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Observer-Based Distributed Fault Detection for Heterogeneous Multi-Agent Systems

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Abstract: This paper solves the distributed fault detection (FD) problem for heterogeneous multi-agent systems (MAS). For a heterogeneous MAS, we adopt a distributed control law to realise cooperative output regulation (COR) when no fault occurs in the MAS, and propose a state-feedback-based FD scheme, where the adopted distributed control law and proposed FD scheme all utilise state information. Furthermore, we consider the condition that state information is unmeasurable, the output-feedback-based distributed FD scheme is proposed, and the adopted distributed control law also utilises measurement output. Finally, two numerical examples are utilised to verify that the proposed distributed FD schemes could locate and remove the faulty agent in time.

Keywords: distributed fault detection (FD); heterogeneous multi-agent systems (MAS); unknown input observer (UIO); cooperative output regulation (COR)

1. Introduction

Recently, a large amount of literature on cooperative control of multi-agent systems (MAS) has emerged, which investigate this problem from different aspects, such as event-triggered and finite-time cooperative control [1,2]. Besides homogeneous MAS, research on heterogeneous MAS is also significant. As an effective method to realise cooperative control of heterogeneous MAS, for instance, a network of unmanned aerial vehicles (UAVs) with different dynamics, cooperative output regulation (COR) has attracted intensive research attention during the past decade [3–6], which was based on output regulation theory, where the influence of mismatched disturbance generated by an exosystem could be completely rejected via converting mismatched forms into matched forms [7]. Specifically, H. Basu and S. Y. Yoon considered the condition that only partial information of an exosystem matrix was accessible to each agent, where a distinct estimator network was proposed to cooperatively estimate the value of the exosystem state [8]. As some agents may destroy other healthy agents due to the influence of unexpected faults, security operation of heterogeneous MAS has attracted some researchers' attention.

Existing security operation schemes of heterogeneous MAS are passive, where the fault tolerant COR is ensured by the designed passive fault tolerant controllers in each agent. In [9], Deng et al. designed a distributed adaptive fault tolerant control law to attenuate partial loss of actuator effectiveness faults. Furthermore, they considered the condition that actuators suffered from both partial loss of effectiveness faults and stuck faults [10,11]. Besides designing passive fault tolerant control laws for each agent, detecting and removing faulty agents is the other effective method to ensure the security operation of MAS. Hence, some literature on FD schemes for MAS has emerged during the past decade. The basic idea is to design an additional FD algorithm for MAS, and run the FD and control algorithms simultaneously; the FD algorithm will locate and remove faulty agents in time, where the FD algorithm could be seen as a redundant algorithm to make the MAS operate well. In [12], N. Meskin et al. proposed a centralised FD scheme for a group of unmanned vehicles, a bank of observers for FD were installed in one vehicle to monitor the remaining vehicles, where the observers needed to utilise all the nodes' information. Furthermore, they considered the condition that agents suffered from external disturbances [13]. In [14], the authors proposed an FD scheme for linear MAS based on sliding mode fault estimators, where the design of fault estimators also utilised all the nodes' input information. In summary, centralised FD schemes require one agent to have ability to obtain all the nodes' information, which is contradicted with the distributed nature of MAS. Hence, researchers start considering distributed FD, where each agent just needs to have ability to obtain itself and its neighbours' measurable information. I. Shames and K. H. Johansson et al. proposed a distributed FD scheme for a group of double integrators based on unknown input observer (UIO) [15,16], where the disturbance decoupling idea was widely utilised [17,18]. The basic idea was to adopt a distributed consensus law for each agent, and construct some FD observers for the MAS closed-loop system in one agent to judge whether a fault occurred in itself or its neighbours, then the utilisation or influence of agents' control inputs for FD results was avoided. Furthermore, the authors proposed a distributed FD scheme for second-order MAS with uncertain communication topology, where the design process of some agent's FD observers just utilised local topology information [19]. As above framework possesses distributed characteristics, some researchers have extended above framework to MAS with different kinds of agent dynamics. Shi et al. designed robust FD observers for a group of discrete-time double integrators with both process and measurement noise [20]. Jia and Wang proposed a distributed anti-disturbance FD scheme for disturbed second-order MAS, where the influence of exogenous disturbances for FD results was actively rejected [21]. Besides MAS with double-integrator dynamics, Liu et al. considered linear MAS with Lipschitz nonlinearity [22], and adopted a reduced-order observer design to lower the computational burden of each agent [23].

To the best of the authors' knowledge, there has been little published research on distributed FD schemes for heterogeneous MAS. M. R. Davoodi et al. designed a distributed FD scheme for heterogeneous MAS [24], where the observer in one agent judged whether a fault occurred in itself and its neighbours or not. However, the above observer was not decoupled from control inputs, which meant that the FD results would be influenced by control inputs. In [25], the authors considered a heterogeneous MAS with sensor faults, where fault estimators were designed in each agent to detect the fault. However, the considered agent dynamics did not contain exogenous disturbances, which were assumed to affect each agent and unknown for most agents in the MAS [3–6], and it seemed costly that all agents were equipped with fault estimators. Motivated by existing works mentioned above, we design a distributed FD scheme for disturbed MAS with heterogeneous dynamics based on UIO. As the exogenous disturbance is unknown for most agents of the MAS [3–6], above agents need to estimate the exogenous disturbance through cooperating with its neighbours, therefore, a bank of UIOs are designed according to the formed closed-loop system with the information of communication topology, which utilise exogenous disturbance estimate as feedback information, i.e., avoids utilising unknown exogenous disturbance. The contributions of this paper are listed as follows: First, a state-feedback-based distributed FD scheme for heterogeneous MAS is proposed in this paper, as well as an output-feedback-based distributed FD scheme. Second, unlike [24], the FD results will not be influenced by agents' control inputs, since FD observers are designed for closed-loop systems. Third, in comparison with [25], the considered agent dynamics contain exogenous disturbances in this paper. FD observers could still be designed for those agents, which cannot obtain exogenous signal directly, and only partial agents need to be equipped with observers.

The rest of this paper is organised as follows: Section 2 gives the preliminaries and problem formulation, where Sections 2.1 and 2.2 introduce relevant knowledge of graph theory and UIO, and Section 2.3 gives the problem formulation. Section 3 gives results of this paper, state-feedback-based and output-feedback-based distributed FD schemes are proposed in Sections 3.1

and 3.2. Section 4 gives two simulation examples to verify that the proposed FD schemes are effective. Finally, Section 5 gives conclusions and future directions.

Notation: Some standard notation will be adopted in this paper. $\mathbb{C}$ and $\mathbb{R}^n$ denote the set of complex numbers and n-dimensional Euclidean space, respectively. I_N and I denote an identity matrix with dimension N and appropriate dimension, respectively. $\otimes$ denotes the Kronecker product. $\mathbf{e}_i$ represents a column with only one nonzero entry '1', which locates in the i-th row. $\mathrm{Re}(\zeta)$ represents the real part of ζ, where $\zeta \in \mathbb{C}$. $\mathrm{diag}(A_1, A_2, \cdots, A_N)$ represents a block-diagonal matrix with matrices A_i, $i = 1, 2, \cdots, N$. $\lambda_i(A)$ denotes the i-th eigenvalue of A. $A > 0$ means that A is positive definite. '!' denotes the factorial of a non-negative integer, and $C_a^b = \frac{a!}{(a-b)!b!}$, where a, b are non-negative integers, and $b \leq a$. The superscript 'T' represents the transpose of a matrix.

