

## Article

# Estimating CPT Parameters at Unsampled Locations Based on Kriging Interpolation Method

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**Abstract:** The cone penetrometer test (CPT) has been widely used in geotechnical investigations. However, how to use the limited CPT data to reasonably predict the soil parameters of the unsampled regions remains a challenge. In the present study, we adopted the Kriging method to obtain the CPT data of an unsampled location in Adelaide, South Australia, based on the collected CPT data from six soundings around this location. Interpolation results showed that the trend of the estimated parameters is consistent with the trend of parameters of the surrounding points. From the Kriging interpolation result, we further carried out axial bearing capacity calculation of a precast concrete pile using the CPT-based direct method to verify the reliability of the method. The calculated bearing capacity of the pile is 99.6 kN which is very close to the true value of 102.8 kN. Our results demonstrated the effectiveness of the Kriging method in considering the soil spatial variability and predicting soil parameters, which is quite suitable for the application in engineering practice.

**Keywords:** CPT; Kriging interpolation method; spatial variability; pile axial bearing capacity



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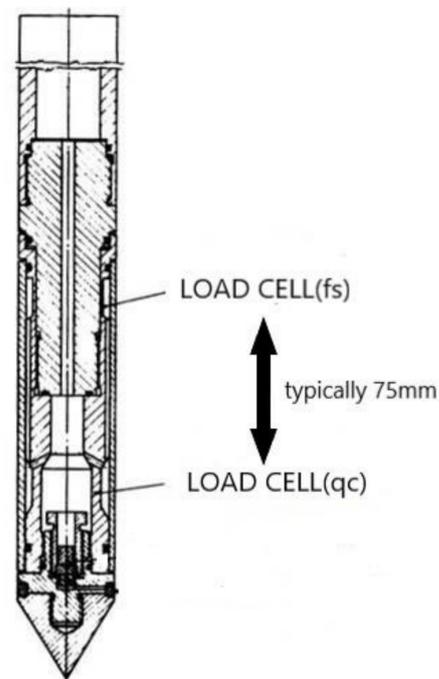
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## 1. Introduction

The cone penetration test (CPT) has been widely used as a quick and reliable soil exploration test that provides subsurface soil properties [1]. As the electric cone penetrometer (Figure 1) advances into the soil with a constant speed, parameters such as the cone tip resistance ( $q_c$ ) and the sleeve friction ( $f_s$ ) are simultaneously measured by the load cells on the penetrometer. Compared with other in situ tests, CPT can provide continuous parameter profiles that can yield much more detailed information [2]. As the two major measured parameters  $q_c$  and  $f_s$  vary significantly with the change of the soil layers, CPT shows advantages in geotechnical investigations to classify soil strata, and several methods have been proposed to classify soils upon CPT data [3–5]. Moreover, CPT can also be used to estimate the strength and deformation characteristics of soils, such as the undrained shear strength of soil  $S_u$  [6]. Generally speaking, CPT is only conducted at necessary locations to obtain the required soil properties, for example, around the underground structure to be constructed. Considering the spatial variability of the soil, the CPT data obtained from adjacent soundings could be quite different, which makes it difficult to estimate the soil properties at the unsampled locations. Therefore, for large areas, how to reasonably estimate the soil properties of the entire area through limited CPT data is of great importance.

Soil properties are regionalized variables; that is, within a soil layer or rock mass, samples that are close to each other indicate a stronger correlation than distant ones [7]. Because CPT parameters directly depend on the soil properties, they are also autocorrelated. In statistics, there are two ways to consider the correlation of the data sequence with distance: one is random field theory, and the other is geostatistics. Random field theory is often used

to analyze time series. Although it has been successfully applied in analyzing soil spatial variability by different researchers [8,9], it is mostly applied to one-dimensional situations.



**Figure 1.** Cone penetrometer of CPT.

Geostatistics often refers to the Kriging method, which has been widely used in the field of geographic science. Different from random field theory, the Kriging method can be applied to multi-dimensional analysis since it is an algorithm for spatial modeling and regression interpolation [10]. It has been used by many researchers to analyze the autocorrelation of geotechnical parameters and achieved promising results [11,12]. However, there are only a few case studies on adopting the Kriging method to predict the soil parameters at the unsampled locations, and the data used in previous studies is often scarce, which is not enough to prove the accuracy and effectiveness of the Kriging method. Currently, there have been several studies using Markov chain and machine learning methods combined with CPT test to predict the soil parameters at the unsampled locations [13,14]; however, they need to meet some strong prerequisites. For example, the Markov chain method is suitable when the unsampled location and the sampled CPT boreholes are located on the same horizontal line, while the machine learning method needs to obtain the soil layer information in advance and perform extensive training to obtain a better prediction result, which is difficult to be applied into practical engineering. In this study, we used CPT parameters with small sampling intervals as the research objects to assess the performance of the Kriging method in predicting spatial soil parameters, thus avoiding data insufficiency due to excessive sample spacing. The study was further supplemented by axial bearing capacity calculation, which we used to examine the reliability of the interpolation results and the applicability of the Kriging method in solving practical engineering problems.

