Review

Topology Optimisation in Structural Steel Design for Additive Manufacturing

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Abstract: Topology Optimisation is a broad concept deemed to encapsulate different processes for computationally determining structural materials optimal layouts. Among such techniques, Discrete Optimisation has a consistent record in Civil and Structural Engineering. In contrast, the Optimisation of Continua recently emerged as a critical asset for fostering the employment of Additive Manufacturing, as one can observe in several other industrial fields. With the purpose of filling the need for a systematic review both on the Topology Optimisation recent applications in structural steel design and on its emerging advances that can be brought from other industrial fields, this article critically analyses scientific publications from the year 2015 to 2020. Over six hundred documents, including Research, Review and Conference articles, added to Research Projects and Patents, attained from different sources were found significant after eligibility verifications and therefore, herein depicted. The discussion focused on Topology Optimisation recent approaches, methods, and fields of application and deepened the analysis of structural steel design and design for Additive Manufacturing. Significant findings can be found in summarising the state-of-the-art in profuse tables, identifying the recent developments and research trends, as well as discussing the path for disseminating Topology Optimisation in steel construction.

Keywords: topology optimisation; topology; steel design; steel structures; optimisation; design methods; design for additive manufacturing; connections; civil engineering

1. Introduction

1.1. The Origins of Topology Optimisation

Notwithstanding historical perspectives, as discussed in [1,2], which root Structural Optimisation and Topology Optimisation (TO) at the very beginning of the classical theory of elasticity, it is usually well accepted that TO had its *de facto* surgency under the name of Optimal Layout Theory and denoted the ability to optimise not only the structural elements shape and size but also its layout. That dates back to successful attempts made by Prager [3,4] and Rozvany [5–8] from the early 1970s to late 1980s, to generalise the 1904s Michell theory for weight optimisation of thin bars (Figure 1a) [9,10], which is based on the Maxwell theorem for frames [11].

Almost simultaneously, Pedersen pioneered the optimal layout design for trusses [12]. In addition, Olhoff [13] managed to optimise Kirchoff equations solutions for finding the plates optimal thickness, leading to ribbed solutions of good value for the aerospace industry. Further developments were endeavoured by Rozvany and Olhoff, among others [14–16].

After these early developments and proven accomplishments in aerospace structures design, TO experienced rapid growth at the beginning of the 1990s as it became easily distinguishable from Shape or Size Optimisation concepts. The latter already well-disseminated the design practice required near-optimal initial topologies and would yield no better than intuitive final layouts, unlike TO [17,18].

Such revolutionary abilities justify the TO massification in several engineering disciplines throughout the product design and manufacture. The automotive industry soon followed the aerospace and applications widened to medical devices and personalised medicine, defence, electronics, several kinds of consumer goods and mechanical engineering endeavours, new materials design, and even arts and architecture. Civil and structural engineering may be latecomers after an auspicious start with truss optimisation investigations [12,19], but also show an accelerating trend in the TO application. Currently, the fourth industrial revolution and its reliance on Additive Manufacturing (AM) processes, on the opposition to standard subtractive processes, face significant challenges, as more advanced, scalable, and user-friendly design methods are required to unleash its incredible potential. TO is undoubtedly an adequate answer for such an enterprise. Its systematic use may overcome the knowledge barriers still moving many practitioners away from AM, as better described by Pradel et al. [20].

1.2. Topology Optimisation Modern Age

Formerly described developments were only made possible by the advances on the homogenisation method, by Bendsøe and Kikuchi in 1988 [21,22] and Suzuki and Kikuchi [23]. In simple terms, the homogenisation method is deemed to solve a material distribution problem, as previously formulated by Kohn and Strang [24–27], considering either only two states: The presence or the absence of the structural material. A more profound explanation and interesting example can be found in Bendsøe and Rozvany books [28,29].

The homogenisation method, along with Svanberg's Method of Moving Asymptotes (MMA) [30], became the basis of most of the following applications, developments, and TO commercial software. However, in 1993, Xie and Steven [31] proposed the so-called Evolutionary Structural Optimisation (ESO) procedure as a possibility for simplifying TO computations by mimicking natural evolutionary processes in Finite Element Analyses (FEA). This led to significant criticism from many other researchers after some shortcomings were identified [32–35].

As the discussion proceeded on the ESO method, justifications were debated, and enhancements have been proposed [36–40]. However, the results of evolutionary methods continue to drive some intense discussions in Academia [41].

Fundamental studies on methods and approaches continued with the work by Duysinx and Bendsøe [42] on the continua TO under stress constraints, as well as in related numerical issues [32,43]. After these fundamental works, TO had a significant growth as a discipline, and the distinction between its two significant sub-fields, the Discrete Optimisation and the Optimisation of Continua became less evident. While the first of those two concepts is deemed to optimise a finite number of known elements and is preferred in several practical applications [44], the second unrestrictedly optimises topology within a solid.

Meanwhile, applications-focused research flourished, with optimisation early works being published in materials design [45], compliant mechanisms [46,47], electronics [48], connections positioning and design [49], buckling phenomena and truss design [50,51] [52], and alternative approaches for the ribbed plate problem [53].

The new millennium brought significant progress in TO algorithms but also in modelling and freely or cheaply accessible software, such as Sigmund's MATLAB code [54,55] or the Karamba plug-in for the Grasshopper environment [56].

Discrete Optimisation had some essential developments, both with algorithms development [57] and practical applications [58,59]. The former depicts an interesting option for Discrete Optimisation in the automotive industry and a good "how-to" example. However, significant numerical problems can arise when node positioning is also considered an optimisable parameter [60].

Within the Optimisation of Continua, many valuable works could be highlighted. Yet, one cannot omit important developments in filters [61,62], projection methods [63,64],

computational methods [65–67], as well as in the controversy evolutionary approach for continua [68]. Furthermore, practical methods, such as Coelho et al.'s useful model with global and local levels of intervention [69], as well as applications in form-finding [70], stiffened plates [71], and TO under load position uncertainty [72], proved the suitability of TO methods for industrial execution. In fact, all the former offer solutions and examples for managing essential aspects of the TO application to structural steel design.

Concomitantly, automotive research centres developed several in-house tools and approaches to introduce TO in industrial Computer-Aided Engineering (CAE). Some examples can be found by Nishigaki et al. [73], Shin et al. [74], Fredricson et al. [44], Aeri and Morrish [75], and Yao et al. [76].

For the TO applications to leverage AM, several notorious works were published from 2010 on. Brackett et al. [77] is an excellent starting point, as it addresses the TO mature approaches, including Solid Isotropic Microstructure (or Material) with Penalisation (SIMP) and Evolutionary Structural Optimisation (ESO), suitable for practical applications. However, much has evolved in the next 10 years, including the consolidation of other approaches, algorithms, and workflows. Therefore, Leary et al. [78] and Nguyen and Vignat [79] worked on design methods offering a good understanding of the theme.

Likewise, insights on AM technologies, as provided in [80,81] are much recommended for fully understanding the role of TO in the fourth industrial revolution.

1.3. Topology Optimisation in Civil and Structural Engineering

Within civil and structural engineering, significant TO applications can be found in several sub-fields. While Discrete Optimisation of truss-like structures is, probably, the most straightforward application—and, indeed, has some early and interesting applications such as the Structural Topology and Shape Annealing (STSA) approach to transmission towers by Shea and Smith [82], the Mixed-Integer Non-Linear Programming (MINLP) approach to industrial steel buildings structural cost optimisation complying with Eurocode 3 (EC3) by Kranvanja and Zula [83], the method for trusses optimisation by Torii et al. [84] or the employment of genetic algorithms by He and Wang [85]—there are some works on the TO of structural systems under seismic loads. Such could be regarded as a surprise, since the seismic design is undoubtedly more complex and demanding compared with static loading common cases but may be explained by the resourcefulness of TO addressing complex issues which, otherwise, would hardly be efficiently solved with analytical means. Among the aforementioned works, one can highlight the steel frames optimisation by Memari and Madhkhan [86], more recent studies of space structures under seismic loads with evolutionary approaches [87,88], a comparison of different soft computing algorithms by Liu and Li, taking infrastructures lifelines as the study object [89], as well as Sarkisian et al.'s well-known mastery for innovative solutions, materialised in the use of TO for meeting significant seismic, aesthetical, budgetary, and regulatory demands for a specific building [90].

The conceptual design of tall buildings gathered practitioners and researchers attention (as depicted in Figure 1b), leading to a more practical design process optimisation [91] or algorithms focused research [92]. Nevertheless, the systematic employment of TO for the conceptual design of tall buildings is particularly evident in the Skidmore, Owings, and Merill experience. As reported by Baker with other SOM engineers and academics [93–97], such solutions were developed for impactful projects and competitions (Figure 1c).

Steel sections and the connections design are two other sub-fields where TO has had an impact. Regarding the former, the Tsavdaridis group (Figure 2a) and Lagaros et al. research on steel beams with web openings [98], Yao et al.'s creative employment of evolutionary algorithms for pre-tensioned cable structures design [99], and Leng's book chapter on cold-formed steel members [100] must be referred. Regarding the latter, it is important to stress out the contributions by Oinonen et al. [101] and Elsabbagh [102] on the bolted steel connections geometric optimisation and stiffeners optimisation, respectively.

The bridge design, on the other hand, has had lesser attention from the TO point of view. Nevertheless, Zhang et al.'s work on bridge design accounting for construction constraints with ESO algorithms [103] and Xie et al.'s Bi-Directional Evolutionary Structural Optimisation (BESO) algorithms application to the bridge conceptual design (Figure 2b) [104] must be mentioned.
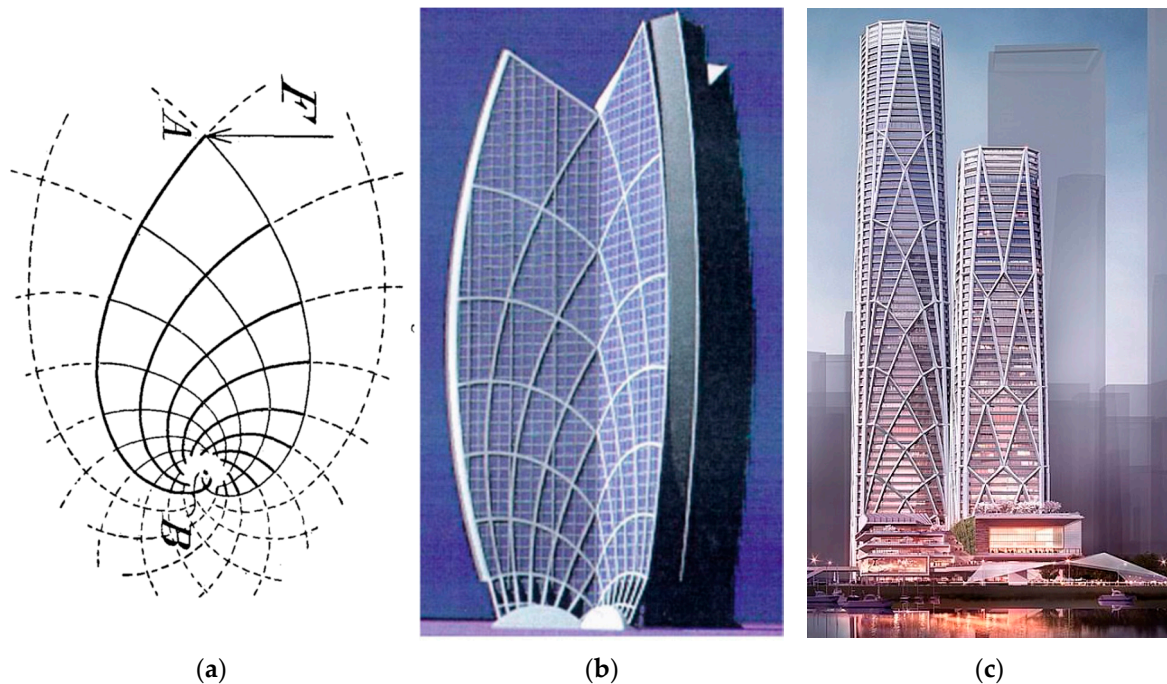

(a)   (b)   (c)

Figure 1. (**a**) Original Michell's minimum frame [9], (**b**) structural design by Zalewski and Zabłocki [105], and (**c**) CITIC financial centre in Shenzhen by SOM [105].

(a)   (b)
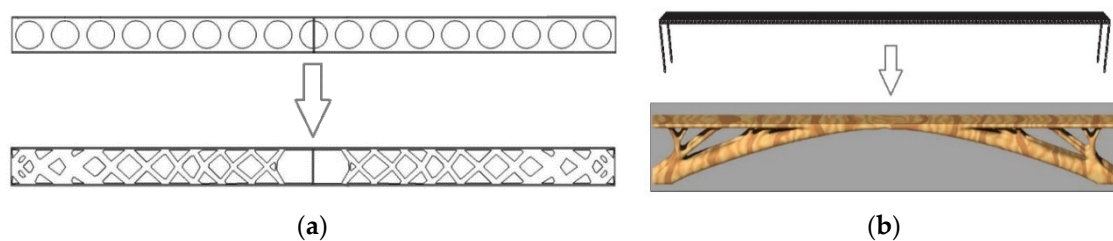
Figure 2. (**a**) From cellular to topology optimised beam [106] (reproduced under the Creative Commons Attribution 4.0 International License, http://creativecommons.org/licenses/by/4.0/ accessed on 25 February 2021) and (**b**) bridge topology optimisation from problem statement to optimal solution [104].

Beyond steel design but still within structural engineering, concrete TO has had some landmarks, including Lee et al.'s work on frame nodes [107], Briseghella et al.'s fresh look into the classical concrete shell supported bridge design [108], Gaynor et al.'s approach to TO for enhancing strut-and-tie models [109], and the use of TO by Chaves and Cunha [110] in the Carbon Fibre Reinforced Polymers (CFRP) reinforcement design for concrete slabs. Other civil engineering sub-fields for TO include the building's thermal behaviour design [111].

1.4. Seminal Works and Systematic Reviews

Most of the former developments are well documented, explained, and exemplified in extensive and, mostly, user-friendly books. That is the case of Haftka and Gurdal's Elements of Structural Optimisation [112] with several editions, the profusely cited Bendsøe

and Sigmund's "Topology Optimisation-Theory, Methods, and Applications" [113] and, more recently, with Arora, Rozvany, and Lewinski books [114,115] and Lewinski et al.'s book [105]. More specific works addressing computational methods [116] or buckling [117] in TO have also recently been made available.

Eschenauer and Olhoff [1] presented an extensive systematic review, encompassing the TO evolution from early structural optimisation to the fundamentals of TO, formulated in Michell structures [9], until the last century's late developments. There, microstructure and macrostructure approaches have been described within a product design context, paving the way for the concepts and approaches developed in the new millennium. The latter include numerical methods such as SIMP, enclosed into the density approaches, and ESO, whose retrospective analysis can be found in another useful Review Article by Rozvany [35]. Moreover, such article is equally crucial for understanding the controversy around ESO (and BESO, its bi-directional development), with several authors referring to it as Sequential Element Rejections and Admissions (SERA) for not employing evolutionary processes nor yielding necessarily an optimal solution.

Before that, Hassani and Hinton [118,119] had summarised the basic concepts and criteria for the emerging TO.

Yet, the history of TO Review Articles is mostly made of sectorial perspectives. Among the first, one can find Fredricson's [120] revision to TO in the automotive industry, focusing on the applications rather than on approaches or methods. Contrariwise, methods-driven reviews have been more frequent, and include assessments about ESO discrete approach developments [121] on Level-Set Methods (LSM or LS) [18]. Broader critical investigations of the former, added to the remaining density approaches, such as Rational Approximation of Material Properties (RAMP), Topological Derivatives (or the Bubble-Method), and Phase-Field Approach, as well as Lagrangian approaches [122,123] are also available.

Optimisation methods in AM have been a theme for a profusion of Review Articles along the last decade. While some assess manufacturing technologies and how optimisation methods can be accounted for in the process [124–128], others analysed the suitability of TO approaches for given AM endeavours, such as construction in Buchanan and Gardner [129], aerospace structures in Plocher and Panesar [130] or the automotive industry in Sehmi et al. [131]. Moreover, optimisation approaches impact the AM customised healthcare, environmental impact, supply chain efficiency, life-cycle analysis, hazards and energy consumption, which is part of Huang et al.'s review [132].

The same decade also brought landmark review articles on particular topics, including TO applications in vibration problems [133], fluid problems [134], and materials design [135]. However, recent reviews on TO-assisted structural engineering are the most meaningful for this article's scope. Those include Kingman et al.'s [136] review on TO employment for perforated steel beams design and tall buildings conceptualisation, a review on buckling by Ferrari and Sigmund [137], Elhegazy's [138] perspective on how TO is a critical part of Value Engineering (VE) and how it benefits the design and life of multi-story buildings, as well as a review by Li and Tsavdaridis on topology optimised and additively manufactured joints for steel structures [139].