2. Preliminaries and Problem Formulation

2.1. Graph Theory

A graph $\mathcal{G} = \{\mathcal{V}(\mathcal{G}),\ \mathcal{E}(\mathcal{G})\}$ could represent communication links among one leader agent and several follower agents, where $\mathcal{V}(\mathcal{G}) = \{\mathcal{V}_0, \mathcal{V}_1,\ \cdots,\ \mathcal{V}_N\}$ and $\mathcal{E}(\mathcal{G}) = \{(\mathcal{V}_i,\ \mathcal{V}_j) : \mathcal{V}_i, \mathcal{V}_j \in \mathcal{V}\} \subseteq \mathcal{V} \times \mathcal{V}$ represent the vertex and edge sets of $\mathcal{G}$, agent i could receive information from agent j if $(\mathcal{V}_i,\ \mathcal{V}_j) \in \mathcal{E}(\mathcal{G})$, where $\mathcal{V}_j$ is also called the neighbour of $\mathcal{V}_i$. If $(\mathcal{V}_i,\ \mathcal{V}_j) \in \mathcal{E}$ means that $(\mathcal{V}_j,\ \mathcal{V}_i) \in \mathcal{E}$, graph $\mathcal{G}$ is called undirected. A path from $\mathcal{V}_i$ to $\mathcal{V}_j$ is a sequence of distinct nodes $\{\mathcal{V}_{k_0},\ \mathcal{V}_{k_1},\ \cdots,\ \mathcal{V}_{k_l}\}$, where $k_0 = i$, $k_l = j$ and $(\mathcal{V}_{k_r},\ \mathcal{V}_{k_{r+1}}) \in \mathcal{E}, 0 \leq r \leq l-1$. An induced subgraph $\mathcal{G}_s$ is a graph such that $\mathcal{V}(\mathcal{G}_s) \subset \mathcal{V}(\mathcal{G})$, and $(\mathcal{V}_i,\ \mathcal{V}_j) \in \mathcal{E}(\mathcal{G}_s)$ indicates that $(\mathcal{V}_i,\ \mathcal{V}_j) \in \mathcal{E}(\mathcal{G})$. Subgraph $\mathcal{G}_s$ with the vertex set $\{\mathcal{V}_1,\ \cdots,\ \mathcal{V}_N\}$ represents the communication relationship among follower agents. The adjacency matrix $\mathcal{A} = [a_{ij}]$ associated with $\mathcal{G}$ is defined as $a_{ii} = 0$ and $a_{ij} = 1$ if $(\mathcal{V}_i,\ \mathcal{V}_j) \in \mathcal{E}(\mathcal{G})$. What is more, define the in-degree of each agent as $d_i = \sum_{j=0}^{N} a_{ij} \geq 1$, and define the Laplacian matrix associated with $\mathcal{G}$ as $\mathcal{L} = [l_{ij}]$, where $l_{ii} = d_i$ and $l_{ij} = -a_{ij}, j \neq i$.

For the convenience of analysis, denote $\mathcal{L}_s$ as the Laplacian matrix associated with $\mathcal{G}_s$, and define $\mathcal{A}_0 = \mathrm{diag}(a_{10}, a_{20}, \cdots, a_{N0})$ and $\mathcal{H} = \mathcal{L}_s + \mathcal{A}_0$. Furthermore, define $\mathcal{N}_i = \{\mathcal{V}_j \in \mathcal{V}(\mathcal{G}_s) : (i,j) \in \mathcal{E}(\mathcal{G}_s), i \neq j\}$ as the neighbour set of node $\mathcal{V}_i \in \mathcal{V}(\mathcal{G}_s)$ in $\mathcal{G}_s$, and define $\bar{\mathcal{N}_i} = \{i\} \cup \mathcal{N}_i$, where $|\bar{\mathcal{N}_i}|$ is the cardinality of $\bar{\mathcal{N}_i}$, and $\{\bar{i}_1, \cdots, \bar{i}_{|\bar{\mathcal{N}_i}|}\}$ are sequence numbers of nodes in $\bar{\mathcal{N}_i}$ from small to large.

Lemma 1 ([26])**.** *If the subgraph $\mathcal{G}_s$ is undirected, and each follower agent has paths to the leader in the graph $\mathcal{G}$, $\mathcal{H}$ is positive definite.*

2.2. Unknown Input Observer

Consider the following system with unknown input:

$$\begin{aligned} \dot{\xi} &= Q\xi + Wu + Yw \\ y &= M\xi \end{aligned} \tag{1}$$

where $\xi \in \mathbb{R}^n$ is the state. $u \in \mathbb{R}^r$ and $w \in \mathbb{R}^s$ are the known and unknown inputs, and W and Y represent their input channels, respectively. $y \in \mathbb{R}^m$ is the measurement output, and H represents the measurement matrix.

In order to estimate the actual state of System (1), the following observer is designed.

$$\begin{aligned} \dot{z} &= Gz + TWu + Ry \\ \hat{\xi} &= z + Hy \end{aligned} \tag{2}$$

where $\hat{\xi} \in \mathbb{R}^n$ and $z \in \mathbb{R}^n$ are the estimated state and observer's state, respectively. Parameter matrices of observer (2) need to be designed to make the state estimation error be not influenced by unknown input w, where the design method is shown as follows:

$$\begin{aligned} &G = TQ - R_1 M, T = I - HM, \\ &(HM - I)Y = 0, R = R_1 + R_2, R_2 = GH \end{aligned} \tag{3}$$

then the state estimation error dynamics is shown as follows:

$$\dot{e}(t) = Ge(t) \tag{4}$$

where $e = \xi - \hat{\xi}$. If the designed R_1 makes G Hurwitz stable, e will converge to zero asymptotically. Observer (2) is usually called unknown input observer (UIO). Lemma 2 gives the existence conditions of a UIO.

Lemma 2 ([16]). *A UIO for System (1) exists if*

(i) $rank(MY) = rank(Y)$

(ii) $\begin{bmatrix} sI - Q & Y \\ M & 0 \end{bmatrix}$ *is of full column rank for* $\forall s \in \mathbb{C}$, $Re(s) \geq 0$, *i.e.,* (TQ, M) *is detectable.*

Remark 1. *The above two conditions guarantee existence of H and R_1 in (3), respectively, where R_1 makes G Hurwitz stable.*

2.3. Problem Formulation

The considered heterogeneous MAS consists of the following N agents:

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i u_i + E_i v + B_{f_i} f_i \\ e_i &= C_i x_i + D_i u_i + F_i v \\ y_{mi} &= C_{mi} x_i, i = 1, 2, \cdots, N \end{aligned} \tag{5}$$

where $x_i \in \mathbb{R}^{n_i}$, $e_i \in \mathbb{R}^{p_i}$, $y_{mi} \in \mathbb{R}^{p_m}$, $u_i \in \mathbb{R}^{m_i}$ and $f_i \in \mathbb{R}^{l_i}$ are the state, tracking error, measurement output, control input and fault signal of the i-th agent, respectively. Columns of B_{f_i} are linearly independent, and (A_i, B_i, C_{mi}) are stabilisable and detectable. $v \in \mathbb{R}^q$ is the reference input to be tracked or the disturbance to be rejected, which is assumed to be generated by the following exosystem:

$$\dot{v} = Sv$$

In this paper, the leader labelled as 0 could represent the exosystem. System (5) could be seen as follower agents, where only some follower agents of the MAS could utilise v directly, i.e., the leader is their neighbour in the graph $\mathcal{G}$, and the remaining follower agents just have paths to the leader. The above two facts mean that the leader has paths to all follower agents in the graph $\mathcal{G}$.

System (5) also needs to satisfy the following assumptions.

Assumption 1. $Re[\lambda_i(S)] = 0, i = 1, \cdots, q.$

Assumption 2. *The following equations*

$$\begin{aligned} X_i S &= A_i X_i + B_i U_i + E_i \\ 0 &= C_i X_i + D_i U_i + F_i, i = 1, 2, \cdots, N \end{aligned} \tag{6}$$

have solution pairs (X_i, U_i), *respectively.*

Remark 2. *According to Theorem 1.9 in [27], solution pairs* (X_i, U_i) *are determined to exist if rank* $\begin{bmatrix} \lambda_i(S)I - A_i & B_i \\ C_i & D_i \end{bmatrix} = n_i + p_i$ *for any* $\lambda_i(S)$*. In fact,* $\begin{bmatrix} sI - A_i & B_i \\ C_i & D_i \end{bmatrix}$ *is of full row rank for* $\forall s \in \mathbb{C}, Re(s) \geq 0$ *if System (5) are minimum phase systems, combined with Assumption 1, matrix Equation (6) will always have solutions if System (5) are minimum phase systems. Furthermore, the solution pair* (X_i, U_i) *will be unique if* $p_i = m_i$*.*

Assumption 3. *There exists only one faulty agent in each* $\bar{\mathcal{N}_i}$*,* $i = 1, \cdots, N$*.*

For System (5), a state-feedback-based distributed control law is proposed in [1].

$$\begin{aligned} u_i &= K_{1i} x_i + K_{2i} \eta_i \\ \dot{\eta}_i &= S\eta_i - c \sum_{j=1}^{N} [a_{ij}(\eta_i - \eta_j) + a_{i0}(\eta_i - v)] \end{aligned} \tag{7}$$

where η_i is a dynamic compensator, c is a positive scalar to design, K_{1i} and K_{2i} are parameter matrices to design. Let K_{2i} be as follows:

$$K_{2i} = U_i - K_{1i} X_i$$

X_i and U_i are determined by matrix equations in Assumption 2. If $A_i + B_i K_{1i}$ are Hurwitz stable, $c > 0$, and the leader has paths to all follower agents, control law (7) will realise COR when there exists no faulty agent [3], i.e., tracking errors e_i converge to zero asymptotically. However, the given exosystem under Assumption 1 just has an unforced purely oscillatory solution, which cannot include some kinds of solutions such as those of a damped differential system or the solution of a forced differential system or dynamic system, i.e., some kinds of practical signals cannot be generated by the given exosystem, where the application prospect is limited. This is due to the fact that the given exosystem needs to ensure that matrix Equation (6) has solutions, and the following Theorems 2 and 3 hold. Furthermore, COR will not realise if some agent suffers from fault signals. Hence, there exists a need to detect the faulty agent.

