

Article

Finite Element Study on Calculation of Nonlinear Soil Consolidation Using Compression and Recompression Indexes

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Abstract: The coefficient of consolidation is traditionally considered as a constant value in soil consolidation calculations. This paper uses compression and recompression indexes to calculate the solution-dependent nonlinear compressibility, thus overconsolidation and normal consolidation are separated during the calculations. Moreover, the complex nonlinear consolidation can be described using the nonlinear compressibility and a nonlinear permeability. Then, the finite element discrete equation with consideration of the time-dependent load is derived, and a corresponding program is developed. Subsequently, a case history is conducted for verifying the proposed method and the program. The results show that the method is sufficiently accurate, indicating the necessity of considering nonlinearity for consolidation calculations. Finally, three cases are compared to reveal the importance of separating the overconsolidation and normal consolidation. Overall, this study concluded that it is inadequate to consider just one consolidation status in calculations, and that the proposed method is more reasonable for guiding construction.

Keywords: compression index; recompression index; nonlinear consolidation; finite element method; time-dependent loading

1. Introduction

Consolidation behavior is a complex phenomenon in clays and is of great importance to most construction activities. The accuracy of consolidation calculations is a key point in the geotechnical field. Terzaghi developed the classical consolidation theory for solving consolidation problems [1]. However, this theory is based on many assumptions, such as constant loading and a constant coefficient of consolidation. In particular, the constant consolidation coefficient will lead to incorrect results in the multilayered soil system [2]. To improve the calculation accuracy, many modifications of the assumptions have been proposed. Past studies mainly investigated the variations of material parameters, such as the permeability and the compressibility [3–10], and many advances related to varying loading, which better reflect real constructions, were obtained [11–15]. As a simplification of inhomogeneity along depth, multilayered soil was studied [16–20]. Other important calculation factors, such as drainage boundary conditions [21–24] and the non-Darcy fluid theory [25–28], were also considered for more reasonable calculations. Although some of these early studies obtained analytical solutions, a majority of them have not been analytically solved. With the development of numerical methods, complex consolidation equations can be solved efficiently, thereby allowing many modifications to be incorporated into the consolidation calculations. The finite element method [2,7,29,30] and the finite difference method [4,27,31–33] are the two most commonly used methods. Moreover, other effective numerical methods were also adopted [34–37].

However, the coefficient of consolidation used in calculation is usually considered as a constant value in aforementioned studies (i.e., a constant ratio of permeability to compressibility), and such a simplification may not be reasonable in some situations. In particular, there is little information available about the solution-dependent behavior of the compressibility, which significantly affects the coefficient of consolidation. Compression and recompression indexes, which are used in calculations of settlement, are used herein for evaluating the nonlinear compressibility. Subsequently, the nonlinear compressibility and a nonlinear permeability are combined to describe the nonlinear consolidation. Using the finite element method, the discrete equation in space and time domains is derived and the corresponding program with consideration of the time-dependent load is developed. A case history for verifying the proposed method and three comparative cases for proving the significance of the separation of overconsolidation and normal consolidation are conducted subsequently.

2. Nonlinear Consolidation

2.1. Nonlinear Compressibility

On the basis of consolidation theory, the relation of void ratio and effective stress in soil is described as [38]

$$\partial e = -a_v \partial \sigma' \tag{1}$$

where e is the void ratio, σ' is the effective stress, and a_v is the compressibility.

According to the relation of e - $\log \sigma'$ [38] shown in Figure 1, the increment of void ratio can be calculated using Equation (2) for overconsolidation and Equation (3) for normal consolidation:

$$de = -C_s d(\log \sigma') = -\frac{C_s}{2.303} \frac{d\sigma'}{\sigma'} \tag{2}$$

$$de = -C_c d(\log \sigma') = -\frac{C_c}{2.303} \frac{d\sigma'}{\sigma'} \tag{3}$$

where de is the increment of void ratio, C_s is the recompression index, C_c is the compression index, and $d\sigma'$ is the increment of effective stress.

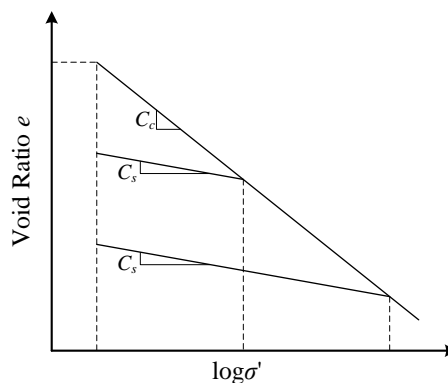


Figure 1. The diagram of e - $\log \sigma'$ relation.