Recent reviews on the use of metaheuristic algorithms in civil engineering, by Yang et al. [140] and Bekdaş et al. [141], do not neglect the genetic algorithms of TOs for bridges, roofs, frames, and trusses design.

1.5. Potential for Topology Optimisation and Additive Manufacturing in Construction

The asymmetry in size between the TO related literature in civil and structural engineering and many other fields made clear by the former brief historical overview, suggests that TO applications in construction are far from meeting its potential. Such a fact can only come as a surprise, given the sector size, with a yearly output of USD 10,800,000,000,000 as of 2017 [142], a tradition in analytical structural optimisation and an urgent need for weight reduction along with stiffness and resistance enhancement, as tools for leveraging

the continuous race towards higher buildings, greater spans in bridges and roofs, more economic efficiency, and more ambitious sustainability goals.

Steel construction, deploying 450,000,000 to 815,000,000 tonnes [2,143,144] of structural steel per year worldwide pre-COVID-19 era, and with a market size over USD 100,000,000,000 in 2019 [145], is involved in a global effort to meet decarbonisation and energy efficiency goals, including the Paris Agreement pledge, European Green Deal objectives, and further commitments, such as the UN-backed carbon neutrality by 2050, to which most EU countries abide [146,147]. The path for achieving this is narrow and relies mostly upon employing less steel in constructions, while manufacturing a higher-end product, to more stringent sustainability demands. As a result, AM and, therefore, TO will be indispensable.

Apart from some slight oscillations due to technical details, it is commonly accepted that steelmaking consumes over 560 kg of Coal Equivalent [148] and produces an average of 1.85 carbon dioxide tonnes per tonne of structural steel [147]. Thus, reported weight reductions of 18% to 75% in steel connections [139,149–151], by employing TO, are expected to have a critical impact in steel construction goals.

Within steel design, connections detailing is one activity where the potential to optimise Topology is more significant. Connections usually account between 12% and 25% of most of the steel structure's total weight and its conceptual design has a tremendous impact on its weight and efficiency, making it especially prone to TO.

1.6. Metrics for Research Output in Topology Optimisation

Performing a data analysis with the Scopus search tool (www.scopus.com) on 28 November 2020, it has been possible to observe that Topology Optimisation has been referred in scientific literature since the early 1980s, but not until the new millennium has the related scientific output been consistently increasing (Figure 3a). This fact can be related to the well-known recent computing power increase and massification, on which TO has a strong dependence. Furthermore, the ascendant trend has increased over the last 5 years, up to almost 1200 documents per year in 2020, during which more scientific and engineering disciplines have reportedly started employing TO more systematically. An interesting etymological approach lies in noticing an early word choice for *Topological Optimisation* over *Topology Optimisation* which, however, was not able to be employed in more than 10% of the analysed documents, over the recent years (Figure 4).

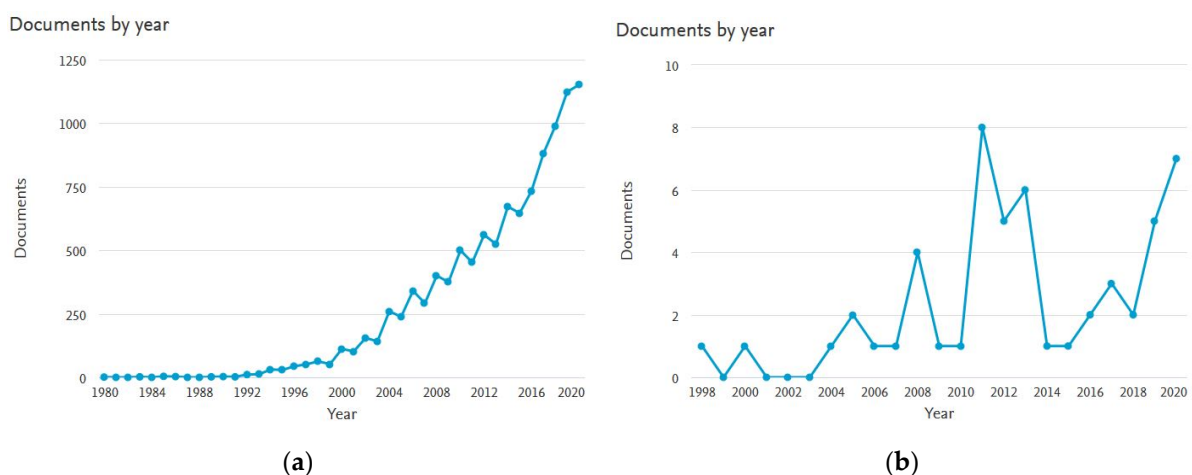(**a**)　　　　　　　　　　　　　　　　　　　　　　　　　　(**b**)

Figure 3. Scopus search data analysis for (**a**) *Topology Optimisation* and (**b**) *Topology Optimisation* AND *Civil Engineering*, in Title, Keywords or Abstract in Research, Review, and Conference articles, on 28 November 2020.
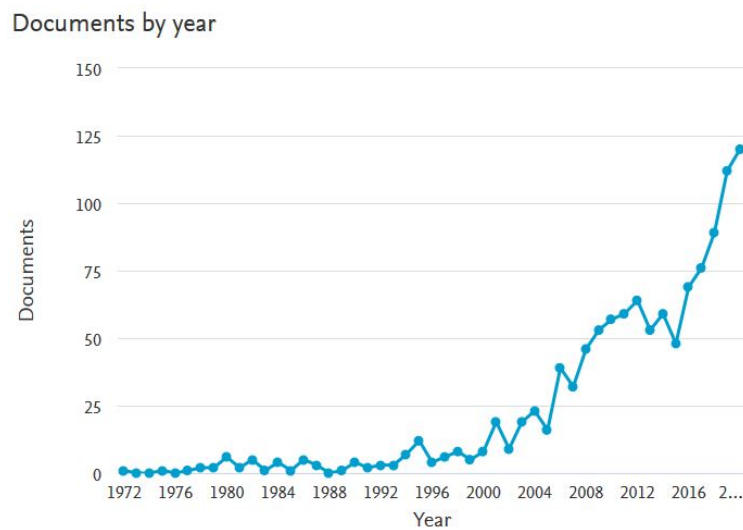
Documents by year

Figure 4. Scopus search data analysis for *Topological Optimisation* in Title, Keywords or Abstract in Research, Review, and Conference articles, on 28 November 2020.

A different perspective is attained analysing the coexistence of TO and *Civil Engineering* (Figure 3b), *Structures* (Figure 5a), and *Steel and Connections or Joints* (Figure 5b) in articles title, keywords or abstract. Among these three, TO and *Structures* is the most common, even if under 2.5% of the total TO articles, and with a steady increasing trend. However, many Mechanical and Industrial Engineering documents use the keyword *Structures*. Furthermore, TO and *Civil Engineering* have been practically not coincident until 2010 and, from there, yield a rather inconstant volume of documents not exceeding 10 per year. A similar conclusion can be drawn for the use of TO and *Steel Connections* (or *Joints*), except for the fact that its modern employment seems to have started in 2005 and that it has not peaked above six documents per year. This suggests that the current TO massification as an advanced engineering design method has not yet been brought to Civil and Structural Engineering comprehensively.
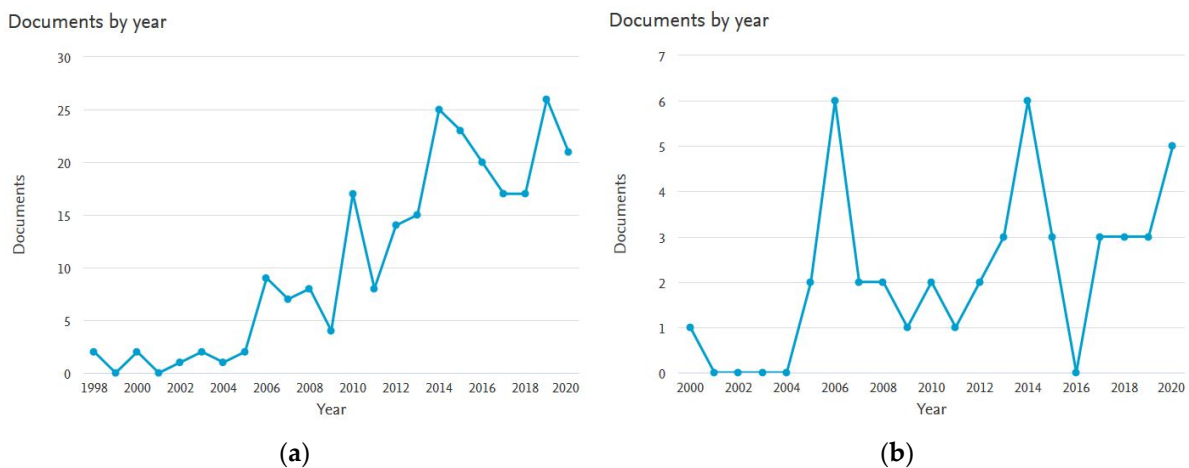
Figure 5. Scopus search data analysis for (**a**) *Topology Optimisation* AND *Structures* and (**b**) *Topology Optimisation* AND *Steel* AND *Connections* OR *Joints*, in Title, Keywords or Abstract in Research, Review, and Conference articles, on 28 November 2020.

Figure 6a as well as Figure 7a,b offer a brief insight on the field leading players. As a result, we observe Sigmund, Nishiwaki, and Xie leading the list of most prolific authors. At the same time, the Dalian University of Technology, Danmarks Tekniske Universitet (DTU) and State Key Laboratory of Structural Analysis for Industrial Equipment are the

most productive research centres. The National Natural Science Foundation of China is the primary funding agent for the research on this topic. It is well ahead of the US National Science Foundation and not matched by the remaining entities.
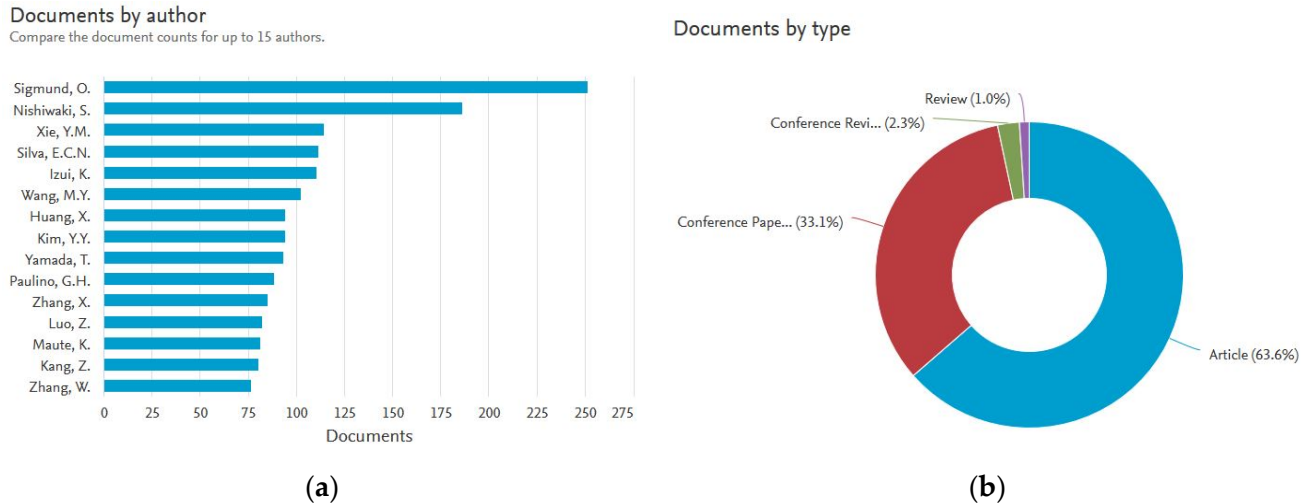
Figure 6. Scopus search data analysis for *Topology Optimisation* in Title, Keywords or Abstract in Research, Review, and Conference articles without date or discipline constraints, on 28 November 2020, organised by (**a**) author and (**b**) document type.

As depicted in Figure 6b, Research Articles account for almost two-thirds of the analysed documents, with Conference Articles making one-third and leaving Conference Reviews and Journal Review Articles with 2.3% and 1.0% of the literature volume, respectively. This suggests a need for systematic reviews on the topic so that the literature is consolidated as a whole and sectorial reviews help each research discipline adapt and incorporate recent advances in TO, which is being developed in other disciplines.
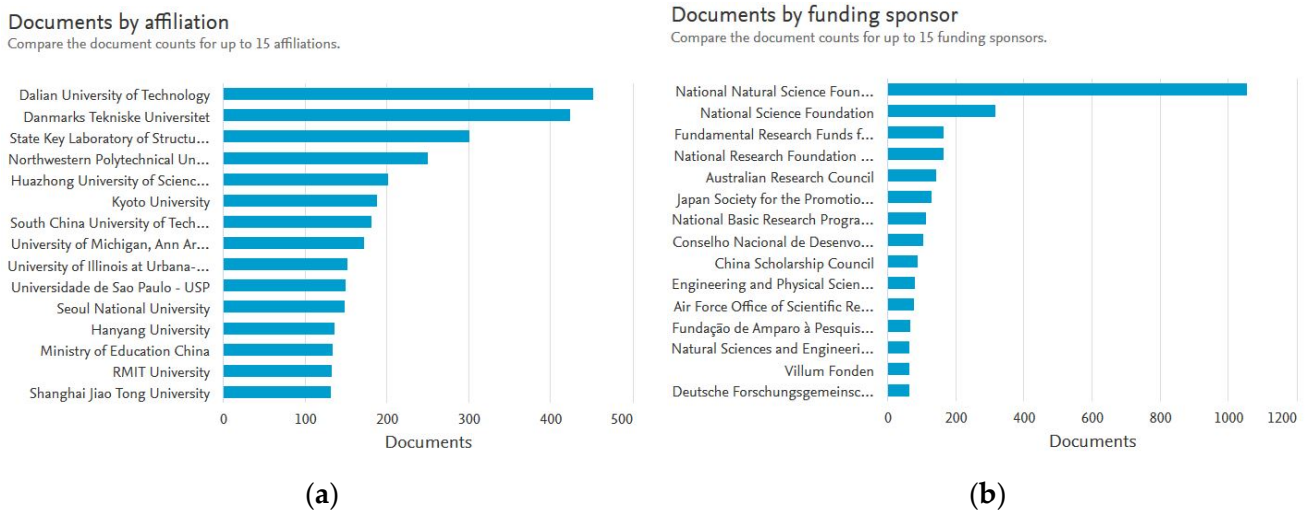
Figure 7. Scopus search data analysis for *Topology Optimisation* in Title, Keywords or Abstract in Research, Review, and Conference articles without date or discipline constraints, on 28 November 2020, organised by (**a**) research institution and (**b**) funding entity.

1.7. Scope of This Document

This work has been developed to systematise recent advances in the Topology Optimisation for structural steel design. It is focused on the 2015–2020 period, apart from this Introductory Section. It is organised with a dual approach, deemed to depict the relatively

modest TO applications in the field in recent years, as well as to systematise significant developments in adjoining fields, which may be inspirational for structural steel design. As a result, it is not expected to collide with any recent Systematic Reviews on TO in other disciplines, while filling the void for a Revision focused on TO recent developments for structural steel design. Therefore, it is aimed to provide a valuable resource and encourage engineers and researchers to embrace TO in a more systematic and sustained manner for structural steel design.

The document structure includes a Methods Section after this Introduction, which will be followed by six sections depicting the literature investigation results, organised in TO fields, approaches, methods, criteria and software, TO in structural steel design, recent advances in other fields with potential for application in structural steel design, TO for AM, and future trends. Afterwards, the most important observations are discussed, and a brief on the attained conclusions is provided.

2. Methods

The literature research on Topology Optimisation was conducted between late November and early December 2020, encompassing Identification, Screening, Sorting, Eligibility Assessment, Information Extraction, Qualitative Synthesis, and Discussion Stages, as better depicted and systematised in Figure 8. While attending to this scientific field's case-specificity, compliance with well-established guidelines for systematic reviews, such as in [152] was pursued. Inspiration was found in other recent Review Articles, such as in [153–156].