This paper aims at adopting a distributed control law for the MAS, and designing distributed observers in some agents to detect the possibly faulty agent. Running the observers and control law simultaneously, the observers could detect the faulty agent if there occurs a fault, and the distributed control law could also realise COR if no fault occurs. It is worth indicating that if the solution pair (X_i, U_i) is not unique, we just need to select one of them to design the control law (7), and design FD observers (15) and (29), where the parameters of FD observers contain the selected solution pairs (X_i, U_i). What is more, the designed control law (7) with any chosen solution pair (X_i, U_i) will realise consensus control when there exists no faulty agent, and existence of the designed FD observers will also hold under any chosen solution pair (X_i, U_i).

3. Results

3.1. State-Feedback-Based Distributed FD

Define y_i as the observer feedback information that agent i's observers for FD could utilise, which is designed to contain state x_i and compensator state η_i of agent i, as well as agent i's neighbours', where agent i's neighbours' compensator state η_i are also utilised in control law (7).

$$y_i = \begin{bmatrix} x_{\bar{i}_1} - X_{\bar{i}_1} \eta_{\bar{i}_1} \\ \vdots \\ x_{\bar{i}_{|\bar{\mathcal{N}}_i|}} - X_{\bar{i}_{|\bar{\mathcal{N}}_i|}} \eta_{\bar{i}_{|\bar{\mathcal{N}}_i|}} \end{bmatrix} \tag{8}$$

Substitute (7) into (5), we obtain

$$\begin{aligned} \dot{x}_i &= (A_i + B_i K_{1i}) x_i + B_i K_{2i} \eta_i + E_i v + B_{f_i} f_i \\ \dot{\eta}_i &= S\eta_i - c\sum_{j=1}^{N}[a_{ij}(\eta_i - \eta_j) + a_{i0}(\eta_i - v)] \end{aligned} \tag{9}$$

Furthermore, let $\theta_i = x_i - X_i v, e_{\eta i} = \eta_i - v$, and according to Assumption 2, we could obatin

$$\begin{aligned} \dot{\theta}_i &= (A_i + B_i K_{1i}) x_i + B_i K_{2i} \eta_i - (A_i X_i + B_i U_i) v + B_{f_i} f_i \\ &= (A_i + B_i K_{1i}) x_i + B_i K_{2i} \eta_i - (A_i X_i + B_i K_{2i} + B_i K_{1i} X_i) v \\ &+ B_{f_i} f_i \\ &= (A_i + B_i K_{1i}) \theta_i + B_i K_{2i} e_{\eta i} + B_{f_i} f_i \\ \dot{e}_{\eta i} &= S e_{\eta i} - c\sum_{j=1}^{N}[a_{ij}(e_{\eta i} - e_{\eta j}) + a_{i0} e_{\eta i}] \end{aligned} \tag{10}$$

Remark 3. *Obviously, it is impossible to design observers for System (10) decoupled from $e_{\eta i}(t)$ if agent i could not obtain exogenous signal $v(t)$ directly. What is more, the evolution of $e_{\eta i}(t)$ is influenced by $e_{\eta j}(t), j \in \mathcal{N}_i$, i.e., $||e_{\eta i}(t)||$ may not be monotonic for $t \geq 0$, therefore, the bound of $||e_{\eta i}(t)||$ is unknown based only on local information, which means that isolation thresholds cannot be selected [28]. Therefore, observers for FD need to be designed for the following closed-loop System (12), where utilisation of exogenous signal v is avoided.*

Denote

$$\begin{aligned} \zeta_i &= \begin{bmatrix} \theta_{\bar{i}_1}^T & \cdots & \theta_{\bar{i}_{|\bar{\mathcal{N}}_i|}}^T \end{bmatrix}^T, \bar{f}_i = \begin{bmatrix} f_{\bar{i}_1}^T & \cdots & f_{\bar{i}_{|\bar{\mathcal{N}}_i|}}^T \end{bmatrix}^T \\ e_\eta &= \begin{bmatrix} e_{\eta 1}^T & \cdots & e_{\eta N}^T \end{bmatrix}^T, \psi_i = \begin{bmatrix} \zeta_i^T & e_\eta^T \end{bmatrix}^T \end{aligned} \tag{11}$$

we obtain System (12) for agent i.

$$\begin{aligned} \dot{\psi}_i &= \mathcal{A}_i \psi_i + \bar{B}_{f_i} \bar{f}_i \\ \mathcal{A}_i &= \begin{bmatrix} \Omega_{1i} & \Omega_{2i} \\ 0 & I_N \otimes S - c\mathcal{H} \otimes I_q \end{bmatrix}, \bar{B}_{f_i} = \begin{bmatrix} \Omega_{3i} \\ 0 \end{bmatrix} \\ \Omega_{1i} &= \mathrm{diag}(A_{\bar{i}_1} + B_{\bar{i}_1} K_{1\bar{i}_1}, \cdots, A_{\bar{i}_{|\bar{\mathcal{N}}_i|}} + B_{\bar{i}_{|\bar{\mathcal{N}}_i|}} K_{1\bar{i}_{|\bar{\mathcal{N}}_i|}}) \\ \Omega_{2i} &= \begin{bmatrix} \mathbf{e}_{\bar{i}_1} \otimes (B_{\bar{i}_1} K_{2\bar{i}_1})^T, \cdots, \mathbf{e}_{\bar{i}_{|\bar{\mathcal{N}}_i|}} \otimes (B_{\bar{i}_{|\bar{\mathcal{N}}_i|}} K_{2\bar{i}_{|\bar{\mathcal{N}}_i|}})^T \end{bmatrix}^T \\ \Omega_{3i} &= \mathrm{diag}(B_{f_{\bar{i}_1}}, \cdots, B_{f_{\bar{i}_{|\bar{\mathcal{N}}_i|}}}) \end{aligned} \tag{12}$$

and (8) can be rewritten as follows:

$$\begin{aligned} y_i &= \mathcal{C}_i \psi_i \\ \mathcal{C}_i &= \begin{bmatrix} I & -\mathcal{C}_{i1} \end{bmatrix} \\ \mathcal{C}_{i1} &= \begin{bmatrix} \mathbf{e}_{\bar{i}_1} \otimes X_{\bar{i}_1}^T & \mathbf{e}_{\bar{i}_2} \otimes X_{\bar{i}_2}^T & \cdots & \mathbf{e}_{\bar{i}_{|\bar{\mathcal{N}}_i|}} \otimes X_{\bar{i}_{|\bar{\mathcal{N}}_i|}}^T \end{bmatrix}^T \end{aligned} \tag{13}$$

Combine (12) and (13), we obtain $|\bar{\mathcal{N}}_i|$ systems as follows:

$$\begin{cases} \dot{\psi}_i = \mathcal{A}_i \psi_i + \bar{b}_{f_{i\bar{i}_k}} f_{\bar{i}_k} + \bar{B}_{f_{-i\bar{i}_k}} \bar{f}_{-i\bar{i}_k} \\ y_i = \mathcal{C}_i \psi_i, k = 1, \cdots, |\bar{\mathcal{N}}_i| \end{cases} \tag{14}$$

where $\bar{b}_{f_{i\bar{i}_k}} = \begin{bmatrix} \mathbf{e}_k^T \otimes B_{f_{\bar{i}_k}}^T & 0 \end{bmatrix}^T$, $\bar{B}_{f_{-i\bar{i}_k}}$ denotes the surplus of $\bar{B}_{f_i}$ after deleting $\bar{b}_{f_{i\bar{i}_k}}$, and

$$\bar{f}_{-i\bar{i}_k} = \begin{bmatrix} f_{\bar{i}_1}^T & \cdots & f_{\bar{i}_{k-1}}^T & f_{\bar{i}_{k+1}}^T & \cdots & f_{\bar{i}_{|\bar{\mathcal{N}}_i|}}^T \end{bmatrix}^T$$

Construct $|\bar{\mathcal{N}}_i|$ observers (15) for all the agents of $\bar{\mathcal{N}}_i$ in agent i, which are labelled as $i\bar{i}_k$, $k = 1, \cdots, |\bar{\mathcal{N}}_i|$.

$$\begin{aligned} \dot{z}_{i\bar{i}_k} &= G_{i\bar{i}_k} z_{i\bar{i}_k} + R_{i\bar{i}_k} y_i \\ \hat{\psi}_{i\bar{i}_k} &= z_{i\bar{i}_k} + Q_{i\bar{i}_k} y_i, k = 1, \cdots, |\bar{\mathcal{N}}_i| \end{aligned} \tag{15}$$

where

$$\begin{aligned} &(Q_{i\bar{i}_k} C_i - I) b_{f_{i\bar{i}_k}} = 0, T_{i\bar{i}_k} = I - Q_{i\bar{i}_k} C_i \\ &G_{i\bar{i}_k} = T_{i\bar{i}_k} A_i - R_{1_{i\bar{i}_k}} C_i, R_{2_{i\bar{i}_k}} = G_{i\bar{i}_k} Q_{i\bar{i}_k} \\ &R_{i\bar{i}_k} = R_{1_{i\bar{i}_k}} + R_{2_{i\bar{i}_k}} \end{aligned} \tag{16}$$

$R_{1_{i\bar{i}_k}}$ is a matrix to make $G_{i\bar{i}_k}$ Hurwitz stable.

