initial void ratio, e_0	1.5
Initial horizontal permeability, k_{h0} ($\times 10^{-9}$ m/s)	2.5
Initial effective stress, σ'_0 (k Pa)	10
Compression index, c_{c}	0.4
$k_{\mathrm{h}}/k_{\mathrm{s}}$	5
Radius of drain well, r_{w} (m)	0.033
$n = r_{\mathrm{e}}/r_{\mathrm{w}}$	12
$s = r_{\mathrm{s}}/r_{\mathrm{w}}$	3
Additional load, q_{t}, (kPa)	100

Figure 5 shows the comparison between the solution of the proposed model and the solutions of Hansbo [3] and Indraratna et al. [17]. With the same consolidation time, the degree of consolidation of Hansbo's solution is smaller than that of the proposed model and Indraratna's solution. The result of the proposed model is close to that of Indraratna's solution, both of which consider the nonlinearity of compression and permeability. Taking into account the influence of nonuniform consolidation (i.e., λ is not a constant value, $\lambda_{\max}$), the proposed model has a faster consolidation rate, which is consistent with the results in Figure 5.

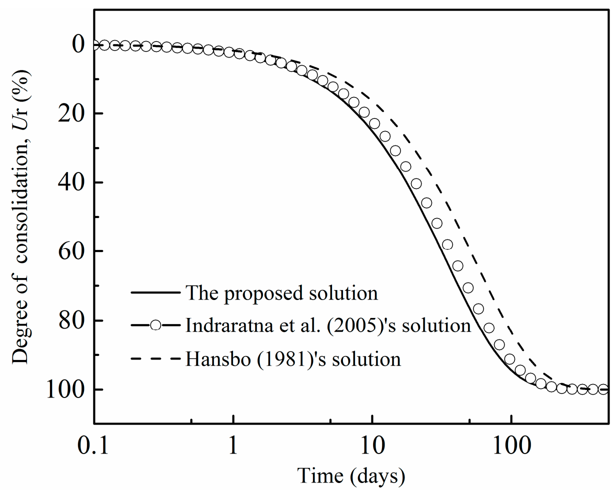

Figure 5. Comparison between the proposed model and solutions from the literature [3,17].

6. Effects of Consolidation Characteristics

Using compression and permeability nonlinearity, the proposed radial nonuniform consolidation with large-strain theory (Equation (26)) includes several parameters, such as λ, s, c_c/c_k, and σ'_0, that affect the consolidation of the vertical drain improved soft foundation. Therefore, the influence of these key parameters on consolidation was studied in this section. During parameter analysis, other specific parameter assumptions were as follows: n = 12; s = 3; c_c/c_k = 0.46; σ'_0 = 15 kPa; $k_{h0} = 2.5 \times 10^{-9}$ m/s; λ_{max} = 5; and q_t = 100 kPa.

6.1. Smear Value of Vertical Drain

The installation of vertical drains squeezes soft soils and reduces the permeability of the surrounding soft soils. The degree of reduction in the permeability coefficient is reflected by λ_{max} in Equation (2). Figure 6 shows that increasing λ_{max} reduces the consolidation rate of soft soils. After 20 days, when the degree of the consolidation of λ_{max} = 2 increased to 78%, it was 48% higher than that of λ_{max} = 10. The analysis showed that the smear effect greatly affected the consolidation rate of the foundation, and it was very important to consider the actual influence of nonuniform consolidation with the smear effect.