## 2. Materials and Method

The CPT data is provided by the ISSMSG TC304 Student Contest Committee. As shown in Figure 2, there are six numbered CPT sampled boreholes randomly distributed on this  $50 \times 40$  m field represented by the black points. The typical drilling depth is 5 m below the mud surface, and the measurement spacing is 5 mm (i.e., a total of 1000  $q_c$  and  $f_s$  would be obtained for each borehole). Figure 3 shows the profiles of  $q_c$  and  $f_s$  values with depth at 6 sampled positions. A precast concrete pile was driven at an unsampled location shown by the red point numbered F5. In order to figure out the soil properties

and evaluate the ultimate bearing capacity of this pile foundation, the Kriging method is adopted to interpret the  $q_c$  and  $f_s$  at the pile location based upon the available CPT data. However, the raw CPT data needs preprocessing before it can be used in the Kriging interpolation program.

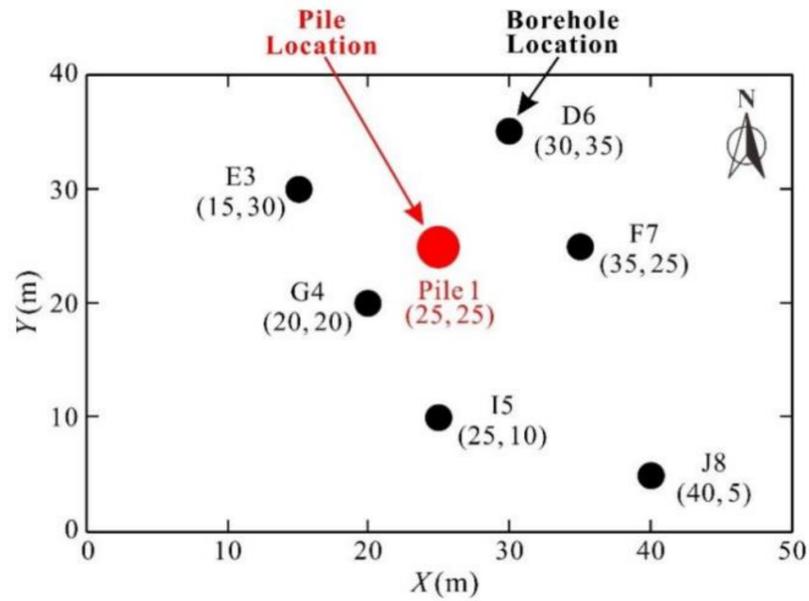


Figure 2. The locations of the sampled CPT boreholes and the pile.

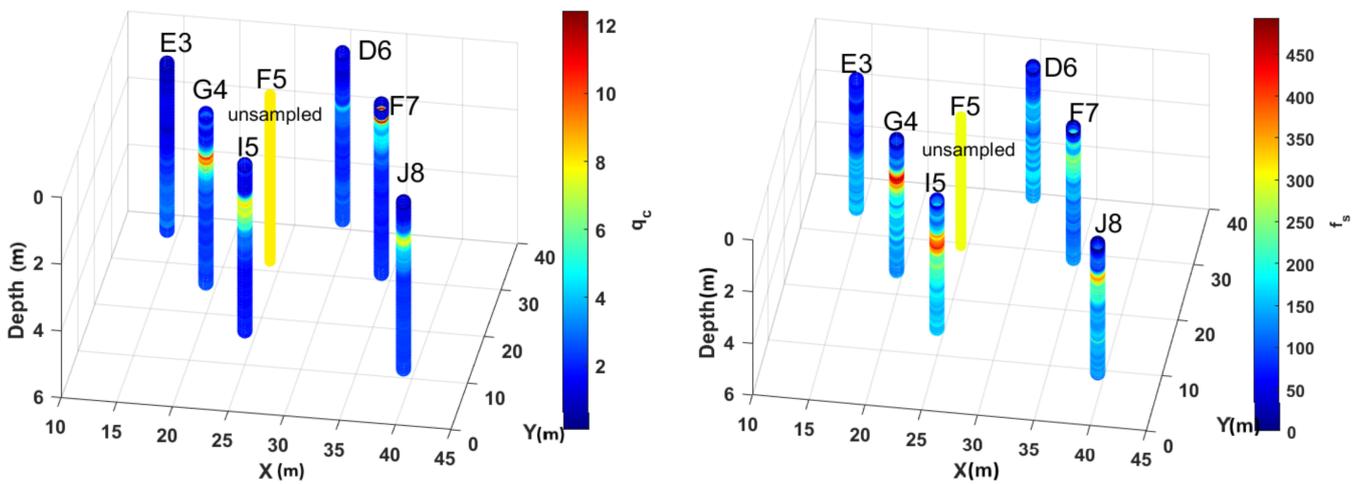


Figure 3. The  $q_c$  and  $f_s$  profiles of the six CPT soundings.

