

## Article

# Optimization of Parameters of a Vertical Ground Heat Exchanger in a Geothermal Heating System

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**Abstract:** This study presents the results of an original study on the influence of selected parameters on the thermal efficiency of a vertical ground heat exchanger (VGHE) in a ground-source heat pump (GSHP) system. The research objective was an analysis of the specific thermal efficiency of a vertical ground heat exchanger  $q$ , received by a U-shaped element made of plastic pipes placed in a borehole, depending on seven direct influencing factors: the ground temperature  $T_g$ ; the soil thermal conductivity coefficient  $\lambda_g$ ; the thermal conductivity coefficient of the well material  $\lambda_m$ ; the temperature of the heating medium (glycol)  $T_w$  at the feed to the ground heat exchanger and its flow rate  $M$ ; the internal diameter of the pipes of the ground heat exchanger  $d_w$ ; and the distances between the external walls of the pipes of the ground heat exchanger  $L$ . The analysis was carried out for the climatic conditions of the Podlasie Voivodeship (Poland). Based on the results of the computational experiment obtained using the TRNSYS numerical environment, a deterministic mathematical model of this relationship was developed, and the effects of the influence of selected factors on the specific thermal efficiency  $q$  of the vertical ground heat exchanger, received by the U-shaped element, were analysed. Based on the model, the contribution of each parameter to the efficiency of the heat exchanger was determined. It turned out that changes in the values of the factors  $T_g$  ( $X_1$ ),  $\lambda_g$  ( $X_2$ ),  $\lambda_m$  ( $X_3$ ),  $M$  ( $X_5$ ),  $d_w$  ( $X_6$ ) and  $L$  ( $X_7$ ) from the lower to the upper level caused an increase in the specific efficiency  $q$  of the heat exchanger by 34.04, 7.90, 15.20, 55.42, 6.58 and 24.26%. Only factor  $T_w$  ( $X_4$ ), with such a change, caused a decrease in the thermal efficiency of the tested heat exchanger by 44.22%. The parameters of the tested element of the geothermal heating system were also optimized according to the energy criterion using a numerical method in the Matlab environment. The information may be useful for scientists, designers, producers and consumers of heating systems based on heat pumps with a vertical ground heat exchanger as the lower heat source.

**Keywords:** geothermal heating system; vertical ground heat exchanger; deterministic mathematical model; optimization



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## 1. Introduction

Energy sources, such as wind, solar and geothermal energy are one of the elements necessary to effectively solve the global energy crisis. Geothermal energy resources are among the most accessible in the world. Ground heat can be and is effectively used as a lower heat source for heat pumps based on ground-source heat pump (GSHP) systems. This technology is very popular in North America and much of Europe. In addition, the targets set for reducing greenhouse gases and increasing the share of energy from renewable sources make these systems an important alternative to traditional heat sources. GSHP systems are widely used in residential and public buildings due to them being maintenance-free and their high energy efficiency. This is due to the fact that below the surface layer of the ground, the temperature of the ground is constant, higher than the air temperature in winter and lower than in the summer [1]. The heat pump can convert the heat stored in the ground and transfer it to the building during low outdoor temperatures,

while in summer, the excess heat can be effectively discharged into the ground. Analyzing the temperature difference of the ground heat exchanger, it is worth noting that from a scientific point of view, cold river water with a temperature of 5 °C (40 °F) contains only 11% less energy compared to hot bath water at 40 °C (105 °F) [2].

When using heat in a shallow geothermal system, with a properly made ground heat exchanger, the impact of this technology on the environment is small [3]. This technology can be used wherever a minimal environmental impact is required. The energy required to power the system is electricity.

The first effective technological system for the use of a ground-source heat pump came from Switzerland in 1912 [4]. The work contains a comprehensive overview of current research on the latest GSHP technologies. It presents a description of the types of GSHP used, such as open and closed GSHPs, as well as vertical and horizontal GSHPs. The influence of applied working fluids and well filling material on the thermal properties of GSHP was discussed. In addition, existing simulation models describing boreholes, such as the Kelvin linear source model, the cylindrical source model, the finite-length linear source method and the Eskilson model, are described. More interest in this technology emerged after World War II, mainly in North America and Europe. However, only the sharp rise in oil prices resulting from the embargo of the countries associated in OPEC in 1973, known today as the oil crisis, caused a significant increase in interest in heat pumps, including GSHPs [5]. According to the World Energy Investment 2023 [6], global sales of heat pumps have seen double-digit growth since 2021. It is estimated that the number of GSHP installations worldwide is growing at a rate of 10–30% per year [7]. The estimated installed geothermal capacity at the end of 2019 was 107,727 MWt. This is an increase of 52% compared to the data for 2015, with an average increase of 8.73% per year. The heat used is 1,020,887 TJ/year (283,580 GWh/year), an increase of 72.3% compared to 2015, at a total rate of 11.5% per year. The share of ground heat pumps in total geothermal energy production is 58.8% [8]. Based on the Direct Utilization of Geothermal Energy 2020 Worldwide Review data from 2021, the equivalent number of units installed, typical for homes in the US and Western Europe with a capacity of 12 kW, is about 6.46 million. This is an increase of 54% compared to 2015, and more than twice as many than in 2010. The power of individual units ranges from 5.5 kW for single-family buildings to large units with a capacity of more than 150 kW for commercial applications [8].

The basic element of the heating system with a lower heat source in the form of soil, having a direct impact on the thermal performance of the heating system, is a ground heat exchanger. The heat resistance of the well made for the needs of a vertical ground heat exchanger (VGHE) affects the energy efficiency of the system and depends on the properties of the soil and the properties of the well filler material in which the heat exchanger loop is located. The hydrogeological conditions and soil properties depend on the location of the VGHE position. The choice of backfill material (BM), its thermal conductivity, the operating parameters of the refrigerant and the geometry of the heat exchanger (the diameter of the exchanger pipes and their location in the well) are the parameters affecting the thermal performance of a VGHE. The BM affects the heat exchange in the heat exchanger environment [9]. The role of BM is to seal the space between the well wall and the exchanger probe pipes. As a result, the thermal resistance is reduced, which has a positive effect on the energy efficiency of the lower source system. In addition, the filling material protects the exchanger from mechanical damage. In the event of possible movements of land masses, it reduces the risk of uncontrolled flows of water between geological layers.

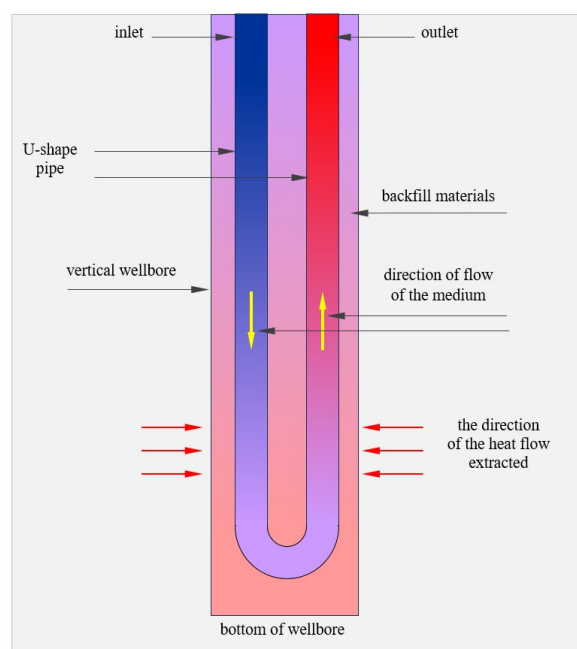
The correct estimation of the thermal efficiency of the ground heat exchanger will ensure the design of a VGHE with the smallest possible length and stable operation. The estimation of the maximum unit thermal efficiency of such an exchanger minimizes the investment outlays for earthworks and material costs associated with the implementation of a VGHE.

There are two basic types of ground heat exchangers: open and closed [10]. In the open system, the heat of the lower source is used directly, without an intermediary medium.

This applies to heat exchangers, where ground or surface water is directly used in the open circuit. In a closed circuit, the heat from the lower heat source, the ground in this study, is taken up by the intermediate medium, through the loop of the heat exchanger, which is made of a durable material with high thermal conductivity.

Depending on the location of heat exchangers underground, we can divide them into two types: vertical and horizontal ground heat exchangers. Types of heat exchangers for heating systems based on heat pumps are presented in [11].

Today's heat exchangers are generally simple U-shapes, made of high-density polyethylene (HDPE) surrounded by groundwater or filler material (Figure 1). The circulating fluid is most often composed of water mixed with a non-freezing agent, such as ethyl alcohol or glycol.



**Figure 1.** Vertical ground-source heat exchangers.

The design and optimization of heating and cooling systems require the use of VGHE modeling. The heating or cooling load of a real facility is a parameter that is difficult to define due to its complexity and dependence on the method of operation of the facility. This parameter, regardless of the thermal properties of the ground, is important for the heat transfer process around the ground heat exchanger. Underestimation of the ground heat exchanger causes a drop in the ground temperature, and in extreme cases, the ground may freeze. For a brief time of soil regeneration, it will not be able to regenerate enough. This will negatively affect the operating costs of the system, as the heat flux will not be transferred in sufficient quantity to the upper heat source. Choosing the correct-length VGHE is crucial for this type of system. Due to the lack of detailed information about processes occurring inside the ground and near VGHE, many effects are omitted in the calculations. This applies to situations such as cracks in the bedrock, which can cause an increased process of convective heat exchange, resulting from the movement of groundwater.

In calculation programs, the heat transfer process between the heating medium in the heat exchanger and the ground is divided into a convective heat transfer process and a heat conduction process. The convection process involves the process of transferring heat from fluid to pipe and the process of conducting heat from the walls of the pipe to the drill holes. The heat conduction process is being analyzed. The way the system is controlled has a huge impact on its performance.

