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International Airline Alliance Network Design with Uncertainty

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Abstract: This paper addresses the alliance route network design problem considering uncertainty in the unit transportation cost. An alliance route network was constructed based on a hub-and-spoke (HS) network, in which airlines could achieve inter-area passenger transport through their international gateways. The design problem was formulated with a robust model containing a set of uncertain cost parameters. The model was established based on a three-subscript model of an HS network. A case study with real-world data was used to test the proposed model. The results showed that this robust solution can reduce the impact of cost uncertainty.

Keywords: alliance route network; network design; hub-and-spoke network; robust model

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1. Introduction

An international airline alliance, which is composed of airlines, is a joint collaboration of airline companies with the aim of establishing a unified global route system for improving competitiveness through code sharing or joint operations. Differently from typical airline operation modes, the airlines in the alliances are coordinated and treated equally to maximize the profits. For the implementation of an international airline alliance, diverse airports provide services to support the alliance, although there are multiple restrictions in practice. Some benefits of having an international alliance, i.e., a collaboration between airline companies among different countries, have been highlighted. For instance, limitations in the regulatory or legal barriers between countries can be released [1], airlines' passenger capacity can be increased, and the chance of improving profits is improved by having a larger number of airline destinations. Moreover, an international airline alliance benefits the passenger experience by way of having access to larger networks, having convenient services, having more choices of carriers, and being connected to more flyer points [2]. An international airline alliance enables an effective coordination of flight schedules so as to minimize the travelers' waiting time and provide a sufficient usage of time between flights [3]. The handover of passenger loads among airports can be simplified, and the same aircraft maintenance services can be shared to reduce the maintenance time within an airline alliance network. An international airline alliance also facilitates the quality of service (QoS)—for instance, by establishing a flexible price negotiation model, optimizing the airline routes, and reducing airlines' running expenses.

For international airline alliances, there are three typical categories, which are grouped in accordance with the topologies among airline members, i.e., Star Alliance, Oneworld, and SkyTeam, which have initial partners located in the major geographic regions and are often involved in bilateral partnerships with other founders. The scheduled passenger volumes of the three major alliances account for more than 50% of the global civil aviation industry [4].

Distinct from domestic airline alliances, international alliances are more challenging due to some practical considerations, such as the large number of uncertain factors, docking through separate gateways, the uncertainties in agile policies, changeable economic and social factors, and the Snyder effect in uncertainties [5].

To address the airline alliance problem, the problem is separated into two specific issues [6], i.e., the selection of an alliance model and the route network design.

Some typical alliance models of the partial connection type have been highlighted, such as competition model and strategic model. The authors of [7] evaluated and discussed the performance of applying a strategic model for the alliance problem, where three alliance types were classified in accordance with their alliance strategies; these were a complementary alliance (also called the vertical competition model), parallel alliance (also called the horizontal competition model), and hybrid alliance. Among them, the complementary model has been widely used for airline alliances owing to its elimination of the negative externalities between routes [8] and the benefits of reducing the connection fare and improvement of social welfare. It is possible to significantly shorten the connection time for flight transfers with the complementary alliance model, leading to the potential possibility of having seamless transfer (one ticket to the end at the place of the departure, with a direct luggage hanging service [9]). Consequently, this paper implements the complementary model as the prototype for addressing the model selection issue.

The route network is the foundation of airline operation. By redesigning and adjusting the route network, the transportation costs of airlines are reduced, resulting in a risk-resistant model for providing reliable services and promising strategies for airline expansion in the future. In particular, the hub-and-spoke (HS) network is a promising network type for formulating route networks due to its feasibility and efficiency in practice. Several research works have been written to address the airline network optimization problem with the HS network. The authors of related articles (Alumur and Kara [10]; Cambell and O'Kelly [11]) examined single allocation, multiple allocation, as well as capacitated, incapacitated, strict, and non-strict models with extensions.

However, the works presented above only work for the common alliance problem, and they assume deterministic alliance models with few constraints introduced [12], which leads to the proposition to exploit uncertain models with complex factors for designing international alliance networks. The common alliance model neglects the uncertainty, which represents unknown situations and may be encountered in practical operations, as well as the flexible parameters that are challenging to estimate. For those flexible parameters, a reasonable interval range or possible probability distribution is obtained by statistics, such as the airport capacity distribution, airline capacity, demands among airports, and unit transportation costs. Furthermore, finding the worst-case scenario with respect to a determined uncertainty set (Bertsimas, Brown, and Caramanis [13]) is another gap that has not been sufficiently investigated in previous research.