#### CPT Data Preprocessing

The first step is to remove outliers from the data. Outliers refer to those abnormal points whose values are obviously different from the surrounding sampled points, which is mainly caused by measurement errors or procedural errors such as rod adding [7]. Outliers do not reflect the actual CPT characteristics of the sampled points but affect the estimation accuracy, so they need to be removed first. This process is accomplished by a Gaussian filter, which replaces the value of the abnormal point with the weighted average of 20 samples around the point. The example of outliers and the smoothed curve is shown in Figure 4.

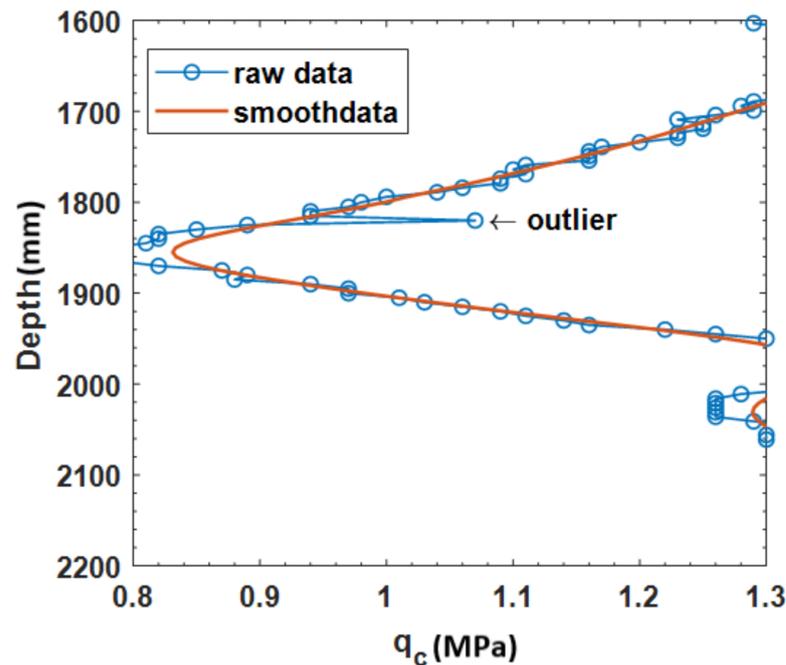


Figure 4. Outlier of raw CPT data.

The second step is to shift the  $f_s$  data to the right place. Schmertmann [15] emphasized that incorrect analysis will be made unless a depth correction, or shift, is applied to the  $f_s$  measurements. This correction is required since the cone tip resistance, and sleeve friction load cells are physically separated by a given distance as shown in Figure 1, and hence measurements of  $q_c$  do not refer to the same soil as that at which measurements of  $f_s$  are taken. Schmertmann [15] recommended that this shift distance should be equal to the distance between the base of the cone and the mid-height of the sleeve, which, for standard electric cone penetrometers, is approximately 75 mm. Campanella et al. [16] argued that the shift distance, also termed the “friction-bearing offset”, is 100 mm and is dependent on the type of soil being penetrated. Actually, when the cone penetrometer is advanced into the subsoil, it will cause a zone of soil to fail and deform plastically, as shown in Figure 5. The  $q_c$  and  $f_s$  are not point values but spatially averaged values of the failure zone [17], and the shift distance would be different for different cases. Jaksa [7] recommended cross-correlation function (CCF) to determine the shift distance because of its superiority in demonstrating the cross-correlation between  $q_c$  and  $f_s$ .

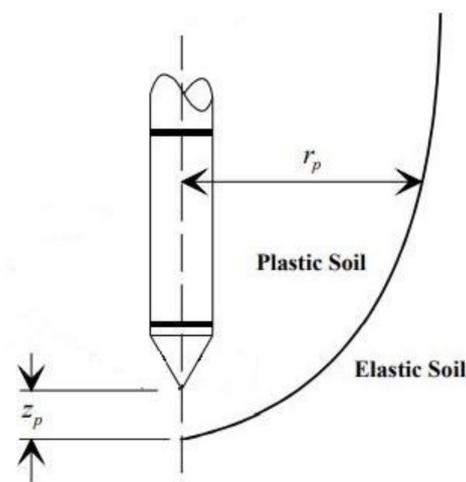


Figure 5. Plastic failure zone caused by the cone penetrometer.

In this study, we adopted the CCF to determine the final shift distance. Taking the  $q_c$  and  $f_s$  data of borehole F7 as an example, the CCF analysis result is shown in Figure 6. It can be seen clearly from Figure 6 that the maximum cross-correlation coefficient occurs at a spacing of 115 mm, which implies that the optimal shift distance is 115 mm, which is higher than the actual physical spacing of 75 mm. The same process is applied to the other five boreholes, and the final shift distance results are shown in Table 1. Based on the result listed in Table 1, the  $f_s$  data of the 6 boreholes will be shifted upward, resulting in a decrease in the maximum available drilling depth of each borehole. The smallest depth is borehole I5, of which the available depth of  $f_s$  is reduced to 4775 mm so that in the interpolation part, the maximum interpolation depth is adjusted to 4775 mm. Meanwhile, in order to solve the problem of missing data at some depth in some boreholes, we use linear interpolation to fill in the missing values, and the details are shown in Appendix A.