Scientific literature has been searched with significant broadness. All published and peer-review items were considered, including Journal Research Articles, Conference Proceedings, Review Articles, Peer-reviewed Book Chapters, and Approved Master and Doctoral Theses. On the other hand, Technical Books, which are not necessarily peer-reviewed, were considered only for framing the research topic in the Introductory Section. Many notable books' unavoidability supports such a decision for defining Topology Optimisation, including Bendsøe and Sigmund's "*Topology Optimisation: Theory, Methods, and Applications*" [113] with 7301 citations according to Google Scholar as of 28 November 2020.
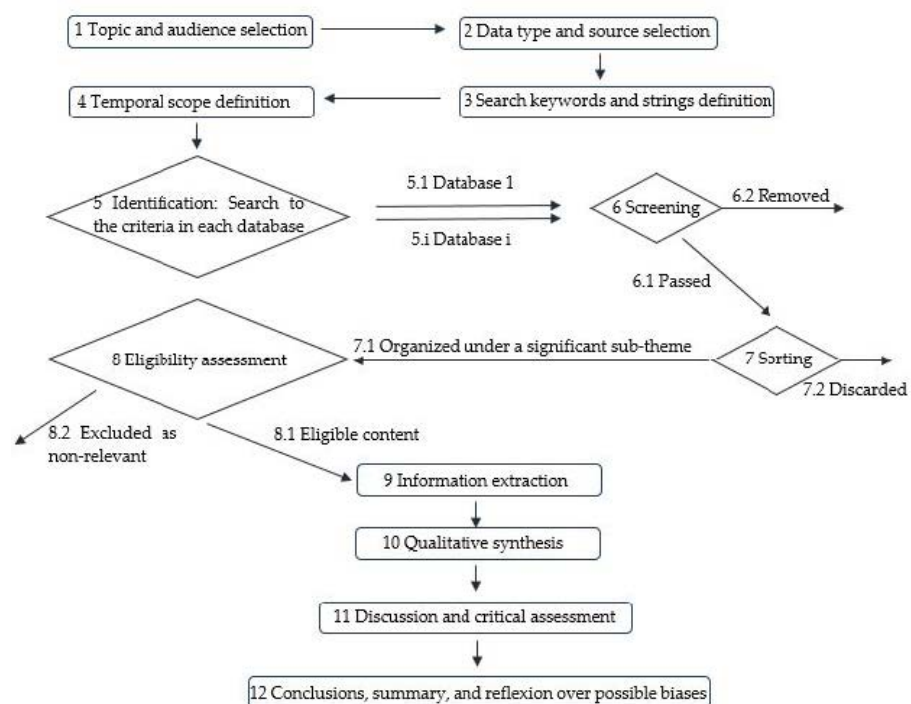

Figure 8. Search and systematic review methodology.

For a more in-depth insight into the current research, industry developments, and future trends, EU-funded research projects as well as European, US, and worldwide patents were also investigated.

The temporal scope for literature search has been set for the most recent 5 years (2015–2020), yet considering the Topology Optimisation novelty as a discipline (illustrated in Figure 3a), previous and very significant research items were investigated and considered for conceiving an explanatory Introduction.

An encompassing fabric of data sources was put together for this endeavour. Thus, Journal Articles and Conference Proceedings were redundantly searched in Mendeley Desktop, Scopus Online, Springer Online, Taylor and Francis Online, and Willey online search engines. Moreover, most active research groups repositories were also investigated, including the DTU TopOpt group, Loughborough University, and the University of Leeds. Several universities Theses repositories were searched. However, literature findings through reading articles was not negligible and added to the former. Patents were found in Google Patents, Espacenet, and USPTO search engines. Research Projects were found in the European Commission projects database, and books were searched using the ProQuest online tool. Table 1 quantifies the search dimensions.

A group of three basic keywords and 12 keyword strings were systematically used in all databases. The former included *"Topology optimization"*, *"Design for Additive Manufacturing"* and *"Topological optimization"*, while the latter are *"Topology optimization"* AND *"Additive Manufacturing"*, *"Topology optimization"* AND *"Steel Design"*, *"Topology optimization"* AND *"Steel Detailing"*, *"Topology optimization"* AND *"Steel Structures"*, *"Topology optimization"* AND *"Structural Engineering"*, *"Topology optimization"* AND *"Connections"*, *"Topology optimization"* AND *"Construction"*, *"Topology optimization"* AND *"Joints"*, *"Topology optimization"* AND *"Review"*, *"Topology optimization"* AND *"Civil Engineering"*, *"Topology optimization"* AND "Multiple *Loading"*, *"Topology optimization"* AND *"Robustness"*. These keywords and strings have been selected to match the articles' Title, Keywords or Abstract, where such option is explicitly available, as it is the case of the Scopus search tool.

Table 1. Search dimensions.

Stage	Included	Excluded
5 Identification	5.1 Mendeley ($n = 68$, of which 27 were eligible) 5.1 Scopus ($n = 292$, of which 253 were eligible) 5.2 Springer ($n = 55$, of which 41 were eligible) 5.3 Taylor and Francis ($n = 21$, of which 15 were eligible) 5.4 Wiley ($n = 24$, of which 16 were eligible) 5.5 Research Groups Repositories ($n = 51$, of which 44 were eligible) 5.6 References found in articles ($n = 181$, of which 174 were eligible) 5.7 Approved Theses Repositories ($n = 7$, of which 4 were eligible) 5.8 Google Patents ($n = 22$, of which 5 were eligible) 5.9 Espacenet ($n = 12$, of which 11 were eligible) 5.10 USPTO ($n = 19$, of which 17 were eligible) 5.11 EU-funded Research Projects ($n = 5$, of which 5 were eligible) 5.12 ProQuest ($n = 10$, of which 10 were eligible)	
6 Screening	$n = 729$	$n = 38$
7 Sorting	$n = 707$	$n = 22$
8 Eligibility	$n = 622$	$n = 85$

The screening criteria for removal included duplicate items, the mismatch between title and content, corrupted files, and the impossibility of accessing the document. Discarded items under the Sorting stage criteria comprised documents whose content do not adhere to any of the sub-themes previously defined as meaningful for this article scope, and better described in Section 3 to Section 8. The eligibility assessment was focused on the document content, excluding items without a particular relevance, without critical information, with any perceived methodological shortcoming or possible strong bias due to funding.

An effort has been endeavoured for performing inclusive research, avoiding the exclusion of less proficiently written articles and language bias. However, some articles written in Chinese and Japanese, for which machine translation was not successful, could not be included.

3. Topology Optimisation—One Concept, Various Fields

3.1. The Mathematical Concept of Topology

Topology, described as General Topology and Algebraic Topology under the Mathematics Subject Classification, studies objects' properties which are subjected to continuous deformations [157–159]. Hence, a topological space (or domain), commonly referred to as a topology, maintains its properties, including dimension, compactness, and connectedness if undergone such deformations. Invariance within topological domains accrues in reversibility under continuous deformations or homomorphism, making this abstract concept crucial for the Topology Optimisation's founding principles, as formulated by Maxwell and Michell.

3.2. Structural Optimisation and Topological Optimisation in the Context of Structural Steel Design

Structural Optimisation and Topological Optimisation concepts have been used with increasingly unrestrained freedom, even in Academia. Thus, it may be helpful to mention that the most rooted nomenclature employs Structural Optimisation as an umbrella for Topology Optimisation, on the one hand, and Shape and Size Optimisation, on the other hand [44,160]. In the context of Structural Steel Design, while seldomly, the use of Structural Topology Optimisation (STO) has been reported [139], referring to TO in this specific field.

Conceptually, Shape and/or Size Optimisation is easy to define, considering its scope, limited to variables as cross-sectional properties, member types and geometry, and rigidly constrained by predefined configurations. In simple terms, Topology cannot be changed and, therefore, this concept is deemed to improve an existing design, in which it very much depends on closeness to optimal.

Topology Optimisation, also named Layout Optimisation by some pioneers, bears the capacity for topological modification. In other words, it is unrestricted in its ability to create voids or add material in the design domain and act upon the structure's connectivity. That said, it is relatively straightforward that TO contains and exceeds the Shape and Size Optimisation and therefore, fulfils all the possible Structural Optimisation scopes.

It should be highlighted that TO is not constrained to materially homogenous volumes. It may be employed as heterogeneous, including composite or microstructurally designed materials, but also in grid-like "ground structures", made of one-dimensional elements.

In practical terms, differences are significant not so much between concepts, but mostly between applications and engineering fields. Concerning structural steel design, the Size, Shape, and Topology Optimisation can be graded into an intervention freedom continuous scale. The practical constraints and one-dimensional members design may limit optimisation to size and shape, even if the possibility for adding or suppressing members in a predefined configuration is available. The optimisation of continua, either in small volumes as joints or macro elements, as a building or bridge geometrical envelope, find a most suitable tool in TO.

3.3. Discrete Optimisation and Optimisation of Continua in the Context of Structural Steel Design

Topology Optimisation applications can be divided into Discrete Optimisation and Optimisation of Continua, based on the Topology considered for the optimisation problem (Figure 9).
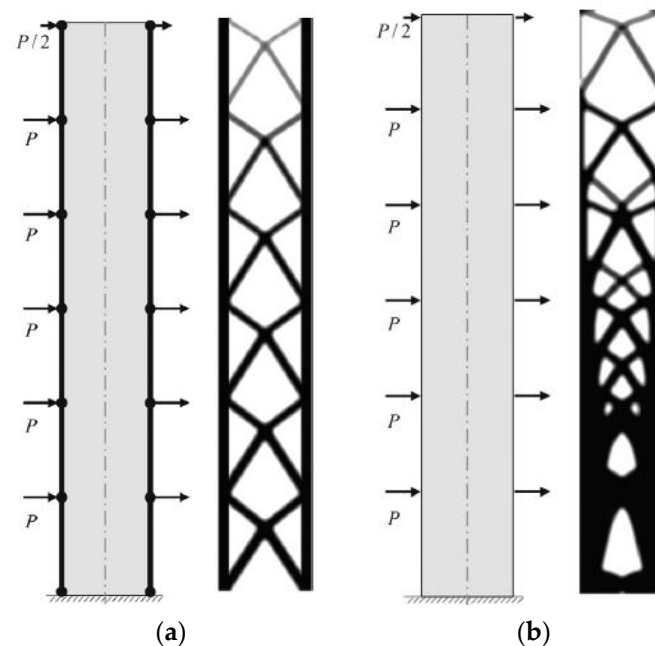
Figure 9. Example of (**a**) problem statement and discrete optimisation [94] and (**b**) problem statement (**left**) and optimisation of continua (**right**) [94].

The optimisation of discrete structures has a history of its own. It has been present in structural engineering from its early days, even if with a different scope, and this reason justifies why some researchers find the roots of TO in the classical theory of elasticity. As suggested by its name, Discrete Optimisation is directed to discrete structures. Its objectives lie in finding the optimal number, location, shape, size, and connectivity for structural elements and nodes coordinates. Therefore, one can understand that most early Discrete Optimisation applications have been limited to Shape and Size Optimisation, rather than TO, due to significant limitations on the optimisable variables' domain. For the very same reason, Discrete Optimisation has been used under the names of Truss Topology Optimisation (TTO) or Topology Optimisation of Skeletal Structures (TOSS) and its early applications to finding optimal layouts led to the name Layout Optimisation, regarded by many as a former name for TO.

However, the Discrete Optimisation scope exceeds one-dimensional members. In fact, since the advances by Prager and Rozvany, which used grid-like structures, TO has grown as a merger of techniques and approaches, deemed to find optimal solutions for specific cases. Continuum structures can be modelled as ground structures—complex systems of one-dimensional elements—and therefore, be subjected to Discrete Optimisation.

A major issue of Discrete Optimisation can be attributed to considering nodes coordinates in or out of the optimisation domain. If coordinates are not possible to optimise, solutions can easily result in a plethora of thin, one-dimensional structural element. Conversely, if the nodes positions vary, node superposition will likely occur, leading to significant computational problems.

The Optimisation of Continua is usually applied to solids, shells, or design envelopes (which, in fact, are solids) and is deemed to optimise the design space external boundaries shape, as well as the internal boundaries shape. The latter refers to the newly created boundaries between the material and void.

4. Approaches, Methods, Criteria, and Software in Topology Optimisation

4.1. Approaches and Optimisation Methods

4.1.1. On the Nomenclature Complexity

An unexpected layer of complexity in TO lies in the terminology. Approaches, methods, methodologies, models, techniques, and algorithms, etc. are used in conflicting ways by an expanding and very diverse research and practice community.

In this article, a choice has been made to refer to the comprehensive strategies for solving a well-defined TO problem as "methods". Such an option adheres to some of the most respected leaders and book-writers in disciplines, such as Bendsøe, Rozvany, and Sigmund, to name a few, with some occasional exceptions. In this manner, the well-known SIMP, RAMP or ESO are considered methods, while others are grouped by similarity. That is the case of OMP and NOM, both nested under the "Homogenisation" methods family. Such an option is, evidently, arguable.

Other options resided in considering six approaches, in which the defining criterion was the ability to group methods by its functional similarity. Thus, Density-Based, Level-Set, Topological Derivatives, Phase-Field, Heuristic and Hybrid approaches were accounted. As a result, other approach classifications, such as Material/Geometrical or Lagrangian/Eulerian, could not be simultaneously considered.

The problem resulting from the possibility of employing different Optimality Criteria (OC) for one given method and possibly using the same Optimality Criterion for different methods could only be solved by creating a diverse group for OC.

However, other problems arise from using different names for the same method or for different authors' applications of the same method. For example, one can consider the ESO method, also referred to as SERA by researchers who were not involved in its original proposition, regarding their views on the method shortcomings. Likewise, the Bubble Method is referred under different names, and recent developments in Level-Set Methods and Topological Derivatives have not yielded universally accepted names for its methods, due to several increments from many researchers and applications to many different problems.

Contrariwise, several well-known methods have remarkable resemblances which could be better regarded as different applications of a single method. That can be found in the extended ESO family, in applying Genetic Algorithms (GA) and Swarm Methods, and in the different options for solving the Hamilton-Jacobi Equation in Level-Set Methods by only a handful of examples. An option was taken to leave all the methods that have some expression in the surveyed literature, regardless of its similitude, and group methods that differ only in using well-known algorithms, such as GA or Swarms.

4.1.2. Framing New Developments into the Body of Knowledge

It was never easy to organise TO in approaches, methods, or sub-fields, as one can observe from the notorious Review Articles' classification discrepancies, such as in [1,35,119,122] or [131]. However, recent research adds complexity in this issue due to so many outputs on particular, yet transversal, issues.

At the current status of accelerated progress and interrelatedness in TO methods and algorithms, it is quite challenging to frame newly published research into categories, so that its value and applicability for one certain problem is evident. For such an end, one can find utility in Table 2, where general approaches and methods were organised to the formerly depicted criteria, and the recent relevant research was inserted. If researchers and practitioners, especially the newly arrived in this field, find it helpful to navigate through the literature and understand the potentially useful contributions, this table will accomplish its purpose.

Table 2. Topology optimisation (TO) approaches, methods, and recent developments.

Approach	References	Non-Exhaustive List of Methods		References	Recent Developments
Density-based (also Material Distribution)	[35,122,161,162]	Homogenisation	Optimal Microstructure with Penalisation (OMP)	[163,164]	[165–200]
			Near-optimal Microstructure (NOM)	[21,201]	
		Rational Approximation of Material Properties (RAMP)		[202]	
		Solid Isotropic Microstructure (or Material) with Penalisation (SIMP)		[22,32,35,42]	
		SINH (due to employing the hyperbolic sine function)		[203]	
		Sum of the Reciprocal Variables (SRV)		[204]	
		Reliability-Based Topology Optimisation (RBTO)		[205–212]	[213,214]
Level-set (LS) methods	[18,215–222]	Conventional LS for solving the Hamilton-Jacobi Equation		[164,217,223]	[170,171,224–233]
		Radial-Basis Functions (RBF) for solving the Hamilton-Jacobi Equation		[234–239]	
		Spectral LS		[240]	
		Non-Linear Programming		[234]	
Topological Derivatives	[241–243]	Bubble Method		[1,244,245]	[246,247]
		Topological Sensitivity		[243,248,249]	
Phase-field approach	[250,251]	Cahn-Hilliard Method		[252–254]	[171,255,256]
		Allen-Cahn Method		[257]	
		Relaxed Phase-Field Methods		[250,252,258]	
Heuristic (also Non-gradient or Evolutionary) [a]	[35,123,141,259–263]	Evolutionary Structural Optimisation (ESO), also Sequential Elements Rejection and Admission (SERA)		[31,35,68,264]	[165,265–287]
		(Hard-kill) Bidirectional ESO (BESO)		[35,121,288–294]	
		Additive ESO (AESO)		[123,131,295]	
		Soft-kill BESO		[121,296,297]	
		Swarms, including Particle Swarm Optimisation (PSO), Fish Swarm Optimisation (FSOA), Ant Colony Optimisation (ACO), Stochastic Diffusion Search (SDS), Artificial Swarm Intelligence (ASI), Multi-Swarm Optimisation, Artificial Bee Colony Algorithm (ABC)		[131,140,298,299]	
		Genetic Algorithms (GA), including Genetic ESO (GESO), Lindenmayer (also map-L) Method		[300–311]	
Hybrid approaches	[122]	Combination of several features and techniques			[312,313]

[a] Another common name for this approach is the "Hard-kill" method. However, it does not account for its current diversity, which includes the "Soft-kill" option.