Many research papers are currently focused on improving the accuracy of GHE modeling. A very thorough introduction to the analysis of heat transfer by ground heat

exchangers, with particular emphasis on analytical methods, was presented in the work of Min Li et al. [12]. Models in different time frames are used, such as heat-source models, short-time models, models for energy piles, in situ thermal-response tests, indoor sandbox experiments, and parameter estimation as an inverse problem. A numerical heat transfer model for a deep borehole ground heat exchanger for extracting geothermal energy for building heating was presented in [13]. The influence of the heat exchanger on the surrounding ground was analysed. The authors proposed a concept of the recommended heat exchanger length for a medium-depth geothermal heat pump system. The thermal performance of borehole thermal energy storage systems was described in [14]. This study presents two complementary approaches. The two methods rely on numerical experiments to obtain the required performance curves. The influence of the filler material in vertical ground heat exchangers on the heat transfer efficiency of heat pump systems was presented in [15]. The thermally enhanced backfill material blended with graphite of high thermal conductivity further improved the heat-transfer performance of the GHE. The effective thermal conductivities of soil increased to 2.10 and 2.17 W/(mK) when 5% and 10% graphite was added, respectively.

Analytical models are based on the infinite linear source model (ILS) [16], finite line model (FLM) or infinite cylindrical model (ICM). In models of an infinite linear and cylindrical source, the temperature around the heat source is determined as a function of distance from the source and time. The well is modelled as a line or roller. The ILS (infinite linear source) model can be expressed as follows (Equation (1)):

$$T(r, t) - T_0 = \frac{q}{4 \pi \lambda} E_i \left[ \frac{r^2}{4 \alpha t} \right] = \frac{q}{4 \pi \lambda} \int_{\frac{r^2}{4 \alpha t}}^{\infty} \frac{e^{-u}}{u} du \quad (1)$$

where  $T_0$  is the undisturbed temperature,  $\lambda$  and  $\alpha$  are the medium thermal conductivity and thermal diffusivity, respectively, and  $E_i$  is the exponential integral function. For the condition Equation (2) is used:

$$\frac{\alpha t}{r^2} > 5 \quad (2)$$

Equation (1) becomes:

$$T(r, t) - T_0 \cong \frac{q}{4 \pi \lambda} \left[ \ln \left( \frac{4 \alpha t}{r^2} \right) - \gamma \right] \quad (3)$$

Equation (3) can be used to evaluate the temperature increase at the borehole wall [17]. Modelling tools for GSHP simulation and design include the following: GLD (ground loop design) version GLDs Build 10.1.18, EED (earth energy designer) (current version is 4.30), GLHEPro (ground loop heat exchanger design software) version 5.0 and TRNSYS version 4.200 [18]. TRNSYS is an environment developed by the University of Wisconsin-Madison Solar Energy Laboratory, and it is an advanced energy simulation software. It has been widely used for building modelling. TRNSYS can also be used for specific heat exchangers. The use of simulation based on TRNSYS software was presented in [19]. The model results were validated through experiments. The model predicts outlet temperatures and energy recovery well with an accuracy of 15% and an average of 4.4% error when compared to existing experimental results, which is acceptable for engineering applications.

In the works available to the authors, no studies were found regarding the optimization of parameters of a VGHE for a geothermal heating system for seven factors. After analyzing the data referred to in the presented literature on the possible impact of heat exchanger parameters and their location on the efficiency of the geothermal heating system, it was considered reasonable to analyse in detail the impact of seven parameters selected by the authors on the unit thermal efficiency  $q$  vertical ground heat exchanger in the form of a

u-image element of plastic pipes, placed in the drilling hole of the geothermal heating system. In addition, the authors plan to obtain the best solution of the tested heat exchanger.

The aim of this study is to develop a model of deterministic dependence of the unit thermal efficiency  $q$  of a vertical ground heat exchanger in the form of a pictorial element of plastic pipes, placed in the borehole of a geothermal heating system, considering the seven most important parameters: ground temperature  $T_g$ ; soil thermal conductivity coefficient  $\lambda_g$ ; thermal conductivity coefficient of the well material  $\lambda_m$ ; temperature of the heating medium (glycol)  $T_w$  at the feed to the ground heat exchanger and its flow rate  $M$ ; internal diameter of the pipes of the ground heat exchanger  $d_w$ ; distances between the external walls of the pipes of the ground heat exchanger  $L$ . Based on the mathematical model, it was planned to estimate the nature and degree of influence of the tested parameters on the unit thermal efficiency of the analyzed heat exchanger and to optimize these parameters according to the energy criterion using the numerical method in the Matlab environment.

This study consisted of the following steps. For the selected seven parameters affecting the unit thermal efficiency of the vertical ground heat exchanger in the form of a U-shape of plastic pipes, located in the borehole of the geothermal heating system, the levels of variability and their values were determined. The next step was deterministic mathematical modelling, based on a seven-factorial symmetric three-level plan, to which the results of simulation calculations obtained using software in the TRNSYS numerical environment were used as a basis. After analyzing the impact of selected factors, their optimization using the numerical method in the Matlab environment was planned. This information could be useful for scientists, designers, manufacturers and consumers of geothermal heating systems.

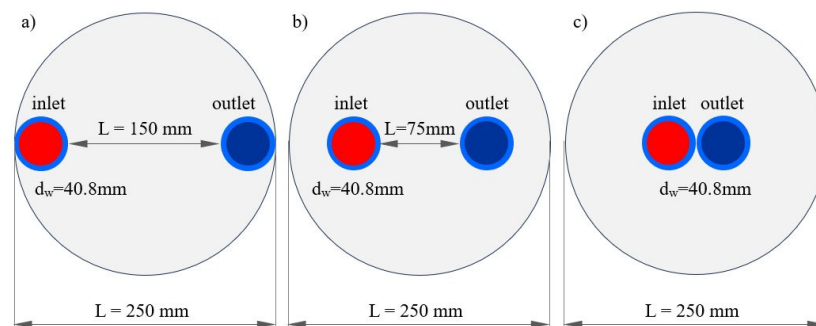
## 2. Materials and Methods

### 2.1. Description of the Geothermal Heating System Component Under Test

In the analyzed model, the studied geothermal element in the form of a U-shape was adopted. The choice of this type of heat exchanger was dictated by the most used solution on the market. The longitudinal section of the ground heat exchanger is shown in Figure 1. In the model, the assumed length of the exchanger is 100 m.

The medium flowing in the vertical ground heat exchanger is a 30% propylene glycol solution that absorbs or gives off heat depending on the fluid temperature and the external environment, the ground temperature and the well mass temperature. The glycol solution corresponds to a solidification temperature of minus 15°C. The upper part of the ground heat exchanger is located 1 m underground. The well is filled with backfill material, with a constant thermal conductivity coefficient along the entire length of the exchanger. The soil was accepted as homogeneous throughout the analyzed control volume.

The position of the pipes in the well for calculation purposes is shown in Figure 2. The distances between the outer walls of the ground heat exchanger pipes are 150, 75 and 0 mm, respectively (Figure 2a–c).

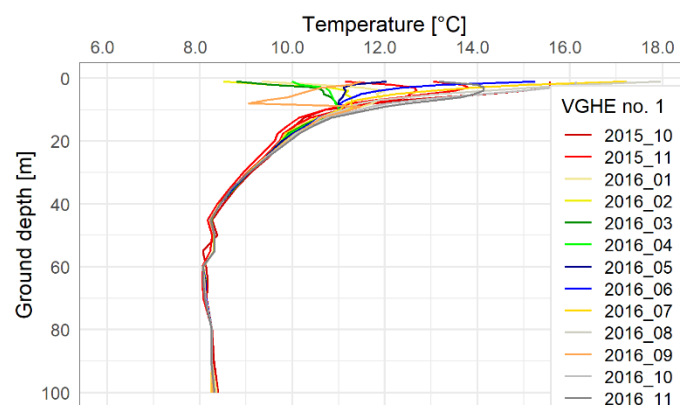


**Figure 2.** The cross-sectional geometry of the pipe position for three different distances between the outer walls. (a): the distance between the pipes is 150 mm. (b): the distance between the pipes is 75 mm. (c): pipes located close to each other.



The pipes of the ground heat exchanger are made of polyethylene class PE 100-RC. The selected material is characterized by high strength, durability and tightness of joints. This material is widely used in the market by designers and contractors of heating systems based on a ground heat exchanger. The heat conduction coefficient of the pipes is 0.42 W/mK.

Based on our own research and measurements in “INNO–EKO–TECH Innovative Research and Didactic Centre for Alternative Energy Sources, Energy Efficient Construction and Environmental Protection” building, in the Bialystok University of Technology, Northeast Poland (latitude 53°11′ N, longitude 23°15′ E), undisturbed temperature fields are presented in Figure 3.