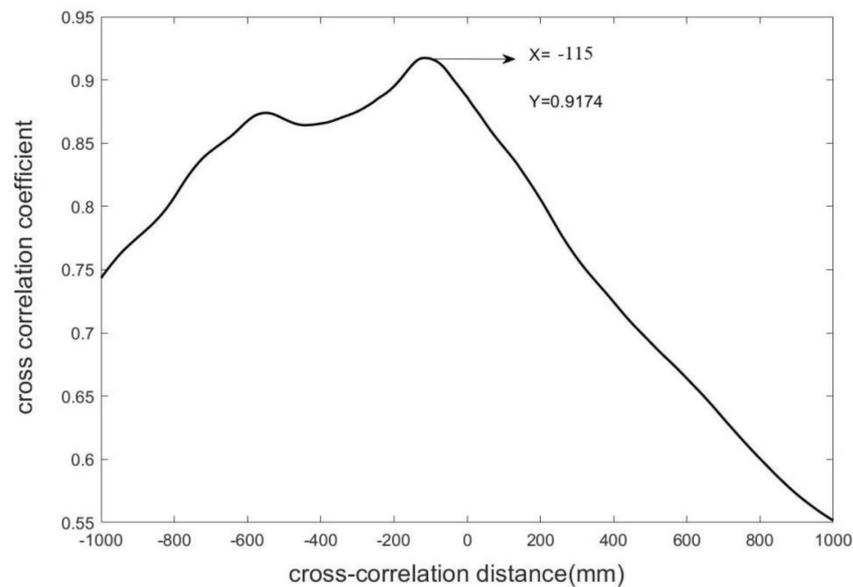


Figure 6. CCF of borehole F7.

Table 1. Shift distance of the six boreholes by CCF.

Number	E3	G4	I5	D6	F7	J8
Shift distance (mm)	0	90	230	0	115	130

After the above preprocessing steps, we divided the soil into a total of 956 sections from the ground down every 5 mm until the depth of 4775 mm, then on each section, we estimated the CPT parameter at the unsampled location using the Kriging method.

### 3. Kriging Interpolation

#### 3.1. Set Up Semivariogram

The core theory of the Kriging method is the semivariogram, which is established to reflect the relationship between distance  $d$  and semivariance  $\gamma_d$  between two sampled points. Generally speaking, the semivariogram is an increasing function. The larger the semivariance, the smaller the spatial dependence and mutual influence between the two sample points. According to the different forms of the semivariogram, different models are selected for fitting. Figure 7 shows some commonly used semivariogram models in the literature, such as the spherical model, exponential model, and Gaussian model. Soulié [18] used semivariogram to analyze the spatial variability of CPT data performed in alluvial deposits of sand and gravel in the Mississippi River flood plain, and the results showed that the spherical model was more proper for fitting the actual curve compared with other models. In addition, previous studies also demonstrated that the spherical model can fit well to the

semivariogram of various soils [19]. So in this study, we also chose a spherical model to fit the semivariogram.

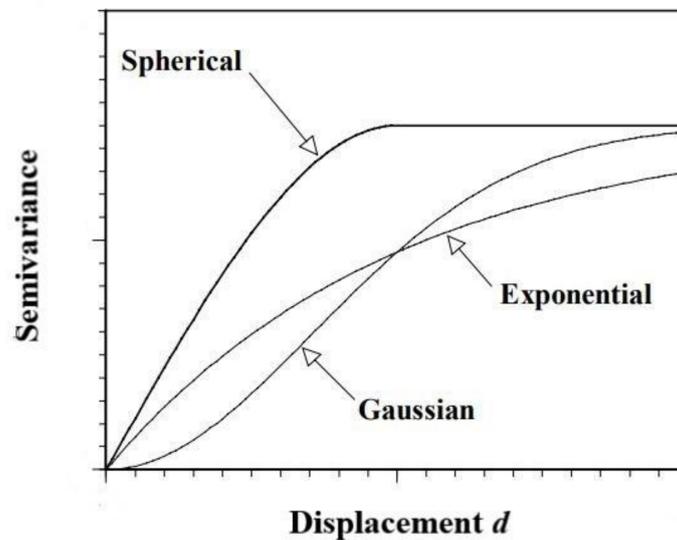


Figure 7. Commonly used semivariogram models.

The semivariogram is defined as follows:

$$\gamma_d = \frac{1}{2}Var[Z_x - Z_{x+d}] \tag{1}$$

where:  $Z_x$  is the variable at location,  $x$ ;  
 $Z_{x+d}$  is the variable at location,  $x + d$ ;  
 $d$  is the distance between the sample pairs;  
 $Var[x]$  refers to the variance of  $x$ .

Equation (1) implies that the nature of the semivariogram function is half the variance of  $Z_x$  and  $Z_{x+d}$ , sample pairs separated by  $d$ , and for a certain semivariogram, the variance is dependent only on  $d$ . This hypothesis, however, is hard to be satisfied for soil, for example,  $Z_{x1}$ ,  $Z_{x2}$ , and  $Z_{x3}$  are samples at different depths in the same CPT borehole, and the distance from  $x_2$  to  $x_1$  and  $x_3$  is the same. The hypothesis means that  $x_2$  should have the same effect on the other two points, but if  $x_2$  and  $x_1$  are in the same layer and  $x_3$  is in another layer, then  $x_2$  will obviously have different effects on the other two points. Although the hypothesis is difficult to meet, the Kriging interpolation is still a reasonable estimation method under the condition that the information of the soil layer at the estimation point is unknown.