Recent research is generally less focused on fundamental and theoretical issues and more attentive to computational issues, application details, and case-specificity. There is also a trend to employ and mix concepts from different methods. For these reasons, framing recent research into approaches and methods is increasingly difficult and potentially erroneous.

One other interesting issue concerns SIMP methods. While such a theme dominated the research output for a long time, new publications devoted to it are relatively decreasing (mostly compared to the escalating numbers of TO related papers). Moreover, many research and industrial endeavours still employ SIMP, even if depicting and discussing it ceased to be considered pivotal.

Among the newer approaches, Level-Set Methods appear to be in accelerated development. This can be explained by the migration of density-based, and especially SIMP, researchers and practitioners to an approach with so many standard features.

On the other hand, Phase-Field Approaches are yet to gain momentum and Topological Derivatives, even if with a significant history, have not much recent research output. The latter, however, continues to provide a background for many developments and comparisons in mainstream methods.

With the Heuristics group, it is quite interesting to observe that the last two decades of profuse output in ESO and BESO methods seem to decrease, while Genetic Algorithms are sharply on the rise.

The working-set approach by Verbart and Stolpe [314] has not been included in the previous table due to its versatility, allowing an easy adaptation to several methods and optimality criteria.

Recent developments, such as the Deformable Simplicial Complex (DSC) Method [315] and the Virtual Scalar Field Method (VSFM), which allows considering a connectivity constraint as a thermal effect [316], show interesting features which substantiate mentioning it in this review. However, it is not yet the time to insert it in the Table 2 classification, as further developments will tell whether specific categories are justified.

Analogously, other methods, such as the Moving Morphable Components (MMC) have been proposed in the past [317,318], as alternatives to the more established aforementioned ones, but its applications beyond these authors' works remain not very profuse. However, promising contributes are regularly obtained by employing it, such as the Virtual Component Skeleton (VCS) method by Wang et al. [319], for controlling topologically optimised boundary smoothness, which deserves to be referred.

For the Discrete Optimisation of trusses, the article by Zhang et al. [320] is fundamental for understanding the Ground Structure Method (GSM), as well as its Voronoi and quadrilateral methods of discretisation.

Recent conclusions on the Equivalent Static Loads Method (ESLM) [321] offered clarity to previously reported findings and highlighted the caution needed for analysing the potential of the methods.

For an interesting discussion on preconditioning, its impact upon computing efficiency, and an example on Preconditioned Conjugate Gradients (PCG), the reader is referred to the work by Kaveh et al. [322].

4.1.3. Heuristics as a Source of Controversy

The use of optimisation methods whose solution is not necessarily optimal, referred to as Heuristics, has been in the centre of discussion on the TO theory, since the first so-called Evolutionary methods were proposed. Without entering a discussion already well depicted in [35,121–123], criticism is mostly due to the reported incapacity of evolutionary methods for attaining convergent optimal solutions, to failing to achieve acceptable solutions in some cases, and to the difficulty in generalising the method for real structures constraints.

It is quite interesting that some other researchers highlight the employment of Heuristics in filters and other techniques deemed to avoid local optima, which are used well beyond the evolutionary methods.

As expected, many of those claims have been rebutted, discussed, but also admitted and led to many of the current developments.

As a result, the current discussion on TO methods in Academia is still centred on the SIMP/BESO antagonism, as well as in the new developments in Level-Set, Topological Derivatives, and Phase-Field Methods, while practical applications are mostly employing SIMP methods.

Considering that Meta-Heuristics (high-level procedures for combining or selecting heuristic methods for one given problem solving or adequate approximation) and Artificial Intelligence (AI) based methods are rapidly spreading within civil and structural engineer-

ing [323,324], due attention will be given to the Heuristics approaches in structural steel design in this article. However, the current imbalance must be highlighted.

4.2. Optimality Criteria

Considering the impracticability in proving that an attained solution is mathematically correct when thousands of variables are involved, the Optimality Criteria (OC) had to be set. For such an end, several precursory intuitive criteria were used, such as the Fully Stressed Design (FSD) and the Simultaneous Failure Mode Design (SFMD) until the so-called rigorous criteria were adopted. The latter epithet is usually given to any criterion complying with Kunh-Tucker optimality conditions.

Table 3 systematises the most common OC, which is applied to the prevalent Density-Based Approach methods.

Table 3. Frequently used optimality criteria methods.

OC Methods		References
	Discretised, Continuum-type Optimality Criteria technique (DCOC)	[119,325]
	Continuum-based Optimality Criteria (COC)	[119,326]
	Iterative COC	[119,327]
	Design Optimisation Tools (DOT)	[123]
	Sparse Nonlinear Optimiser (SNOPT)	[328]
General optimisation codes	Interior Point Optimiser (IPOPT)	[329]
	Convex Linearisation Method (CONLIN)	[42,330,331]
	Method of Moving Asymptotes (MMA)	[30,332]
	Globally Convergent Method of Moving Asymptotes (GCMMA)	[333]

4.3. Practical Methodologies

Unsurprisingly, the literature devoted to TO is profuse in depicting theoretical approaches and methods, as well as in validating it with well-known or trivial cases, but much scarcer in providing comprehensive and holistic methodologies (which contain practical aspects) for implementing a TO strategy into the Engineering design. This is related not only to researchers' tendency to publish their work and industrial practitioners and developers not doing so, but also to the fact that TO is still in a stage of developing and stabilising methods before a universal application by non-experts. Moreover, Topology Optimisation is still much more complex than the regular structural analysis and design in most engineering fields, requiring a significant time, studying and computational resources that limit the number of large-scale projects currently being developed.

One further issue lies in TO objectives. While academic developments must seek assurance of optimisation to the theoretical extrema, industrial applications are usually comfortable with optimisation to a certain pre-defined threshold and value reliability, predictability, reproducibility and, frequently, computing efficiency above all. However, avoiding local extrema is a common goal.

Nevertheless, some examples were found in a recently published literature, which can be referred to as interesting examples for conceiving case-specific methodologies for the systematic application of TO in structural steel design. Table 4 summarises those findings.

Table 4. Methodologies or strategies for practical TO implementation in engineering design.

Methodology/Strategy	Reference
Method for the TO of frame structures with flexible joints	[44]
Optimisation for Manufacture (OFM) methodology for introducing manufacturing time and cost into the TO problem	[334]
Axiomatic Design Method for AM	[335]
TO-directed manufacturing methods	[221]
Using surrogate models for conceptual design	[336]
TO method to mitigate AM-induced anisotropy	[337]
Methodology for introducing AM time and cost into the TO problem	[183]
Lumped Parameter Model (LPM) for multiphysics problems	[338]
Fail-Safe Methodology	[339,340]
Sectional Optimisation Method (SOM)	[341]
An AM-focused TO strategy using the SIMP method (for automotive parts)	[342]
Integrated design optimisation by Skidmore, Owings, and Merrill LLP (SOM)	[343]
Methodology for the TO of cellular structures for AM	[344]
Projection-based Ground Structure TO Method (P-GSM) as an advance in Ground Structure Methods (GSM) for addressing the issue of complex geometries, as well as small, disconnected, buckling-prone, and non-manufacturable elements	[194]
The TO method accounts for AM geometrical, mechanical, and machining constraints	[345]

4.4. Computer Programmes

The availability of computational resources is paramount for ensuring TO applications beyond the research community, which has the ability to produce their software. That is undoubtedly the case of Structural Steel Design researchers and professionals, to whom software development may not be the primer priority.

Fortunately, both the commercial software and code provided by researchers and developers are available. However, while the former frequently comes as a "black-box" and can even be challenging to be aware of the employed approaches and methods, the latter may lack user-friendliness, require pre- and post-processing, lack graphical interfaces, and not provide an adequate tool for applications more complex than trivial examples.

Considering the relevance of computer programmes for developing TO strategies, a review of the currently available and reportedly more used commercial software and computer programmes is summarised in Table 5.

Table 5. TO commercial software and computer programmes.

Programme	Nature	Runs on	Approaches/Methods	Reference	Advantages	Disadvantages
Altair HyperWorks platform, including OptiStruct solver and Inspire interface	Commercial	Independent	SIMP and LS methods	[35,130,131,345–353]	User-friendliness. Broadness of use and industrial testing.	Cost. Difficulty in controlling the process. Impossibility in modifying the code.
NASA's NASTRAN code	Freeware	Written in Fortran, can be adapted to the user's resources	Density-based approaches	(a)	Freeware. Explicit control over the results and code. The oldest and more tested code for industrial applications.	Laborious input and output. Lack of user-friendliness.
MSC NASTRAN	Commercial	Independent	SIMP, Density-based approaches	[35], (a)	Uses the oldest and more tested code for industrial applications.	Cost. Difficulty in controlling the process. Impossibility in modifying the code.

Table 5. *Cont.*

Programme	Nature	Runs on	Approaches/ Methods	Reference	Advantages	Disadvantages
Simcenter 3D (former NX NASTRAN)	Commercial	Independent	Density-based approaches	[354], (a)	User-friendliness. Uses the oldest and more tested code for industrial applications.	Cost. Difficulty in controlling the process. Impossibility in modifying the code.
Autodesk Inventor NASTRAN (former NEi NASTRAN)	Commercial	Independent	SIMP	(a)	Uses the oldest and more tested code for industrial applications.	Cost. Difficulty in controlling the process. Impossibility in modifying the code.
Autodesk Within (former Within Enhance)	Commercial	Independent	Uses both the Within Enhance and NEi NASTRAN solvers	[355], (a)	User-friendliness.	Cost. Difficulty in controlling the process. Impossibility in modifying the code.
ANSYS Mechanical	Commercial	Independent	SIMP	[35,130,356]	User-friendliness. Pre-defined options. Integration in a very reliable FEA package.	Cost. Lesser propensity for user-defined options. Optimisation algorithms lesser disclosure. Impossibility in modifying the code.
TOSCA Structure	Commercial	Dassault Systèmes' Abaqus; Dassault Systèmes' SOLIDWORK; ANSYS; MSC Nastran	Formerly ESO, SIMP + MMA in recent editions	[35,345,349,357–360]	User-friendliness. Integration in very reliable FEA packages.	Cost. Difficulty in controlling the process.
GENESIS	Commercial	Either independent or for ANSYS	SIMP, RBTO	[35,130], (a)	User-friendliness. Profusely used in industrial applications.	Cost. Difficulty in controlling the process. Impossibility in modifying the code.
Intes PERMAS	Commercial	Independent	SIMP, RAMP	[130], (a)	User-friendliness. Attentive support. Profusely used in industrial applications.	Cost. Difficulty in controlling the process. Impossibility in modifying the code.
Samtech Boss Quattro	Commercial	Independent	Density-based approaches and Genetic Algorithms	[130,361]	User-friendliness. Profusely used in industrial applications.	Cost. Difficulty in controlling the process. Impossibility in modifying the code.
COMSOL	Commercial	Independent	Density-based approaches and LS methods	[130,362,363]	User-friendliness. Profusely used in industrial applications. Cost.	Cost. Difficulty in controlling the process.
Karamba3D	Commercial	Rhinoceros or Grasshopper	Genetic Algorithms	[56,364]	User-friendliness. Already used in some Architecture and Structural Engineering applications.	Difficulty in controlling the process. Impossibility in modifying the code.

Table 5. *Cont.*

Programme	Nature	Runs on	Approaches/ Methods	Reference	Advantages	Disadvantages
DTU TopOpt app	Freeware	Grasshopper	SIMP	(a)	Freeware. User-friendliness. Developed and tested by the scientific community.	Impossibility in modifying the code.
DTU TopOpt programme	Freeware	Web browser	SIMP	[132,365]	Freeware. User-friendliness. Developed and tested by the scientific community.	Impossibility in modifying the code.
DTU TopOpt Portable and Extendable Toolkit for Scientific Computing (PETSc)	Freeware	Portable code, which can be implemented in Windows, Linux, etc.	Customisable	[366]	Freeware. Very useful when employing significant computational resources. Developed and tested by the scientific community.	Much more difficult to intervene over the code compared to other DTU's freeware codes. Case-specific.
DTU TopOpt mobile app	Freeware	Android, iPhone	SIMP	[123,367]	Freeware. User-friendliness. Developed and tested by the scientific community.	Impossibility in modifying the code.
DTU TopOpt Shape mobile app	Freeware	iPhone	Hybrid	[312]	Freeware. User-friendliness. Developed and tested by the scientific community.	Impossibility in modifying the code.
DTU Sigmund SIMP code for MATLAB	Freeware	MATLAB	SIMP	[54], Appendix of [122,359]	Freeware. Developed and tested by the scientific community.	Code modification is needed for most real cases. Less user-friendliness. More prone to user errors. Difficult to apply in complex geometries.
DTU Sigmund SIMP code for MATLAB new (2020) generation	Freeware	MATLAB	SIMP	[186]	Freeware. Developed and tested by the scientific community.	Code modification is needed for most real cases. Less user-friendliness. More prone to user errors.
DTU Andreassen SIMP code for MATLAB	Freeware	MATLAB	SIMP	[55], Appendix of [122,359]	Freeware. Developed and tested by the scientific community.	Code modification is needed for most real cases. Less user-friendliness. More prone to user errors. Difficult to apply in complex geometries.
DTU Andreassen LS code for MATLAB	Freeware	MATLAB	LS methods	[232], (a)	Freeware. Developed and tested by the scientific community.	Code modification is needed for most real cases. Less user-friendliness. More prone to user errors.

Table 5. *Cont.*

Programme	Nature	Runs on	Approaches/ Methods	Reference	Advantages	Disadvantages
Python alternatives to DTU (Sigmund, Andreassen et al.) MATLAB codes	Freeware	Python	SIMP	[a]	Freeware. Developed and tested by the scientific community.	Code modification is needed for most real cases. Less user-friendliness. More prone to user errors.
Zuo and Xie's BESO code for Python	Freeware	Python	BESO	[359,368]	Freeware. Developed and tested by the scientific community.	Code modification is needed for most real cases. Less user-friendliness. More prone to user errors. Difficult to apply in complex geometries. Some researchers still contest the approach.
Liu and Tovar's SIMP code for MATLAB	Freeware	MATLAB	SIMP	[359,369]	Freeware. Developed and tested by the scientific community.	Code modification is needed for most real cases. Less user-friendliness. More prone to user errors. Difficult to apply in complex geometries.
Huang and Xie's BESO code for MATLAB	Freeware	MATLAB	Soft-kill BESO	[121]	Freeware. Developed and tested by the scientific community.	Code modification is needed for most real cases. Less user-friendliness. More prone to user errors. Some researchers still contest the approach. Difficult to apply in complex geometries.
Suresh's Pareto-optimal tracing code for MATLAB	Freeware	MATLAB	Topological Sensitivity	[370]	Freeware. Developed and tested by the scientific community.	Code modification is needed for most real cases. Less user-friendliness. More prone to user errors. Difficult to apply in complex geometries.
Challis' LS code for MATLAB	Freeware	MATLAB	LS Methods	[371]	Freeware. Developed and tested by the scientific community.	Code modification is needed for most real cases. Less user-friendliness. More prone to user errors. Difficult to apply in complex geometries.
TOBS (Topology Optimisation of Binary Structures) code for MATLAB	Freeware	MATLAB	Gradient-based method with binary variables	[372]	Freeware. Developed and tested by the scientific community.	Code modification is needed for most real cases. Less user-friendliness. More prone to user errors. The approach undergone recent developments and it is not yet clear if it will gather acceptance.

Table 5. *Cont.*

Programme	Nature	Runs on	Approaches/ Methods	Reference	Advantages	Disadvantages
Wei et al.'s LS code for Matlab	Freeware	MATLAB	LS methods	[373]	Freeware. Developed and tested by the scientific community.	Code modification is needed for most real cases. Less user-friendliness. More prone to user errors. Difficult to apply in complex geometries.
Wang et al.'s TOPLSM for MATLAB	Freeware	MATLAB	LS methods	[369,373]	Freeware. Developed and tested by the scientific community.	Code modification is needed for most real cases. Less user-friendliness. More prone to user errors. Difficult to apply in complex geometries.
Schmidt and Schulz's SIMP code for MATLAB	Freeware	MATLAB	SIMP	[373,374]	Freeware. Developed and tested by the scientific community.	Lengthy code, which hinders its edition. Code modification is needed for most real cases. Less user-friendliness. More prone to user errors. Difficult to apply in complex geometries.
Polytop for MATLAB	Freeware	MATLAB	SIMP	[373,375]	Freeware. Developed and tested by the scientific community.	Code modification is needed for most real cases. Less user-friendliness. More prone to user errors. Difficult to apply in complex geometries.
Zhou et al.'s BESO code for MATLAB	Freeware	MATLAB	BESO	[373,376]	Freeware. Developed and tested by the scientific community.	Code modification is needed for most real cases. Less user-friendliness. More prone to user errors. Difficult to apply in complex geometries. Some researchers still contest the approach.
Otomori et al.'s LS code for MATLAB	Freeware	MATLAB	LS methods	[373,377]	Freeware. Developed and tested by the scientific community.	Code modification is needed for most real cases. Less user-friendliness. More prone to user errors. Difficult to apply in complex geometries.
Xia and Breitkopf's Homogenisation code for MATLAB	Freeware	MATLAB	Homogenisation	[373,378]	Freeware. Developed and tested by the scientific community.	Code modification is needed for most real cases. Less user-friendliness. More prone to user errors. Difficult to apply in complex geometries.