From Equations (1)–(3), the nonlinear compressibility, which can be calculated using the compression index or the recompression index, is obtained and expressed as

$$a_v = \begin{cases} \frac{C_s}{2.303\sigma'} & \text{Overconsolidation} \\ \frac{C_c}{2.303\sigma'} & \text{Normal Consolidation} \end{cases} \tag{4}$$

2.2. Nonlinear Permeability

For the nonlinear permeability, the kind of e - $\log k$ relation [31], which is mathematically described in Equation (5), is adopted:

$$\log k = \log k_0 - \frac{e_0 - e}{C_k} \tag{5}$$

where k_0 is the initial permeability, e_0 is the initial void ratio, k is the current permeability corresponding to current void ratio e , and C_k is the permeability index.

2.3. Degree of Consolidation

The average degree of consolidation used in this paper is defined as [38]

$$u_{av} = \frac{q_t - (1/H) \int_0^H u dz}{q_c} \tag{6}$$

where u_{av} is the average degree of consolidation, u is the excess pore water pressure, q_t is the current load at time t , q_c is the ultimate load when t approaches to ∞ , and H is the calculation depth.

For multilayered soil, Equation (7) should be used:

$$u_{av} = \frac{q_t - (1/H) \sum_{i=1}^n \int_{z_{i-1}}^{z_i} u_i dz}{q_c} \tag{7}$$

where n is the number of layers, or the number of elements in the finite element analyses.

2.4. Settlement

Settlement of any depth at any time can be evaluated using Equation (8):

$$S(t, z) = \sum_{i=1}^t \left[\int_0^z \frac{-\Delta e_i}{1 + e_0} dz \right] \tag{8}$$

3. Finite Element Solution

3.1. Finite Element Equations

The governing equation of small strain problems in the consolidation theory [38] is written as

$$\frac{\partial}{\partial z} \left(\frac{k}{\gamma_w} \frac{\partial u}{\partial z} \right) = \frac{a_v}{1 + e_0} \left(\frac{\partial u}{\partial t} - \frac{\partial \sigma}{\partial t} \right) \tag{9}$$

where σ is the total stress, γ_w is the unit weight of water, t is the time, and z is the depth.

Using the Galerkin weighted residual method [30], Equation (9) can be converted to Equation (10):

$$[\mathbf{k}_c]\{\mathbf{u}\} + [\mathbf{m}_m] \left\{ \frac{\partial \mathbf{u}}{\partial t} \right\} = \{\mathbf{f}\} \tag{10}$$

where $[\mathbf{k}_c]$ is the fluid conductivity matrix, $[\mathbf{m}_m]$ is the mass matrix, $\{\mathbf{u}\}$ is the excess pore water pressure vector, $\{\partial \mathbf{u} / \partial t\}$ is the derivative of excess pore water pressure vector with respect to time, and $\{\mathbf{f}\}$ is the external total load vector. For a one-dimensional line element with two nodes, expressions of the matrices and the vector are Equations (11)–(13).

$$[\mathbf{k}_c] = \frac{k}{\gamma_w L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{11}$$

$$[\mathbf{m}_m] = \frac{a_v L}{1 + e_0} \begin{bmatrix} 1/3 & 1/6 \\ 1/6 & 1/3 \end{bmatrix} \tag{12}$$

$$\{\mathbf{f}\} = \frac{a_v L}{2(1 + e_0)} \frac{\partial \sigma}{\partial t} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \tag{13}$$

where L is the length of the line element. Then, a kind of interpolation in the time domain using the θ method [2] made here is as follows:

$$\{\mathbf{u}\}_1 = \{\mathbf{u}\}_0 + \Delta t \left[(1 - \theta) \left\{ \frac{\partial \mathbf{u}}{\partial t} \right\}_0 + \theta \left\{ \frac{\partial \mathbf{u}}{\partial t} \right\}_1 \right] \tag{14}$$

where 0 is the subscript indicating the beginning of a time increment, 1 is the subscript indicating the end of a time increment, Δt is the value of the time increment, and θ is the weighted coefficient ($0 \leq \theta \leq 1$).