The variables used to plot the semivariogram should satisfy the following equation:

$$E[Z_x - Z_{x+d}] = 0 \quad \forall x, d \tag{2}$$

where  $E[x]$  refers to the expectation of  $x$ .

As shown in the above equation, all the variables within the study area should have the same expectations, which are always called stationary variables. Figure 8 shows the  $q_c$  profiles of borehole E3, and it is clear from Figure 8 that CPT data are not stationary but with a specific trend. In order to satisfy Equation (2), ordinary least squares regression is performed to estimate the trend. After that, the trend component will be removed from the original CPT data so that the residuals would be stationary, and the semivariogram could be established upon it. The process is shown in Figure 8.

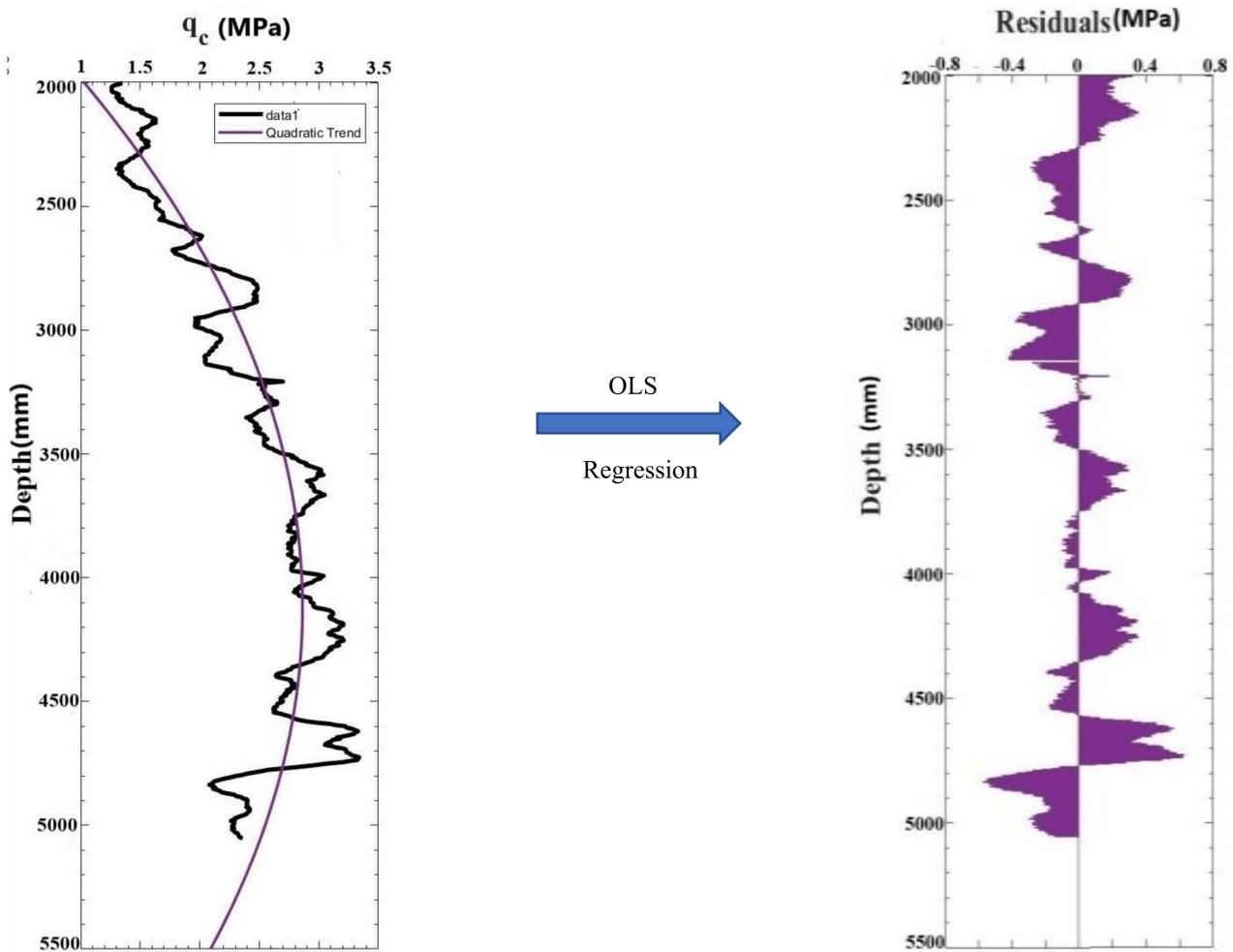


Figure 8. Example of the CPT data detrend.