Table 5. *Cont.*

Programme	Nature	Runs on	Approaches/Methods	Reference	Advantages	Disadvantages
Zhang et al.'s MMC code for MATLAB	Freeware	MATLAB	MMC	[373,379]	Freeware. Developed and tested by the scientific community.	Code modification is needed for most real cases. Less user-friendliness. More prone to user errors. Difficult to apply in complex geometries.
OpenMDAO	Freeware	Independent (Platform for modular code)	SIMP and LS methods	[380,381]	Freeware. Very useful for developing and adapting code with modularity.	It is not a TO programme and code needs to be developed. Suitable for research and learning, not so much for industrial applications.
Allaire's LS code for Scilab	Freeware	Scilab	LS Methods	[373]	Freeware. Developed and tested by the scientific community.	Code modification is needed for most real cases. Less user-friendliness. More prone to user errors. Difficult to apply in complex geometries. Many researchers are not used to Scilab environment.
FreeFem + +	Freeware	Can be adapted to the user's resources	LS Methods	[123,382]	Freeware. Developed and tested by the scientific community.	Code modification is needed for most real cases. Less user-friendliness. More prone to user errors.
Chisari's TOSCA (Tool for Optimisation in Structural and Civil engineering Analyses)	Freeware	Independent	Genetic Algorithms	[383]	Freeware. User-friendliness.	Difficulty in controlling the process. Recent and not profusely tested by the community.
Lagaros' C# code for SAP2000	Freeware	SAP2000 open application programming interface (OAPI)	SIMP	[192,359]	Freeware. Very useful for civil and structural engineering practitioners. Developed and tested by the scientific community.	Limited user-friendliness for the target group. Once implemented in SAP2000, process control and code modification are limited.
He's script for adaptive layout optimisation of trusses	Freeware	Python	Discrete Optimisation; Heuristics	[384]	Freeware. User-friendliness. Allows member adding. Developed and tested by the scientific community.	Constrained to the optimisation of ground structures.

Table 5. *Cont.*

Programme	Nature	Runs on	Approaches/ Methods	Reference	Advantages	Disadvantages
GRAND (Ground structure analysis and design)	Freeware	MATLAB	Discrete Optimisation	[385,386]	Freeware. User-friendliness. Developed and tested by the scientific community.	Constrained to the optimisation of ground structures. Does not allow Member Adding.
Sokól's truss optimisation code for Mathematica	Freeware	Mathematica	Discrete Optimisation	[387]	Freeware. Developed and tested by the scientific community.	Constrained to the optimisation of ground structures. Does not allow Member Adding. Many researchers are not used to Mathematica programming.

[a] Based either on publicly available information or on personal communications with the companies' contacts (which can be disclosed).

A remark shall be made concerning the *Advantages* and *Disadvantages* columns. Not only does the information rely on the consulted literature and commercial software technical detailing, but it also compares very different programmes. Commercial software is fundamentally different from free codes. Therefore, the advantages and disadvantages are essentially focused on each programme's nature and much less on its quality. Regarding the latter, we remain neutral and only reported (mostly) successful applications depicted in the published literature.

5. Topology Optimisation in Structural Design of Steel Elements and Joints

5.1. Steel Elements Design

Structural design has been pushed into Topology Optimisation for several reasons. Not only does the technology availability in much user-friendlier tools [388] play a critical role in facilitating the centuries-old task of optimising structural design, but also external pressures drove structural engineers into TO. Such pressures have been found both upstream, with an architectural demand for shapes that can only be optimised with extreme computational resources [389,390] and downstream, with the need for design processes able to foster Additive Manufacturing [391].

The former reasoning also explains why structural steel design has shown a particular prospect for successful TO applications [392]. The range of the applications includes Shape and Size Optimisation for steel members, as well as Topology Optimisation for the whole structural envelope. Furthermore, and unlike most of the other engineering disciplines, structural steel engineering still finds a preferred tool in Discrete Optimisation, over the Optimisation of Continua, for several problems with the macro-structures conceptual design.

Among the members design, perforated beams and shear walls are particularly prone to TO. The former has been studied in depth by Tsavdaridis' group at the University of Leeds [346], and the latter had a recent development in Bagherinejad [360], using commercial software. In both cases, a preliminary design with circular holes has been shown to evolve to a lattice-like geometry, with significant material subtraction in well-known less stressed areas. In a larger scale, floor diaphragm members have also been studied by Fischer et al. [393], to understand the optimisation possibilities under in-plane loading.

Still, within the sectional optimisation domain, the Free Material Optimisation (FMO) method extension [394] provided a critical tool for plates, shells, and member's parts design, and an impressive Academia-Industry joint effort allowed developing the Sectional Optimisation Method (SOM), which enabled the design of optimised aluminium members, accounting for fabrication constraints, standards regulations, and local instability phenomena [341].

Concerning structural systems and trusses optimisation, recent improvements include multi-objective optimisation techniques [395], Differential Evolution Algorithms [267], quantile regression for fostering the use of (discontinuous) I-beam cross-sections as design variables [396], and developments with the Interior Point method for non-linear and non-convex truss optimisation problems [397]. However, one of the most impressive developments can be found by Larsen et al. [398], a near-optimal truss design based on the homogenisation-based continuum TO. Practical applications of truss optimisation to bracing the systems design have some interesting contemporary examples in [399–403].

Therefore, high-rise buildings are an ideal ground for employing truss optimisation methods [106]. Specific issues of tall buildings have been addressed by recent TO studies, including the development of a genetic algorithm-based method for optimising outrigger systems [404], using swarm optimisers [279], considering linearised buckling in the TO process [405], developing a method for optimising bracing systems under adaptive multimodal load patterns [406,407], and conceiving structural systems for tall buildings based on the Optimisation of Continua using Reissner–Mindlin (or Mindlin–Reissner) shell elements [407].

Loading is, undeniably, a major challenge for TO in buildings structural systems, especially when multiple actions, combinations, and modes are considered and more so when the structure-load interaction is strong and influences the latter intensity. That is the case of seismic loading, in which specific, and usually extensive, code prescriptions apply. Subjected to such constraints, research in Topology Optimisation of seismically loaded structures is still exiguous. Nevertheless, the works of Kaveh's group on shear walls [408] and on different ductility steel Moment Resisting Frames (MRF) [409] can be highlighted as well as Qiao et al.'s [410] braces optimisation to the non-linear dynamic analyses of earthquakes time-histories.

A similar complexity can be found in fixed offshore structures for oil and gas or wind energy production. Even though such structural systems usually have a simple and intelligible conceptual design, wave and extreme loading require rather complex engineering [411]. For those reasons, recent developments in the conceptual design optimisation of jackets [412], including geotechnical aspects [413,414] and fatigue design [415] are very significant. Likewise, the method by Cicconi et al. [416] for multicriteria optimisation of modular steel towers is expected to have an impact on the industry.

Long-span structures have also been an object of study in TO, even if the research output is scarce in quantity. Within this subject, one can mention the Topology Optimisation of domes through a Colliding Bodies Optimisation (CBO) method by Kaveh [417] and the general space-frame steel roof optimisation method with Genetic Algorithms by Kociecki and Adeli [265]. Concerning bridge engineering, the spotlight is in the DTU TO group steel girder optimisation for super-long suspended spans, employing computational morphogenesis [418] (Figure 10).

Other significant advances, which may have an impact on structural steel design optimisation, include Christensen's work on TO under extensive and non-linear deformations [165] and Kristiansen et al.'s [419] studies on contact pressure and friction.

As steel design and detailing is deeply affected by fabrication and erection procedures, which usually constrain engineering options in a more extensive manner compared to construction with other common materials, a further investigation into recent research papers devoted to considering such aspects into TO is due. For such an end, and regardless of a deeper review on Additive Manufacturing for TO in Section 7, one shall refer to recent reviews [420,421], and keynote [422] on employing welding robotics in the so-called Wire and Arc Additive Manufacturing (WAAM) of topology optimised steel members and connections. However, the most common methods of creating steel in AM use powder as a feedstock and laser or electron beams as binding mechanisms [423]. Even if high-quality surfaces and accurate geometries are achievable with such means [424], steel properties are TO variables with broader uncertainty, due to its complex and repeated heating and cooling cycles [425]. Control, inspection, and testing will have an increasingly paramount

role in enabling steel fabrication processes reliant on Topology Optimisation [129] and will add to the well-known hindrances of cost, aversion to change, and lack of technical knowledge [426].
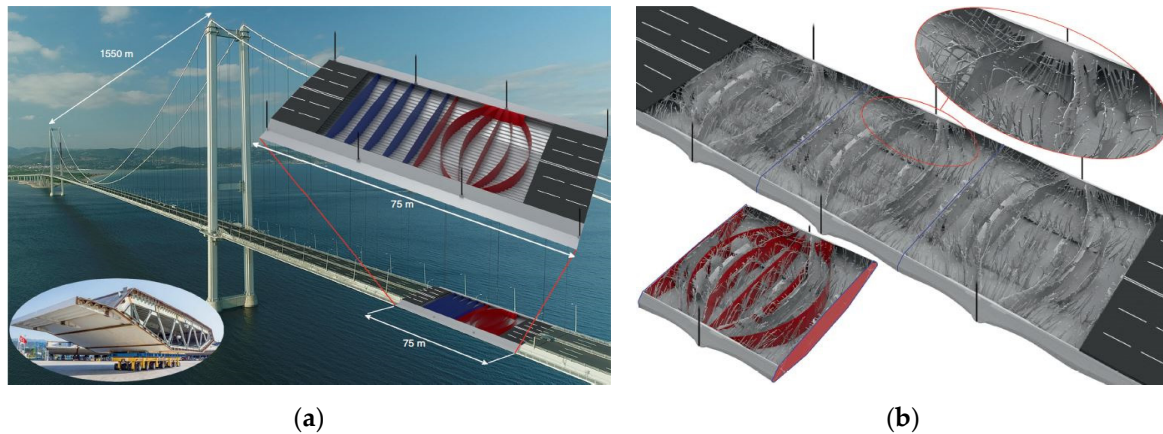

Figure 10. Super-long suspended spans optimisation with computational morphogenesis. (**a**) Case-study initial (blue) and interpreted (red) design [418] and (**b**) computational morphogenesis result [418]; (reproduced under the Creative Commons Attribution 4.0 International License, http://creativecommons.org/licenses/by/4.0/ accessed on 25 February 2021).

5.2. Joints Detailing

In the current state-of-the-art steel connections (or joints, since both names are interchangeably used in structural engineering), TO is built of many mechanical engineering originated research endeavours and some developments made in the context of structural engineering research projects. Within the latter domain, innovative connections have been prototyped, breaking barriers of conventional manufacture constraints and delivering outstanding weight reduction [139]. Moreover, joints compactness, as regularly achieved by such designs, is critical for structural steel detailing, which is usually heavily constrained by space limitations.

Some recent examples of topologically optimised and additively manufactured joints can be found in Figure 11. Herein, new concepts and interesting solutions for space structures can be found [422], yet the significant predominance of applications for lattice, reticulated, and generally tensegrity structures is evident. Such shortcoming is mostly due to the limited quantity of load combinations which can reasonably be considered for the optimisation process. Should multi-axial bending and shear add to the tension and compression stresses, the TO procedure would escalate several levels of complexity and yield less intuitive results, in which validation would become a critical issue [427]. In fact, experimental testing of these joint details under a multitude of load cases is an inevitable step towards the broader employment of TO in steel joints detailing [139,427].

A further step towards the application of TO in joints design can be envisaged in Wang et al.'s comprehensive method for optimizing and fabricating joints in three-like structures [428]. As such, the nature-inspired structural concept is particularly prone to TO, in which a solution for optimising and manufacturing its joints is paramount. Other interesting developments in the TO of joints among tubular elements can be found in the work by Kanyilmaz and Berto [429,430]. Concerning the optimisation of spherical nodes in space frames, the work by Hassani et al. [364] is particularly relevant for providing a holistic guideline, including fabrication concerns, practical issues with Grasshopper modelling, as well as results post-processing and interpretation. Likewise, the work by Alberdi et al. [431] on the connections topology optimisation in Moment Resisting Frames (MRF) is deemed to assist structural designers in the task of managing the recursive task of designing frames and its joints.

Nevertheless, several and significant factors still hinder joints TO. Some of the most apparent problems lie in the materials properties uncertainty and fabrication cost. With a strong relation to AM techniques [139], those can only be solved by sensible technology developments, financial investments in the industrial capacity, and workforce know-how, as well as a stronger standards framework.

Problems with the optimisation process have an outstanding issue in the incapacity of some algorithms for accounting non-linearity [422]. This is critical in several connection types, including the bolted ones, where plasticity plays a major role. However, significant advances have been made in the modelling of bolted connections, holes, bolt-hole contact, and friction in the context of TO [432–435].

Furthermore, a very significant research published in the context of mechanical engineering systems is general enough to have a profound impact also on steel joints detailing. That is undoubtedly the case of the technique for creating idealized bolts with the topological derivative approach proposed by Rakotondrainibe et al. [247] in a Renault associated research. Other examples can be found in the synthesis method for mechanisms proposed by Kang et al. [436] with a possible application to the pinned and sliding joints in structures.
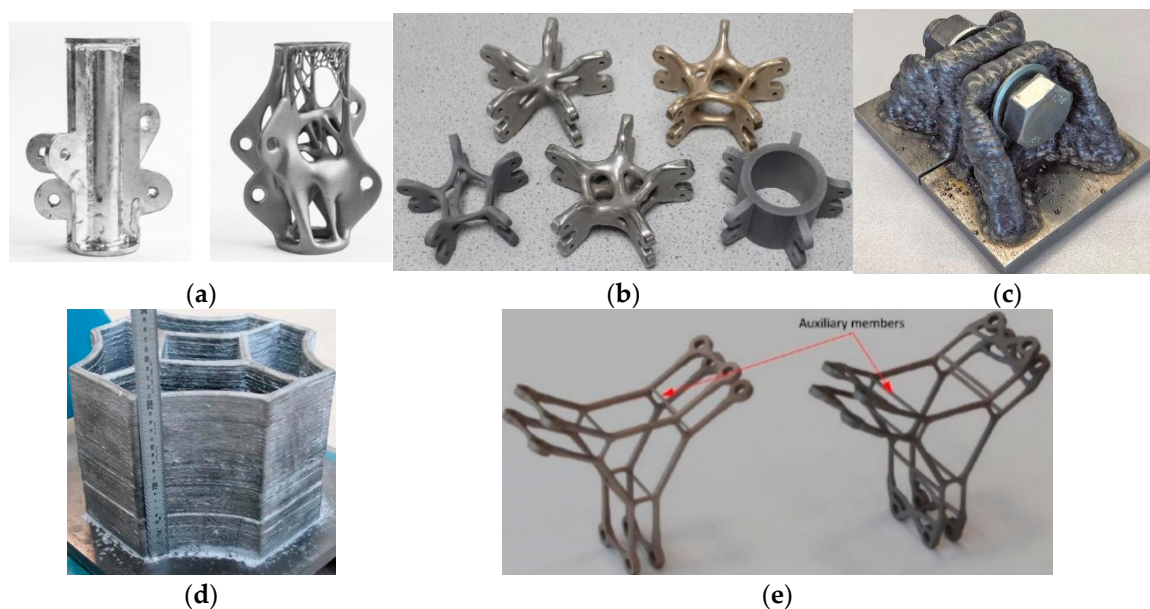

Figure 11. Examples of topologically optimised and additively manufactured metallic connections for civil engineering structures. Left to right and top to down: (**a**) Non-TO and TO joint specimens by ARUP [355]; (**b**) several joints designed in the University of Leeds [139]; (**c**) wire and arc additive manufacturing (WAAM) printed bolted joint in the Technical University of Darmstadt [422] (reproduced under the Creative Commons Attribution-NonCommercial-NoDerivs License); (**d**) WAAM printed node in the Technical University of Darmstadt [422] (reproduced under the Creative Commons Attribution-NonCommercial-NoDerivs License); (**e**) joint specimens topologically optimized with the bi-directional evolutionary structural optimisation (BESO) method for axial loads and bending in the RMIT University [427].