Combining Equations (10) and (14) with the method introduced by Huang and Griffiths [2], the finite element discrete equation, both in space and time domains, is derived as

$$\{[\mathbf{m}_m] + \theta \Delta t [\mathbf{k}_c]\} \{\mathbf{u}\}_1 = \{[\mathbf{m}_m] - (1 - \theta) \Delta t [\mathbf{k}_c]\} \{\mathbf{u}\}_0 + \Delta t [(1 - \theta) \{\mathbf{f}\}_0 + \theta \{\mathbf{f}\}_1] \tag{15}$$

3.2. Calculation Flow

On the basis of the equations stated above, the transient nonlinear consolidation problems can be solved using the finite element method, and a corresponding program was developed with FORTRAN. Using this program, the excess pore water pressure, the average degree of consolidation, and the settlement can be obtained simultaneously. The detailed calculation flow is illustrated in Figure 2.

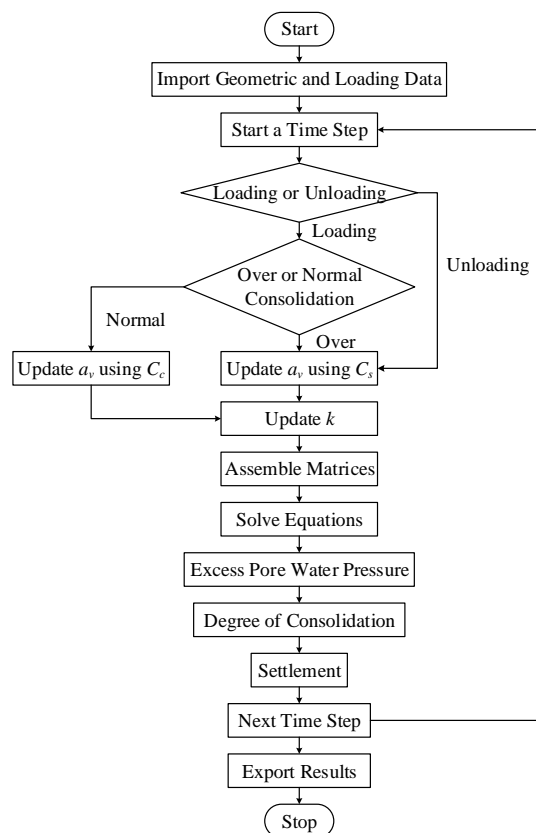


Figure 2. The calculation flow chart based on the finite element method.

4. Case Verification

Case history II from Mesri and Choi [31] is used to verify the proposed method. It was an embankment project, and the construction of this project was finished after 25 days. The final load at the top surface of the foundation is 40.6 kPa, the loading process can be dealt with as a linear load (Figure 3). The calculation height of the soil is 14 m and it is divided into 12 layers; the parameters of all layers are listed in Table 1. Both top and bottom surfaces are permeable. The calculation time length is 12,000 days, and the unit weight of water is 10 kN/m³.

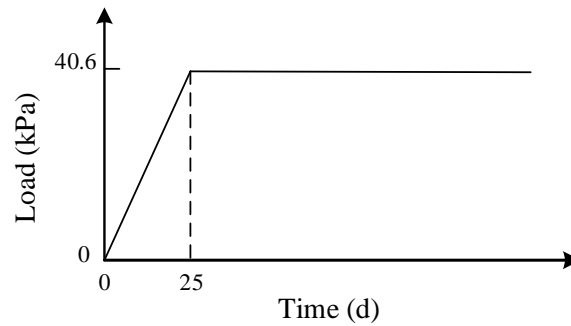


Figure 3. The diagram of the loading process.

Table 1. Recompiled soil parameters after Mesri and Choi [31].