Then, construct $|\bar{\mathcal{N}}_i|$ residual generators $r_{i\bar{i}_k}$ as follows:

$$r_{i\bar{i}_k} = y_i - C_i \hat{\psi}_{i\bar{i}_k}, k = 1, \cdots, |\bar{\mathcal{N}}_i|.$$

Theorem 1. *Suppose that observers (15) exist for each agent in $\bar{\mathcal{N}}_i$, and Assumption 3 is satisfied. If $\lim_{t\to\infty} r_{i\bar{i}_k}(t) = 0$, and $\lim_{t\to\infty} r_{ij}(t) \neq 0$, where $j \in \bar{\mathcal{N}}_i$ and $j \neq \bar{i}_k$, there occurs a fault in agent $\bar{i}_k$.*

Proof of Theorem 1. The UIO error (residual) dynamics are shown as follows:

$$\begin{aligned} \dot{e}_{i\bar{i}_k}(t) &= G_{i\bar{i}_k} e_{i\bar{i}_k}(t) + T_{i\bar{i}_k} \bar{B}_{f_{-i\bar{i}_k}} \bar{f}_{-i\bar{i}_k}, \\ r_{i\bar{i}_k}(t) &= C_i e_{i\bar{i}_k}(t), \end{aligned} \tag{17}$$

where $e_{i\bar{i}_k} = \psi_i - \hat{\psi}_{i\bar{i}_k} = T_{i\bar{i}_k}\psi_i - z_{i\bar{i}_k}$. Therefore, residual generators will only be influenced by $f_{-ik}(t)$. Assumption 3 indicates that $\bar{f}_{-i\bar{i}_k}(t) = 0$ if $f_{\bar{i}_k}(t) \neq 0$, therefore, $\lim_{t\to\infty} r_{i\bar{i}_k}(t) = 0$ and $\lim_{t\to\infty} r_{ij}(t) \neq 0$, $j \neq \bar{i}_k$ indicates that agent $\bar{i}_k$ is faulty. □

Remark 4. *Assumption 3 could be relaxed if the number of faulty agents is known, and all the faulty agents locate in the same set $\bar{\mathcal{N}}_i$. Assume the number of faulty agents as $h < |\bar{\mathcal{N}}_i|$, then construct $C_{|\bar{\mathcal{N}}_i|}^h$ systems as follows:*

$$\begin{cases} \dot{\psi}_i = A_i \psi_i + \tilde{b}_{f_{ik}} \tilde{f}_{ik} + \tilde{B}_{f_{-ik}} \tilde{f}_{-ik} \\ y_i = C_i \psi_i, k = 1, \cdots, C_{|\bar{\mathcal{N}}_i|}^h \end{cases}$$

where $\tilde{f}_{ik} = \begin{bmatrix} \tilde{f}_{ik_1}^T & \tilde{f}_{ik_2}^T & \cdots & \tilde{f}_{ik_h}^T \end{bmatrix}^T$, i.e., h elements chosen from $\{f_{\bar{i}_1}, f_{\bar{i}_2}, \cdots, f_{\bar{i}_{|\bar{\mathcal{N}}_i|}}\}$, then $\tilde{f}_{ik}$ has $C_{|\bar{\mathcal{N}}_i|}^h$ combinations, as well as $\tilde{b}_{f_{ik}}$. $\tilde{B}_{f_{-ik}}$ is the surplus of $\bar{B}_{f_i}$ after deleting $\tilde{b}_{f_{ik}}$.

Design $C_{|\bar{\mathcal{N}}_i|}^h$ observers and residual generators in agent i for above systems following the same steps. Finally, the only residual generator containing all the faults converges to zero. For $h = |\bar{\mathcal{N}}_i| - 1$, if all the residual generators will not converge to zero, there exists no healthy agent in $\bar{\mathcal{N}}_i$.

Theorem 2. *If Assumption 1 is satisfied, $c > 0$, and each follower agent has paths to the leader in the graph $\mathcal{G}$, there exists an observer (15) for any $\bar{i}_k \in \bar{\mathcal{N}}_i$.*

Proof of Theorem 2. For $\bar{i}_k \in \bar{\mathcal{N}}_i$, Lemma 2 indicates that there exists an observer (15) if $rank(C_i \bar{b}_{f_{ik}}) = rank(\bar{b}_{f_{ik}})$, and $\mathcal{M}_{i\bar{i}_k} = \begin{bmatrix} sI - A_i & \bar{b}_{f_{ik}} \\ C_i & 0 \end{bmatrix}$ is of full column rank for $\forall s \in \mathbb{C}$, $\mathrm{Re}(s) \geq 0$.

Firstly, we verify the first condition, where

$$C_i\bar{b}_{f_{ik}} = \begin{bmatrix} I & -C_{i1} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{e}_k \otimes B_{f_{\bar{i}_k}} \\ 0 \end{bmatrix} = \mathbf{e}_k \otimes B_{f_{\bar{i}_k}} \tag{18}$$

It is easy to verify that coulmns of $C_i\bar{b}_{f_{ik}}$ are linearly independent, where the first condition holds.

Then, we need to verify that $\mathcal{M}_{i\bar{i}_k}$ is of full column rank for $\forall s \in \mathbb{C}$, $\mathrm{Re}(s) \geq 0$, which is shown as follows:

$$\begin{bmatrix} sI - \Omega_1 & -\Omega_2 & \mathbf{e}_k \otimes B_{f_{\bar{i}_k}} \\ 0 & sI - (I_N \otimes S - c\mathcal{H} \otimes I_q) & 0 \\ I & -C_{i1} & 0 \end{bmatrix} \tag{19}$$

Let $\nu_1 \in \mathbb{R}^{\sum_{k=1}^{|\bar{\mathcal{N}}_i|} n_{\bar{i}_k}}$, $\nu_2 \in \mathbb{R}^{Nq}$ and $\nu_3 \in \mathbb{R}^{l_{\bar{i}_k}}$ satisfy (20).

$$\begin{bmatrix} (sI - \Omega_1) \cdot \nu_1 - \Omega_2 \cdot \nu_2 + (\mathbf{e}_k \otimes B_{f_{\bar{i}_k}}) \cdot \nu_3 \\ [sI - (I_N \otimes S - c\mathcal{H} \otimes I_q)] \cdot \nu_2 \\ \nu_1 - C_{i1} \cdot \nu_2 \end{bmatrix} = 0 \tag{20}$$

$\mathcal{H}$ is positive definite according to Lemma 1, then, there exists a Y_1 to make $Y_1\mathcal{H}Y_1^{-1} = \Lambda_1 = \mathrm{diag}[\lambda_1(\mathcal{H}), \cdots, \lambda_N(\mathcal{H})]$, where $\lambda_1(\mathcal{H}), \cdots, \lambda_N(\mathcal{H}) > 0$. For the exosystem matrix S, a unitary matrix Y_2 could make $Y_2SY_2^H = \Lambda_2$, where Λ_2 is an upper triangular matrix with diagonal elements $\{\lambda_1(S), \cdots, \lambda_q(S)\}$ [29]. Then, we obtain eigenvelus of $I_N \otimes S - c\mathcal{H} \otimes I_q$ via pre-multiplying by $(I_N \otimes Y_2) \cdot (Y_1 \otimes I_q)$ and post-multiplying by $(Y_1^{-1} \otimes I_q) \cdot (I_N \otimes Y_2^H)$ for $I_N \otimes S - c\mathcal{H} \otimes I_q$, which are shown as follows:

$$\{\lambda_i(S) - c\lambda_j(\mathcal{H}) : i = 1, \cdots, q, j = 1, \cdots, N\}$$