5.3. Buckling and Local Instability Phenomena

Member buckling phenomena has been addressed in Discrete Optimisation for a long time. However, within the Optimisation of Continua, multiple global and local, real and fictional, buckling modes hampered progress after precursory works undertaken in Instituto Superior Técnico [437]. Some notorious problems have been found in the appearance of buckling associated with low-density regions, high local stresses, which add to the repetition of modes, convergence problems, and the need for an extensive computational capacity for dealing with so many modes [137,388,438].

Recent advances already allow accounting for buckling in TO endeavours in a computationally feasible manner [438–442], mitigating some of the aforementioned problems.

Though, those methods rely on linearized buckling, which is generally regarded as an inadequate simplification for many engineering problems [137,443,444] and therefore, face significant opposition from many researchers. Adding to such a discussion, a recent research paper using non-linear pre-buckling analyses in the context of microstructural design [445] found that under certain loading conditions, linear and non-linear buckling analyses yield similar results. Considering the knowledge gathered by structural engineers in linear and non-linear buckling over time, it is expected that such a conclusion may be proven valid for several other cases.

On the other hand, advances have been made in the last few years on the non-linear buckling of topologically optimized continua [446,447]. Such remarkable achievements employ non-incremental analyses and recursive design.

Despite the mentioned problems, under which buckling phenomena can only be regarded as a mostly unsolved problem in the TO of continua, some practical applications found simpler or more sophisticated strategies for modelling instabilities in optimisation methods. That is the case of Tsavdaridis' work on local instability in optimised aluminium cross-sections [341].

Regarding Discrete Optimisation, methods for considering buckling also evolved. Among a recent contribution, one can highlight the work by Weldeyesus and Tugilimana on trusses [397,448], Xu et al. suggested a practical approach for TO in tensegrity structures [449,450], as well as the research by Zhao et al. [451] on methods for mitigating member instability in reticulated structures.

5.4. Structural Design Codes Compliance

Bridging research and practice in structural engineering faces some hindrances beyond the simple transfer of knowledge. Unlike many other fields, where product design is strongly bounded with Research and Development, since testing, compliance, and certification will follow pilot production, the design of building and bridge structures must comply with an extensive set of rules, codes of practice, and standards beforehand. Those documents are typically reviewed in a pluriannual basis and not necessarily include the most recent research, since a broad and heterogeneous community of practitioners is not expected to radically change the design methods frequently.

As a result, many calculation approaches based on non-constant members or employing advanced sectional analyses, even if practical and validated, may take long until explicitly defined in structural standards [452]. This is certainly the case for widely adopting TO in civil and structural engineering and, also, a reason underlying the scarce number of recent publications in TO to specific standards prescriptions.

Within the recent literature pertaining to optimisation programmes, exceptions to the former can be found in Tsavdaridis' optimisation of aluminium cross-sections [341], as well as in works of the Discrete Optimisation of trussed structures, including truss design to Eurocode 3 (EN1993-1-1 or, simply, EC3) with Differential Evolution Algorithms [267] and bracing systems optimisation with GA to the AISC-LRFD American standard [402,453].

5.5. Multiple Loading and Robustness

Significant obstacles still limit the broader employment of TO in structural design. One of the most important is the susceptibility of optimisation results to loading patterns. Albeit, recent advances onto robust solutions, which show endurance to loading scenarios multiplicity [454] and extreme degrees of TO in structural design usually result in members that efficiently withstand a finite number of loading patterns used in the design, but may be inefficient for other loading patterns, even resulting in less intuitive and less visually appealing topologies. In fact, real structural design optimisation problems are deemed to fulfil the requirements of multiple objective functions [455–458].

Under these circumstances, the TO use for structural engineering is impaired by three reasons: First, loading patterns can be very profuse, up to hundreds or thousands in complex structures. This is not necessarily a research problem, but a practical one since

common computational methods are still not suited for delivering TO results for many loading patterns in a reasonable time.

The second problem is related to the absence of accidental and unconventional loading patterns in regular structural design. Without it, the redundancy of structural members may become negligible and an unexpected collapse under non-conventional loading may occur.

The last, and possibly most severe, set of problems is related to the current methods for TO under multiple alternating loads. Notwithstanding some recent advances, current methods still face the inconvenience of non-unique solutions [459] and local extrema, as well as the difficult to account for loads with very different scales of intensity [458].

Multiple-loading and multi-objective optimisations have been handled with a plethora of methods. Both deterministic or physically accurate, and uncertainty-based methods, including notorious fuzzy approaches, have been used [458]. Arguably, the Kreisselmeier–Steinhauser function (KS) stood out in the past decades, mostly in aeronautical engineering studies.

Among recent advances, one can highlight the efforts to account for uncertainties in the loading intensity and position with the method by Wang and Gao [460], and the compliance-function-shape-oriented approach by Csébfalvi [461] and precursory work [462], the generalized material interpolation scheme by Chan et al. [463], as well as the RBTO approach by Nishino and Kato [464].

Previously, Li et al. proposed one more option for multi-objective TO, employing the Normalized Exponential Weighted Criterion (NEWC) and the Fuzzy Multiple-Attribute Group Decision-Making (FMAGDM) theory [458]. Likewise, Yi and Sui introduced the use of the Transplanting Independent Continuous and Mapping Ideas into Materials with the Penalisation (TIMP) method for the TO of plate structures under multiple loading [465]. Its interesting feature lies in adding one more penalty function to a SIMP-like method.

One very different advance can be found in the approach by Tang et al. [466] to the wind loading complexity through integrating Computational Fluid Dynamics (CFD) models into a BESO method for TO.

Much of this research is related to contemporary multiple loading. The problem of non-simultaneous multiple loading implies significant computational efforts to extend the analysis scope, integrate TO results, and deal with very significant practical issues.

Works on dealing with alternating loads can be found early in the TO literature, including the 1970s Prager work [467], but hardly can be considered solutions for the complexity of the currently analysed problem. Recent works with alternating loads include Alkalla et al.'s Revolutionary Superposition Layout (RSL) method [468], as well as Lógó and Pintér contributions, [459,467]. Furthermore, Tsavdaridis et al. managed to use a method to examine and overly stress paths and compose comprehensive layouts, optimized for several sets of loads [341], [469].

As the way ahead is likely to be facilitated by the admirable rate of global computational power increase, only a few TO research centres [169] already possess the means for dealing with multiple and alternating loads and load combinations in more-than-trivial problems.

However, structural engineering has deployed solutions for dealing with uncertain loading and enhancing reliability. One solution lies in the current "Capacity Design" philosophy, favoured in Structural Eurocodes for seismic actions [470,471]. Employing it in TO could provide a minimum threshold for providing the versatility to the structural design of elements.

6. Recent Advances in Related Fields with Applicability in Structural Steel Design

Owing to the broadness of Topology and Topology Optimisation concepts, applications are primarily cross-disciplinary within scientific, engineering, and even graphical fields. Thus, neighbouring fields have been thoroughly investigated for recent developments with a perceived or expected potential for usefulness in structural steel design.

Nevertheless, one shall refer to the fact that only a fraction of what lies within the TO umbrella is of value for the theme under scrutiny. Suitability depends mostly on the driven objectives towards optimisation. Therefore, the following synthesis is not deemed to assess TO in other fields, but to identify advances in TO which can be adopted in TO endeavours for structural steel design.

6.1. A Broader Look into the Construction Industry

Newly developed tools for cross-disciplinarity, namely involving architectural design into the structural design efforts for TO [343], will have an impact on structural design. One other interesting investigation analyses incremental loads and structural layouts in TO [472]. Even if it is directed towards incremental concrete bridges, its methods can be useful for dealing with steel structures staged construction and, in a broader sense, assist in TO with multiple loading and evolutive layouts. Furthermore, advances in soil-structure interaction for geotechnical structures optimisation [473] may have an impact on the moving loads' issue.

6.2. Concrete Structural Design

The recent "Digital Concrete" conference [474] contains strong proofs of concrete design research and practice engagement in TO. While it is undeniable that the current progress is more strongly bounded to concrete Additive Manufacturing, under the name of 3D Concrete Printing (3DCP) [475], Topology Optimisation in concrete structures already advanced far beyond the unreinforced concrete, where AM is already flourishing, paving the way for interesting developments in reinforcement design models.

More than 120 years after the work by Ritter [476] and Mörsch, as well as over 30 years past Schlaich et al.'s notorious contribution [477], the Strut-and-Tie Models (STM) still govern the concrete design with remarkable resemblances to what previous generations of structural engineers mastered. That may well be subject to change as new developments in TO are deemed to reshape our understanding of reinforced concrete design, with contributions by Zhou et al.'s [478], Yang et al.'s [479], Jewett and Carstensen's [480], and Xia et al.'s [481]. However, the promise of fundamental innovation beyond the aforesaid applications remains restricted to only a few distinct approaches. One of those is Pastore et al.'s [482] risk-factor to replace the Von Mises stress criteria for optimizing heavily constraint structural elements.

Other applications of optimisation in structural concrete design include targeting seismic performance objectives [483–485], the optimisation of prestressed concrete members [486], and defining critical concrete structures general topology, from the Optimisation of Continua, as performed by Wu and Wu for bridge pylons [487]. These works will, most certainly, have a contribution also for steel and concrete composite structures.

6.3. Aerospace and Defence Industries

Aerospace engineering, with its utter need for weight reduction and frequently generous funding, has been a source of inspiration for TO breakthrough innovations. Over the years, sensible contributions to critical issues in TO, such as fastener design and dynamic analyses, now used in structural steel design, came from this field [488].

Recent advances in TO from this field include optimisation methodologies for additively manufactured parts with enhanced accuracy to strain and displacement [489], experimental analysis of several topologically optimised micro and macro-structural systems for ribs [490], optimisation to cumulative mechanical and thermal loads [491], and advanced materials design for high-quality AM [492]. All these contributions can have a deep impact on the quality of topologically optimised and additively manufactured alloys, leveraging its rapid application also in the construction industry.

6.4. Automotive Industry

Practical and fabrication-oriented methods are regularly deployed by the automotive industry. Hence, this field has been a continuous source of inspiration for framing TO developments in the track of meaningful advances. Novel methods include Mantovani et al.'s guidelines for integrating AM requirements into TO, as well as for successfully processing and managing the TO results [493]. Similarly relevant are the studies by Van de Ven et al. [494] and Mass and Amir [495] for mitigating the impact of overhangs for AM as design constraints. Limiting or conveniently positioning overhangs is paramount not only for subtracting complexity to the numerical models, but also may prevent failure mechanisms in steel alloys, such as fatigue-related. Such developments are applicable to the most additively manufactured parts, not only in the automotive industry.

Detailed procedures for optimising vehicles parts can be found in Topaç et al.'s [496] reduction of 63% of a mechanical component mass in Kumar and Sharma [497], in Mantovani et al.'s [498] reduction of a steering column support mass in almost 50% in Li and Kim's approach to topologically optimise a car part with limited information, manufacturability concerns to leverage extrusion and casting processes, as well as a post-processing method for geometry reinterpretation, while achieving almost 40% of weight reduction [499].

6.5. New Materials, Composites, and Polymers Design

Materials design has been taking advantage of TO at a microstructural level and, conversely, promoting its development. As Osanov and Guest [135] formulated it, the fundamental question in architected material design optimisation can be resumed to finding which microstructure will deploy sensible enhancements to macrostructural properties. The answer is complex and involves entrenched unit cell modelling, upscaling, and repetition [184,500,501]. Yet, diverse and frequently remarkable solutions can be attained, from useful elastic properties, including auxetic (NPR) materials (those with a negative Poisson's ratio), as popularized in Sigmund's work [502] and currently drawing much attention from the scientific community [503], to extreme thermal properties, optimised fluid permeability, and materials governed by non-linear mechanics for utmost energy absorption.

Within the aforementioned exciting framework, recently published works include modelling methods for designing hierarchical structures employing non-uniform topologically optimised lattices [504], serving as enhanced energy absorbers in sandwich sheets [505] or facilitating manufacturing [506], even with new methods for mitigating non-smooth surfaces, as the Bézier Skeleton Explicit Density (BSED) Representation Algorithm [507]. The microstructural design for avoiding stress peaks has also been recently addressed [508], as uncertainty-resilient design for inter-diffusion interface issues has been brought forward [509], and advances in buckling of microstructures were published by Bluhm et al. [510].

The fabrication of these architected materials with AM techniques is yet another concern. Not seldomly published methods can be incomplete or vague, as Huang et al.'s review pertinently points out, and offers mitigation by congregating some state-of-the-art answers [511].

Even though new materials and, specifically, new material microstructures may still be far from promoting a change in steel alloys, structural steel design can find many important lessons in the formerly cited research. Both the tools and methods created for microstructural optimisation can be employed in the multi-purpose TO, but also solutions developed for a particular material microscale may have an employment in construction macrostructures composed of smaller members. That is also the case of recently developed methods for efficiently handling buckling in polymers optimisation [512], as well as for multimaterial TO (MMTO) [513,514], with its specific problems, as extensive local extrema, and techniques designed to overcome such obstacles. Furthermore, many important advances in TO methods and approaches are being developed within the materials design field, making it especially relevant, also for TO in civil and structural steel design.

At a coarser scale, construction composite materials can also be enhanced under the assistance of TO. That is the case of the types of cement with an enclosure of rubber waste [515] or Carbon Fibre Reinforced Plastics (CFRP) [516].

6.6. Industrial Design, Mechanical Engineering, and Multiphysics Endeavours

One of the most prolific research lines in TO relates to compliant mechanisms design. These flexible structures, which are ubiquitous in most high and low technology consumable products nowadays, have been a perfect ground for TO. Owing to its reliance in parts flexibility rather than hinged joints, these components rigid body design had been complex, frequently erroneous, and very dependent on prototype testing, whereas compliant mechanisms TO bridges most of those shortcomings by efficiently addressing the target flexibility.

Upscaling from small consumables, in electronics for example, larger components such as grippers have been produced using the TO of compliant mechanisms [517]. Furthermore, new methods for addressing the manufacture uncertainty and stress constraints [518] show a new maturity level for compliant mechanisms design, which may lead to its use in more perennial applications and structures.

The mechanical design of diverse components and structures has also faced recent developments in TO related subjects. If, on the one hand, the TO of vibration problems remains largely constrained to small amplitudes [133], on the other hand, techniques are being developed for introducing High-Cycle fatigue as an optimisation criterion [519], [198], and concerns over accidental actions are being addressed within the ship design industry [520].

Multiphysics problems in optimisation are usually deemed to address thermal and mechanics conditions [521]. Recently, Cheng et al. employed the Lattice Structure Topology Optimisation (LSTO) to design a cooling channel system [522] and Perumal et al. found a technique to mitigate thermal accumulations by optimising lattice structures locally where such concentrations are more prone due to the metal AM process with a powder-bed fusion [523].

Another very interesting branch lies in the TO of fluid-based problems. Although recent, it congregates the optimisation of flow contacting surfaces, solids transport, heat transfer or porous media [134,363]. Shortly, the herein developed techniques may be efficiently integrated into CFD analyses [524] and significantly enhance the building structures design and optimisation to wind effects. Furthermore, the fascinating "*poor man's*" approach for natural convection problems [525] can be very useful for optimisation in porous media, including geosciences, geotechnics, and hydrogeology.

6.7. Medical Devices and Personalised Medicine

The medical devices industry has shown a remarkable appetite for creating high-value products with newly available technologies for the very demanding health sector. Thus, it was no surprise that AM was mastered by customised medical devices from its early days.

Currently, the capacity for three-dimensional printing solutions with high accuracy, efficiency, and adhering to the patient needs is already well-rooted, and that provides practical lessons for many other fields where TO is being used for enabling AM, including structural steel design. Recent examples include Rapid Prototyping (RP) of plastic casts made with human limbs scan data as an efficient replacement for the plaster casts method [526], with further TO for improving resistance, rigidity, and ventilation, as well as personalized aneurysm implants [527], which are expected to be topologically optimised and additively manufactured with NPR materials, so that the clinical use with significant advantages for the patients is attained.

Understandably, the medical devices industry needs to be at the forefront in developing high-quality surfaces for the additively manufactured parts. A recent contribution can be found in Chen et al.'s [528] technique to minimize the number of fabrication supports accounting for the main printing direction.