This paper explores the methods for narrowing the gaps in airline network optimization with uncertainties. We applied the uncertainty optimization method to optimize the design of an alliance route network, and we present an international route network with both economy and risk resistance for airlines. Specifically, we propose an approach that is able to use the complementary alliance model with the HS route method for the purpose of optimization. An incapacitated multiple p-hub median problem (UMpHMP)-based alliance model is proposed, along with the integration of uncertain factors. The network optimization problem was addressed by developing a three-subscript model and was measured on the worst-case scenario. Compared with typical network model structures [12], the proposed three-subscript optimization model is applicable for more than two airline alliances and multiple gateway numbers, and the uncertain factors can be integrated to meet the practical operation requirements.

The rest of this paper is organized as follows. Section 2 provides a brief review of the literature on airline alliances and HS networks. Section 3 details the construction of an alliance route network based on an HS network and proposes a mathematical model to formulate the alliance route network design problem with uncertain costs. A case is provided in Section 4 to illustrate the proposed model. Section 5 presents the conclusion and suggestions for future research.

2. Literature Review

Regarding the multiple airline alliance models, several works have been done. Oum, Park, and Zhang [14] investigated alliances using a complementary and a parallel model, with the conclusion of better economic performance with the complementary alliance model. Zhang [7] examined hybrid alliances whose members have complementary and overlapping routes, and discussed the implications of hybrid alliances for international airline alliances. From the alliance cooperation perspective, Oum, Park, Kim, and Yu [15] grouped alliances into high-level and low-level categories according to the degree of participation in the cooperation. A high-level alliance involves network-level collaboration, representing that the allied airlines connect their route networks, while the low-level collaboration considers the route-level cooperation, neglecting the network topology considerations.

Some research has been done on designing and optimizing airline alliance networks. Wen and Hsu [16] took factors of the flight frequency and cost into consideration for the alliance network design, where a multi-objective function was presented based on the code sharing among member airlines for the purpose of maximizing the overall profits. Adler and Smilowitz [12] investigated international alliances and mergers in a competitive environment. The results revealed that the optimal international gateway choices vary depending on the number of competitors remaining in the market. Lordan, Sallan, and Simo [17] focused on a reliability analysis of the three typical network models for an alliance network, and proposed an alternative node selection strategy to evaluate their robustness and vulnerability. Lordan and Klophaus [18] analyzed the vulnerability of member airlines for exiting the alliance. Their results suggested that Oneworld is the most vulnerable alliance, with SkyTeam ranked as the second and Star Alliance as the third.

For designing optimized networks, some works have been done. O'Kelly [19] firstly applied an HS network to address the minimization of the cost of designing hub networks. Campbell [20,21] proposed an integer-linear-program-based method with the consideration of single and multiple allocation schemes. Ernst and Krishnamoorthy [22,23] proposed variants oriented from hub-location-allocation problems with fewer variables. The above typical linear formulations were adjusted according to unique specifications, along with the processing of some distinguished features. Among them, the parameter uncertainty was studied in order to address HS network design problems. After their investigations, Fageda and Flores-Fillol obtained the conclusion that the hub-and-spoke network structure remains advantageous with higher congestion costs in the hubs [24]. To build route networks with the consideration of market competition, Jiang and Zhang considered the long-term impact resulting from the competition of high-speed rail with airlines, and developed an analytical model to investigate the airlines' impact on networks, as well as the impacts on markets when the competition of high-speed rail in trunk lines exists [25]. Babić and Kalić discussed airline models operating in a competitive environment in order to select a network structure. In order to capture the interaction between competing airlines in the selection, the impacts of price, flight frequency, seat accessibility, and route length on product differentiation were thoroughly studied [26].

For network optimization with uncertainties, some studies were carried out. Averbakh [27] formulated the deviation and robust optimization problem and designed an optimization algorithm to convert the original problem, whose objective function was MinMax, into a deterministic objective function. Wang and He [28] presented a robust optimization model with the regret model format for the purpose of the central localization of logistics in an uncertain environment; the proposed model outperformed the stochastic optimization model in their simulations. Szucs [29] took the costs of network elements as the uncertain factors and proposed a solution enabled by the Dempster–Shafer theory and Dijkstra's algorithm for planning routes in a transport network. Shahabi and Unnikrishnan [30] formulated a robust model for the hub localization problem with features of considering incapacitated single- and multiple-allocation schemes and the uncertain demands in advance. The robust model was transformed from a mixed-integer nonlinear program into a mixed-integer conic quadratic program in their propositions.