Considering Equation (2), Equation (1) can be simplified as:

$$\gamma_d = \frac{1}{2} E [(Z_{x+d} - Z_x)^2] \tag{3}$$

In practice, the semivariogram must be estimated from the available discrete data, i.e., the theoretical expectations could be estimated by sample mean:

$$\gamma_d^* = \frac{1}{2N} \sum_{i=1}^N [(Z_{i+d} - Z_i)^2] \tag{4}$$

where  $N$  is the number of point pairs that are separated by distance  $d$ .

### 3.2. Build Weight Matrix

After the determination of the semivariogram based on existing data, it is feasible to estimate the CPT parameters at the unsampled location. Specifically, the Kriging method assigns different weights to the sampled CPT parameters at different locations, then linearly combines these parameters with their weights to obtain the estimation as shown by Equations (5) and (6). Where  $e$  indexes the unsampled location,  $i$  indexes the  $i$ th sampled location,  $Z$  is the CPT parameter (i.e.,  $q_c$  or  $f_s$ ),  $\lambda$  is the corresponding weight, and  $n$  is

the number of samples. The weight  $\lambda_i$  could be obtained by solving matrix presented in Equations (7) and (8).

$$Z_e = \lambda_1 Z_1 + \lambda_2 Z_2 + \lambda_3 Z_3 + \dots + \lambda_n Z_n \tag{5}$$

$$\sum_{i=1}^n \lambda_i = 1 \tag{6}$$

$$K\lambda = D \rightarrow \lambda = K^{-1}D \tag{7}$$

$$K = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \dots & \gamma_{1n} & 1 \\ \gamma_{21} & \gamma_{22} & \dots & \gamma_{2n} & 1 \\ \dots & \dots & \dots & \dots & 1 \\ \gamma_{n1} & \gamma_{n2} & \dots & \gamma_{nn} & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}, \lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \dots \\ \lambda_n \\ \mu \end{bmatrix}, D = \begin{bmatrix} \gamma_{1e} \\ \gamma_{2e} \\ \dots \\ \gamma_{ne} \\ 1 \end{bmatrix} \tag{8}$$

where  $\mu$  is Lagrange constant and  $\gamma_{ij}$  is the value of semivariogram corresponding to distance between  $i$ th and  $j$ th location. The Kriging method is unbiased implying that the expectation of the estimator obtained from the above equations is equal to the actual value (i.e., the average of the estimation errors will be zero). The final  $q_c$  and  $f_s$  profiles of the target location F5 obtained by the Kriging method are shown in Figure 9, accompanied by data from the other six sampled boreholes. The detailed program implementation using MATLAB is shown in Appendix A.

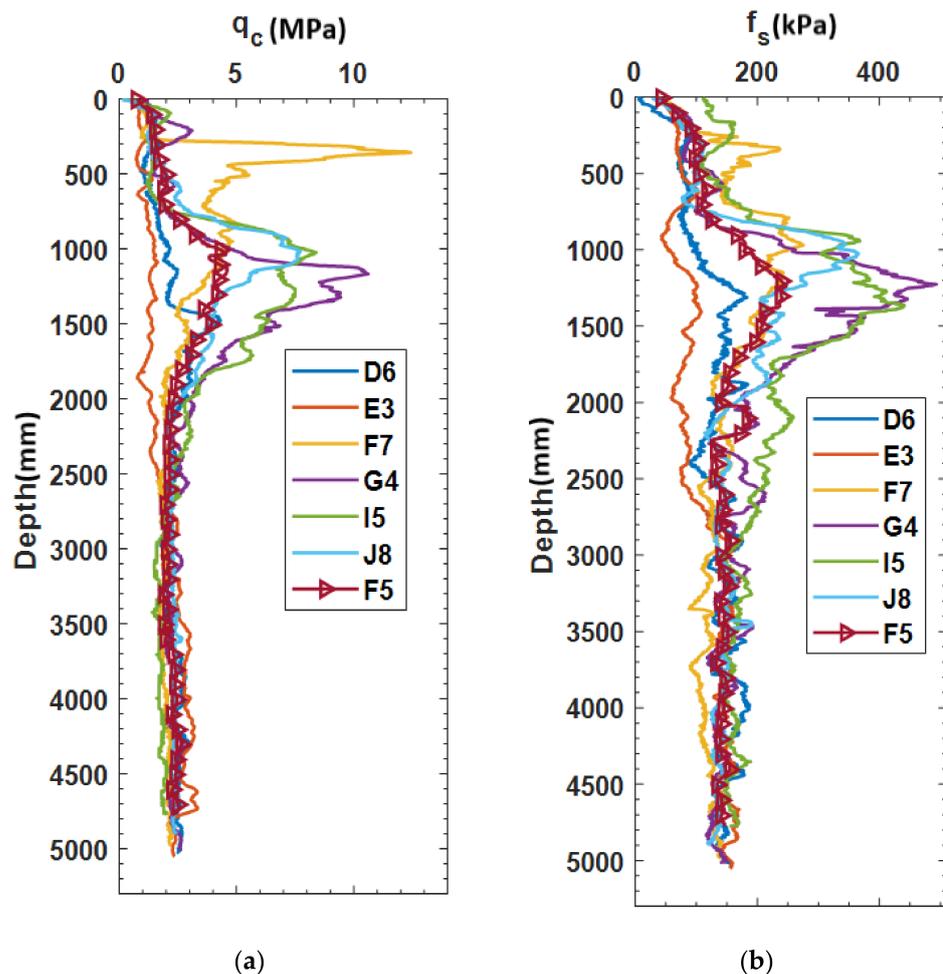


Figure 9. The 2D and 3D Kriging interpolation results of the unsampled location F5: (a) 2D; (b) 3D.