According to Assumption 1, we have $\mathrm{Re}[\lambda_i(S)] = 0$, i.e., $\mathrm{Re}[\lambda_i(I_N \otimes S - c\mathcal{H} \otimes I_q)] < 0$. Hence, columns of $sI - (I_N \otimes S - c\mathcal{H} \otimes I_q)$ are linearly independent for $\forall s \in \mathbb{C}$, $\mathrm{Re}(s) \geq 0$, i.e., $\nu_2 = 0$, as well as ν_1. As columns of $B_{f_{\bar{i}_k}}$ are linearly independent, we have $\nu_3 = 0$, therefore, (19) is of full column rank for $\forall s \in \mathbb{C}, \mathrm{Re}(s) \geq 0$. Above conditions guarantee existence of observers (15) for any node in $\bar{\mathcal{N}}_i$. □

Remark 5. *If compensator state $\eta_{\bar{i}_1}, \eta_{\bar{i}_2}, \cdots, \eta_{\bar{i}_{|\bar{\mathcal{N}}_i|}}$ in y_i are replaced by exosystem state v, existence of the observers for FD is still ensured. In practical operation, only partial follower agents could obtain exosystem state v, but each follower agent has access to itself and its neighbours' compensator state, i.e., the estimate of v, therefore, feedback information y_i is designed as (8).*

Remark 6. *Establishment of Theorem 1 depends on Assumption 3 and existence of observers for FD, where existence of the observers for FD is proved in Theorem 2. Then, Theorems 1 and 2 together guarantee that the faulty agent will be detected.*

Remark 7. *Existence of FD observers just requires $c > 0$, which is the same as realisation conditions of COR, i.e., control law (7) and observers (15) are able to run simultaneously. What is more, if the remaining follower agents still have paths to the leader after removing the faulty agent, the remaining follower agents could still realise COR.*

According to residual dynamics (17), residual generators will not converge to zero until time approaches infinity, therefore, appropriate isolation thresholds need to be set [21]. Then, the following location algorithm is given to locate the possibly faulty agent in $\bar{\mathcal{N}}_i$.

Algorithm 1 Faulty agent location algorithm

1. In agent i, construct residual generators $r_{i\bar{i}_k}$ and set appropriate thresholds $\Theta_{i\bar{i}_k} > 0, k = 1, \cdots, |\bar{\mathcal{N}}_i|$.
2. Run residual generators $r_{i\bar{i}_k}, k = 1, \cdots, |\bar{\mathcal{N}}_i|$.

(1) Check $r_{i\bar{i}_k}, k = 1$. If $||r_{i\bar{i}_k}|| < \Theta_{i\bar{i}_k}$, and $||r_{ij}|| \geq \Theta_{ij}, j \neq \bar{i}_k$, stop and remove the faulty node. Otherwise, go to step (2).

$$\vdots$$

($|\bar{\mathcal{N}}_i|$) Check $r_{i\bar{i}_k}, k = |\bar{\mathcal{N}}_i|$. If $||r_{i\bar{i}_k}|| < \Theta_{i\bar{i}_k}$, and $||r_{ij}|| \geq \Theta_{ij}, j \neq \bar{i}_k$, stop and remove the faulty node.

Remark 8. *A simple selection method of isolation thresholds Θ_{ik} is shown as follows. Assume that the initial error $e_{i\bar{i}_k}(0) \leq$ is bounded, i.e., $||e_{i\bar{i}_k}(0)|| \leq \epsilon$. Then, one can compute the threshold by considering an upper estimate of the error expression (17):*

$$\Theta_{i\bar{i}_k} = \Gamma_{i\bar{i}_k}(t)||C_i||\epsilon$$

where $\Gamma_{i\bar{i}_k}(t)$, which can be obtained by Jordan decomposition, is such that $||e^{G_{i\bar{i}_k}t}|| \leq \Gamma_{i\bar{i}_k}(t)$.

What is more, selection of isolation thresholds is also related with trade-offs between false alarm and misdetection rate, among others [30], where more details could be found in [24] and references there-in.

3.2. Output-Feedback-Based Distributed FD

As state information is difficult to obtain, an output-feedback-based control law is designed here, as well as an output-feedback-based distributed FD scheme, where matrices A_i, B_{fi} and C_{mi} need to satisfy the following assumption.

Assumption 4. *$C_{mi}B_{fi}$ and $\begin{bmatrix} sI - A_i & B_{fi} \\ C_{mi} & 0 \end{bmatrix}$ are of full column rank for $\forall s \in \mathbb{C}, Re(s) \geq 0, i = 1, 2, \cdots, N$.*

Inspired by Reference [2], we design a dynamic output-feedback-based distributed control law:

$$\begin{aligned} u_i &= K_{1i}\hat{x}_i + K_{2i}\eta_i \\ \dot{\hat{x}}_i &= A_i\hat{x}_i + B_iu_i + E_i\eta_i + L_i(C_{mi}\hat{x}_i - y_{mi}) \\ \dot{\eta}_i &= S\eta_i - c\sum_{j=1}^{N}[a_{ij}(\eta_i - \eta_j) + a_{i0}(\eta_i - v)] \end{aligned} \tag{21}$$

where c and K_{1i}, K_{2i} and L_i are the scalar and parameter matrices to design, respectively. Let K_{2i} be designed as follows:

$$K_{2i} = U_i - K_{1i}X_i$$

X_i and U_i are still determined by the matrix equations introduced in Assumption 2.

Remark 9. *Control law (23) consists of two observers, i.e., the compensator η_i and state observer $\hat{x}_i$, which aim at estimating exosystem and system states, respectively.*

y_i is designed as follows:

$$y_i = \begin{bmatrix} y_{m\bar{i}_1} - C_{m\bar{i}_1}\hat{x}_{\bar{i}_1} \\ \vdots \\ y_{m\bar{i}_{|\bar{\mathcal{N}}_i|}} - C_{m\bar{i}_{|\bar{\mathcal{N}}_i|}}\hat{x}_{\bar{i}_{|\bar{\mathcal{N}}_i|}} \end{bmatrix} \tag{22}$$

which contains measurement output and state estimate of agents in $\bar{\mathcal{N}}_i$.

Substitute (23) into (5), we have

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i K_{1i}\hat{x}_i + B_i K_{2i}\eta_i + E_i v + B_{f_i} f_i \\ \dot{\hat{x}}_i &= A_i \hat{x}_i + B_i u_i + E_i \eta_i + L_i (C_{mi}\hat{x}_i - y_{mi}) \\ \dot{\eta}_i &= S\eta_i - c\sum_{j=1}^{N}[a_{ij}(\eta_i - \eta_j) + a_{i0}(\eta_i - v)] \end{aligned} \tag{23}$$

let $\theta_i = x_i - X_i v$, $e_{xi} = x_i - \hat{x}_i$, $e_{\eta i} = \eta_i - v$, and according to Assumption 2, we could obatin (24):

$$\begin{aligned} \dot{\theta}_i &= A_i x_i + B_i K_{1i}\hat{x}_i + B_i K_{2i}\eta_i + B_{f_i} f_i + (E_i - X_i S)v \\ &= A_i x_i + B_i K_{1i}\hat{x}_i + B_i K_{2i}\eta_i + B_{f_i} f_i - (A_i X_i + B_i U_i)v \\ &= A_i x_i + B_i K_{1i}(x_i - e_{xi}) + B_i K_{2i}\eta_i + B_{f_i} f_i - (A_i X_i \\ &+ B_i K_{2i} + B_i K_{1i} X_i)v \\ &= (A_i + B_i K_{1i})\theta_i - B_i K_{1i} e_{xi} + B_i K_{2i} e_{\eta i} + B_{f_i} f_i \\ \dot{e}_{xi} &= (A_i + L_i C_{mi}) e_{xi} - E_i e_{\eta i} + B_{f_i} f_i \\ \dot{e}_{\eta i} &= S e_{\eta i} - c\sum_{j=1}^{N}[a_{ij}(e_{\eta i} - e_{\eta j}) + a_{i0} e_{\eta i}] \end{aligned} \tag{24}$$