3. Airline Alliance Network

In this section, we present mathematical formulations for the alliance route network design problem with consideration of the uncertainty of cost parameters. Three objectives are considered in the model: (1) determining the optimal locations of international gateways; (2) determining the network configuration, including transport paths for demands and the flow volume on each path; (3) making the international gateway location decisions risk-resistant in terms of the unit transportation cost.

3.1. Problem Formulation

An example of an alliance route network is displayed in Figure 1.The nodes of the network include spokes and hubs. As a complex HS route network, this alliance route network architecture has the capability of connecting the networks of different airlines. The hubs are separated into regional hubs (R) and international gateways (K). Regional hubs connect local airports (i) within an area, while international gateways have connections with different areas. Alliance airlines may choose one or more international gateways from regional hubs to transfer passengers from different areas.

Compared with an HS network, the constructed alliance route network is characterized below:

1. The cooperation between alliance partners should meet the limitations of traffic rights.
2. The round-trip passenger flow of international routes usually differs greatly and is asymmetric.
3. Usually, the domestic network of an airline is relatively complete before the airline joins an alliance, and the regional hubs can be given. However, airlines need to choose their international gateways from the set of regional hubs through optimization.
4. One or more international gateways coexist with each area for each airline, and all gateways are interconnected across international areas.

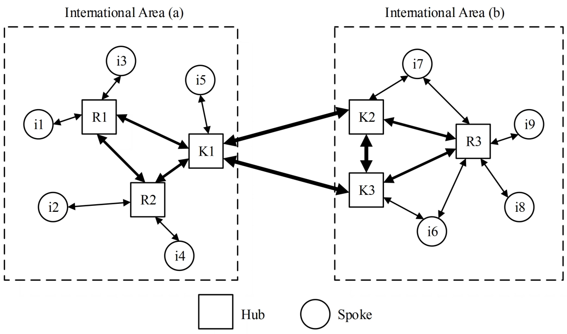

Figure 1. Alliance route network configuration.

In order to reasonably reflect the practice and simplify the problem, the proposed model is subject to the following assumptions:

1. All international gateways are interconnected across international areas. Due to the economy of scale in which passengers converge at gateway airports, a discount factor α is incorporated into the cost between international gateways.
2. According to the actual transportation situation, international transportation usually does not exceed two transits, so airports other than the international gateways are regarded as "spokes" of international routes. Multi-allocation connections between "spokes" and international gateways are adopted; χ is the discount factor from

"spokes" to international gateways, and δ is the discount factor from international gateways to "spokes". Generally, $0 \leq \alpha < \chi, \delta \leq 1$.
3. The regional hubs within each area are given, and airlines choose their international gateways from this subset of regional hubs.
4. Inter-area journeys are limited to three legs, that is, if both the origin and destination nodes are "spokes", traveling across international areas will necessarily involve a three-leg journey. For example, to travel from i5 to i7 would involve one leg from i5 to K1, a second leg from K1 to K2, and, finally, a third leg from K2 to i7. On the other hand, inter-area journeys contain at least one leg, and this happens when both the originating and destination nodes are the international gateways.
5. Inter-area traffic must all be transported from the originating node to the destination node.
6. The round-trip passenger demand of international routes is usually asymmetric; $W_{ij} \neq W_{ji}$.

3.2. Alliance Route Network Model

Sets, parameters, and decision variables are introduced before describing the mathematical formulation.

Sets and parameters:

A is a set of international areas that need to establish alliances with airlines; $a \in A$.

N is a set of all nodes in the network; $i \in N$, $j \in N$.

N^a is a set of nodes in the network of area $a \in A$; $N^a \subseteq N$.

H is a set of all regional hubs in the network; $r \in H$, $k \in H$, $m \in H$, $t \in H$, $H \subseteq N$.

H^a is a set of regional hubs within area $a \in A$; $H^a \subseteq H$.

S is a set of scenarios for the uncertain transportation cost; $s \in S$.

W_{ij} is the passenger demand from origin $i \in N^a$ to destination $j \in N \backslash N^a$.