Layer	<i>h</i> (m)	<i>k</i> ₀ (m/d)	<i>C</i> _k	<i>e</i> ₀	$\sigma_{0'}$ (kPa)	$\sigma_{p'}$ (kPa)	<i>C</i> _s	<i>C</i> _c
1	1	8.67×10^{-5}	1.16	2.51	7.03	31.72	0.15	1.43
2	1	1.73×10^{-4}	1.34	2.91	12.11	21.98	0.19	1.89
3	0.5	1.73×10^{-4}	1.44	3.21	16.52	24.22	0.22	2.39
4	1	9.57×10^{-5}	1.34	2.99	20.14	25.43	0.14	2.40
5	1.5	1.12×10^{-4}	1.18	2.61	26.11	32.44	0.24	2.48
6	2	1.12×10^{-4}	1.18	2.61	33.39	38.14	0.27	2.63
7	0.5	9.59×10^{-5}	1.08	2.41	39.15	44.71	0.17	2.10
8	1.5	9.64×10^{-5}	0.93	2.08	46.20	54.67	0.11	2.14
9	2	9.59×10^{-5}	0.95	2.14	55.20	59.86	0.20	1.83
10	0.5	8.61×10^{-5}	1.10	2.41	59.06	72.67	0.19	2.70
11	1.5	1.29×10^{-4}	0.70	1.56	66.44	86.94	0.12	2.81
12	1	1.30×10^{-4}	0.61	1.21	74.45	89.84	0.10	2.75

Figures 4 and 5 show comparisons between calculated and measured excess pore water pressure, along time and depth, respectively. While Figures 6 and 7 compare calculated and measured settlement along time and depth, respectively. It is clear from these figures that the calculated results are in good agreement with the measurements, proving the accuracy of the method proposed in this study. Meanwhile, the comparison underscores the necessity of considering the nonlinear permeability and the nonlinear compressibility in calculations, and demonstrates that using compression and recompression indexes for evaluating the nonlinear compressibility is effective and stable.

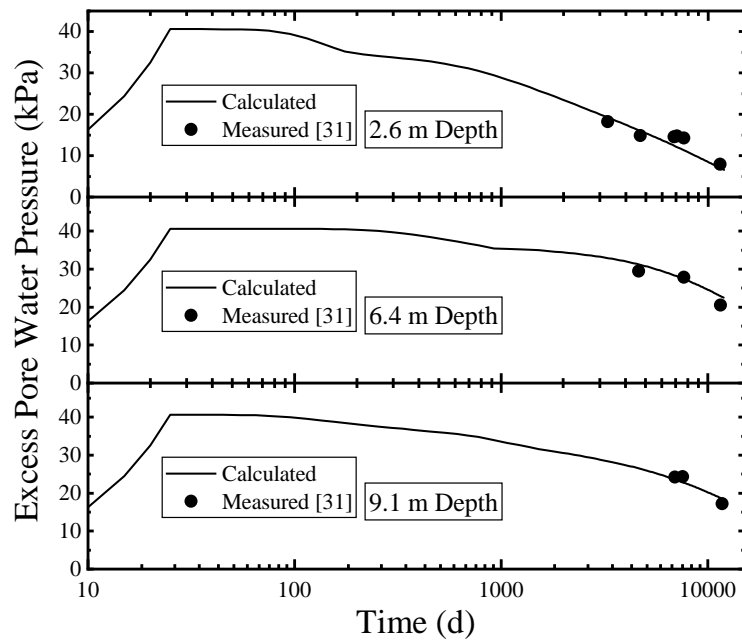


Figure 4. Comparisons of calculated and measured excess pore water pressure along time.

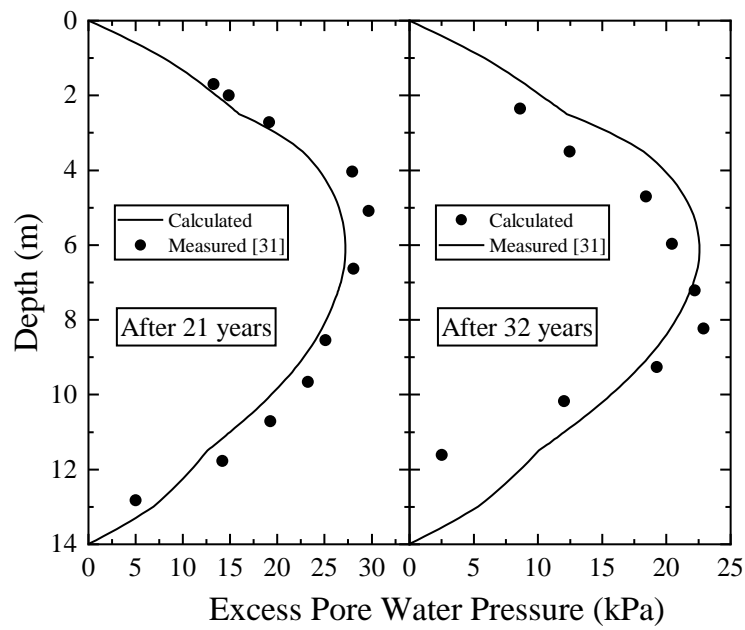


Figure 5. Comparisons of calculated and measured excess pore water pressure along depth.