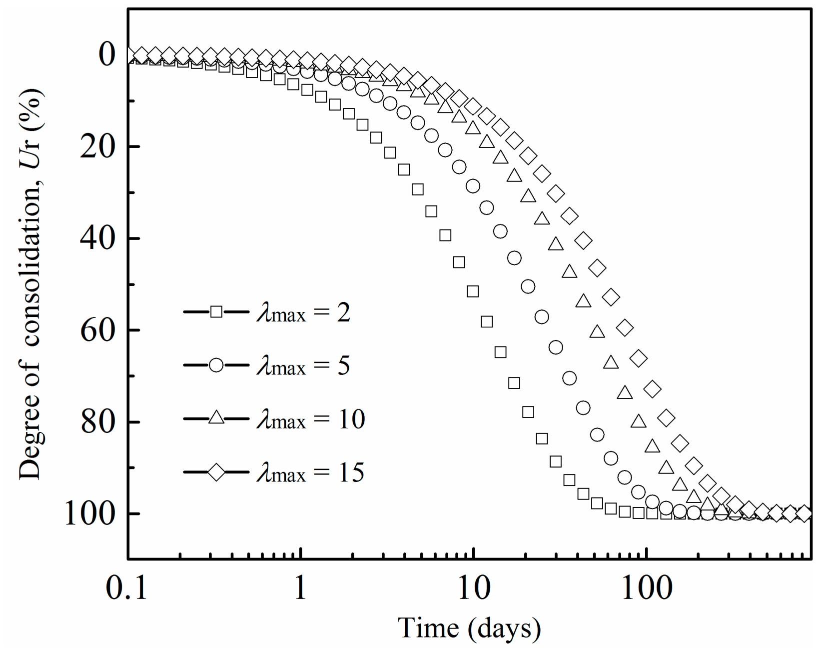

Figure 6. Average degree of the proposed model with variation in λ_{max}.

The thickness of the compacted soft soil layer formed by extrusion is the smear radius. As shown in Figure 7, the consolidation rate decreased as s increased. When s = 2, the smear zone was small, consolidation took only 22 days, and the degree of consolidation reached 60%. It required 41 days for the same degree of consolidation with s = 6. However, when s was greater than four, with s increasing, the rate of the decrease in the consolidation rate decreased. This showed that the consolidation rate was mainly affected by the soft soil layer near the vertical drain. Notably, with the continuous increase in s, the downwards trend of the consolidation rate decreased rapidly. Therefore, a smear ratio of 2–4 was reasonable. The results also showed that even when the smear layer was very thin, it greatly affected the consolidation rate.

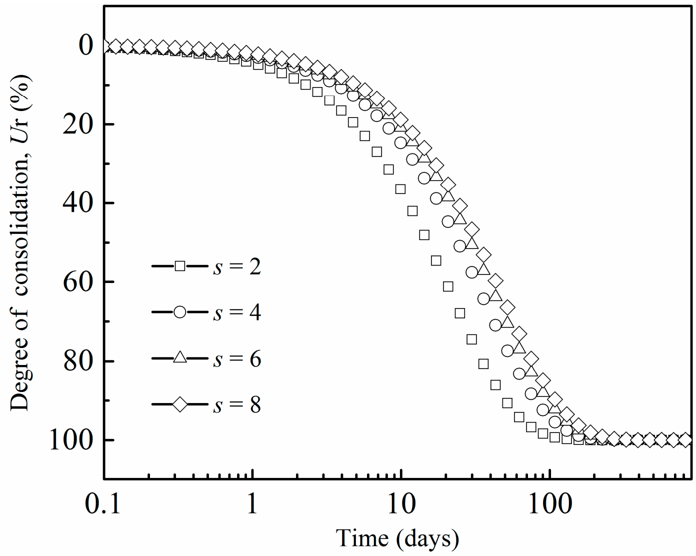

Figure 7. Average degree of the proposed model with a variation in *s*.

6.2. Drain Diameter Ratio of the Vertical Drain

The radius range of the vertical drain foundation treatment is not infinite, especially when the treatment needs to be carried out in a reasonable amount of time. Therefore, it is necessary to explore the reasonable range for foundation treatment through different values of *n*. Figure 8 shows that the greater *n* is, the smaller the consolidation rate. However, with the increase in *n*, the decreasing rate of the consolidation rate decreases only slightly. The range of the foundation treatment is positively correlated with the consolidation rate.

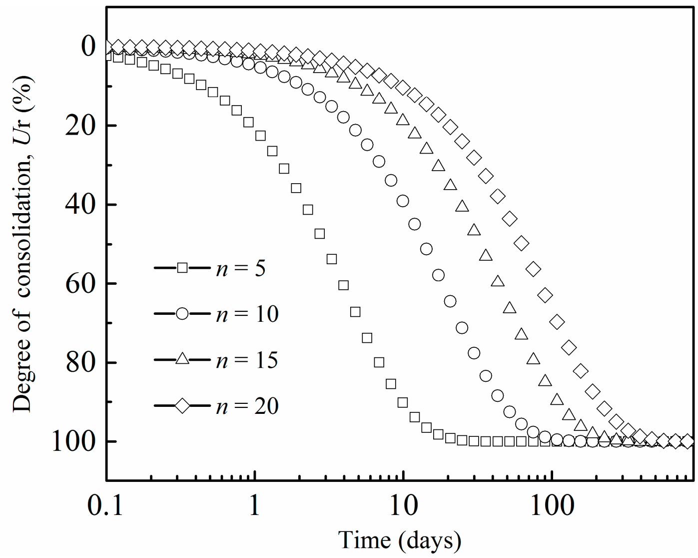

Figure 8. Average degree of the proposed model with a variation in *n*.