O_i is the total traffic flow starting from node $i \in N^a$; $O_i = \sum_{j \in N \backslash N^a)} W_{ij}$.

$C_{ik}(s)$ is the unit transportation cost from "spoke" $i \in N^a$ to international gateway $k \in H^a$ under scenario $s \in S$.

$C_{km}(s)$ is the unit transportation cost from international gateway $k \in H^a$ to international gateway $m \in H \backslash H^a$ under scenario $s \in S$.

$C_{mj}(s)$ is the unit transportation cost from international gateway $m \in H \backslash H^a$ to "spoke" $j \in N^{(m)}$ under scenario $s \in S$, where $N^{(m)}$ denotes the set of nodes in the area containing gateway $m \in H \backslash H^a$.

Decision variables:

h_k is 1 if an international gateway is located at node $k \in H^a$, and 0 otherwise.

Z_{ik} is the total amount of flow from regional hub $i \in N^a$ to international gateway $k \in H^a$.

Y^i_{km} is the amount of flow from regional hub $i \in N^a$ to international gateway $k \in H^a$ that arrives at the gateway $m \in H \backslash H^a$.

X^i_{mj} is the amount of flow from regional hub $i \in N^a$ to node $j \in N \backslash N^a$ that goes through the international gateway $m \in H \backslash H^a$.

$Z^*(s)$ is the minimum total transportation cost of an alliance network constructed under scenario s.

Given the uncertain transportation effects, an alliance route network model is proposed as:

$$min\ \lambda,$$

which is subject to:

$$\sum_{\alpha \in A} \left[\sum_{i \in N^{\alpha}} \left(\sum_{k \in H^{\alpha}} \chi C_{ik}(s) Z_{ik} + \sum_{k \in H^{\alpha}} \sum_{m \in H \backslash H^{\alpha}} \alpha C_{km}(s) Y_{km}^{i} + \sum_{m \in H \backslash H^{\alpha}} \sum_{j \in N^{(m)}} \delta C_{mj}(s) X_{mj}^{i} \right) \right] \leq (1+\lambda) Z^{*}(s), \forall s \in S \tag{1}$$

$$\sum_{\alpha \in A} \sum_{k \in H^{\alpha}} h_k = p \tag{2}$$

$$\sum_{k \in H^{\alpha}} Z_{ik} = O_i; \forall i \in N^{\alpha}, \alpha \in A \tag{3}$$

$$\sum_{m \in H^{(j)}} X_{mj}^{i} = W_{ij}; \forall i \in N^{\alpha}, j \in N \backslash N^{\alpha}, \alpha \in A \tag{4}$$

$$\sum_{m \in H \backslash H^{\alpha}} Y_{km}^{i} = Z_{ik}; \forall i \in N^{\alpha}, k \in H^{\alpha}, \alpha \in A \tag{5}$$

$$\sum_{j \in N^{(m)}} X_{mj}^{i} = \sum_{k \in H^{\alpha}} Y_{km}^{i}; \forall i \in N^{\alpha}, m \in H \backslash H^{\alpha}, \alpha \in A \tag{6}$$

$$Z_{ik} \leq O_i h_k; \forall i \in N^{\alpha}, k \in H^{\alpha}, \alpha \in A \tag{7}$$

$$X_{mj}^{i} \leq W_{ij} h_m; \forall i \in N^{\alpha}, m \in H \backslash H^{\alpha}, j \in N^{(m)}, \alpha \in A \tag{8}$$

$$Z_{ik}, Y_{km}^{i}, X_{mj}^{i} \geq 0; \forall i \in N^{\alpha}, k \in H^{\alpha}, m \in H \backslash H^{\alpha}, j \in N^{(m)}, \alpha \in A \tag{9}$$

$$h_k \in \{0,1\}, \forall k \in H^{\alpha}, \alpha \in A. \tag{10}$$

Formula (1) is the requirement of relatively robust optimization, that is, for each design of the alliance route network, the relative deviation between the total transportation cost and the optimal transportation cost under each scenario is calculated, and the maximum relative deviation must be minimized. The total transportation cost in parentheses includes the collection cost, transfer cost, and distribution cost. Constraints (2) indicate that the number of international gateways for an alliance network must be exactly p. Constraints (3) guarantee that all of the traffic flow should be shipped out from the originating city. Constraints (4) guarantee that all of the traffic flow should be delivered to the destination city. Constraints (5) and (6) are passenger flow balance constraints. Constraints (7) and (8) ensure that the flow of transport through the international gateway is possible only if that gateway is open. Constraints (9) require that all of the flow variables are non-negative, and Constraints (10) specify that the international gateway selection variables are binary.