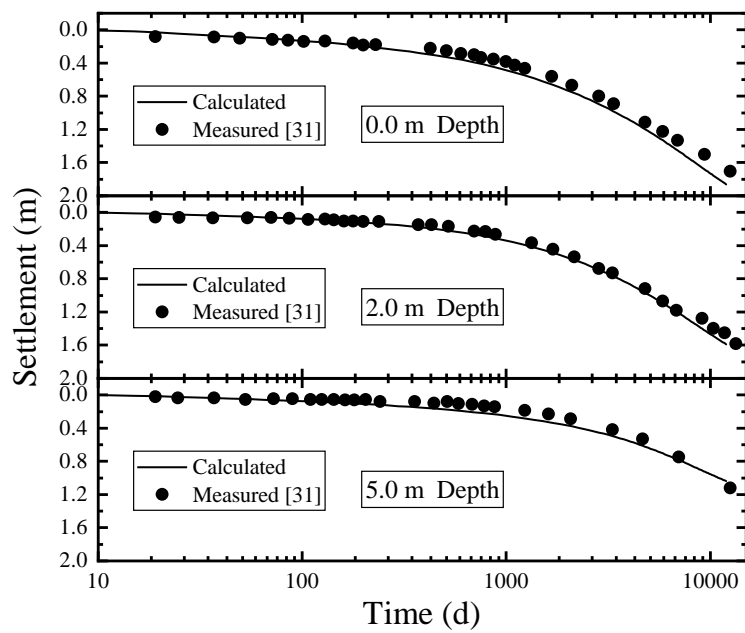


Figure 6. Comparisons of calculated and measured settlement along time.

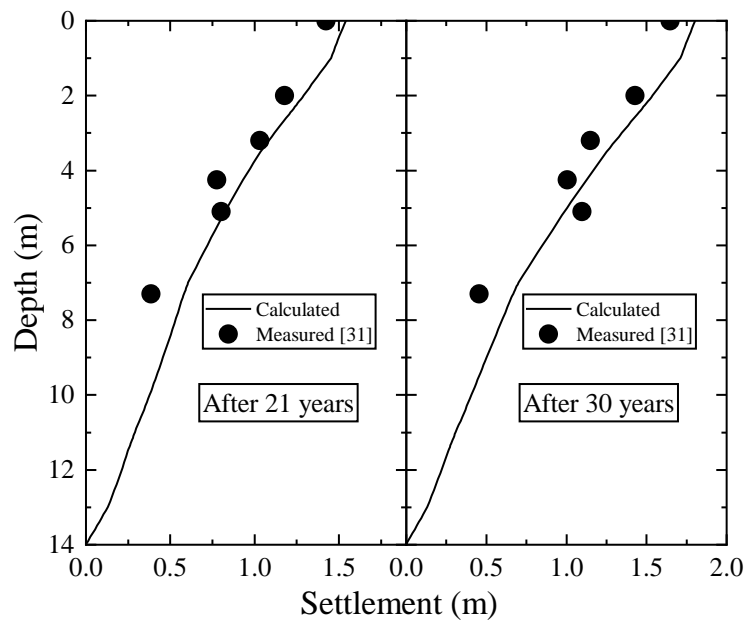


Figure 7. Comparisons of calculated and measured settlement along depth.

In the research of Mesri and Choi [31], the detailed variation of the average degree of consolidation was not given. Only the value of 82% after 32 years was presented according to the real settlement, which is almost the same as the calculated average degree of consolidation in this study (Figure 8).