6.3. The c_c/c_k Ratio

The permeability coefficient and volume compressibility often affect the consolidation rate of soft soils. As shown in Equations (4) and (5), the change in the permeability coefficient is reflected by c_k, while the change in the volume compressibility is reflected by c_c. Berry and Wilkinson [26] pointed out that c_c/c_k can be used to consider the influence of the void ratio change in the consolidation characteristics of soft soils. For most soils, the c_c/c_k ratio ranges from 0.5 to 2 [27]. Therefore, taking c_c/c_k = 0.5, 1, 1.5, and 2, the influence of c_c/c_k on vertical drain consolidation was analyzed. Figure 9 shows that the consolidation rates increased as c_c/c_k decreased because as c_c/c_k decreased, the change rate of permeability decreased. It should be noted that when c_c/c_k is greater than one, the decrease in the consolidation rate is greater than that when c_c/c_k is less than one. The analysis shows that different values of c_c/c_k have a great influence on the consolidation rate. The importance of considering the nonlinear variation in compression and permeability is explained.

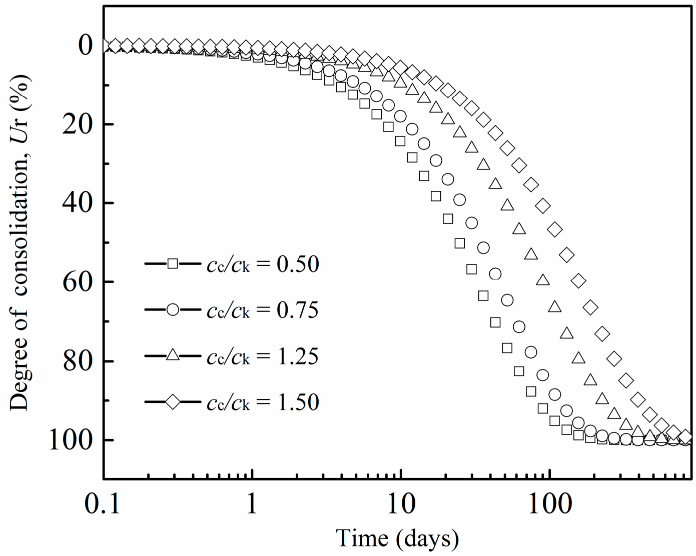

Figure 9. Average degree of the proposed model with a variation in c_c/c_k.

6.4. Initial Effective Stress

A decrease in the permeability coefficient reduces the consolidation settlement rate, and an increase in compressibility also reduces the consolidation settlement rate, which is related to the stress history, i.e., initial effective stress σ'_0. As shown in Figure 10, the greater the value of σ'_0 is, the greater the consolidation rate. When σ'_0 = 5 kPa, the initial effective stress was very small, and 53 days were required for the degree of consolidation to reach 60%. However, for σ'_0 = 25 kPa, it took only 27 days to achieve the same degree of consolidation. With the increase in σ'_0, the rate of increase in the consolidation rate decreased.

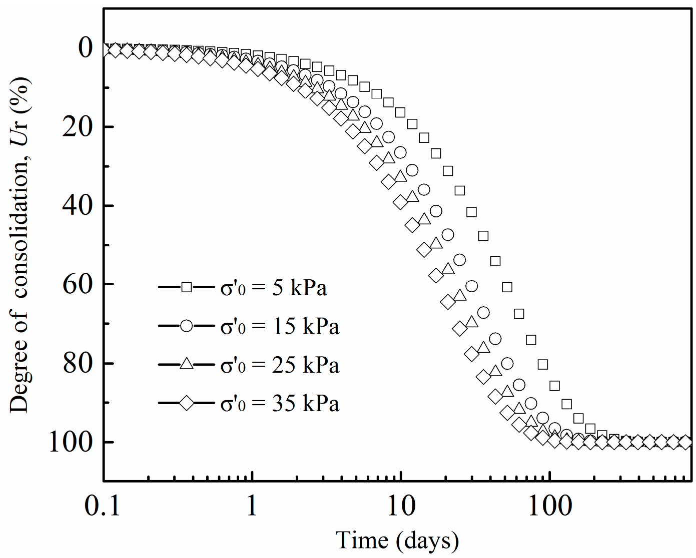

Figure 10. Average degree of the proposed model with a variation in σ'_0.