**Figure 3.** Undisturbed temperature field for each month of the year.

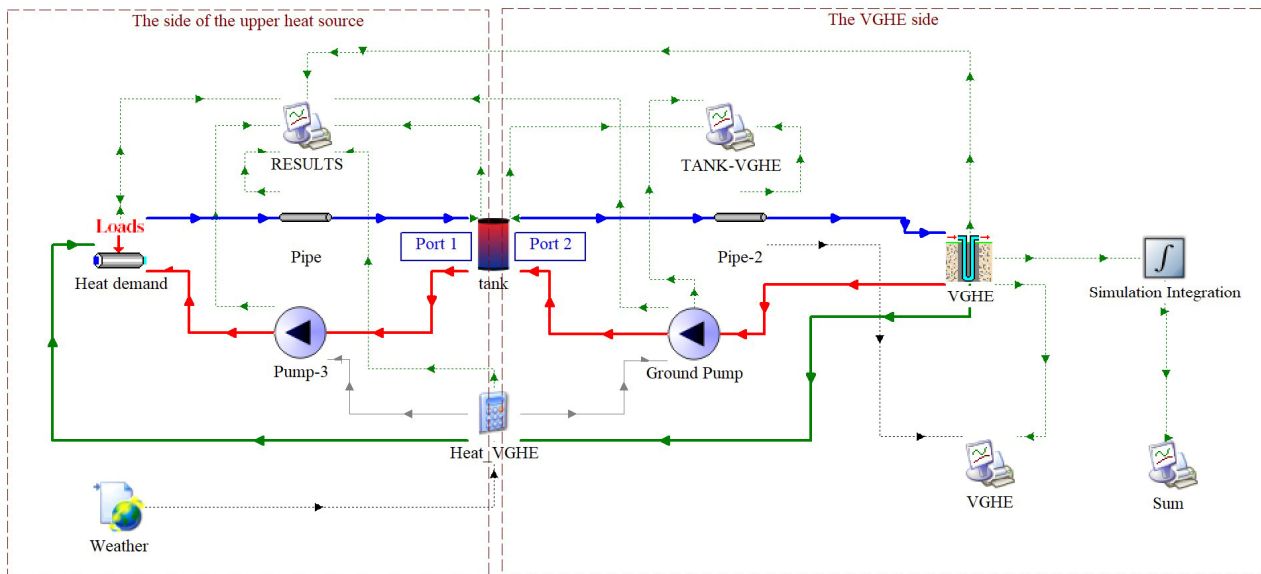
## 2.2. Technique for Estimating the Heat Flow Received by the Tested Component of the Geothermal Heating System

In order to perform numerical simulations for the deterministic model, a numerical model was developed in the TRNSYS environment.

The need for repeatability of numerical calculation parameters led to the choice of environment. The physical model analyzed in this work requires a nine-fold change in the geometry of the physical model. This is due to a three-fold change in pipe diameter and three different pipe distances in the well (Figure 2). Using mesh methods, such as ANSYS Fluent 2021 R1 environment, a nine-fold model construction is required, and a mesh generation is required for each model. Different mesh can generate different numerical errors, which will affect the results of the obtained mathematical model. The difference in the results obtained in the numerical simulation process for different mesh and different initial-shore conditions is a derivative of the change in these conditions and the change in the generated numerical mesh. Estimating the impact of changing the generated mesh on the result of calculations is difficult to estimate.

Figure 4 shows the model with a ground heat exchanger made in the TRNSYS environment. The system consists of two independent circuits: the hydraulic circulation of the upper heat source and the hydraulic circulation of the vertical ground heat exchangers. In Figure 4, both are marked as the side of the upper heat source and the VGHE side. The first circuit simulates the operation of the heating system. The heating factor is water. This circuit is called the upper heat source because the heat flux is transferred to the central heating system. The upper heat source circuit includes the following main components: heat demand, tank and pump-3. In the analyzed system, the heat receiving element is the Heat\_demand component. This component imposes the heat load, heating or cooling, which is equal to the heat flux received or transferred to the ground. The second circuit simulates the operation VGHE. This circuit is known as lower heat source circuit and it is realized by VGHE, Ground\_Pump and tank components. The heat flux from the ground simulates the heat load, which is balanced by the heat stored in the buffer tank—tank component. The tank component acts as a heat storage buffer tank and a heat exchanger. The heating factor in circulation is glycol with a density of 1038 kg/m<sup>3</sup>. The VGHE component

models the operation of a vertical ground heat exchanger that thermally interacts with the ground. The heat flux received from the ground to the heating medium in the circulation is transferred to the buffer tank—the tank component. The heating medium after cooling in the buffer tank is again directed to the ground heat exchanger.



**Figure 4.** System model with VGHE in TRNSYS environment.

The system operates only during the heating season. It was assumed that the heating season, according to the climate data of the city of Bialystok (Poland), lasts until 10 May and from 10 September inclusive. The heat load in the Heat\_demand component is equal to the heat flux drawn from the ground. This simulates the operation of the exchanger at its maximum load during its operation.

The applied model allows for determining the energy efficiency of vertical ground heat exchangers. The operating time of each system, regardless of the geometry of the heat exchanger and the initial-shore conditions, is equal. The imposed heat load by the HVAC system, as is the case in engineering practice, will not give full information about the performance of the heat exchanger, because due to different conditions, initially, the shore working time of these systems will be different. The conversion rate of the heat pump will then also vary. This will disturb the information about the actual energy efficiency of the ground heat exchanger itself. The model proposed in this work (Figure 4) does not have these limitations.

Two circulation systems, heat load—buffer tank and buffer tank system—and ground heat exchangers, are controlled by the Heat\_VGHE component and can adopt two operating states: on and off.

The ground heat exchanger model is implemented by the type 557 component of the ANSYS environment. The program component calculates the thermal resistance between the factor flowing in the exchanger and the ground based on the geometry and thermal conductivity of the pipes and the material filling the well. For the assumed deterministic model, the distance between the feed pipe and the return exchanger is important, which considers the adopted model. In the model represented by the component type 557, component VGHE, the wells are distributed evenly in the cylindrical volume of the well. The model adopted one ground heat exchanger with a fixed length of 100 m. The diameter of the well is fixed and is 250 mm. The well is backfilled with the assumed thermal conductivity coefficient  $\lambda$ , respectively, equal to 0.6; 1.3; 2.0 W/mK. In the model, the ground was adopted as homogeneous with values of  $\lambda$  as 1.55; 1.72; 1.89 W/mK. The soil temperature used for the calculations is 8.2, 9.4 and 10.3 °C, respectively.

During the summer, the vertical ground heat exchanger does not work, and during this time, the ground regeneration process takes place. This is the case in most heat pump systems with ground exchangers in climates such as Poland or in solar collectors. In the model used in this work, the heat exchanger works only during the heating season. For the climatic conditions of Bialystok, the northeastern part of Poland, the normal duration of the heating season is 231 days, from 21 September to 10 May. In the model, the beginning of the system is the beginning of the year. The control of the hydraulic system of the heat exchanger ground circuit and the heat collection system affects the instantaneous reaction of the temperature of the medium in the VGHE component system and the change in the heat load of the receiving system.

Table 1 presents a detailed description, which was selected from the data included with the TRNSYS software.

**Table 1.** Geometry of the vertical ground heat exchanger and initial-shore conditions.

Parameter	Value	Unit
<b>Soil</b>		
Cylindrical volume of soil	6500	m <sup>3</sup>
Thermal conductivity coefficient of the ground including the volume of the warehouse	1.55	W/(mK)
The thermal capacity of the land including the storage volume	2016	kJ/(m <sup>3</sup> K)
Thermal conductivity of the material in the gap between the U-tube pipes and the filling material	1.0	W/(mK)
Specific heat of fluid flowing ground heat exchanger—glycol	3.38	kJ/(kgK)
<b>Cylindrical Storage Tank</b>		
Tank volume	500	dm <sup>3</sup>
Fluid specific heat	4.19	kJ/(kgK)
Fluid density	1000	kg/m <sup>3</sup>
Fluid thermal conductivity	0.609	W/(mK)
<b>Vertical Ground Heat Exchanger</b>		
Ground heat exchanger medium—glycol	1038	kg/m <sup>3</sup>
Phase delay (to the temperature field)	90	days
Phase delay of air temperature for preheating calculations	240	days
Air temperature amplitude during the year	15	K
Thermal conductivity of a specific vertical layer At the outer limit of the warehouse volume	4.68	W/(mK)
Flow rate of water density through the circulation of the heating system	1000	kg/m <sup>3</sup>
Specific heat of the water flowing through the heating system circuit	4.19	kJ/(kgK)

### 3. Mathematical Modelling of the Dependence of the Unit Thermal Efficiency of a Vertical Ground Heat Exchanger on Selected Parameters

According to the purpose of the test, the unit thermal efficiency was adopted as the function of the target  $q$  (function  $Y$ ) of a vertical ground heat exchanger in the form of a u-image element of plastic pipes, placed in the drilling hole of the geothermal heating system. We examined the impact on the function of seven parameters, which, according to the authors, can significantly change the amount of heat received by the considered



u-image element of the geothermal heating system: ground temperature  $T_g$  (factor  $X_1$ ); soil thermal conductivity coefficient  $\lambda_g$  (factor  $X_2$ ); thermal conductivity coefficient of the well material  $\lambda_m$  (factor  $X_3$ ); the temperature of the heating medium (glycol) on the supply to the ground heat exchanger  $T_w$  (factor  $X_4$ ) and flow of rate  $M$  (factor  $X_5$ ) internal diameter of the pipes of the ground heat exchanger  $d_w$  (factor  $X_6$ ); distances between the external walls of the pipes of the ground heat exchanger  $L$  (factor  $X_7$ ). The selected factors are classified into two groups:

- Factors characterizing the properties of the surrounding soil (boundary conditions):  $X_1, X_2$ ;
- Factors characterizing the heat exchanger parameters of the heating system:  $X_3, X_4, X_5, X_6, X_7$ .

By selecting target functions and influencing factors, the authors thoroughly analyzed their compliance with the requirements of the applied mathematical modelling method. According to [20], the function of the target should be quantitatively measurable; have a clear physical sense; be unambiguous, not specified in percentage; informative; statistically effective. In contrast, the factors should be controllable; measurable; mutually independent; non-contradictory; unambiguous; direct influence; every single row more accurate than the function.

The choice of the purpose function was related to the authors' desire to obtain a practical indicator of the heat efficiency of the exchanger, which would allow us to evaluate and compare the performance of similar heating systems. For this, the unit thermal efficiency  $q$  was selected, characterizing the heat exchanger's ability to receive heat on each 1 running meter of its length. Thus, the unit for  $q$  was taken as [W/m].

The choice of factors was related to the goal set by the authors to detect the possible effects of the impact, including the following: properties of the surrounding soil, parameters of the geothermal heat exchanger heating system and the process of extracting energy from the ground through a heat exchanger. As usual, the situation is complicated due to the fact that these parameters not only directly affect the efficiency of the heating system but also interact with each other. The use of optimal values of these factors, according to the authors' assumption, can ensure maximum efficiency of the heating system under study. When choosing the range of variability of factors, the author's design experience, physical properties of land and commonly used materials and parameters of some products in construction were considered.