3.3. Optimal Solution

The robust optimization of the alliance route network design model with uncertainty is to find minimum λ satisfying the constraint condition and route network design plan (selection of gateway and route, arrangement of OD flow) of minimum λ avoiding risks to the maximum extent. The solution principle is to give a relatively small constant value of λ (λ can be solved by adding iterative calculation.), calculate the model (11) of each scenario with different gateway combination, and update λ to approach the minimum value of λ which the network design is continuously improved.

The model of each scenario with different gateway combination can be formulated.

$$Z(s) = min \sum_{\alpha \in A} \left[\sum_{i \in N^{\alpha}} \left(\sum_{k \in H^{\alpha}} \chi(s) C_{ik} Z_{ik}(s) + \sum_{k \in H^{\alpha}} \sum_{m \in H \setminus H^{\alpha}} \alpha(s) C_{km} Y^{i}_{km}(s) + \sum_{m \in H \setminus H^{\alpha}} \sum_{j \in N^{(m)}} \delta(s) C_{mj} X^{i}_{mj}(s) \right) \right] \tag{11}$$

which is subject to:

$$\sum_{\alpha \in A} \sum_{k \in H^{\alpha}} h_k(s) = p \tag{12}$$

$$\sum_{k \in H^{\alpha}} Z_{ik}(s) = O_i(s); \forall i \in N^{\alpha}, \alpha \in A \tag{13}$$

$$\sum_{m \in H^{(j)}} X^{i}_{mj}(s) = W_{ij}(s); \forall i \in N^{\alpha}, j \in N \setminus N^{\alpha}, \alpha \in A \tag{14}$$

$$\sum_{m \in H \setminus H^{\alpha}} Y^{i}_{km}(s) = Z_{ik}(s); \forall i \in N^{\alpha}, k \in H^{\alpha}, \alpha \in A \tag{15}$$

$$\sum_{j \in N^{(m)}} X^{i}_{mj}(s) = \sum_{k \in H^{\alpha}} Y^{i}_{km}(s); \forall i \in N^{\alpha}, m \in H \setminus H^{\alpha}, \alpha \in A \tag{16}$$

$$Z_{ik}(s) \leq O_i(s) h_k(s); \forall i \in N^{\alpha}, k \in H^{\alpha}, \alpha \in A \tag{17}$$

$$X^{i}_{mj}(s) \leq W_{ij}(s) h_m(s); \forall i \in N^{\alpha}, m \in H \setminus H^{\alpha}, j \in N^{(m)}, \alpha \in A \tag{18}$$

$$Z_{ik}(s), Y^{i}_{km}(s), X^{i}_{mj}(s) \geq 0; \quad \forall i \in N^{\alpha}, k \in H^{\alpha}, m \in H \setminus H^{\alpha}, j \in N^{(m)}, \alpha \in A \tag{19}$$

$$h_k(s) \in \{0, 1\}, \forall k \in H^{\alpha}, \alpha \in A \tag{20}$$

The proposed solver is presented as follows in Algorithm 1.

Algorithm 1 An Iterative Optimization Algorithm for the Alliance Route Network

1: Initialization $Z^*(s) = +\infty, s = 1, \ldots, S$, $\lambda = T$, T is a small positive number.
2: Select p regional hubs as international gateways. The possible combinations corresponding to p international gateways are $C^{a'}_{H^a} \times C^{b'}_{H^b} \times C^{c'}_{H^c} \times \ldots$, one of the combination is represented by H, H^a represents the set of optional hubs in area a. $\{a, b, c, \ldots\} = A$, $a' + b' + c' + \cdots = p$, $a', b', c' \geq 1$.
3: For the combination H, use the model (11) to find the solution under scenario s, $s = 1, \ldots, S$, obtain the total transportation cost $Z(H, s)$.
4: For scenario s, if $Z(H, s) \geq Z^*(s)$, turn to Step 5, else $Z(H, s) < Z^*(s)$, renew $Z^*(s) = Z(H, s)$, turn to Step 5.
5: If $\frac{Z(H,s) - Z^*(s)}{Z^*(s)} \leq T$ for all $s = 1, \ldots, S$, output $\lambda = T$, $H^* = H$, H^* is the optimal international gateway combination, else turn to Step 6.
6: if $\frac{Z(H,s) - Z^*(s)}{Z^*(s)} \geq T$ for some s, turn to Step 2.