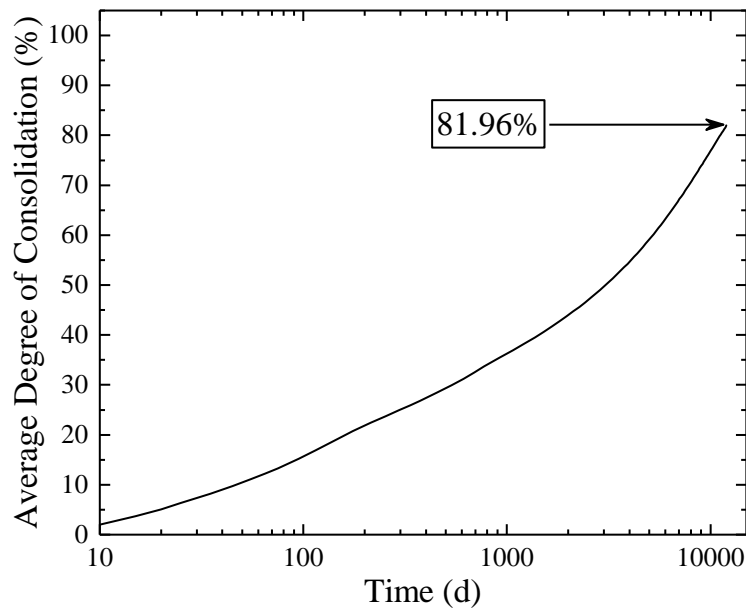


Figure 8. The variation of average degree of consolidation along time.

5. Influence of Consolidation Status

Three comparative cases are conducted here to show the importance of the separation of overconsolidation and normal consolidation: (1) Class A: complete overconsolidation, the initial preconsolidation pressure is 100 kPa; (2) Class B: overconsolidation and normal consolidation, the initial preconsolidation pressure is 50 kPa; (3) Class C: complete normal consolidation, the initial preconsolidation pressure is 10 kPa.

Figure 9 is the calculation diagram for all three cases, and the soil is vertically divided into 100 line elements. Assuming that the soil is homogeneous, and top and bottom surfaces are permeable, the total calculation time length is 1200 days. Table 2 lists the other parameters. The load is mathematically described in Equation (16), and the coefficient of consolidation is calculated using Equation (17).

$$q(t) = \begin{cases} 70t/60kPa & t \leq 60days \\ 70kPa & t > 60days \end{cases} \tag{16}$$

$$C_v = \frac{k(1 + e_0)}{\gamma_w a_v} \tag{17}$$

where C_v is the coefficient of consolidation.

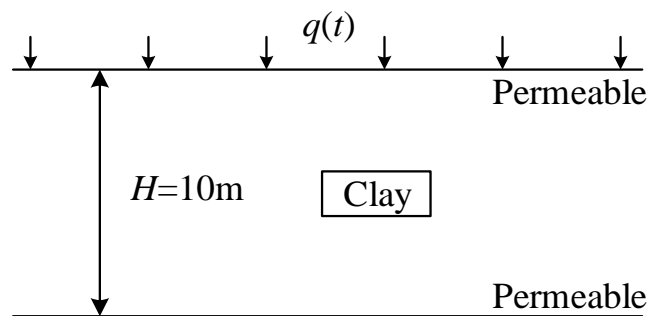


Figure 9. The calculation diagram for all three cases.

Table 2. Calculation parameters.

k_0 (m/d)	C_k	e_0	C_s	C_c	$\sigma_{0'}$ (kPa)	H (m)	γ_w (kN/m ³)
0.00008	1.5	0.8	0.1	0.5	10	10	10

Figure 10 shows the variations of consolidation coefficients evaluated using Equation (17), during calculations at 2.5–2.6 m depth. As can be seen, the coefficients of consolidation remain stable for a period when the excess pore water pressure dissipates slowly under external loadings, (Figure 11) and hence, the effective stress maintains the same level. Subsequently, the coefficients of consolidation of Classes A and B increase nonlinearly as a result of the growth of the effective stress in soil, while that of Class C still remains stable due to the slow dissipation of the excess pore water pressure. Therefore, the transition from overconsolidation to normal consolidation significantly affects the development of the coefficient of consolidation.