The most important factor of the studied system is the ground temperature because heat from the ground is a source of energy. Unfortunately, this factor, as determined above, can be characterized only by the average ground temperature, which significantly changes in different localities. For the climatic conditions of the Podlasie Voivodeship (Poland), the average ground temperature is 9.40 °C. From the average temperature, deviations, up to 8.4 °C in the lower direction and up to 10.3 °C in the higher direction, take place [1]. Considering such a temperature distribution for factor  $X_1$  (soil temperature  $T_g$  °C), three levels were adopted: 8.4 (−1), 9.4 (0) and 10.3 (+1) °C. This did not allow us to detect the effects of changes in ground temperature throughout Poland, but it gave the opportunity to determine the effects of fluctuations in some properties of land in the Podlasie Voivodeship.

Factor  $X_2$  (soil heat conduction coefficient  $\lambda_g$ , W/mK) also characterizes the physical properties of the soil, which randomly change in Poland. We found that for land in Podlasie Voivodeship, the values of coefficient  $\lambda_g$  have an average value of 1.720 W/mK with deviation up to 1.550 W/mK lower and up to 1.890 W/mK towards the higher end. Thus, the value of factor  $X_2$  (the soil heat conduction coefficient  $\lambda_g$ , W/mK) adopted the following levels: 1.550 (−1), 1.720 (0) and 1.890 (+1) W/(mK).

We assume that in the developed model, only the effects of fluctuations in the coefficient  $\lambda_g$  of land in the Podlasie Voivodeship will be detected.

Other factors concerning the parameters of the heat exchanger  $X_3, X_4, X_5, X_6, X_7$  were adopted as a group, useful for the implementation of the procedure for optimizing the characteristics of the tested exchanger. Factor  $X_3$  values (that is, the heat conduction

coefficient of the material of the BM  $\lambda_m$ , W/(mK) characterized the properties of the selected material to fill the well, which, from the experience of the authors, at individual levels were adopted as follows: 0.600 (−1), 1.300 (0) and 2.000 (+1) W/(mK). Factor  $X_4$  (the temperature of the heating medium (glycol) on the supply to the ground heat exchanger  $T_w$ , °C) was adopted, respectively: 1.0 (−1); 3.0 (0); 5.0 (+1), °C. For factor  $X_5$  (that is, the expenditure of heating medium (glycol)  $M$ , kg/s), 0.059 (−1), 0.118 (0) and 0.173 (+1), kg/s were adopted. Factor  $X_6$  (meaning the inner diameter of the ground pipes of the heat exchanger  $d_w$ , mm) was adopted considering the available market series of pipes at levels 26.0 (−1), 32.6 (0) and 40.8 (+1), mm. The last factor,  $X_7$  (concerning the distance between the outer walls of the ground pipes of the  $L$ , mm heat exchanger), was taken at levels 0 (−1); 75 (0); 150 (+1), mm. In parentheses, next to the natural values of factors, their coded values are given.

It is assumed that the dependency  $Y = f(X_1, X_2, X_3, X_4, X_5, X_6, X_7)$  can describe a polynomial of the second-degree  $k + 1 = 36$  unknown coefficients in the following form (Equation (4)):

$$Y = a_0 + a_1 X_1 + a_2 X_2 + a_3 X_3 + a_4 X_4 + a_5 X_5 + a_6 X_6 + a_7 X_7 + a_{12} X_1 X_2 + a_{13} X_1 X_3 + a_{14} X_1 X_4 + a_{15} X_1 X_5 + a_{16} X_1 X_6 + a_{17} X_1 X_7 + a_{23} X_2 X_3 + a_{24} X_2 X_4 + a_{25} X_2 X_5 + a_{26} X_2 X_6 + a_{27} X_2 X_7 + a_{34} X_3 X_4 + a_{35} X_3 X_5 + a_{36} X_3 X_6 + a_{37} X_3 X_7 + a_{45} X_4 X_5 + a_{46} X_4 X_6 + a_{47} X_4 X_7 + a_{56} X_5 X_6 + a_{57} X_5 X_7 + a_{67} X_6 X_7 + a_{11} X_1^2 + a_{22} X_2^2 + a_{33} X_3^2 + a_{44} X_4^2 + a_{55} X_5^2 + a_{66} X_6^2 + a_{77} X_7^2 \quad (4)$$

To obtain a database for the modelling and description of this relationship, a seven-factor active computational experiment was conducted according to the second-degree plan (Table 2). In an active experiment, factors take on specific values that are invariable in each trial. These experiments are carried out according to optimal plans, the quality of which is confirmed by criteria calculated using computers. To obtain information about the object, a limited number of data points is needed. By choosing a plan in this experiment, nine plans out of seven factors were analyzed. A three-level plan was selected for 40 trials, with a high D-criterion value of  $-e^{(D)} = 0.910$  (Equation (5)).

**Table 2.** Natural and coded meanings of selected factors.

Designations and Names of Natural Factors	Natural Meanings $\dot{X}_{imin}$ , $\dot{X}_{i0}$ , $\dot{X}_{imax}$ , That Correspond to Coded Meanings $X_{imin}$ , $X_{i0}$ , $X_{imax}$ ,			
	$X_{imin} = -1$	$X_{i0} = 0$	$X_{imax} = +1$	$\Delta X_i$
$\dot{X}_1$ —ground temperature $T_g$ , °C	8.20	9.40	10.30	1.20
$\dot{X}_2$ —soil thermal conductivity coefficient $\lambda_g$ , W/(mK)	1.550	1.720	1.890	0.170
$\dot{X}_3$ —thermal conductivity coefficient of the well material $\lambda_m$ , W/(mK)	0.600	1.300	2.000	0.700
$\dot{X}_4$ —temperature of the heating medium (glycol) at the inlet of VGHE $T_w$ , °C	1.00	3.00	5.00	2.00
$\dot{X}_5$ —the flow rate of the heating medium (glycol) $M$ , kg/s	0.059	0.118	0.177	0.059
$\dot{X}_6$ —internal diameter of the pipes of the ground heat exchanger $d_w$ , mm	26.0	32.6	40.8	7.2
$\dot{X}_7$ —distances between the external walls of the pipes of the ground heat exchanger $L$ , mm	0	75	150	75

In optimal plans, coded factor values are used. The usual planning area is a hyper cube, and the values of the factors  $|X_i| \leq 1$ , (−1, 0, +1) [21]. Natural and coded factors are shown in Table 2. Coding (transition from natural values  $\dot{X}_i$  to coded  $X_i$ ) factors are performed according to Equation (5):

$$X_i = \left[ 2\dot{X}_i - (\dot{X}_{imax} + \dot{X}_{imin}) \right] / (\dot{X}_{imax} - \dot{X}_{imin}) \quad (5)$$

where:  $\dot{X}_i$ ,  $\dot{X}_{imax}$ ,  $\dot{X}_{imin}$ —accordingly: current, maximum and minimum importance of the natural  $i$ -th factor.

The implementation of computational experiments assumes the calculation of the function of the  $Y_i$  target on a determined theoretical model in the form of an analytical apparatus, formally arranged by computer programs. Since, in this case, there is a mutually unambiguous agreement between the external impact on the modelled system and its response to this impact, only one experiment is carried out at each point of the plan, and  $Y_i$  is calculated.  $Y_i$  was used to develop and test the model at every point in the plan.

Determination of the unit thermal efficiency  $q$ , W/m of the VGHE in the form of a u-shape element of plastic pipes, placed in the well, was made considering the climatic conditions of Bialystok based on our own research [22]. The results of simulation calculations of heat received by the u-shape element of the geothermal heating system  $q_i$  ( $Y_i$ ) in 40 analyzed samples according to a computational experiment, obtained using the numerical environment TRNSYS, are presented in Table 3.

**Table 3.** Plan of calculation experiment for seven variables on  $N = 40$  samples and results of simulation calculations of values  $q_i$  ( $Y_i$ ), W/m in individual variants ( $\hat{Y}_i$ —calculated by model), where:  $T_g$ , °C;  $\lambda_g$ , W/(mK);  $\lambda_m$ , W/(mK);  $T_w$ , °C;  $M$ , kg/s;  $d_w$ , mm;  $L$ , mm—natural factors;  $X_1, X_2, X_3, X_4, X_5, X_6, X_7$ —coded factors.

Case	$X_1$ $T_g$ [°C]	$X_2$ $\lambda_g$ [W/mK]	$X_3$ $\lambda_m$ [W/mK]	$X_4$ $T_w$ [°C]	$X_5$ $M$ [kg/s]	$X_6$ $d_w$ [mm]	$X_7$ $L$ [mm]	$Y_i$ $q_i$ [W/m]	$\hat{Y}_i$ $q_i$ [W/m]
1.	+1 10.3	−1 1.55	+1 2.0	−1 1.0	+1 0.177	+1 40.8	+1 150	18.9	18.84
2.	+1 10.3	+1 1.89	−1 0.6	−1 1.0	+1 0.177	+1 40.8	+1 150	20.6	20.59
3.	+1 10.3	+1 1.89	+1 2.0	+1 5.0	+1 0.177	−1 26.0	+1 150	10.9	11.01
4.	−1 8.2	+1 1.89	+1 2.0	−1 1.0	+1 0.177	−1 26.0	+1 150	15.3	15.29
5.	−1 8.2	−1 1.55	+1 2.0	+1 5.0	−1 0.059	−1 26.0	+1 150	3.6	3.48
6.	+1 10.3	+1 1.89	+1 2.0	−1 1.0	+1 0.177	+1 40.8	−1 0	16.8	16.80
7.	+1 10.3	−1 1.55	+1 2.0	+1 5.0	−1 0.059	+1 40.8	−1 0	5.4	5.50
8.	−1 8.2	−1 1.55	−1 0.6	+1 5.0	−1 0.059	+1 40.8	−1 0	2.7	2.68
9.	+1 10.3	−1 1.55	+1 2.0	+1 5.0	+1 0.177	−1 26.0	−1 0	8.8	8.66
10.	+1 10.3	−1 1.55	+1 2.0	−1 1.0	−1 0.059	−1 26.0	0 75	10.5	10.62
11.	0 9.4	+1 1.89	+1 2.0	−1 1.0	−1 0.059	+1 40.8	+1 150	10.5	10.61
12.	0 9.4	−1 1.55	−1 0.6	−1 1.0	+1 0.177	−1 26.0	+1 150	15.2	15.14
13.	0 9.4	+1 1.89	−1 0.6	+1 5.0	−1 0.059	−1 26.0	+1 150	4.8	4.84
14.	−1 8.2	−1 1.55	0 1.3	−1 1.0	−1 0.059	−1 26.0	−1 0	7.0	7.05

Table 3. Cont.