#### 4. Bearing Capacity Calculation

In order to examine the reliability of the Kriging interpolation results and its applicability to solve practical engineering problems, we use the interpolated CPT data of F5 to calculate the bearing capacity of a driven precast pile. The length and diameter of the pile are 4.5 and 0.3 m, respectively.

Based on previous studies, researchers held the point that the cone penetrometer can be considered as a mini-pile foundation, whereby the measured tip resistance ( $q_c$ ) and sleeve resistance ( $f_s$ ) correspond to the pile end bearing ( $q_b$ ) and the component of side friction ( $f_p$ ). So far, many CPT-based axial bearing capacity calculation methods of pile foundation have been proposed [20]. Cai et al. [21] evaluated and graded 10 CPT-based methods by comparing the measured capacity from static load tests with the estimated result from different pile capacity prediction methods, and the results showed that the CPT-based methods had reliability and the Laboratoire Central des Ponts et Chaussées method (LCPC, Bustamante and Gianeselli) [22] is one of the most accurate prediction methods. Another research conducted by Abu-Farsakh and Titi [1] obtained the same result that the LCPC method was relatively better when applied to predict the ultimate bearing capacity of square precast prestressed concrete (PPC) piles driven into Louisiana soils. Moreover, Jaksa [7] also recommended LCPC methods as the best prediction method for the soil in Adelaide.

For the LCPC method, the unit tip bearing capacity of the pile ( $q_b$ ) is predicted from the following equation:

$$q_b = k_b q_{eq} \tag{9}$$

where  $k_b = 0.45$  for driven precast piles in clay or silt with  $q_c$  values ranging from 1 to 5 MPa,  $q_{eq}$  is equivalent average cone tip resistance from 1.5 D above the pile tip to 1.5 D below the pile tip as shown in Figure 10.

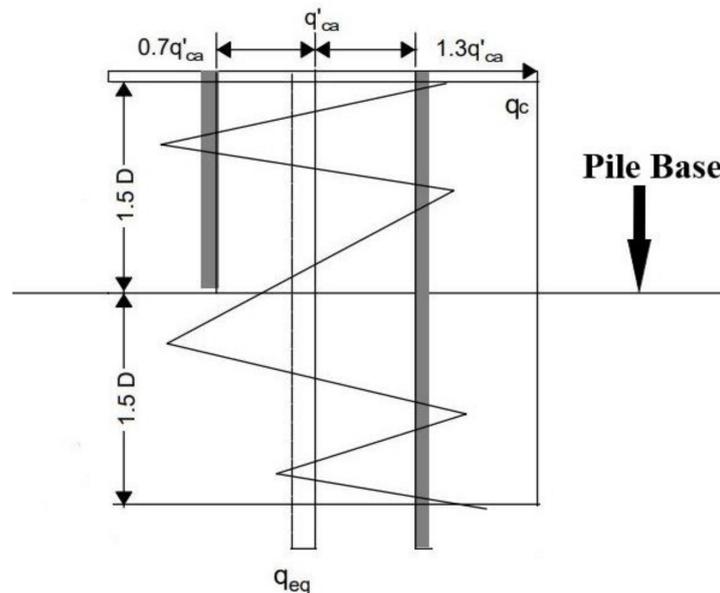


Figure 10. Calculation of the equivalent average tip resistance for the LCPC method.

Firstly, the average value  $q'_{ca}$  in the upper and lower 1.5 D range is determined. Then the values higher than  $1.3 q'_{ca}$  or lower than  $0.7 q'_{ca}$  are eliminated. After that, the  $q_{eq}$  could be obtained by averaging the remained  $q_c$  values over the same zone. In this study, due to data insufficiency and stability of  $q_c$  values around the pile tip, as shown in Figure 9,  $q_{eq}$  is determined by averaging the  $q_c$  values from 1.5 D above the pile tip to the last available value, and the result of  $q_b$  is 1.089 MPa. Now the end bearing capacity can be determined as:

$$Q_b = q_b A_b \tag{10}$$

where:  $A_b$  is the end area of the pile, and the calculation result is 77 kN.