Denote

$$\begin{aligned} \zeta_i &= \begin{bmatrix}\theta_{\bar{i}_1}^T & \cdots & \theta_{\bar{i}_{|\bar{\mathcal{N}}_i|}}^T\end{bmatrix}^T, \bar{e}_{xi} = \begin{bmatrix}e_{x\bar{i}_1}^T & \cdots & e_{x\bar{i}_{|\bar{\mathcal{N}}_i|}}^T\end{bmatrix}^T \\ e_\eta &= \begin{bmatrix}e_{\eta 1}^T & \cdots & e_{\eta N}^T\end{bmatrix}^T, \psi_i = \begin{bmatrix}\zeta_i^T & \bar{e}_{xi}^T & e_\eta^T\end{bmatrix}^T \\ \bar{f}_i &= \begin{bmatrix}f_{\bar{i}_1}^T & \cdots & f_{\bar{i}_{|\bar{\mathcal{N}}_i|}}^T\end{bmatrix}^T \end{aligned} \tag{25}$$

then we obtain the following MAS closed-loop system:

$$\begin{aligned} \dot{\psi}_i &= \mathcal{A}_i \psi_i + \bar{B}_{fi}\bar{f}_i \\ \mathcal{A}_i &= \begin{bmatrix}\Omega_{11i} & \Omega_{12i} & \Omega_{13i} \\ 0 & \Omega_{22i} & \Omega_{23i} \\ 0 & 0 & I_N \otimes S - c\mathcal{H}\otimes I_q\end{bmatrix}, \bar{B}_{f_i} = \begin{bmatrix}\Omega_{3i} \\ \Omega_{3i} \\ 0\end{bmatrix} \\ \Omega_{11i} &= \mathrm{diag}(A_{\bar{i}_1} + B_{\bar{i}_1} K_{1\bar{i}_1}, \cdots, A_{\bar{i}_{|\bar{\mathcal{N}}_i|}} + B_{\bar{i}_{|\bar{\mathcal{N}}_i|}} K_{1\bar{i}_{|\bar{\mathcal{N}}_i|}}) \\ \Omega_{12i} &= \mathrm{diag}(-B_{\bar{i}_1} K_{1\bar{i}_1}, \cdots, -B_{\bar{i}_{|\bar{\mathcal{N}}_i|}} K_{1\bar{i}_{|\bar{\mathcal{N}}_i|}}) \\ \Omega_{13i} &= \begin{bmatrix}\mathbf{e}_{\bar{i}_1} \otimes (B_{\bar{i}_1} K_{2\bar{i}_1})^T, \cdots, \mathbf{e}_{\bar{i}_{|\bar{\mathcal{N}}_i|}} \otimes (B_{\bar{i}_{|\bar{\mathcal{N}}_i|}} K_{2\bar{i}_{|\bar{\mathcal{N}}_i|}})^T\end{bmatrix}^T \\ \Omega_{22i} &= \mathrm{diag}(A_{\bar{i}_1} + L_{\bar{i}_1} C_{m\bar{i}_1}, \cdots, A_{\bar{i}_{|\bar{\mathcal{N}}_i|}} + L_{\bar{i}_{|\bar{\mathcal{N}}_i|}} C_{m\bar{i}_{|\bar{\mathcal{N}}_i|}}) \\ \Omega_{23i} &= \begin{bmatrix}-\mathbf{e}_{\bar{i}_1} \otimes E_{\bar{i}_1}^T, \cdots, -\mathbf{e}_{\bar{i}_{|\bar{\mathcal{N}}_i|}} \otimes E_{\bar{i}_{|\bar{\mathcal{N}}_i|}}^T\end{bmatrix}^T \\ \Omega_{3i} &= \mathrm{diag}(B_{f_{\bar{i}_1}}, \cdots, B_{f_{\bar{i}_{|\bar{\mathcal{N}}_i|}}}) \end{aligned} \tag{26}$$

and (24) could be changed to the following form:

$$\begin{aligned} y_i &= \mathcal{C}_i \psi_i \\ \mathcal{C}_i &= \begin{bmatrix}0 & \bar{C}_i & 0\end{bmatrix}, \bar{C}_i = \mathrm{diag}(C_{m\bar{i}_1}, \cdots, C_{m\bar{i}_{|\bar{\mathcal{N}}_i|}}) \end{aligned} \tag{27}$$

Combine (26) and (27), we obtain $|\bar{\mathcal{N}}_i|$ systems as follows:

$$\begin{cases} \dot{\psi}_i = A_i\psi_i + \bar{b}_{f_{i\bar{i}_k}} f_{\bar{i}_k} + \bar{B}_{f_{-i\bar{i}_k}} \bar{f}_{-i\bar{i}_k} \\ y_i = C_i\psi_i, k = 1, \cdots, |\bar{\mathcal{N}}_i| \end{cases} \tag{28}$$

where $\bar{b}_{f_{i\bar{i}_k}} = \begin{bmatrix} \mathbf{e}_k^T \otimes B_{f_{\bar{i}_k}}^T & \mathbf{e}_k^T \otimes B_{f_{\bar{i}_k}}^T & 0 \end{bmatrix}^T$, $\bar{B}_{f_{-i\bar{i}_k}}$ is the component of $\bar{B}_{f_i}$ after deleting $\bar{b}_{f_{i\bar{i}_k}}$.

Follow the same steps in Section 2.1, $|\bar{\mathcal{N}}_i|$ observers are designed in agent i.

$$\begin{aligned} \dot{z}_{i\bar{i}_k} &= G_{i\bar{i}_k} z_{i\bar{i}_k} + R_{i\bar{i}_k} y_i \\ \hat{\psi}_{i\bar{i}_k} &= z_{i\bar{i}_k} + Q_{i\bar{i}_k} y_i, k = 1, \cdots, |\bar{\mathcal{N}}_i| \end{aligned} \tag{29}$$

where the design of parameter matrices is the same as (16). Next we just need to prove existence of observers for FD.

Remark 10. *In comparison with y_i in (8), (22) contains measurement output and state estimate of one agent's and its neighbours'. In the following, Theorem 3 will prove that existence of the above observers is still ensured, where y_i does not contain state information. The objective proposed in Section 1 will realise under the condition that state information is unmeasurable.*

Theorem 3. *If Assumptions 1 and 4 are satisfied, $c > 0$, $A_{\bar{i}_k} + B_{\bar{i}_k} K_{1\bar{i}_k}$ are Hurwitz stable, $k = 1, \cdots, |\bar{\mathcal{N}}_i|$, and each follower agent has paths to the leader in the graph $\mathcal{G}$, there exists an observer (29) for any agent $\bar{i}_k \in \bar{\mathcal{N}}_i$.*

Proof of Theorem 3. For $\bar{i}_k \in \bar{\mathcal{N}}_i$, Lemma 2 indicates that there exists an observer (29) if $rank(C_i\bar{b}_{f_{i\bar{i}_k}}) = rank(\bar{b}_{f_{i\bar{i}_k}})$, and $\mathcal{M}_{ik} = \begin{bmatrix} sI - A_i & \bar{b}_{f_{i\bar{i}_k}} \\ C_i & 0 \end{bmatrix}$ is of full column rank for $\forall s \in \mathbb{C}$, $\mathrm{Re}(s) \geq 0$.

Firstly, we verify the first condition, for $k = 1$, we have

$$C_i\bar{b}_{f_{i\bar{i}_1}} = \begin{bmatrix} 0 & \mathrm{diag}(C_{m\bar{i}_1}, \cdots, C_{m\bar{i}_{|\bar{\mathcal{N}}_i|}}) & 0 \end{bmatrix} \cdot \begin{bmatrix} \mathbf{e}_1 \otimes B_{f_{\bar{i}_1}} \\ \mathbf{e}_1 \otimes B_{f_{\bar{i}_1}} \\ 0 \end{bmatrix} = C_{m\bar{i}_1} B_{f_{\bar{i}_1}} \tag{30}$$

where $C_{m\bar{i}_1} B_{f_{\bar{i}_1}}$ is of full column rank according to Assumption 4. Furthermore, for $k = 2, \cdots, |\bar{\mathcal{N}}_i|$, columns of $C_i\bar{b}_{f_{i\bar{i}_k}}$ are always linearly independent, where the first condition holds.