Case	$X_1$ $T_g$ [°C]	$X_2$ $\lambda_g$ [W/mK]	$X_3$ $\lambda_m$ [W/mK]	$X_4$ $T_w$ [°C]	$X_5$ $M$ [kg/s]	$X_6$ $d_w$ [mm]	$X_7$ $L$ [mm]	$Y_i$ $q_i$ [W/m]	$\hat{Y}_i$ $q_i$ [W/m]
15.	+1 10.3	0 1.72	−1 0.6	+1 5.0	−1 0.059	+1 40.8	+1 150	5.9	5.78
16.	−1 8.2	0 1.72	−1 0.6	+1 5.0	+1 0.177	−1 26.0	+1 150	4.9	4.93
17.	−1 8.2	0 1.72	+1 2.0	+1 5.0	+1 0.177	+1 40.8	−1 0	5.7	5.64
18.	+1 10.3	0 1.72	−1 0.6	−1 1.0	+1 0.177	−1 26.0	−1 0	10.6	10.72
19.	+1 10.3	+1 1.89	+1 2.0	+1 5.0	−1 0.059	−1 26.0	−1 0	5.4	5.39
20.	−1 8.2	+1 1.89	0 1.3	+1 5.0	−1 0.059	+1 40.8	+1 150	3.7	3.74
21.	+1 10.3	+1 1.89	0 1.3	−1 1.0	−1 0.059	−1 26.2	+1 150	11.2	11.06
22.	−1 8.2	−1 1.55	−1 0.6	0 3.0	−1 0.059	−1 26.2	+1 150	5.5	5.50
23.	+1 10.3	−1 1.55	−1 0.6	0 3.0	+1 0.177	+1 40.8	−1 0	9.3	9.34
24.	+1 10.3	−1 1.55	−1 0.6	0 3.0	−1 0.059	+1 40.8	−1 0	6.5	6.58
25.	−1 8.2	+1 1.89	−1 0.6	0 3.0	+1 0.177	−1 26.0	−1 0	6.1	5.93
26.	−1 8.2	−1 1.55	+1 2.0	+1 5.0	+1 0.177	+1 40.8	+1 150	6.1	6.31
27.	+1 10.3	−1 1.55	−1 0.6	−1 1.0	−1 0.059	+1 40.8	+1 150	10.7	10.94
28.	−1 8.2	+1 1.89	−1 0.6	−1 1.0	0 0.118	+1 40.8	−1 0	8.9	8.93
29.	−1 8.2	−1 1.55	−1 0.6	+1 5.0	0 0.118	−1 26.0	−1 0	3.2	3.24
30.	+1 10.3	+1 1.89	−1 0.6	+1 5.0	+1 0.177	0 32.6	−1 0	6.4	6.48
31.	−1 8.2	−1 1.55	−1 0.6	−1 1.0	+1 0.177	0 32.6	−1 0	8.7	8.78
32.	−1 8.2	−1 1.55	+1 2.0	−1 1.0	−1 0.059	0 32.6	−1 0	7.5	7.67
33.	−1 8.2	−1 1.55	+1 2.0	−1 1.0	−1 0.059	+1 40.8	0 75	8.3	8.01
34.	+1 10.3	−1 1.55	−1 0.6	+1 5.0	−1 0.059	−1 26.0	0 75	5.1	5.17
35.	−1 8.2	+1 1.89	−1 0.6	−1 1.0	−1 0.059	−1 26.0	0 75	7.4	7.45
36.	0 9.4	0 1.72	0 1.3	0 3.0	0 0.118	0 32.6	0 75	10.4	10.45
37.	+1 10.3	−1 1.55	−1 0.6	−1 1.0	−1 0.059	0 32.6	−1 0	8.0	7.62

Table 3. Cont.

Case	$X_1$ $T_g$ [°C]	$X_2$ $\lambda_g$ [W/mK]	$X_3$ $\lambda_m$ [W/mK]	$X_4$ $T_w$ [°C]	$X_5$ $M$ [kg/s]	$X_6$ $d_w$ [mm]	$X_7$ $L$ [mm]	$Y_i$ $q_i$ [W/m]	$\hat{Y}_i$ $q_i$ [W/m]
38.	+1 10.3	+1 1.89	+1 2.0	+1 5.0	0 0.118	+1 40.8	+1 150	9.6	9.48
39.	0 9.4	−1 1.55	−1 0.6	+1 5.0	+1 0.177	+1 40.8	+1 150	7.6	7.46
40.	−1 8.2	0 1.72	−1 0.6	−1 1.0	+1 0.177	+1 40.8	+1 150	13.6	13.58

Then, on the basis of the obtained results of calculations (Table 3) using the least squares method [17], a model (Equation (6)) in the form of a dependency regression equation was developed  $Y = f(X_1, X_2, X_3, X_4, X_5, X_6, X_7)$ :

$$\begin{aligned}
 Y = & 10.45 + 1.45 X_1 + 0.42 X_2 + 0.72 X_3 - 2.87 X_4 + 2.20 X_5 + 0.33 X_6 + 1.03 X_7 + \\
 & 0.16X_1 X_2 - 0.06 X_1 X_3 - 0.16X_1 X_4 + 0.29X_1 X_5 + 0.25X_1 X_6 + 0.29X_1 X_7 - \\
 & 0.16X_2 X_3 - 0.04 X_2 X_4 + 0.09X_2 X_5 + 0.36X_2 X_6 + 0.18X_2 X_7 + 0.01X_3 X_4 + \\
 & 0.45X_3 X_5 - 0.26X_3 X_6 - 0.46X_3 X_7 - 0.76X_4 X_5 - 0.13X_4 X_6 - 0.49X_4 X_7 + \\
 & 0.06X_5 X_6 + 0.63X_5 X_7 - 0.04X_6 X_7 - 0.48X_1^2 + 0.60X_2^2 - 0.25X_3^2 - \\
 & 0.34X_4^2 - 0.31X_5^2 - 0.09X_6^2 - 0.93X_7^2
 \end{aligned} \quad (6)$$

In order to check the accuracy, model adequacy testing was carried out. It was considered that deterministic models are characterized by a mutually unambiguous correspondence between external interaction and the response to this impact. For this reason, only one experience was made at each point of the plan. For testing in this case, the Fisher criterion is used, which shows how many times the spread of the regression equation is reduced compared to the spread of the mean equation (Equation (7)) [23]:

$$F = \frac{S_y^2(f_1)}{S_r^2(f_2)} \quad (7)$$

where:

$S_y^2$ —mean variance;  $S_r^2$ —residual variance;

$f_1, f_2$  number of degrees of freedom;  $f_1 = (N - 1) = 40 - 1 = 39$ ;  $f_2 = (N - N_b) = 40 - 36 = 4$ .

$N = 40$ —number of calculations performed.

$N_b = 36$ —number of factors in the regression equation.

In practice, it was assumed that the regression equation describes the results of calculations adequately if the value of  $F$  is much greater than the tabular value of  $F_t$  at the level of significance  $p$  and degrees of freedom  $f_1$  and  $f_2$ . As is clear from the calculations,  $F = 17.7081/0.1499 = 118.1581$ , the tabular value  $F_t = F_{0.05;39,4} = 5.725$  [19]. Thus, the value of  $F$  repeatedly exceeds  $F_t$ , which means that the model is adequate and useful for further analysis. Its high quality is also confirmed by the coefficient of determination  $R^2 = 0.9991$ .

#### 4. Analysis of the Examined Relationship and Interpretation of the Results

The average temperature of the disturbed temperature field near the well, in the center, on the edge of the area and the heat extracted by the heat exchanger in individual time steps were determined through calculations. The area should be understood as the volume of the cylindrical-shaped area in which the ground heat exchanger is located. In the model, the assumed volume of the area is  $6500 \text{ m}^3$ . All simulations were performed with a time step equal to 1 h. Knowing the hourly efficiency of the ground heat exchanger  $E$  [kWh/h], the average unit efficiency of the exchanger was calculated from (8):

