Supplementary Material

Novel hydroxyapatite beads for the adsorption of radionuclides from the decommissioning site nuclear power plants

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Table 1. Adsorption isotherm models.

Isotherms models	Equations
Langmuir	$q_e = q_{\max} \frac{K_L C_e}{1 + K_L C_e}$
Freundlich	$q_e = K_F C_e^{1/n}$
Sips	$q_e = q_{max} \frac{K_S C_e^{n_s}}{1 + K_S C_e^{n_s}}$
Tempkin	$q_e = \frac{RT}{b_T} ln\, A_T C_e$

where q_e (mg/g) is the quantity of adsorbate adsorbed on a unit mass of the b-HdA; q_{max} (mg/g) is the maximum adsorption capacity; K_L (L/mg) is the Langmuir adsorption constant; K_F (mg/g (L/mg)$^{1/n}$) is the Freundlich constant; $1/n$ is a measurement of the adsorption intensity; R (8.314 J/mol K) is the gas constant; T is the absolute temperature (K); n_s is the Sips model exponent of the adsorbent, and K_S (1/mg) is the Sips constant; b_T (J/mol) is the Tempkin constant associated to the heat of adsorption; A_T (L/mg) is the Tempkin equilibrium constant.

Table 2. Thermodynamic analysis.

$K_0 = \frac{a_s}{a_e} = \frac{\gamma_s}{\gamma_e} \frac{C_s}{C_e}$
$\Delta G^0 = -RT \ln(K_0)$
$\ln(K_0) = -\frac{\Delta H^0}{R} \frac{1}{T} + \frac{\Delta S^0}{R}$

where a_s and γ_s are the activity and activity coefficient of adsorbed Co(II), respectively; a_e and γ_e are the activity and activity coefficient of Co(II) in the solution at equilibrium, respectively; C_s and C_e (mg/L) are the adsorbed and equilibrium concentration of Co(II), respectively; K_0 is the thermodynamic equilibrium constant; ΔG^0 (kJ/mol) is the standard Gibbs free energy change; (ΔH^0) is the standard enthalpy change; ΔS^0 (J/mol K) is the standard entropy change.

Table 3. Adsorption kinetic models.

Kinetic models	Equations
Pseudo first order	$\log(q_e - q_t) = \log q_e - \frac{k_1}{2.303}t$
Pseudo second order	$\frac{t}{q_t} = \frac{1}{k_2 q_e^2} + \frac{1}{q_e}t$
Intra-particle diffusion	$q_t = k_{id}t^{0.5} + I$
Three-stage	$\frac{C_t}{C_o} = \frac{(1-\xi_1)(1-\xi_1-\beta\xi_2)}{(1-\xi_1-\beta\xi_2 e^{-\gamma t})}$ $\xi_1 = \frac{mq_1(\infty)}{VC_o}$ $\xi_2 = \frac{mq_2(\infty)}{VC_o}$ $\gamma = \frac{(1-\xi_1-\beta\xi_2)\alpha}{\beta\xi_2}$

where q_e and q_t (mg/g) are the adsorption capacity at equilibrium conditions and at contact time t (h), respectively; k_1 (1/h) and k_2 (g/mg h) are the pseudo-first-order and pseudo-second-order rate constant, respectively; I (intercept) is an arbitrary constant, and k_{id} (mg/g $h^{0.5}$) is the rate constant; C_t (mg/L) are the concentrations of adsorbate at contact time t; C_t/C_0 is the relative aqueous concentration; and $q_1(\infty)$ and $q_2(\infty)$ are the adsorption capacity for the instantaneous adsorption and the interior sites at infinite time, respectively; ξ_1 and ξ_2 are the fractions between the amount of the adsorbate adsorbed onto the external surface and into the interior adsorptive sites, respectively, with the total quantity of the initial adsorbate at the infinite time; α (1/h) is the rate constant; β is a limiting factor for $q_2(\infty)$ ($0 < \beta \leq 1$).

Table S4. Chemical properties of selected radionuclides.

Ions	Ionic radius (nm) [1]	Hydration radius (nm) [2,3]	Pauling's Electro-negativity [1]	Hydration enthalpy (kJ/mol) [1]	Charge density (C/mm^3) [4]
Ca(II)	0.100	0.412	1.00	-1600	52
Co(II)	0.075	0.423	1.88	-2035	155
Ni(II)	0.069	0.404	1.91	-2115	134
Sr(II)	0.125	0.412	0.95	-1470	33
Cs(I)	0.170	0.329	0.79	-280	6
U(VI)	0.175	0.251	1.38	-1360	348

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