Then, we need to verify that $\mathcal{M}_{ik}$ is of full column rank for $\forall s \in \mathbb{C}$, $\mathrm{Re}(s) \geq 0$, which is shown as follows:

$$\begin{bmatrix} sI - \Omega_{11i} & -\Omega_{12i} & -\Omega_{13i} & \mathbf{e}_k \otimes B_{f_{\bar{i}_k}} \\ 0 & sI - \Omega_{22i} & -\Omega_{23i} & \mathbf{e}_k \otimes B_{f_{\bar{i}_k}} \\ 0 & 0 & sI - (I_N \otimes S - c\mathcal{H} \otimes I_q) & 0 \\ 0 & \bar{C}_i & 0 & 0 \end{bmatrix} \tag{31}$$

Consider (31), let $\nu_1, \nu_2 \in \mathbb{R}^{\sum_{k=1}^{|\bar{\mathcal{N}}_i|} n_{\bar{i}_k}}$, $\nu_3 \in \mathbb{R}^{Nq}$ and $\nu_4 \in \mathbb{R}^{l_k}$ satisfy (32).

$$\begin{bmatrix} (sI - \Omega_{11i}) \cdot \nu_1 - \Omega_{12i} \cdot \nu_2 - \Omega_{13i} \cdot \nu_3 + (\mathbf{e}_k \otimes B_{f_{\bar{i}_k}}) \cdot \nu_4 \\ (sI - \Omega_{22i}) \cdot \nu_2 - \Omega_{23i} \cdot \nu_3 + (\mathbf{e}_k \otimes B_{f_{\bar{i}_k}}) \cdot \nu_4 \\ [sI - (I_N \otimes S - c\mathcal{H} \otimes I_q)] \cdot \nu_3 \\ \bar{C}_i \cdot \nu_2 \end{bmatrix} = 0 \tag{32}$$

It has been proved in Theorem 2 that columns of $sI - (I_N \otimes S - c\mathcal{H} \otimes I_q)$ are linearly independent for $\forall s \in \mathbb{C}$, $\mathrm{Re}(s) \geq 0$, we have $\nu_3 = 0$. Therefore, $\nu_2 = \nu_4 = 0$ if (33) is of full column rank for $\forall s \in \mathbb{C}, \mathrm{Re}(s) \geq 0$,

$$\begin{bmatrix} sI - \Omega_{22i} & \mathbf{e}_k \otimes B_{f_{\bar{i}_k}} \\ \bar{C}_i & 0 \end{bmatrix} \tag{33}$$

where (33) could be transformed to (34) through appropriate row and column transformation, the rank of which is equal to (33),

$$\begin{bmatrix} sI - \mathrm{diag}(A_{\bar{i}_1}, \cdots, A_{\bar{i}_{|\bar{\mathcal{N}}_i|}}) & \mathbf{e}_k \otimes B_{f_{\bar{i}_k}} \\ \mathrm{diag}(C_{m\bar{i}_1}, \cdots, C_{m\bar{i}_{|\bar{\mathcal{N}}_i|}}) & 0 \end{bmatrix} \tag{34}$$

according to Assumption 4, (34) is of full column rank for $\forall s \in \mathbb{C}, \mathrm{Re}(s) \geq 0$, i.e., $\nu_2 = \nu_4 = 0$. As Ω_{11i} is Hurwitz stable, $\nu_1 = 0$. Hence, (31) is of full column rank for $\forall s \in \mathbb{C}, \mathrm{Re}(s) \geq 0$. Finally, existence of observers (29) for any node in $\bar{\mathcal{N}}_i$ is established. □

Remark 11. *In comparison with the output-feedback-based COR problem in Reference [4], where the authors design two control laws, which correspond to two kinds of follower agents in the MAS, the first kind are called informed agents, i.e., measurement output y_{mi} contains exogenous signal v, then v is detectable from y_{mi}. The second kind are called uninformed agents, where measurement output y_{mi} does not contain exogenous signal v. As Section 2 just considers that all the follower agents are uninformed, i.e., a special case of literature [2], a single control law (21) will realise COR if no fault occurs, where $c > 0$, and the designed K_{1i} and L_i make $A_i + B_i K_{1i}$ and $A_i + L_i C_{mi}$ Hurwitz stable, which is not contradicted with the existence conditions of observers (29), i.e., control law (21) and observers (29) could run simultaneously.*

Remark 12. *The faulty agent location algorithm for the output-feedback-based distributed FD scheme is the same as Algorithm 1 and omitted here.*

4. Simulation Example

In this section, we will provide an example to illustrate the effectiveness of the two proposed distributed FD schemes.

Consider the following agent dynamics in Reference [31]:

$$\begin{aligned} \dot{x}_i &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & c_i \\ 0 & -d_i & a_i \end{bmatrix} x_i + \begin{bmatrix} 0 \\ 0 \\ b_i \end{bmatrix} u_i + \begin{bmatrix} -0.5 & 0 \\ -1 & 0.5 \\ 0 & 0 \end{bmatrix} v \\ e_i &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x_i + \begin{bmatrix} -1 & 0 \\ -0.5 & -1 \end{bmatrix} v \\ y_{mi} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x_i \end{aligned} \tag{35}$$

where the parameters $\{a_i, b_i, c_i, d_i\}$ are modelled as $\{1, 1, 1, 1\}$, $\{10, 2, 1, 1\}$, $\{2, 1, 1, 10\}$ and $\{2, 1, 1, 1\}$, respectively.

Assume the communication graph $\mathcal{G}$ among all the follower agents and the exosystem can be described by Figure 1, where node 0 represents the exosystem and the other nodes represent four follower agents, where agent 2 is assumed as faulty, it can be observed that only agent 3 can access the exosystem state v, v is generated by the exosystem $\dot{v} = Sv$, where

$$S = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Then, it can be verified that Assumptions 1–3 are satisfied, (A_i, B_i) are stabilisable, (A_i, C_{mi}) are detectable, and four follower agents have paths to the leader in the graph $\mathcal{G}$. What is more,

$$B_{f_1} = B_{f_2} = B_{f_3} = B_{f_4} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$$

which satisfy Assumption 4. The solutions of (6) are given by

$$X_i = \begin{bmatrix} 1 & 0.5 & 0 \\ 0 & 1 & 0 \end{bmatrix}^T, U_i = \begin{bmatrix} \frac{0.5d_i}{b_i} & \frac{d_i}{b_i} \end{bmatrix}$$

where $i = 1, 2, 3, 4$.

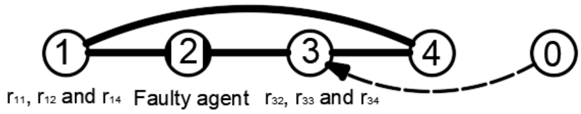

Figure 1. Communication topology of the multi-agent system (MAS).

Choose the coupling gain coefficient c, parameter matrices of control laws as Theorems 2 and 3, which correspond to the state-feedback-based and output-feedback-based distributed FD schemes, respectively, as well as parameter matrices of the observers for FD.

For the condition that state information is measurable, run control law (7) and observers (15) for FD.

The fault is assumed to be a constant and occur in the first element of x_2, i.e., $f_2 = 3.5$, which occurs after 25 s. Residual generators r_{11}, r_{12} and r_{14} in agent 1 are shown as Figure 2, which are represented by 2-norm type. According to Remark 8, we could choose a positive scalar ϵ, which is larger than $||e_{21}(0)||$, $||e_{22}(0)||$ and $||e_{24}(0)||$, and $e^{\sigma t}$, which is larger than $||e^{G_{21}t}||$, $||e^{G_{22}t}||$ and $||e^{G_{24}t}||$, combined with $||C_2|| = 1.64$. Then, the isolation threshold could be set as $1.64\epsilon \cdot e^{\sigma t}$.

It is shown that above residuals converge to little enough values before the fault occurs, then r_{11} and r_{14} fluctuate when the fault occurs at the time of 25 s. However, r_{12} does not fluctuate as r_{11} and r_{14} when the fault occurs, then according to Algorithm 1, the fault occurs in agent 2.

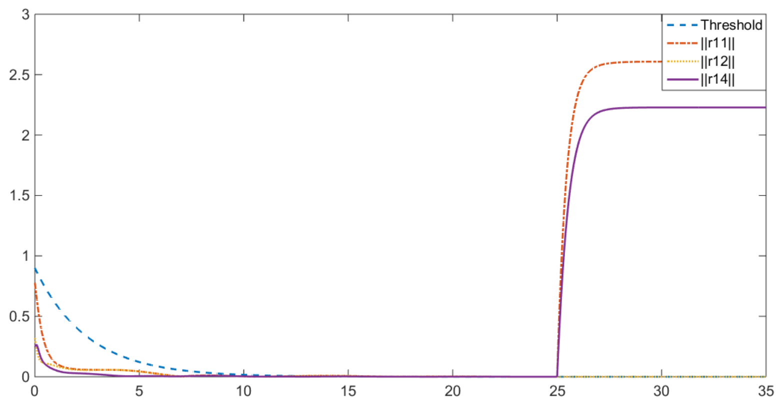

Figure 2. Residual generators in agent 1 under state feedback.

For the condition that state information is unmeasurable, run control law (21) and observers (29) for FD.

The fault signal is the same as the state feedback case, as well as the isolation threshold selection method. Simulation results of residual generators r_{11}, r_{12} and r_{14} in agent 1 are shown as Figure 3, and still represented by 2-norm type. It is shown that r_{12} remains converging after the fault occurs, where r_{11} and r_{14} fluctuate; therefore, the fault occurs in agent 2.