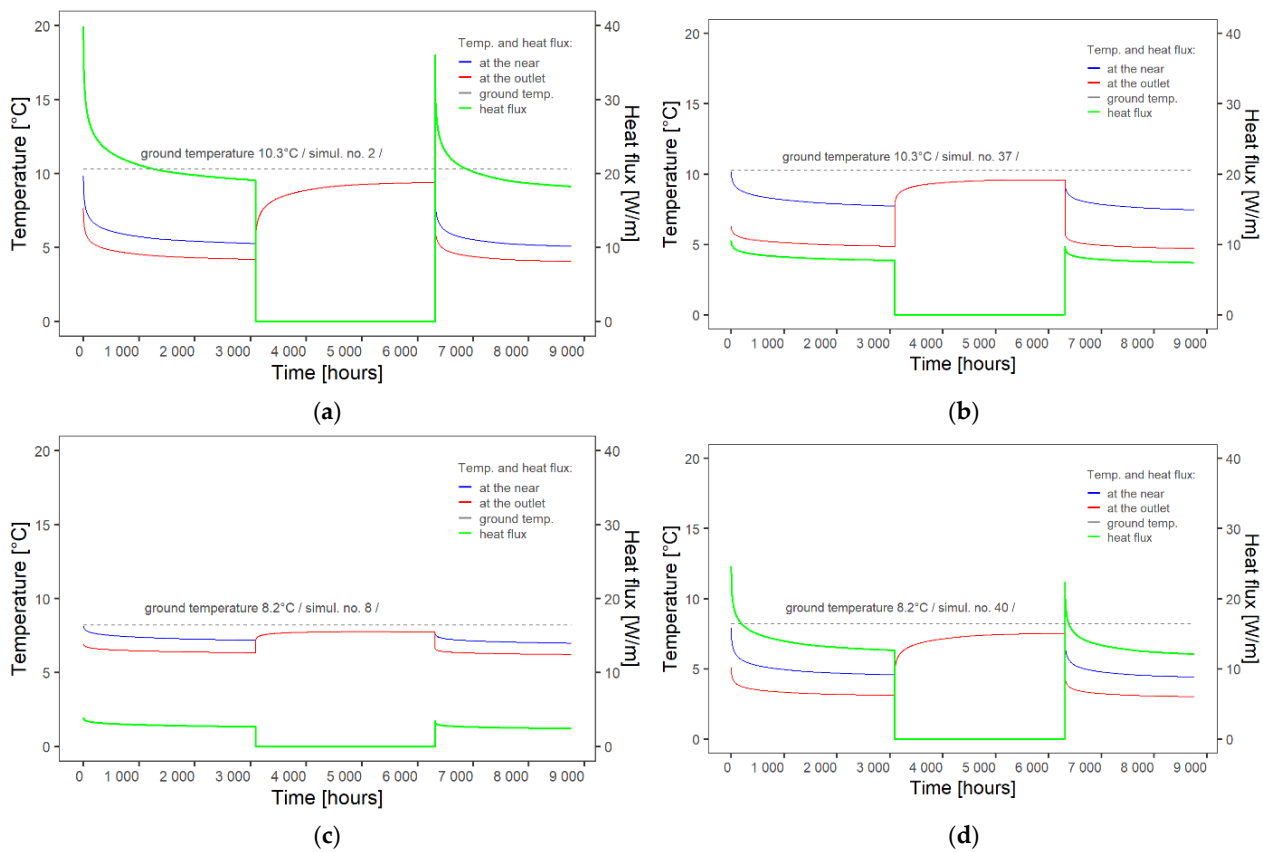


$$q = \frac{1}{K} \sum_{k=1}^K E_k \cdot 1000 \cdot \frac{1}{L_{VGHE}} \left[ \frac{Wh}{h} \frac{1}{m} = \frac{W}{m} \right] \quad (8)$$

where:

$q$ —average unit power output of a ground heat exchanger, W/m,  
 $K$ —number of time steps during operation of the heat exchanger, h,  
 $E_k$ —exchanger efficiency in individual time steps, kWh/h,  
 $L_{VGHE}$ —length of heat exchanger, m.

Example calculation results for selected initial-shore conditions, for extreme and average unit heat output values of the ground heat exchanger, are presented in Figure 4. The unit heat output values of the ground heat exchanger are, respectively, as follows: simulation no. 2—maximum value—18.94 W/m; simulation no. 40—13.56 W/m; simulation no. 37—7.99 W/m; simulation no. 8—minimum value—2.75 W/m. A summary of all results is shown in Figure 5.



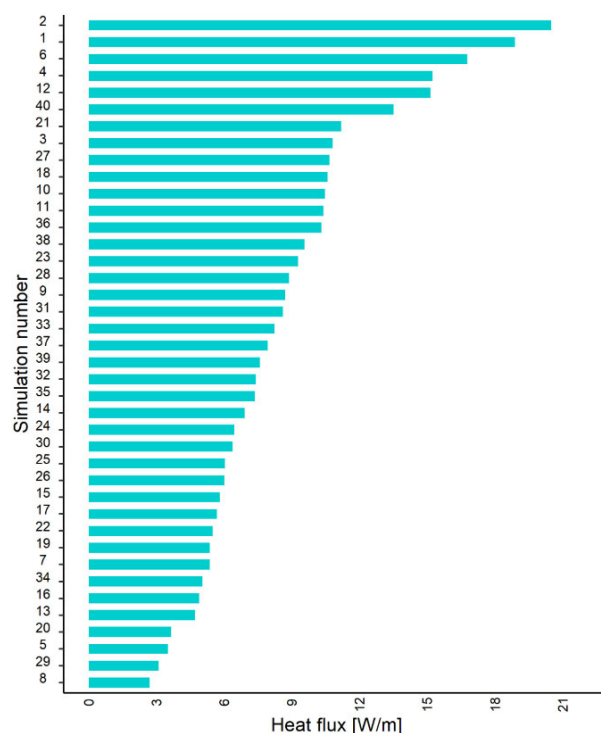
**Figure 5.** Temperature values of the heating medium and the heat flux transmitted from VGHE to the system within a year. (a) Simulation no. 2—maximum unit power of VGHE; (b) simulation no. 37; (c) simulation no. 8—minimum unit power of VGHE; (d) simulation no. 40.

The graphs (Figure 5a–d) show the operating hours of the system. When the ground temperature field is undisturbed, the value of the heat flux received by VGHE and transferred to the system reaches maximum values (left axis on the graph). Over time, the value decreases, because the heat exchange intensity decreases. This is due to the decrease in the temperature difference between the working environment of the ground heat exchanger, well and ground, at a constant supply temperature of the heat exchanger with a heating medium. This value stabilized relatively quickly. The middle part of the graph, where the stream transmitted to the system is 0 W/m (the right axis of the graph), represents the summer period. At this time, the system does not operate. You can clearly see the

increase in the temperature of the medium, red colored, to the value of the undisturbed temperature field. This value is not achieved because the temperature around the ground heat exchanger is disturbed. The ground is in the process of regeneration. When the system starts working again, the cycle repeats. For a larger temperature difference with the other operating parameters kept constant, the heat flux is greater, followed by a nonlinear decrease in the heat flux, which reaches its minimum in infinity. The calculation did not consider the heat loss of the buffer tank.

The model methodology, of course, after proper extension with the necessary additional elements of the heating system, can be effectively used to model the thermal field of the ground temperature.

The ordered values of heat transferred to the system from the lowest to the highest value are presented in Figure 6. There are big differences visible between individual results for individual operating parameters.

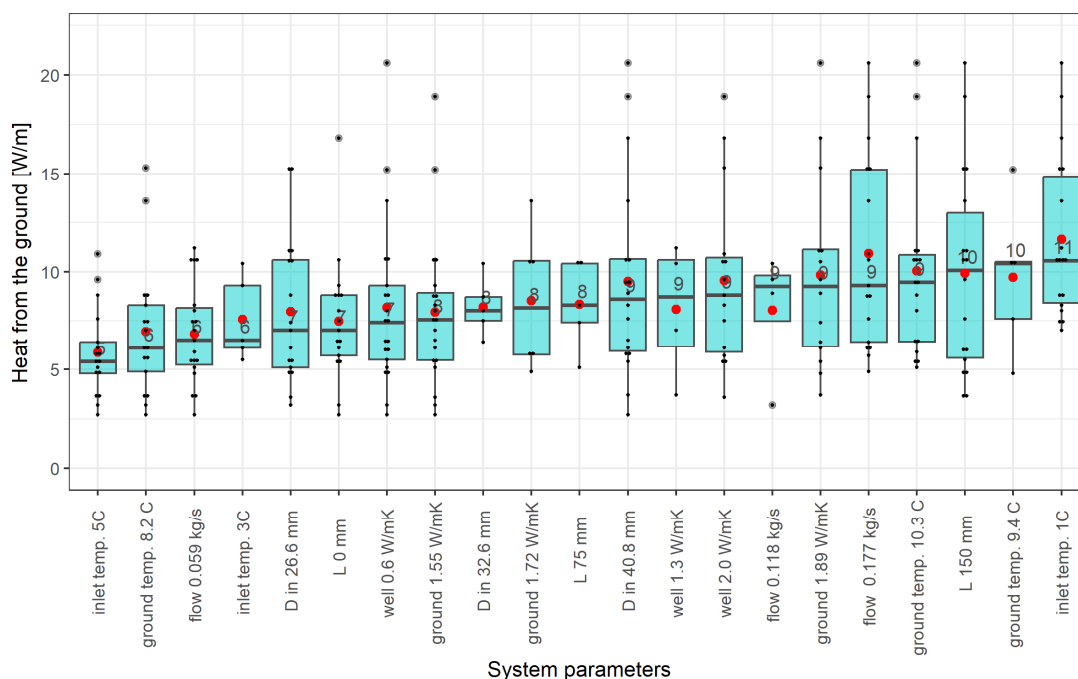


**Figure 6.** The ordered values of the unit thermal efficiency of the ground heat exchanger for each simulation.

For additional visual interpretation of the calculation results, regardless of the deterministic model, the simulation results are presented in a box-and-whisker plot (Figure 7). For each of the analysed parameters, the full range of results was presented. In the graph, the area delimited from the top and bottom by the rectangle area represents the first and third quartiles. The values between them represent 50% of the results. The median represents the center of the results. In the graph, the results are presented in an increasing order of median values. Red color represents average values.

The influence of the tested factors on the unit thermal efficiency of  $q$ , W/m of the vertical ground heat exchanger received by the u-image element of plastic pipes, placed in the drilling hole, was analyzed in a mathematical model [21].

Discussion of the results to ensure better clarity was made on natural variables. In addition, the word connections “beneficial effect” and “beneficial factor” mean that with a change in the factor from the lower to the upper level, the value of the unit thermal efficiency  $q$ , W/m vertical ground heat exchanger through the u-shape heating system element is growing. Conversely, the effect or factor is unfavorable when  $q$  decreases.