7. Application of the Proposed Model to a Case Study

The following case history is about the foundation treatment of an embankment built on the Muar Plain in Malaysia [17]. Figure 11 shows a brief description of the project, including the vertical cross-section of the embankment, load action, and prefabricated vertical drain (PVD). Indraratna et al. explained the details of the first stage of embankment loading in the original paper. Table 2 shows the relevant properties of the soft soil, including compressibility indices, soft soil unit weights, initial void ratios, preconsolidation pressures, and permeability coefficients. In addition, as suggested by Tavenas et al. [28], the slope of e-lgk_h was expressed by c_k = 0.5e_0. The 16 m long PVD was installed in a triangular pattern with a spacing of 1.3 m. The embankment was lifted to a height of 2.57 m within 14 days. To simplify the model calculation, it was considered that the application of the load was completed instantaneously, as shown in Figure 11. The project continuously monitored the settlement at the embankment centerline for 105 days.

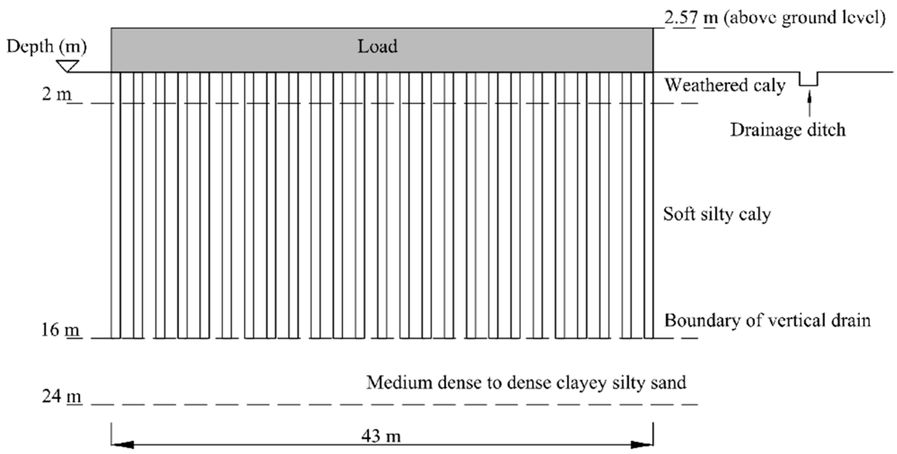

Figure 11. Project details of the Muar clay embankment in Malaysia [17].

Table 2. Soft soil parameters for Muar embankments.

Depth (m)	Initial Void Ratio, e_0	Total Unit Weight of Soil, γ (kN/m^3)	Initial Effective Stress, σ'_0 (kPa)	Initial Horizontal Permeability, k_{h0} ($\times 10^{-9}$ m/s)	Compression Index, c_c
0.00–1.75	3.10	16.5	4.88	6.4	0.71
1.50–2.50	3.10	15.0	12.25	5.2	0.71
2.50–5.50	3.00	15.0	22.25	5.2	0.38
5.50–6.50	3.00	15.5	32.5	3.1	1.38
6.50–8.00	1.95	15.5	39.38	3.1	0.71
8.00–10.00	1.82	16.0	49.50	1.3	0.71
10.00–12.00	1.86	16.0	61.50	0.6	0.83
12.00–14.00	1.89	16.0	73.50	0.6	0.83
14.00–16.00	1.86	16.0	85.50	0.6	0.83

Combined with the settlement measurement data, the degree of consolidation of the project was calculated by the three-point method [1]. To simplify the calculation, the load was considered to be an instantaneous load (Figure 12). In the theoretical calculation of the proposed model, the equivalent diameter of the vertical drain was 0.07 m. Depending on the type of drainage and installation procedure, the value of k_h/k_s on site may vary from 1.5 to 5 [17]. Therefore, λ_{max} (i.e., the value of k_h/k_s) of this case study was 2, and the smear ratio was s = 3. As shown in Figure 13, the degree of consolidation calculated by the proposed model was in good agreement with the degree of consolidation obtained from the field test results, which showed the rationality of the proposed model.

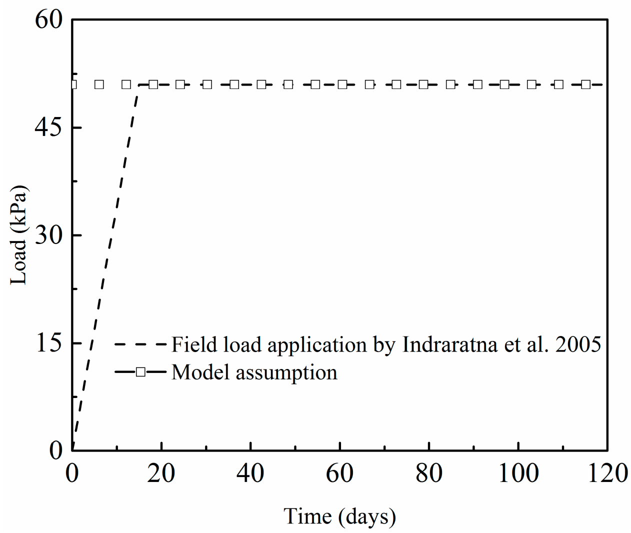

Figure 12. Loading of the Muar clay embankment in Malaysia [17].