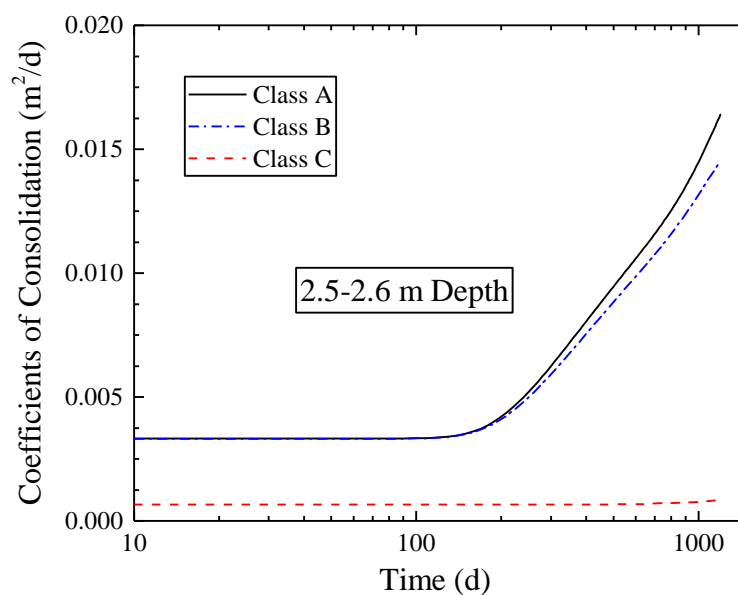


Figure 10. Comparisons of coefficients of consolidation at 2.5–2.6 m depth.

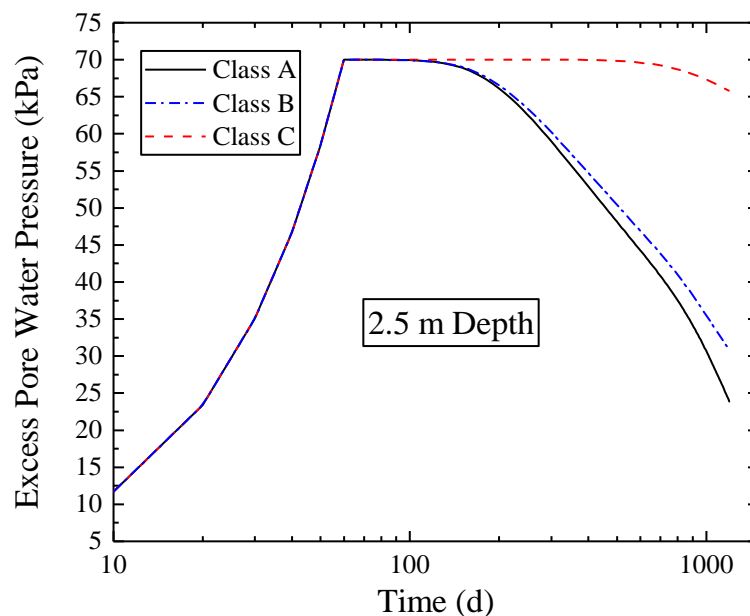


Figure 11. Comparisons of excess pore water pressure at 2.5 m depth.

The excess pore water pressure of Class A at 2.5 m depth under normal consolidation is apparently larger than that under overconsolidation with the same external loadings (Figure 11). It follows that the compressibility under normal consolidation is larger according to Equation (4). As a result, the settlement under normal consolidation increases faster, being in accordance with the results shown in Figure 12. The results are in agreement and reveal that the separation of overconsolidation and normal consolidation during calculations has a significant influence on the accuracy of results.

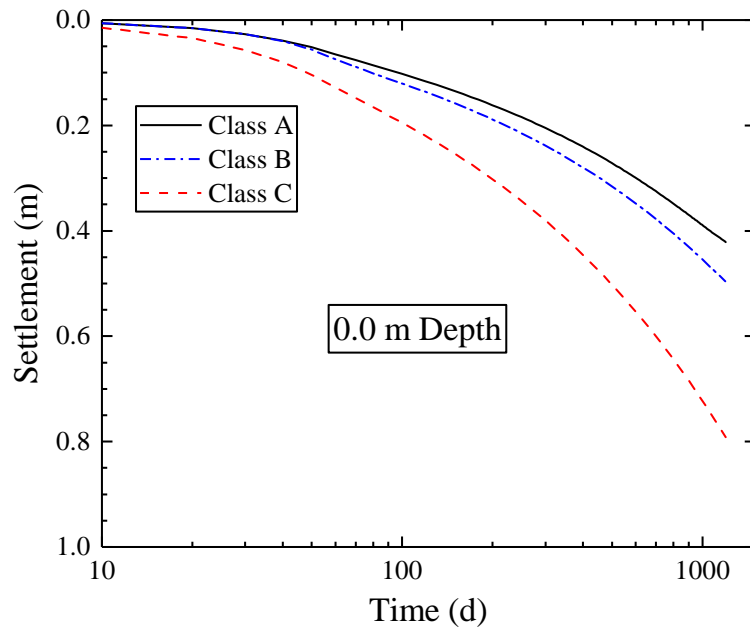


Figure 12. Comparisons of settlement at the top surface.