**Figure 7.** Box graph of the entire representation of the results of the analyzed model for the adopted parameters. Red dots indicate the arithmetic mean of the simulation results. Black dots show simulation results.

From the developed model [21], we detected the  $G_p$  center of a multifactorial space, which is characterized by coordinates corresponding to the average level of all factors: ground temperature  $T_g$  ( $\bar{X}_1$ ) = 9.4 °C; ground heat conduction coefficient  $\lambda_g$  ( $\bar{X}_2$ ) = 1.72 W/(mK); heat conduction coefficient of the well material  $\lambda_m$  ( $\bar{X}_3$ ) = 1.30 W/(mK); heating medium (glycol) temperature on the inlet  $T_w$  ( $\bar{X}_4$ ) = 3 °C; flow rate heating fluid (glycol) expenditure in the system  $M$  ( $\bar{X}_5$ ) = 0.118 kg/s; the inner diameter of the ground heat exchanger pipes  $d_w$  ( $\bar{X}_6$ ) = 32.6 mm; distances between the outer walls of the ground heat exchanger pipes  $L$  ( $\bar{X}_7$ ) = 75 mm, the unit size of the thermal efficiency  $q$ , W/m VGHE by U-shape element of plastic pipes, placed in the drilling hole. For the selected climatic conditions of Podlasie Voivodeship (Poland),  $q = 10.45$  W/m.

Using the  $G_p$  point as a reference point, the influence of individual factors on the unit thermal efficiency of  $q$ , W/m of the tested heat exchanger by the u-shape element of plastic pipes was then estimated. It turned out that almost all selected factors, in addition to the temperature of the heating medium (glycol) on the inlet  $T_w$  ( $X_4$ ), show beneficial effects and increase the size of  $q$ . The effects of their influence on the  $q$  size when changing factors from the lower (−1) to the upper (+1) level are, respectively,  $T_g$  ( $X_1$ ): from 8.52 to 11.42 W/m, i.e., +34.04%;  $\lambda_g$  ( $X_2$ ): from 10.63 to 1.47 W/m, i.e., +7.90%;  $\lambda_m$  ( $X_3$ ): from 9.48 to 10.92 W/m, i.e., +15.20%;  $M$  ( $X_5$ ): from 7.94 to 12.34 W/m, i.e., +55.42%;  $d_w$  ( $X_6$ ): from 10.03 to 10.69 W/m, i.e., +6.58%;  $L$  ( $X_7$ ): from 8.49 to 10.55 W/m, i.e., +24.26%.

The only unfavorable factor was the temperature of the heating medium (glycol) at the inlet of VGHE  $T_w$  ( $X_4$ ). When changing from the lower to the upper level of this factor, a decrease in the unit thermal efficiency of the vertical ground heat exchanger from 12.98 to 7.24 W/m, i.e., −44.22%, occurred.

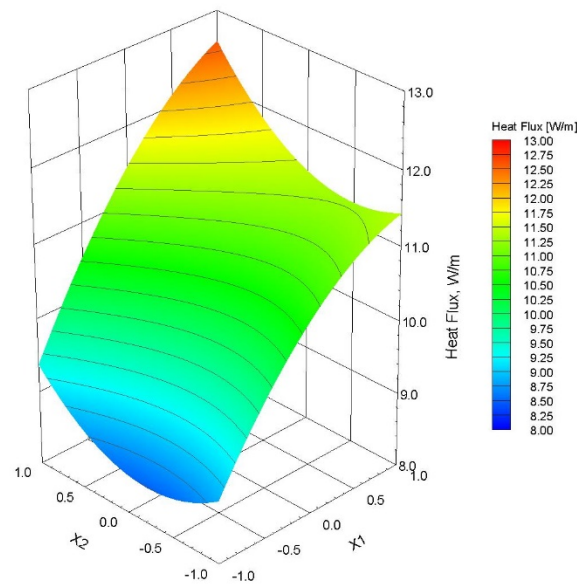
As can be seen from the analysis, the highest beneficial effect of +55.42% was obtained from the expenditure of the heating medium (glycol) in the system  $M$  ( $X_5$ ), corresponding to the nature of the process; after all, the expenditure determines the intensity of the heat transfer process. In second place, with an effect of +34.04%, is the ground temperature factor  $T_g$   $X_1$ . This confirms the high potential of this factor, because even the impact of fluctuations in the ground temperature in relation to the average value in one province

results in such a significant effect. In third place, with an effect of +24.26%, there is a factor distance between the outer walls of the ground heat exchanger pipes  $L$  ( $X_7$ ).

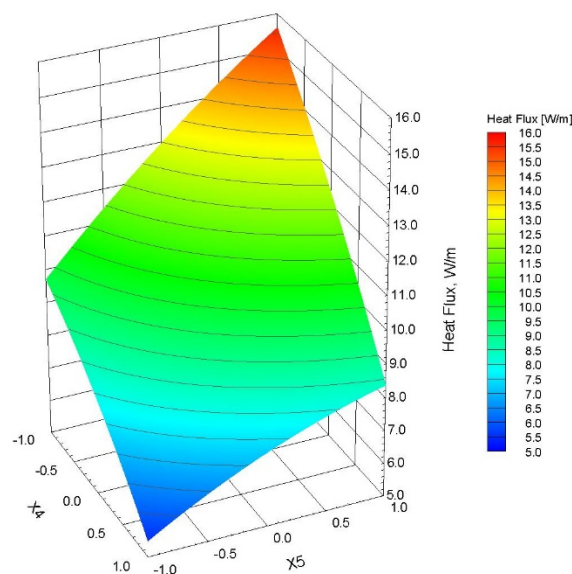
Auxiliary model for the chart:

$$Y_1 = 10.45 + 1.45 X_1 + 0.42 X_2 + 0.16X_1 X_2 - 0.48X_1^2 + 0.60X_2^2$$

The nature of the influence of some of the analyzed factors on the performance of  $q$  is illustrated in Figures 8 and 9. Figure 8 shows the dependence of the unit yield  $q$  of the heat exchanger on two factors characterizing the boundary conditions:  $T_g$  ( $X_1$ )—ground temperature and  $l$  ( $X_2$ )—ground conduction coefficient. In contrast, Figure 9 shows the dependence of the unit efficiency  $q$  of the heat exchanger on two factors of the second group, characterizing the parameters of the heat exchanger: heating medium (glycol) temperature on  $T_w$  ( $X_4$ ) and heating medium (glycol) in system M ( $X_5$ ) system.



**Figure 8.** Dependence of  $q$  on factors characterizing soil properties:  $T_g$  ( $X_1$ ) soil temperature, °C and  $\lambda_g$  ( $X_2$ ) ground heat conduction coefficient, W/(mK) (other factors were taken at the average level).



**Figure 9.** Dependence of  $q$  on factors: heating medium (glycol) temperature at the inlet  $T_w$  ( $X_4$ ), °C and flow rate of heating medium (glycol)  $M$  ( $X_5$ ), kg/s (other factors were taken at the average level).

Auxiliary model for the chart:

$$Y_2 = 10.45 - 2.87 X_4 + 2.20 X_5 - 0.76X_4 X_5 - 0.34X_4^2 - 0.31X_5^2$$

The presented analysis and the detected factors with significant effects confirmed the expectations of the authors on the possibility of improving the thermal efficiency of the vertical ground heat exchanger by optimizing its parameters.

## 5. Optimization of Heat Exchanger Parameters According to the Energy Criterion

After the analysis of the influence of the tested factors on the unit thermal efficiency of  $q$ , W/m ( $Y$ ) heat exchanger by the u-image element of plastic pipes, a procedure was performed to optimize the parameters of this exchanger on the mathematical model according to the energy criterion. The method of iterative (numerical) searching of the adequate area in the Matlab environment was used, which consists of detecting the extreme values of functions by checking the entire tested area with the appropriate sampling step of individual input factors.

In the implementation of the optimization procedure, the values of the heat exchanger parameters, ensuring the maximum unit thermal efficiency, were searched. However, the developed mathematical model [21] contains two groups of factors: characterizing boundary conditions—ground temperature ( $X_1$ ) and ground conductivity coefficient ( $X_2$ ); characterizing the heat exchanger solution—heat conduction coefficient of the well material ( $X_3$ ), heating medium (glycol) temperature on the power supply ( $X_4$ ), heating medium (glycol) expenditure in the system ( $X_5$ ), internal diameter of the ground heat exchanger pipes ( $X_6$ ), distance between outer walls of ground heat exchanger pipes ( $X_7$ ). In order to optimize the parameters of the heat exchanger, the factors characterizing  $X_1$ ,  $X_2$  boundary conditions had to be eliminated from the model. This was possible by adopting several variants of boundary conditions with selected values of factors  $X_1$ ,  $X_2$ , including, in the model [21], the adopted values of these factors; appropriate improvement and simplification of models after eliminating factors  $X_1$ ,  $X_2$ .

In this study, the following three boundary condition variants for the heat exchanger were adopted:

- Variant 1—factors characterizing the properties of the soil at the average level:  $X_1 = 0$ ,  $X_2 = 0$ ;
- Variant 2—factors characterizing the properties of the soil at the lower level:  $X_1 = -1$ ,  $X_2 = -1$ ;
- Variant 3—factors characterizing the properties of the soil at the upper level:  $X_1 = +1$ ,  $X_2 = +1$ .