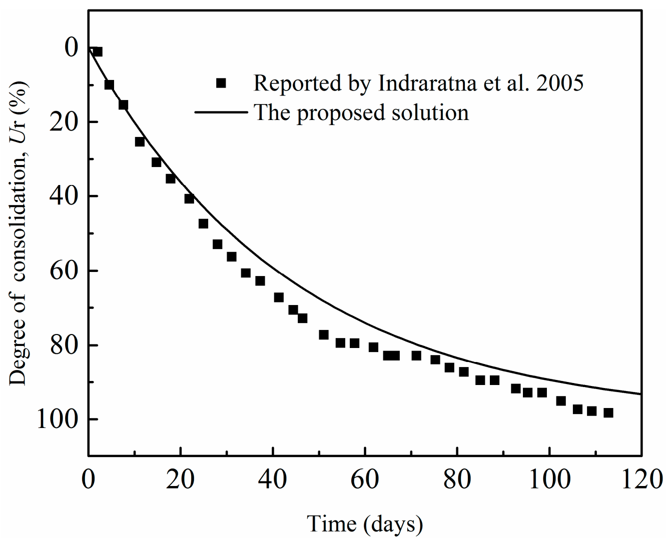

Figure 13. Comparison between the proposed model and field measurements [17].

8. Summary and Conclusions

In this study, the nonuniform consolidation characteristics of the soft geological environment treated by vertical drains were investigated, and a mathematical model was developed to describe them. Based on the large-strain and double-layer models, an analytical model was proposed that took into account nonuniform variations in consolidation and correct calculations under the influence of the nonlinear relationships of soil parameters. The main conclusions of this study can be summarized as follows.

(1) Based on current studies, a modified model considering nonuniform variations and nonlinear relationships for soil parameters was proposed for vertical drain consolidation.
(2) A mathematical expression for nonuniform variations in consolidation was proposed, and nonlinear relationships for soil parameters were introduced into the mathematical model. In addition, a simplified calculation method considering the nonlinear variation in the soil parameters is proposed for the convenience of engineering application and promotion.
(3) The results calculated by the proposed model were similar to the results of a field test. Based on the parametric analysis, it was concluded that the consolidation rate

decreased with an increasing smear value, drain diameter, and c_c/c_k and increased with increasing initial effective stress.

The calculation can be simplified by refining the proposed model by considering additional factors related to actual situations, e.g., free strain and stress history. The proposed model provides ideas and basic models for considering other problems, and it will support future scientific research.

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Nomenclature

c_c	compression index
c_k	permeability index
d	drain spacing (m)
d_c	diameter of smear zone (m)
d_e	diameter of influence zone (m)
d_w	equivalent drain diameter (m)
e	void ratio
e_o	initial void ratio
k_{h0}	initial permeability coefficient (m/s)
k_h	horizontal permeability coefficient in undisturbed zone (m/s)
k_s	horizontal permeability coefficient in the smear zone (m/s)
l	length of drain (m)
m_{vo}	coefficient of volume compressibility for one-dimensional compression (m^2/kN)
n	ratio r_e/r_w
q_t	additional load (kPa)
r	radius (m)
r_s	radius of smear zone (m)
r_e	radius of influence zone (m)
r_w	radius of drain well (m)
s	ratio r_s/r_w
t	time (days)
u	excess pore-water pressure (kN/m^2)
u_s	excess pore pressure in the smear zone
u_r	excess pore pressure in the undisturbed zone
u_w	excess pore pressure in the well
$\overline{u}_t$	average excess pore-water pressure for the unit cell (kN/m^2)
U_r	degree of consolidation (%)
V	volume of soil mass (m^3)
W_L	liquid limit (%)
W_P	plastic limit (%)
z	depth (m)
γ_w	unit weight of water (kN/m^3)
ε	vertical strain
λ	ratio k_h/k_s
μ	parameters representing the geometry of the vertical drain system
σ'_0	initial effective stress

σ'_{v} effective stress (kN/m^2)
$\overline{\sigma'}_{v}$ average effective stress (kN/m^2)
ω water content (%)
$\Delta\sigma$ additional load (kN/m^2)

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