7. Designing for Additive Manufacturing with Topology Optimisation

Metallic materials conventional design has been mostly based on subtractive techniques for centuries. Cutting and drilling flat sheets of various thicknesses and laminated profiles have been the rule in steel construction, in which joining by bolting and welding became a ubiquitous counterpart. As a result, the design is limited to geometrical bounds, stockage is profuse, waste is significant, the material is not necessarily located where it contributes the most, and the whole process is very labour intensive. A notorious exception can be found in casted elements, which had its broadest use in the iron construction and now is mostly limited to steel nodes. However, the need for moulds makes this technique only viable when many similar products must be produced. Such lack of versatility usually limits this option to large space structures with equal nodes.

Contrarily to the former, recent techniques allow producing metallic alloys by addition. Despite some critical technological differences, these techniques have been nested under the informal name of 3D Printing in the context of the current fourth industrial revolution. Other terms include Additive Manufacturing [130], Additive Layer Manufacturing (ALM) [131], to allude to the layer-by-layer nature of the process [423,425] or Solid Freedom Fabrication (SFF) [126].

While AM has been commercially used for more than 20 years, typically for rapid prototyping without commercialisation purposes [354], only now the industrial capacity for widespread and reliable manufacturing of steel, titanium, and aluminium has been achieved [423], offering a tremendous opportunity for the construction industry [129].

Making AM a reality in construction plants and yards will depend on several economic and technical factors. Among the latter, one can highlight the ability to introduce robotics in construction [421] as with Large-Scale Prefabrication (LSP) [426], having consistent and usable-by-practitioners design tools [130,388,529], as well as achieving commercial maturity, including reliability and scale in AM techniques.

Plentiful AM techniques, diverging solely from its name or commercial branding to its nature fundamentally, hinder non-experts the understanding of the available options for manufacturing. Even if an encompassing knowledge on the matter requires consulting comprehensive Review Articles, such as [129,423,425,530].

Table 6, which adheres to the American Society for Testing and Materials (ASTM) classification of AM techniques, may assist in the task. It shall also be noted that distinctions among categories and techniques are usually made concerning its feedstocks and binding mechanisms [423].

However, not all those categories bear AM processes currently suitable for manufacturing of steel alloys. Some, as depicted, do not allow using metal powder or wire, being limited to polymers, ceramics, and other low-strength materials. Regarding the need for post-processing, DED-PA and DED-GMA metal production require machining, DED-EB usually needs surface grinding, DED-L may entail surface treatments, while PBF-L and PBF-EB are less likely to be post-processed [425]. Therefore, PBF and some DED processes are commonly designated direct-to-metal.

Table 6. Additive manufacturing (AM) categories and techniques.

AM Categories to ASTM's Committee F42, ASTM F2792-12a [531] (Withdrawn) and ISO/ASTM 52900:2015 [532] Definitions	AM Techniques	Remarks	References
Vat Photopolymerisation	Stereolithography (SLA)	A liquid resin is solidified by Ultraviolet (UV) light exposure.	[126,530]
Material Jetting	Polyjet	Polymer or wax drops are jetted through a nozzle and cured with UV light	[126,530]
Binder Jetting	Indirect Inkjet Printing	A print head jets powder-based materials and fluid binder layers. It can be used for metals, but only high-porosity products are usually produced	[126,530]
Material Extrusion	Fuse Deposition Modelling (FDM)	In FDM, material is heated and continuously expelled through a nozzle	[126,530,533]
	Contour Crafting		
Powder Bed Fusion (PBF) Used in metals	PBF-L, also known as Laser Beam Melting (LBM), Direct Metal Laser Sintering (DMLS), Selective Laser Melting (SLM), Laser Metal Fusion (LMF), LaserCUSING, or industrial 3D printing	Divided by heat source to liquify the powder: L for laser, EB for electron beam. It is essential to mention that while some authors regard DMLS and SLM similarities as sufficient for considering the terms as synonyms, others prefer to separate it. SLS is used for polyamides and polymers. Generally, PBF techniques allow producing metals with both good accuracy and mechanical properties	[126,129,421,423,425,530,534–537]
	PBF-EB, also known as Electron Beam Melting (EBM)		
	Selective Laser Sintering (SLS)		
Sheet Lamination	Ultrasonic Additive Manufacturing (UAM). Used in metals	Low temperature joining of metal sheets by ultrasonic welding	[126,129,530]
	Laminated Object Manufacturing (LOM)	Usually with paper and glue	
Directed Energy Deposition (DED) Used in metals. In such a case the term Metal Deposition (MD) has been employed	DED-L, also known as Laser Metal Deposition (LMD), Laser Engineered Net Shaping (LENS), Direct Metal Deposition (DMD), Laser Engineered Net Shaping (LENS), laser deposition welding or laser cladding	Divided by heat source: L for laser, EB for electron beam, PA for plasma arc, and GMA for gas metal arc. Powder or wire material sources can be used. WAAM technique is similar to conventional automatic welding procedures, using wire and an electric arc, as Plasma Arc Welding (PAW), Gas Metal Arc Welding (GMAW), and Gas Tungsten Arc Welding (GTAW). As a result, WAAM may be nested under the other DED techniques.	[126,129,420,421,423,425,530]
	DED-EB		
	DED-PA		
	DED-GMA		
	Wire and Arc Additive Manufacturing (WAAM), also known as 3D welding, Shape Metal Deposition (SMD), Shape Welding (SW), Shape Melting (SM), Rapid Prototyping (RP) or Solid Freeform Fabrication (SFF)		

Other aspects of paramount importance for assessing the suitability for steel manufacturing include the fabrication speed, properties reliability, and the alloy cooling rate. The latter will have a profound impact not only in residual stresses but also in the steel composition, where the control of martensite and retained austenite is crucial [423].

The former aspects are not always easy to assess, mostly since the equipment, context, and several other industrial manufacturing details play a significant role. Nevertheless, a choice has been made to limit the Table 6 scope to proven and industrially viable technologies, even if not all of those are applicable to the metal alloys manufacture. Thus, emerging technologies such as Electrochemical Additive Manufacturing (ECAM) [129,538] was not included.

On the opposite field, some AM techniques have long been proposed before the concept of AM was defined. That is the case of WAAM, which dates back to 1925 [424] and is now highly productive. Hence, some researchers regard WAAM as a leading option for steels manufacture [421], provided that shortcomings with high residual stresses, material and geometrical properties are successfully addressed (as can be seen in Figure 12). On the other hand, PBF-L (also in Figure 12), PBF-EB, and DED-L have been mentioned as the most relevant technologies for producing metallic alloys in the current context of AM [423].
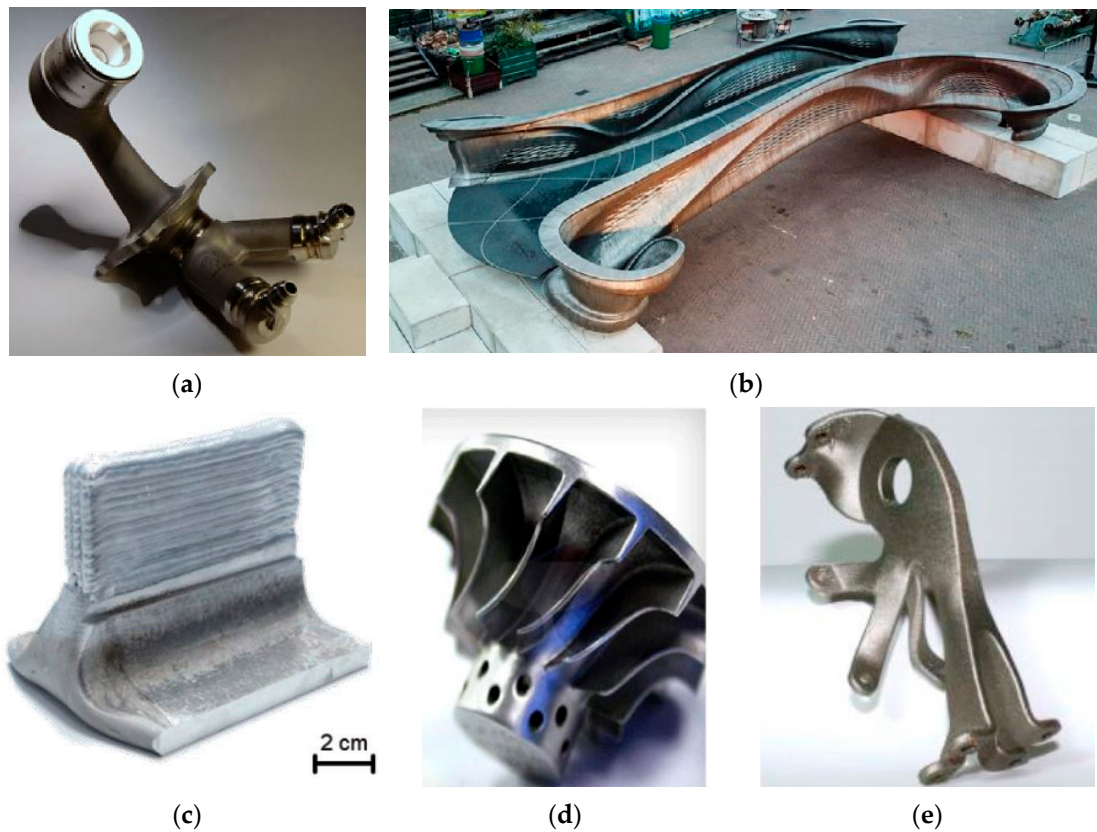
Figure 12. Examples of additively manufactured metallic parts. Left to right and top to down: (**a**) Nozzle manufactured with PBF-L [423]; (**b**) MX3D bridge manufactured with WAAM [129,420]; (**c**) rib manufactured with WAAM [420,539] (reproduced under the Creative Commons Attribution 4.0 International License, http://creativecommons.org/licenses/by/4.0/ accessed on 25 February 2021); (**d**) turbine part manufactured with PBF-L [530]; (**e**) aerospace structure component manufactured with PBF-L [540].

The role of TO as an enabler for AM justifies the unparalleled growth of both [130]. Currently, the Design for Additive Manufacturing (DfAM), which succeeded the Design for Manufacturing (DfM) traditional methods [541,542], employs more or less advanced optimisation tools, spanning practical lattice-based ones, as depicted by Chen et al. [543] and illustrated in Figure 13, to complex approaches [544,545], and is already paving its way into structural steel connections [546,547], albeit mostly for tension-only nodes.
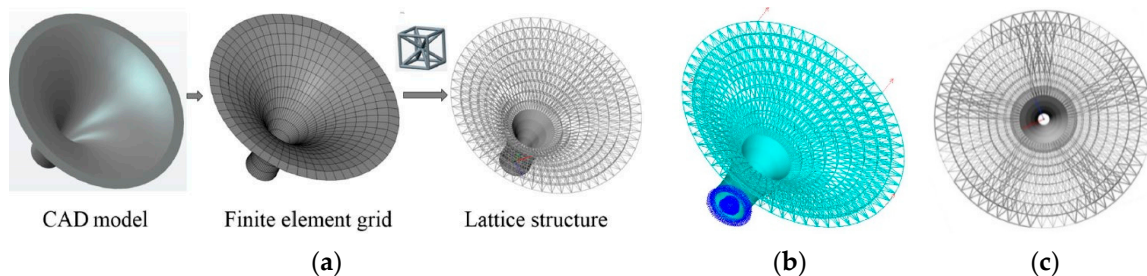

Figure 13. Lattice-based design for additive manufacturing (DfAM). From left to right: (**a**) CAD model; FEM; lattice model; (**b**) boundaries and loading definition; and (**c**) final optimized structure [543] (reproduced under the Creative Commons Attribution 4.0 International License, http://creativecommons.org/licenses/by/4.0/ accessed on 25 February 2021).

As highlighted in recent Review Articles [130,424,425,530] as well as Buchanan and Gardner 2019 [421], TO is yet to overcome important challenges to meet AM requirements. A stronger link between the DfAM and production is needed to overcome the current

bottleneck, including better accounting for AM metallurgical aspects, such as residual stresses, internal defects, interlayer bonding conditions, and anisotropy [536,548]. In fact, composite materials TO for AM is still afar from the industrial practice.

Other challenges for the current DfAM can be found in the difficulty to avoid overhangs and general support structures [549], which promote waste, require labour, time, and cost to post-process and hinder surfaces' quality. Furthermore, for the AM process convenience, the present DfAM makes extensive employment of open-walled infills which, amidst important contributions for material reduction, have a frequently detrimental effect on the product stiffness [550], and especially on the torsional stiffness.

Another important and often overlooked problem lies in the difficulty in testing additively manufactured parts, especially by Non-Destructive Testing (NDT) techniques [551], which result in a poor understanding of material properties yielded by different AM techniques [552].

On the other hand, recent research brought innovative design techniques to assist AM. Table 7 summarises some selected recent developments, which add to the advances in TO depicted in the previous chapters.

Table 7. Recent research on design techniques for AM.

Development	Reference
Multi-Material Topology Optimisation (MMTO) by generalising the single-material Pareto tracing method.	[553]
Guidelines for integrating TO and AM to restrain support structures.	[246]
A TO approach to minimising support material by optimising an allowable self-supporting angle.	[554]
A TO approach to designing self-supporting structures.	[555]
Guidelines for TO regarding AM process parameters.	[556]
Guidelines for enabling AM newcomers to produce better quality designs. The proposed worksheet has a useful "pass-or-fail" arrangement.	[557]
A method for topologically optimised infills in additively manufactured parts. By fostering non-uniform infills, better results have been attained.	[558]
TO for AM with EBM (or PBF-EB).	[559]
AM-fabricated Materials with Site-Specific Properties (MSP). This design approach allows designing materials with heterogeneous properties, where local material enhancement is possible without requiring an increasing fabrication time and cost for the whole element.	[560]
TO approach to minimise overhangs, assisted by its sensitivity analysis.	[561]
TO approach to minimise overhangs and generating self-supporting structures.	[562]
TO approach to minimise support structures and thin features.	[563]
Optimisation of support structures design for AM.	[564]
A multi-objective TO approach to minimise supports material consumption and removal time and cost, while minimising parts deformation. For such an end, a repulsion index (RI) is used.	[565]
A design method for additively manufacturable free-form periodic metasurfaces.	[566]
A multiscale method which includes both TO macro-scale optimisation and AM layers mesoscale.	[567]
A new version for the MMC method based on AM data.	[568]
A TO model to account for thermal residual stresses and deformations due to the AM processes.	[569]
TO approach to minimise overhangs in compliant mechanisms, using the Smallest Univalue Segment Assimilating Nucleus algorithm.	[570]
TO approach to minimise overhangs and assure a minimum length scale, both for enhanced printability. Application to a tensegrity connection.	[547]
TO for AM with SLM (or PBF-L).	[571]
A Non-Probabilistic Reliability-Based Topology Optimisation (NRBTO) method to account for additively manufactured materials' properties uncertainty.	[572]
A TO method to account for AM cost and fabrication time.	[183]
A TO density-based approach with one field for design parameters and another field for support layout optimisation.	[573]
A TO approach for designing and re-designing additively manufacturable parts.	[574]

Table 7. *Cont.*

Development	Reference
A new perspective on overhang control, taking into account AM techniques specific features with a skeleton-based structure decomposition approach.	[575]
TO approach to minimise supports in enclosed voids. Employs a Nonlinear Virtual Temperature Method (N-VTM) to find enclosed voids.	[576]
A TO approach for re-designing additively manufacturable parts. Integration with a commercial computer-aided design software is discussed, which enforces the study's practical applicability.	[577]
A TO approach for parts repair or upgrade with AM after subtractive machining.	[578]
A TO approach for merging structural optimisation and WAAM deposition sequence.	[579]
TO approach for taking into account the assembly design.	[580]
A multi-objective minimisation TO approach which account for AM fabrication time and cost.	[581]
Multiresolution Topology Optimisation (MTOP) method for high resolution AM with overhang and minimum length control.	[582]
An NRBTO method to account for additively manufactured materials' properties local uncertainty. For such an end, Uncertainty Regions (UR) are defined within the design domain.	[583]
TO with kinetic analysis added to the common FEA.	[584]
Solid Anisotropic Material with the Penalisation (SAMP) technique better integrating process parameters into TO.	[585]
TO of multi-material infills with a systematic multi-phase design method.	[586]
Poisson's equation-based scalar field constraint to suppress enclosed voids in TO.	[587]

The path between a topologically optimised model and a manufacturing input can pose significant obstacles, even if the smoothing of TO model boundaries has already been accomplished [588]. Not only several fabrication parameters must be accounted, but also data transfer can be a complex task. The TO model must be exported to a CAD three-dimensional format, suited to the manufacturing process and sliced into thin layers [423]. Moreover, many available approaches have not been broadly and satisfactorily tested for manufacturability [589].