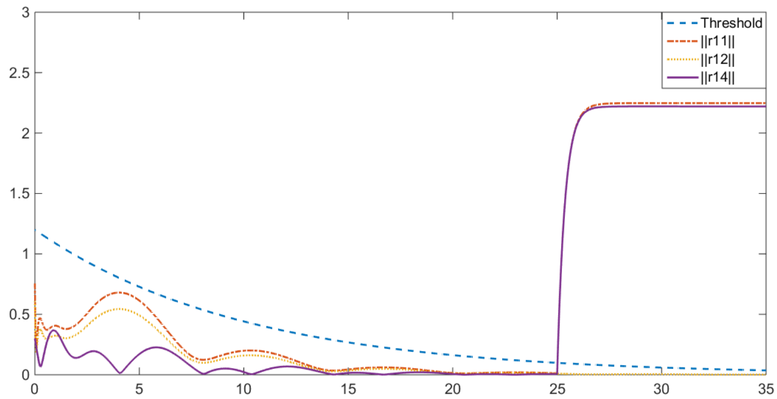

Figure 3. Residual generators in agent 1 under output feedback.

5. Conclusions

Two distributed FD schemes for heterogeneous MAS are proposed in this paper. A state-feedback-based distributed control law is adopted to realise COR when there does not occur any fault, where state-feedback-based observers for FD are designed, and existence of the designed observers is also proved. Furthermore, we consider the condition that state information is unmeasurable, an output-feedback-based distributed control law is designed, as well as output-feedback-based observers for FD, where existence of the above observers are ensured through appropriately designed feedback information. Finally, two simulation examples verify the effectiveness of the proposed FD schemes. However, the above distributed control laws and FD schemes require one agent to obtain information from its neighbours, such as state, compensator state, measurement output and state estimate, which will exert a heavy burden on communication networks.

Possible future work includes considering the condition that agents suffer from disturbances and faults simultaneously, as well as reducing communication burden.

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Abbreviations

The following abbreviations are used in this manuscript:

MAS	Multi-agent systems
FD	Fault detection
COR	Cooperative output regulation
UIO	Unknown input observer
UAV	Unmanned aerial vehicles

References

1. Hu, W.F.; Liu, L. Input time delay margin in event-triggered consensus of multiagent systems. *IEEE Trans. Cybern.* **2019**, *5*, 1849–1858.

2. De la Sen, M.; Alonso-Quesada, S. On finite-time consensus objectives in time-varying interconnected discrete linear dynamic systems under internal and external delays. *Adv. Mech. Eng.* **2018**, *7*, 1–24. [CrossRef]
3. Su, S.; Huang, J. Cooperative output regulation of linear multi-agent systems. *IEEE Trans. Autom. Control* **2012**, *4*, 1062–1066.
4. Su, S.; Huang, J. Cooperative output regulation of linear multi-agent systems by output feedback. *Syst. Control Lett.* **2012**, *61*, 1248–1253. [CrossRef]
5. Li, Z.; Chen, M.Z.Q.; Ding, Z.T. Distributed adaptive controllers for cooperative output regulation of heterogeneous agents over directed graph. *Automatica* **2016**, *68*, 179–183. [CrossRef]
6. Lv, M.; Liu, L. Cooperative output regulation of linear multi-agent systems by a novel distributed dynamic compensator. *IEEE Trans. Autom. Control* **2017**, *62*, 6481–6488.
7. Hashikura, K.; Kojima, A.; Suzuki, T.; Yamada, K. Exosystem-observer approach to H_2 control with output regulation constraint. *IET Control Theory Appl.* **2019**, *16*, 732–745. [CrossRef]
8. Basu, H.; Yoon, S.Y. Robust cooperative output regulation under exosystem detectability constraint. *Int. J. Control* **2020**, *5*, 1102–1114. [CrossRef]
9. Deng, H.C.; Yang, G.H. Cooperative adaptive output regulation for linear multi-agent systems with actuator faults. *IET Control Theory Appl.* **2017**, *11*, 2396–2402. [CrossRef]
10. Deng, H.C.; Yang, G.H. Distributed adaptive fault-tolerant control approach to cooperative output regulation for linear multi-agent systems. *Automatica* **2019**, *103*, 62–68. [CrossRef]
11. Deng, H.C. Cooperative fault-tolerant output regulation of linear heterogeneous multiagent systems under directed network topology. *IEEE Trans. Syst. Man Cybern. Syst.* **2019**, *2019*, 2944254. [CrossRef]
12. Meskin, N.; Khorasani, K. Actuator fault detection and isolation for a network of unmanned vehicles. *IEEE Trans. Autom. Control* **2009**, *4*, 835–840. [CrossRef]
13. Meskin, N.; Khorasani, K.; Rabbath, C.A. A hybrid fault detection and isolation strategy for a network of unmanned vehicles in presence of large environmental disturbances. *IEEE Trans. Control Syst. Technol.* **2009**, *6*, 1422–1429. [CrossRef]
14. Menon, P.P.; Edwards, C. Robust fault estimation using relative information in linear multi-agent networks. *IEEE Trans. Autom. Control* **2014**, *2*, 477–482. [CrossRef]
15. Shames, I.; Teixeira, A.M.H.; Sandberg, H.; Johansson, K.H. Distributed fault detection for interconnected second-order systems. *Automatica* **2011**, *47*, 2757–2764. [CrossRef]
16. Darouach, M.; Zasadzinski, M.; Xu, S.J. Full-order observers for linear systems with unknown inputs. *IEEE Trans. Autom. Control* **1994**, *39*, 606–609. [CrossRef]
17. Emami, S.A.; Araujo, R.; Asvadi, A. Distributed simultaneous estimation of states and unknown inputs. *Syst. Control Lett.* **2020**, *138*, 104660. [CrossRef]
18. Hou, M.; Muller, P.C. Fault detection and isolation observers. *Int. J. Control* **1994**, *5*, 827–846. [CrossRef]
19. Teixeira, A.; Shames, I.; Sandberg, H.; Johansson, K.H. Distributed fault detection and isolation resilient to network model uncertainties. *IEEE Trans. Cybern.* **2014**, *11*, 2024–2037. [CrossRef]
20. Shi, J.; He, X.; Wang, Z.D.; Zhou, D.H. Distributed fault detection for a class of second-order multi-agent systems: an optimal robust observer approach. *IET Control Theory Appl.* **2014**, *8*, 1032–1044. [CrossRef]
21. Jia, W.; Wang, J.Z. Partial-Nodes-Based distributed fault detection and isolation for second-order multi-agent systems with exogenous disturbances. *IEEE Trans. Cybern.* **2020**. [CrossRef] [PubMed]
22. Liu, X.; Gao, X.; Han, J. Observer-based fault detection for high-order nonlinear multi-agent systems. *J. Frankl. Inst.* **2016**, *353*, 72–94. [CrossRef]
23. Gao, X.; Liu, X.; Han, J. Reduced order unknown input observer based distributed fault detection for multi-agent systems. *J. Frankl. Inst.* **2017**, *354*, 1464–1483. [CrossRef]
24. Davoodi, N.M.R.; Khorasani, K.; Talebi, H.A.; Momeni, H.R. Distributed fault detection and isolation filter design for a network of heterogeneous multiagent systems. *IEEE Trans. Control Syst. Technol.* **2014**, *3*, 1061–1069. [CrossRef]
25. Qin, L.; He, X.; Zhou, D.H. Fault-tolerant cooperative output regulation for multi-vehicle systems with sensor faults. *Int. J. Control* **2017**, *10*, 2227–2248. [CrossRef]
26. Ni, W.; Cheng, D. Leader-following consensus of multi-agents systems under fixed and switching topologies. *Syst. Control Lett.* **2010**, *59*, 209–217. [CrossRef]
27. Huang, W.J. Linear output regulation. In *Nonlinear Output Regulation: Theory and Applications*; King, B., Ed.; SIAM: Philadelphia, PA, USA, 2004; pp. 3–15.

28. Ding, X. Norm-based residual evaluation and threshold computation. In *Model-Based Fault Diagnosis Techniques: Design Schemes, Algorithms and Tools*, 2nd ed.; Grimble, M.J., Johnson, M.A., Eds.; Springer: London, UK, 2013; pp. 285–314.
29. Horn, W.R.A.; Johnson, C.R. Unitary and real orthogonal triangularizations. In *Matrix Analysis*, 2nd ed.; Cambridge University Press: New York, NY, USA, 2013; pp. 94–101.
30. Frank, H.P.M.; Ding, S.X. Survey of robust residual generation and evaluation methods in observer-based fault detection systems. *J. Process Control* **1997**, *6*, 403–424. [CrossRef]
31. Hu, W.; Liu, L. Cooperative output regulation of heterogeneous linear multi-agent systems by event-triggered control. *IEEE Trans. Cybern.* **2017**, *1*, 105–116. [CrossRef]