Substituting the values of  $X_1$ ,  $X_2$  factors to the model [21] and performing simplifications, for each of the variants of boundary conditions, an appropriate model of the dependence of the unit thermal efficiency  $Y$  of the heat exchanger tested on five factors was obtained, characterizing the heat exchanger solution, i.e., dependency models  $Y_{1,2,3} = f(X_3, X_4, X_5, X_6, X_7)$ . These models allowed us to perform the procedure of optimizing the heat exchanger parameters. The models have the following form:

- Variant I ( $X_1 = 0$ ,  $X_2 = 0$ ):

$$Y_1 = 10.45 + 0.72 X_3 - 2.87 X_4 + 2.20 X_5 + 0.33 X_6 + 1.03 X_7 + 0.01X_3X_4 + 0.45X_3 X_5 - 0.26X_3X_6 - 0.46X_3 X_7 - 0.76X_4X_5 - 0.13X_4 X_6 - 0.49X_4X_7 + 0.06X_5 X_6 + 0.63X_5X_7 - 0.04X_6 X_7 - 0.25X_3^2 - 0.34X_4^2 - 0.31X_5^2 - 0.09X_6^2 - 0.93X_7^2. \quad (9)$$

- Variant II ( $X_1 = -1$ ,  $X_2 = -1$ ):

$$Y_2 = 8.86 + 0.94 X_3 - 2.67 X_4 + 1.82 X_5 - 0.28 X_6 + 0.56X_7 + 0.01X_3 X_4 + 0.45X_3 X_5 - 0.26X_3 X_6 - 0.46X_3 X_7 - 0.76X_4 X_5 - 0.13X_4 X_6 - 0.49X_4 X_7 + 0.06X_5 X_6 + 0.63X_5 X_7 - 0.04X_6 X_7 - 0.25X_3^2 - 0.34X_4^2 - 0.31X_5^2 - 0.09X_6^2 - 0.93X_7^2. \quad (10)$$

- Variant III ( $X_1 = +1$ ,  $X_2 = +1$ ):

$$Y_3 = 12.60 + 0.50 X_3 - 3.07 X_4 + 2.58 X_5 + 0.94 X_6 + 1.50X_7 + 0.01X_3 X_4 + 0.45X_3 X_5 - 0.26X_3 X_6 - 0.46X_3 X_7 - 0.76X_4 X_5 - 0.13X_4 X_6 - 0.49X_4 X_7 + 0.06X_5 X_6 + 0.63X_5 X_7 - 0.04X_6 X_7 - 0.25X_3^2 - 0.34X_4^2 - 0.31X_5^2 - 0.09X_6^2 - 0.93X_7^2. \quad (11)$$



Using models (9)–(11), a procedure was performed to optimize the parameters of the heat exchanger. The results of the optimization are given in Table 4.

**Table 4.** Optimal values of parameters of the selected geothermal heat exchanger of the heating system with optimization against the energy criterion.

Variant No.	$q$ , W/m	$\lambda_m$ ( $X_3$ )	$T_w$ ( $X_4$ )	$M$ ( $X_5$ )	$d_w$ ( $X_6$ )	$L$ ( $X_7$ )
1 $X_1 = 0, X_2 = 0$	$q_{1max} = 17.44$	1.97 (0.95)	1.0 (−1)	0.177 (1)	40.8 (1)	142.5 (0.9)
2 $X_1 = -1, X_2 = -1$	$q_{2max} = 15.28$	2.00 (1)	1.0 (−1)	0.177 (1)	26.0 (−1)	127.5 (0.7)
3 $X_1 = 1, X_2 = 1$	$q_{3max} = 21.10$	1.62 (0.45)	1.0 (−1)	0.177 (1)	40.8 (1)	150 (1)

As can be seen from Table 3, the highest unit heat efficiency of the heat exchanger through the u-shape element of plastic pipes with a height of 21.10 W/m can be achieved by laying it in soils with elevated temperature to the upper limit of  $T_g = 10.2$  °C and the ground heat conduction coefficient at the level of  $=1.89$  W/(m<sup>2</sup>K). The optimal parameters of the heat exchanger in such a system should be taken as follows: the heat conduction coefficient of the material well 1.62 W/(m<sup>2</sup>K); the temperature of the heating medium (glycol) at the inlet  $T_w = 1.0$  °C; flow rate of heating medium (glycol) in system  $M = 0.177$  kg/s; internal diameter of ground heat exchanger pipes  $d_w = 40.8$  mm; distance between external walls of ground heat exchanger pipes  $L = 150$  mm.

After analyzing the contributions of individual parameters and optimizing the selected groups of parameters to ensure the maximum unit thermal efficiency of the heat exchanger, the purpose of this study was achieved.

## 6. Summary and Conclusions

On the basis of the results of the calculation experiment for the climatic conditions of the Podlasie voivodship, deterministic mathematical models of the dependence of the unit thermal efficiency of the heat exchanger by the u-image element of plastic pipes  $q$ , W/m were developed (function Y) from seven factors: ground temperature  $T_g$  ( $X_1$ ) and ground conductivity coefficient ( $X_2$ ); heat conduction coefficient of well material ( $X_3$ ), etc, heating medium (glycol) temperatures at the inlet  $T_w$  ( $X_4$ ), flow rate heating medium (glycol) in the system  $M$  ( $X_5$ ), internal diameter of the ground pipes of the heat exchanger  $d_w$  ( $X_6$ ), distance between outer walls of ground heat exchanger tubes  $L$  ( $X_7$ ). The obtained model analyzed the degree and nature of the influence of factors and performed optimization of heat exchanger parameters by the u-image element of plastic pipes according to the energy criterion.

The developed deterministic mathematical model allowed us to detect that, when changing from the lower to the upper level of factors,  $T_g$  ( $X_1$ ),  $\lambda_g$  ( $X_2$ );  $\lambda_m$  ( $X_3$ ),  $M$  ( $X_5$ ),  $d_w$  ( $X_6$ ),  $L$  ( $X_7$ ), the value of the unit thermal efficiency of the vertical heat exchanger by the u-image element of plastic pipes  $q$  (Y) increased by, respectively: 34.04; 7.90; 15.20; 55.42; 6.58; 24.26%. The only unfavorable factor was the temperature of the heating medium (glycol) at the inlet to VGHE  $T_w$  ( $X_4$ ). When changing from the lower to the upper level of this factor, a decrease in the unit thermal efficiency of the tested heat exchanger, by  $-44.22\%$ , was seen.

After performing the numerical optimization procedure, it was found that for the basic option 1 for boundary conditions (ground temperature  $T_g$  ( $X_1$ ) = 9.4 °C and the ground conduction coefficient  $\lambda_g$  ( $X_2$ ) = 1.72 W/(mK)), the maximum unit value of the heat output of a vertical heat exchanger through a u-shape plastic pipe element  $q_{1max}$  (Y) = 17.44 W/m can be achieved with the following heat exchanger parameters: heat conduction coefficient of the well material  $\lambda_m$  ( $X_3$ ) = 1.97 W/(mK), temperature of heating medium (glycol) at the inlet  $T_w$  ( $X_4$ ) = 1 °C; flow rate in the heating factor (glycol) in the system  $M$  ( $X_5$ ) = 0.177 kg/s;

internal diameters of the ground heat exchanger pipes  $d_w$  ( $X_6$ ) = 40.8 mm and the distance between the outer walls of the ground heat exchanger pipes  $L$  ( $X_7$ ) = 142.5 mm.

The highest unit thermal efficiency of the heat exchanger under test  $q_{3max}$  ( $Y$ ) = 21.10 W/m can be achieved at the location of the exchanger in the land of the Podlasie voivodship with upper values of ground temperature  $T_g$  ( $X_1$ ) = 10.3 °C and the ground conductivity coefficient  $\lambda_g$  ( $X_2$ ) = 1.89 W/(mK)). Then, the values  $q_{3max}$  ( $Y$ ) = 21.10 W/m can be achieved with the following heat exchanger parameters: heat conduction coefficient of the well material  $\lambda_m$  ( $X_3$ ) = 1.62 W/(mK); by temperature of heating medium (glycol) at the inlet  $T_w$  ( $X_4$ ) = 1 °C; flow rate in the heating factor (glycol) in the system  $M$  ( $X_5$ ) = 0.177 kg/s; internal diameters of the ground heat exchanger pipes  $d_w$  ( $X_6$ ) = 40.8 mm; and the distance between the outer walls of the ground heat exchanger pipes  $L$  ( $X_7$ ) = 150 mm.

The authors plan to confirm the detected regularities on other types of heat exchangers and detect the optimal parameters for these exchangers. Since we considered the factors affecting the unit thermal efficiency of the heat exchanger, with the appropriate selection of materials and elements of the geothermal heating system, significant reserves of energy generation can be detected; this is important in an energy crisis.

In further scientific research, the authors intend to utilize their experience to enhance the performance of ground heat exchangers in conjunction with the building's heating system. In a cold climate, some small changes in the ground heat exchanger system bring real effects and benefits.

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## Abbreviations

### Nomenclature

$T_g$	ground temperature, °C
$T_w$	temperature of the heating medium (glycol), °C
$d_w$	internal diameter of the pipes of the ground heat exchanger, mm
$\lambda_g$	soil thermal conductivity coefficient, W/(mK)
$\lambda_m$	thermal conductivity coefficient of the well material, W/(mK)
$q$	average unit power output of a ground heat exchanger, heat flux, W/m
$K$	number of time steps during operation of the heat exchanger, h
$E_k$	exchanger efficiency in individual time steps, kWh/h
$L$	distances between the external walls of the pipes of the ground heat exchanger, m

### Nomenclature for mathematical modelling

$\dot{X}_1$	ground temperature $T_g$ , °C
$\dot{X}_2$	soil thermal conductivity coefficient $\lambda_g$ , W/(mK)
$\dot{X}_3$	thermal conductivity coefficient of the well material $\lambda_m$ , W/(mK)
$\dot{X}_4$	temperature of the heating medium (glycol) at the inlet of VGHE $T_w$ , °C
$\dot{X}_5$	the flow rate of the heating medium (glycol) $M$ , kg/s
$\dot{X}_6$	internal diameter of the pipes of the ground heat exchanger $d_w$ , mm
$\dot{X}_7$	distances between the external walls of the pipes of the ground heat exchanger $L$ , mm
$Y_i$	heat flux, results of simulation $q$ , W/m
$\hat{Y}_i$	heat flux, results calculated by model $q$ , W/m

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