Regarding this issue, one may refer to the work by Zegard and Paulino [97], yielding TOPslicer, a MATLAB code for transforming matrices or arrays with density values into additively manufacturable digital outputs. Furthermore, ontologies databases may assist designers in enhancing DfAM. Within this domain, Dinar and Rosen [590] contributed to systematise a knowledge base, and to infer the design rules using the Web Ontology Language (OWL)/Resource Description Framework (RDF) Protégé tool must be highlighted.

8. Future Trends

In the aftermath of the latest World Congress on Structural and Multidisciplinary Optimisation, held in 2019, surrogate-based optimisation and optimisation under uncertainty were highlighted as two trending issues for the future of TO [591]. In fact, the latter has a significant expression in recent studies by some of the leading researchers in the field, as one can regard in Da Silva et al.'s work with the Augmented Lagrangian method for computing loading uncertainty [214,454].

The prospective dissemination of TO in structural design, based on increasingly user-friendly and reliable software [2], is quite an undisputed prediction. However, the means to enforce such development are subjected to different views. As Sangree et al. reported the experience of inserting TO in engineering education as soon as in freshman levels at Johns Hopkins University both as a design tool and as a mean for understanding the force flows [592], Lagaros [2] alluded to the role of the State to force practitioners to apply new technologies. Concurrently, Gao et al. [530] pointed out to open academic research platforms and intellectual property expiry, but also governmental investments and expertise as catalysts for enforcing the application of emerging standards and fostering DfAM and AM processes integration and interconnectivity, potentially opposed by larger revenue companies.

DfAM or AM-Oriented TO (MOTO) is envisaged to undertake the systematic study of trade-off relationships between TO and AM, unlike what has mostly happened to date [593].

This may also include considering fabrication models which integrate more conventional processes [530], as well as advancing in multi-material products, assisted by progress in simulation algorithms [530]. Nevertheless, an increasingly closer relationship between TO and AM is perceived as vital for the industry's development [594].

On the other hand, significant hurdles are yet to overcome in TO for AM. Meng et al. reported the need for more advanced TO methods for multifunctional products, as well as for progress in multiscale TO [594] and Lim and Wong focused on the need for enhancing TO performance regarding aerodynamics problems [352].

Conclusions drawn from structural steel TO suggest that a higher optimisation grade and further proofs of safe applicability of additively manufactured connections are the future paths for achieving economic viability, compared with labour-intensive traditional manufacturing [355].

Focusing on the AM process, Bañón and Raspall [81] concluded that future directions in three-dimensional printing for architectural purposes include large-scale printing, Artificial Intelligence (AI) embedment into the design process, and computerized assembly [81]. In addition, Liu et al. [595] mentioned the accounting for metal AM residual stresses and the costly post-machining as issues yet to be solved, as Meng et al. [594] stressed out material performance assurance and fabrication speed and resolution as future research lines.

Future research on AM methods is deemed to comprise solutions for lightweight cellular materials design [594] or the validation for AM purposes of several innovative TO approaches [595].

Another perspective on future trends can be attained from analysing current funded projects and recent patents on a subject. Searching within the EU projects database, one can find two projects concerned with TO. An already finished project studied "*Optimization Techniques for MIMO Radar Antenna Systems*" [596] and therefore, is not related to this review scope. On the other hand, the project "*Innovative Re-Design and Validation of Complex Airframe Structural Components Formed by Additive Manufacturing for Weight and Cost Reduction*" has been funded since 2017 until the current year under the framework of "*Design Guide Lines and Simulation Methods for Additive Manufactured Titanium Components*" and deployed significant publications [127,178,597,598]. While the research has been conducted with titanium as the primary material, the innovative approaches are applied in any engineering field where weight reduction is a major objective.

Concerning recent Topology Optimisation patents with the world, Europe, or US coverage, software developers have registered several computational methods. Those include Livermore Software general design methods EP2251805A2 [599] and US8126684 [600], the numerical derivates method US0160078161 [601], and enhanced global design variables method to account for impact events US0170255724 [602]. In addition, Dassault Systèmes' method for designing a mechanical part with TO EP3502931A1 [603], US20190197210A1 [604], EP3647973A1 [605], and JP2020071887A [606], as well as autodesk TO for subtractive manufacturing techniques US 2018/0349531 [607] and Altair's Failsafe TO method US10354024 B2 [608].

Other recently patent-protected design methods include patents US20160140269 [609], US8335668 [610], and WO2020215533A1 with a material-field reduction series expansion [611], EP3285189A1 for flexible hinges [612], EP3292657A1 and US0180139130 for the multi-layer network TO [613,614], US0170161405 employing reduced length boundaries methods [615], US0200134918 for cellular structures [616], and US010613496 for Additive Manufacturing [617].

Unlike what has been depicted for published research, patents show a significantly growing impetus for practical methods, especially in automotive, composite materials manufacturing, and aerospace industries. In fact, while the 2010 to 2015 period has some of the most significant TO industrial patents in Caterpillar's US0100274537 stress-based TO method [618] and US0140156229 fatigue-based TO method [619], the last 5 years brought several and significant advances, such as Toyota's methods for TO with a member-

ship variable WO2019152596A1 [620] and US0090326138 with shape transformation [621]. In addition, the GM US0100035974 system for a plurality of materials [622], several additive manufacturing methods, including microstructure-based US0200180228 [623], Freespace Composites' US0150239178 [624] and US9789652 [625] manufacturing systems, Thales Alenia Space Italia EP3545443A1 adaptive TO for layer AM [626] and Siemens' WO2015106021A1 and US009789651 method for additively manufactured lattice structures [627,628], WO2019178199A1 multi-physics applications [629], WO2020160099A1 machine learning-based TO [630], as well as WO2020159812A1 TO method for additively manufactured thermoelastic structures [631].

9. Discussion and Research Perspectives

In opposition to the previous section, where future trends have been collected from distinguished experts' opinions and research articles, the current section aims to analyse the whole revision work and, from a critical assessment, depict a current status and infer future tendencies.

Adding to relatively scarce previous Review Articles, the current article was able to provide an embracing picture of TO, and DfAM developments occurred in the last 5 years. Considering the concentration of cutting-edge research in a few research centres, as well as a significant asymmetry between the leading funding agencies and the remaining ones, the study of patents in TO was regarded as an indispensable step for understanding the role of the industry in pushing TO forward.

Nevertheless, it also has been found that the current surge in TO is partly owed to many newcomers entering the field. Other reasons can be attributed to the close link between TO and AM and the later significant emergence.

Approaches and methods in TO have been found to compose a broad, yet heterogeneous, fabric of resources. Those encompass and blend techniques originated in Discrete Optimisation and Optimisation of Continua and offer an increasing palette of the forthcoming TO massification options. Nevertheless, the latest research trends suggest a post-SIMP era, where such a method's current predominance is now challenged both by very interesting research on Level-Set methods, as well as evolutionary approaches maturity and pervasiveness in many engineering applications, especially fuelled by Genetic Algorithms.

Arguably, this trend may be related to the significant controversy over the ESO and BESO methods, which, despite continuous enhancements, seem to be deprecated to GA methods as the first choice in many new practical applications.

Observing the most prolific research centres recent output, it is possible to speculate that research in TO methods and algorithms is likely to evolve towards computationally demanding solutions, such as multiscale projection and giga-resolution solutions, as well as further developments in surrogate-based optimisation, more complex techniques for addressing stress constraints and optimisation under uncertainty.

Bringing TO into the engineering practice is another issue, as non-experts are particularly dependent on existing programmes, codes, and commercial software. Regarding this matter, the SIMP method is expected to remain preponderant, considering its current dominance in commercial software. However, the herein depicted investigation showed a clear future tendency for the software market leaders to offer hybrid approaches and to patent their own methods.

Another observation from the current revision on programmes and codes is the unveiling of a more profuse set of alternatives, compared to what is frequently referred. Such information may assist both practitioners and early-stage researchers to find convenient alternatives and means for benchmarking solutions.

In structural steel design, TO is now mostly constricted to prototyping and nodes in tensegrity structures. Its large-scale employment depends on several factors, including reliability in additively manufactures alloys properties, the ability to account for multiple alternate loading and multiple local instabilities in TO, better addressing the non-linearity

and the existence of comprehensive and code-compliant practical methodologies for practitioners.

Yet, it is interesting to observe that while many researchers find the mass reduction in steel members and connections due to the employment of TO extremely rewarding and unattainable otherwise, others still put the economic viability threshold further than what has been attained so far. Arguably, the type of member and connection seems to play a decisive role in the economic viability of TO.

Notwithstanding, researchers are almost unanimous in pointing significant benefits in using TO for AM in steel structures, namely the waste reduction, sustainability, global weight reduction, which may enhance the performance of big span and earthquake-prone structures, as well as erection speed.

Contributions from other disciplines are envisaged to foster sensible advances in TO for structural steel design. That is the case of more advanced alloys and composites either due to fabrication with leading AM processes provided its cost is reduced, or attained from TO architected microstructures, as well as better procedures and guidelines offered by the industrial practice.

However, the most significant driving force for implementing TO in the design practice is the prospective use of AM. DfAM is reliant on TO, and the massification of AM can only push TO.

Recent advances in DfAM have been mostly centred in taking AM requirements into consideration for TO. Likewise, impressive efforts have been made to attain designs with better surfaces quality, less post-fabrication machining, and avoiding overhangs and general supports.

Notwithstanding some meritorious exceptions, mentioned throughout the text, as one assesses recent research in TO it is evident that Cohn and Dinovitzer's diagnosis of 25 years, mentioning that profuse advances in mathematical algorithms using simple examples, rather than formulating methodologies for real structural engineering problems, led to practitioners' lack of interest in TO [632], still holds its validity.

As any other systematic review, this article is subjected to the risk of unintended bias, incompleteness, etc. To mitigate such risks, efforts were undertaken in performing inclusive research, considering all the possible viewpoints, valuing equally more or less proficiently written articles, and remaining neutral in opinions.

10. Conclusions

This article's contribution to the current body-of-knowledge lies in offering a pervasive review of TO methods and applications, developments in the past 5 years, and research trends. It is focused on structural steel design and detailing but encompasses all the adjoining domains, including other fields, the optimisation software, and Additive Manufacturing processes.

Therefore, it is hoped that it may encourage researchers and practitioners, especially newcomers into the field, to endeavour research in a field where it is usually very time-consuming to enter.

Among the herein depicted review work conclusions, one can highlight that SIMP is still the leading method for TO. However, research trends suggest an escalating importance of Level-Set Methods and Genetic Algorithms. On the other hand, commercial software for TO is deemed to continue as SIMP-based, while a trend to offer hybrid and in-house developed methods can be regarded.

Employing TO in steel construction is a clear future trend, either fostered by AM massification or due to its significant benefits in waste reduction, weight reduction, sustainability or as the ultimate optimisation tool.

However, for that to happen, significant advances will be required in the alloys properties quality and reliability, alternate loading, local instabilities, and non-linearity accounting into the design methods, as well as the creation of a holistic and code-compliant practical methodology for practitioners.

Some of the much-needed solutions are expected to be brought from other engineering fields, such as aerospace, automotive, materials or medical devices engineering.

As a further recommendation for the research community, we suggest creating a classification scheme, for example, similar to the Mathematics Subject Classification, in order to better organise and order TO in sub-fields, approaches, and methods.

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Abbreviations

3DCP	3D Concrete Printing
ABC	Artificial Bee Colony Algorithm
ACO	Ant Colony Optimisation
AESO	Additive Evolutionary Structural Optimisation
AI	Artificial Intelligence
ALM	Additive Layer Manufacturing
AM	Additive Manufacturing
ASI	Artificial Swarm Intelligence
ASTM	American Society for Testing and Materials
BESO	Bidirectional Evolutionary Structural Optimisation
BSED	Bézier Skeleton Explicit Density
CAE	Computer-Aided Engineering
CBO	Colliding Bodies Optimisation
CFD	Computational Fluid Dynamics
CFRP	Carbon Fibre Reinforced Plastics or Carbon Fibre Reinforced Polymers
COC	Continuum-Based Optimality Criteria
CONLIN	Convex Linearisation Method
DCOC	Discretized Continuum-Type Optimality Criteria Technique
DED	Directed Energy Deposition
DED-EB	Electron Beam Directed Energy Deposition
DED-GMA	Gas Metal Arc Directed Energy Deposition
DED-L	Laser Directed Energy Deposition
DED-PA	Plasma Arc Directed Energy Deposition
DfAM	Design for Additive Manufacturing
DfM	Design for Manufacturing
DMD	Direct Metal Deposition
DMLS	Direct Metal Laser Sintering
DOT	Design Optimisation Tools
DSC	Deformable Simplicial Complex
EBM	Electron Beam Melting
EC3	EN1993-1-1 or Eurocode 3
ECAM	Electrochemical Additive Manufacturing

ESLM	Equivalent Static Loads Method
ESO	Evolutionary Structural Optimisation
FDM	Fuse Deposition Modelling
FEA	Finite Element Analyses
FMAGDM	Fuzzy Multiple-Attribute Group Decision-Making
FSD	Fully Stressed Design
FSOA	Fish Swarm Optimisation
GA	Genetic Algorithms
GCMMA	Globally Convergent Method of Moving Asymptotes
GESO	Genetic Evolutionary Structural Optimisation
GMAW	Gas Metal Arc Welding
GRAND	Ground Structure Analysis and Design
GSM	Ground Structure Method
GTAW	Gas Tungsten Arc Welding
IPOPT	Interior Point Optimiser
KS	Kreisselmeier–Steinhauser Function
LBM	Laser Beam Melting
LENS	Laser Engineered Net Shaping
LMD	Laser Metal Deposition
LMF	Laser Metal Fusion
LOM	Laminated Object Manufacturing
LPM	Lumped Parameter Model
LS	Level-Set Methods
LSM	Level-Set Methods
LSP	Large-Scale Prefabrication
LSTO	Lattice Structure Topology Optimisation
MD	Metal Deposition
MINLP	Mixed-Integer Non-Linear Programming
MMA	Method of Moving Asymptotes
MMC	Moving Morphable Components
MMTO	Multi-Material Topology Optimisation
MOTO	Additive Manufacturing-Oriented Topology Optimisation
MRF	Moment Resisting Frames
MSP	Materials with Site-Specific Properties
MTOP	Multiresolution Topology Optimisation
NDT	Non-Destructive Testing
NEWC	Normalised Exponential Weighted Criterion
NOM	Near-Optimal Microstructure
NPR	Negative Poisson's Ratio Materials
NRBTO	Non-Probabilistic Reliability-Based Topology Optimisation
N-VTM	Nonlinear Virtual Temperature Method
OAPI	Open Application Programming Interface
OC	Optimality Criteria
OFM	Optimisation for Manufacture
OMP	Optimal Microstructure with Penalisation
OWL	Web Ontology Language
PAW	Plasma Arc Welding
PBF	Powder Bed Fusion
PBF-EB	Electron Beam Powder Bed Fusion
PBF-L	Laser Powder Bed Fusion
PCG	Preconditioned Conjugate Gradients
PETSc	Portable and Extendable Toolkit for Scientific Computing
P-GSM	Projection-Based Ground Structure Topology Optimisation Method
PSO	Swarms including Particle Swarm Optimisation
RAMP	Rational Approximation of Material Properties
RBF	Radial-Basis Functions
RBTO	Reliability-Based Topology Optimisation
RDF	Resource Description Framework

RI	Repulsion Index
RP	Rapid Prototyping
RSL	Revolutionary Superposition Layout
SAMP	Solid Anisotropic Material with Penalisation
SDS	Stochastic Diffusion Search
SERA	Sequential Elements Rejection and Admission
SFF	Solid Freeform Fabrication or Solid Freedom Fabrication
SFMD	Simultaneous Failure Mode Design
SIMP	Solid Isotropic Microstructure (or Material) with Penalisation
SLA	Stereolithography
SLM	Selective Laser Melting
SLS	Selective Laser Sintering
SM	Shape Melting
SMD	Shape Metal Deposition
SNOPT	Sparse Nonlinear Optimiser
SOM	Sectional Optimisation Method
SOM	Skidmore Owings and Merrill LLP
SRV	Sum of the Reciprocal Variables
STM	Strut-and-Tie Models
STO	Structural Topology Optimisation
STSA	Structural Topology and Shape Annealing
SW	Shape Welding
TIMP	Transplanting Independent Continuous and Mapping Ideas into Material with Pe-nalisation
TO	Topology Optimisation
TOBS	Topology Optimisation of Binary Structures
TOSS	Topology Optimisation of Skeletal Structures
TTO	Truss Topology Optimisation
UAM	Ultrasonic Additive Manufacturing
UR	Uncertainty Regions
UV	Ultraviolet Light
VCS	Virtual Component Skeleton
VE	Value Engineering
VSFM	Virtual Scalar Field Method
WAAM	Wire and Arc Additive Manufacturing

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