Figure 13 illustrates the evolutions of the average degree of consolidation, which is an important indicator for assessing the development of consolidation. The curves reveal that the average degree of consolidation is significantly influenced by different consolidation statuses, and the trends are consistent with those in Figure 11. In addition, the consolidation rate under normal consolidation is slower than that under overconsolidation.

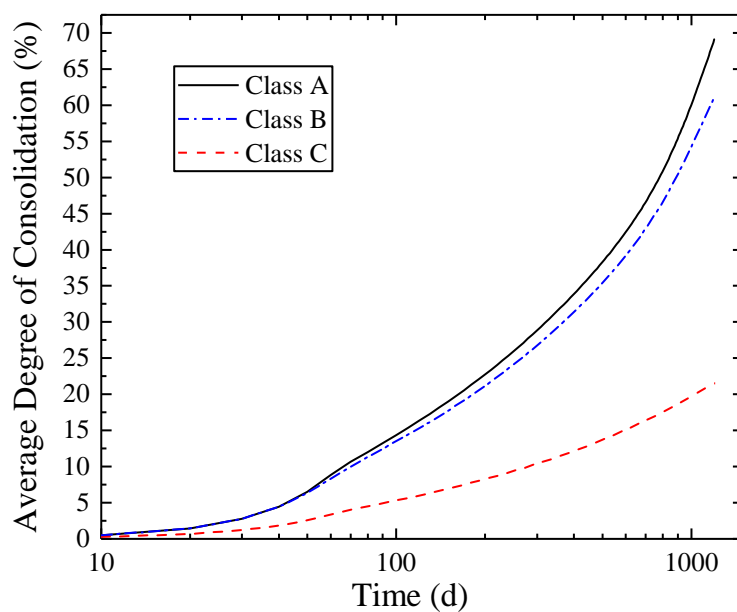


Figure 13. Comparisons of average degree of consolidation.

To investigate the errors between the results obtained using nonlinear and constant coefficients, three control cases, using constant compressibility and permeability, were conducted. Table 3 lists the errors of settlement and the excess pore water pressure (EPWP) evaluated at 2.5 m depth at typical time points. The results show the nonignorable influence of nonlinearity of coefficients, and the errors are larger at longer elapsed times. Furthermore, the errors under normal consolidation are lower than those under overconsolidation.

Table 3. Errors between results obtained using nonlinear and constant coefficients at 2.5 m depth.

Time (Day)	Class A		Class B		Class C	
	Settlement (%)	EPWP (%)	Settlement (%)	EPWP (%)	Settlement (%)	EPWP (%)
200	24.30	4.61	26.67	4.02	7	0
400	28.68	16.17	29.91	14.22	7.06	0.02
600	29.62	21.94	30.67	19.31	7.25	0.24
800	30.02	27.10	30.99	23.36	7.58	0.71
1000	30.25	35.01	31.27	28.45	7.98	1.34
1200	30.10	45.59	31.32	33.44	8.37	2.02

6. Conclusions

In this paper, the compression index and the recompression index are used for evaluating solution-dependent nonlinear compressibility, and the e - $\log k$ relation is used to calculate nonlinear permeability. The complex nonlinear consolidation is described using these two nonlinear coefficients. Moreover, the finite element method is adopted to solve the nonlinear consolidation equation suitable for small deformation problems with consideration of the time-dependent load. The main findings are summarized below:

- (1) The coefficient of consolidation varies nonlinearly during the development of consolidation, due to the change of the effective stress in soil. In particular, the transition from overconsolidation to normal consolidation will exert a significant influence on the coefficient of consolidation;
- (2) The two statuses of overconsolidation and normal consolidation should be separately dealt with in consolidation calculations, and using only one kind of consolidation status in calculations fails to accurately reflect real cases;
- (3) The nonlinear variations of coefficients in soil exhibit a slighter influence on results under normal consolidation compared with the influence on results under overconsolidation;
- (4) The solution presented in this paper is verified through a published engineering case. It is proved that the method is effective, stable, and accurate for meeting engineering requirements.

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