The pile unit skin friction is estimated by:

$$f_p = \frac{q_c}{k_s} \quad (11)$$

where  $k_s$  is the skin friction coefficient ranging from 30 to 150 and equals 40 for driven procast piles in moderately compact clay ( $q_c$  range: 1 MPa–5 MPa). Bustamante and Gianeselli [22] also imposed different upper limits for  $f_p$  depending on pile and soil typology as well as installation methods. In this case, the upper limit is 35 kPa. According to Equation (11), the  $q_c$  data is first transformed to  $f_p$ , and the data over 35 kPa is replaced with 35 kPa, then the data is used to calculate the total pile skin friction by Equation (12).

$$Q_s = \sum_{i=1}^n f_{pi} A_{si} \quad (12)$$

where  $A_{si}$  is the side area of the  $i$ th layer, and the  $Q_s$  is calculated to be 148 kN. Finally, Bustamante and Gianeselli [20] suggested that the allowable design load of the pile is defined by the following equation:

$$Q_u = \frac{Q_b}{3} + \frac{Q_s}{2} \quad (13)$$

Through Equation (13), the final allowable design load is 99.6 kN. Jaksa [7] calculated the axial bearing capacity of the pile under the same condition through a 3D finite element model, of which the soil parameters came from the laboratory test on soil samples of the study area. The result given by Jaksa [7] is 102.8 kN, which is very close to our result.

## 5. Discussion

The Kriging method treats soil parameters as regionalized variables and considers the mutual influence by assigning different weights according to the semivariogram. In this study, we use the Kriging method combined with CPT data to make a reasonable estimation of the soil parameters at unsampled locations. Since CPT data is continuous and sufficient, our method can estimate parameters of locations within a certain scope and overcome the data insufficiency problem caused by relatively large sample spacings of the previous studies. As shown in Figure 9, line F5 roughly reflects a similar trend as the other six sampled lines, with a fluctuation between depth 700–2000 mm for both  $q_c$  and  $f_s$ , and the data tends to keep stable under 2.5 m. The bearing capacity analysis further proved the effectiveness of the Kriging method. However, there are also some limitations for this study: The Kriging method relies on semivariograms, which need to be figured out on parameters at sampled locations. However, the number of available CPT soundings is limited, leading to instability of the result. For each cross-section, we only used six known  $q_c$  and  $f_s$  to set up the experimental semivariogram, which could not be quite accurate. More CPT tests are required for a better estimation. Due to the shift of  $f_s$ , the maximum interpolation depth is 4775 mm, which cannot meet the requirement of calculating depth of the LCPC methods, so the calculation may have errors. What is more, the LCPC method poses an upper limit for  $f_s$ , which may cause underestimation of the total bearing capacity, and the influence distance above and below the pile tip used in  $q_b$  calculation is different for different methods [20]. Cai et al. [21] suggested that the influence distance given by the original method may be improper, and further study is needed to revise it depending on the soil type.

## 6. Conclusions

In summary, our research demonstrates the feasibility of using the Kriging method to consider the spatial variability of soil and provides a reliable estimation of soil parameters. Kriging methods can be combined with in situ tests and further applied into both geotech-

nical investigation and the analysis of the underground structures such as piles, bringing new directions and broad prospects in solving practical engineering problems.

**Author Contributions:** J.L. (Jinhao Liu): Material and Method, Writing—original draft. J.L. (Jinming Liu): Conceptualization, Writing—review and editing. Z.L.: Kriging interpolation, Program implementation. X.H.: Bearing capacity calculation. G.D.: Review and editing, Validation. All authors have read and agreed to the published version of the manuscript.

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## Appendix A

Linear interpolation program code using MATLAB

```
depth = [0 : 5 : 4775] %% from the ground down every 5 mm until the depth of 4775mm
qce3 = interp1(E3(:,1), E3(:,2), depth) %% E3() stands for the preprocessed CPT data of borehole E3
fse3 = interp1(E3(:,1), E3(:,3), depth)
```

The same process is applied to the other five numbered boreholes to divide the soil into slices.

Kriging method program code

This paper uses the DACE toolbox developed based on MATLAB to complete the Kriging interpolation estimation of CPT data. Lophaven et al. (2002) established this toolbox to easily estimate the values of unsampled variables based on sampled data. This toolbox consists of two main functions for building the Kriging interpolation model and using the model to estimate the values of unsampled variables. The program code is as follows:

$$[dmodel, perf] = dacefit(S, Y, regr, corr, theta(), lob, upb) \quad (A1)$$

$$y = predictor(x, dmodel) \quad (A2)$$

The description of each item in codes (A1) and (A2) is listed in Table A1. The code (A1) is used to build the model, which is used by the code (A2) to predict the result. As stated before, regpoly2 was used as a regression model to remove the quadratic trend of qc and fs, and corrspherical, that is, the spherical model was adopted to fit the semivariogram.

**Table A1.** Description of code (8).

Name	Description	Options
S	Location of sampled CPT borehole	
Y	Value of sampled CPT borehole	
regr	Regression model	* regpoly0, regpoly2 and regpoly3
corr	Correlation model	* correx, correpg, corrgauss, corrlin, corrspherical and corrspline
theta0	Initial guess of correlation parameter	default
lob	Lower limit of correlation parameter	default
upb	Upper limit of correlation parameter	default
dmodel	DACE model	
perf	Information about the optimization	
x	location of unsampled point	

\* Note: regpoly0, regpoly2, and regpoly3 mean zero-order polynomial, first polynomial, and second-order polynomial; correx, correpg, corrgauss, corrlin, corrspherical, and corrspline mean exponential model, generalized exponential model, Gaussian model, linear model, spherical model, and cubic spline model.

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