4. Case Study

4.1. Data Settings

In this section, we selected a Chinese-based and US-based airline alliance network as a general example to examine the performance of the proposed models; the use case can be easily extended to other scenarios. We selected 10 cities in China and 8 cities in the US to form the route network and implement the airline alliance. The distribution of airports and relevant parameter configurations are presented in Table 1.

Table 1. Citys airport.

Node	Airport	Node	Airport	Node	Airport
1	PEK*\Beijing	7	CKG\Chongqing	13	DTW*\Detroit
2	CAN\Guangzhou	8	XIY*\Xi'an	14	LAX*\Los Angeles
3	PVG*\Shanghai	9	WUH*\Wuhan	15	MSP*\Minneapolis
4	CTU\Chengdu	10	NKG\Nanjing	16	SFO\San Francisco
5	SZX\Shenzhen	11	ATL*\Atlanta	17	SEA\Seattle
6	KMG*\Kunming	12	JFK*\New York	18	ORD\Chicago

The city airports marked with * are the regional hubs of the corresponding airlines. Transportation between these regional hubs is assumed to meet the traffic regulations. The crucial data, such as the passenger demands, are sourced from the Market Information Data Transfer Database and segment distance data [31].

We applied the unit transportation cost function to measure the distance between the nodes (with $C_{rr} = 0$). We used the cost per available seat kilometer (CASK) with the great circle distance between the relevant nodes for measuring the cost parameters [32] due to the variations in the CASK values according to the distance. Moreover, the uncertain factors could be conveniently integrated into CASK as the main indicator. For the three published aircraft categories, i.e., wide body, narrow body, and regional jets, the average CASK values were computed accordingly. The wide-body aircraft serve long-distance markets (distance $>$ 5000 km); narrow-body aircraft serve distances between 1000 and 5000 km; regional jets are utilized in short-distance markets (distance $<$ 1000 km). Similarly to Adler et al. (2018) [32], the CASK values used in this research are presented in Table 2.

Table 2. Cost per available seat kilometer.

	Short Haul	Medium Haul	Long Haul
CNY per ASK	0.3724	0.2894	0.2941

To assess the impact of uncertainty in transportation cost on the design of an alliance network, we assume that the values of the CASK fluctuate within the interval $[-20\%, +20\%]$. We generated several scenarios by changing the CASK magnitudes, as shown in Table 3. As risk control is critical in the robust optimization, a boundary scenario should be considered in the robust optimization. There is no special requirement for the number of scenarios.

As shown in Table 3, we generated nine different scenarios for a combination of the three CASK values, among which scenario 1 is called the base case. In each scenario $s \in S$, we can get a set of values of the unit transportation cost.

Based on the model formulation in Section 3.3, we know that this is a linear program. The model was coded on the AIMMS platform and solved by CPLEX12.5 with the CPLEX options set to their default values. All tests were executed using a personal computer running the Microsoft Windows 7 operating system and equipped with an Intel Core i5 CPU 6500 with 3.20 GHz and 4 GB RAM.

Table 3. Scenarios.

Scenario Number	CASK-SH Deviation (%)	CASK-MH Deviation (%)	CASK-LH Deviation (%)
1	0	0	0
2	20	20	20
3	−20	−20	−20
4	20	20	−20
5	20	−20	20
6	−20	20	20
7	20	−20	−20
8	−20	20	−20
9	−20	−20	20

4.2. Computational Results

In this section, we present a computational analysis of the robust optimization model of the alliance route network design problem in order to assess the effects of uncertainty in the unit transportation cost on the resulting solutions. We assumed that the number of international gateways selected from the two areas was three. The collection and distribution discount factors were taken to be equal to one; i.e., $\chi = \delta = 1$. For the alpha value, on the other hand, we let $\alpha \in 0.2, 0.4, 0.6, 0.8$, as is customarily done in the literature.

We solved the problem separately for each scenario with the goal of minimizing the total transportation cost and solving the robust model based on the nine scenarios. The results of the optimal international gateway locations and the total transportation costs are presented in Table 4.

Table 4. Results with the uncertain unit transportation costs.

	$\alpha = 0.2$		$\alpha = 0.4$		$\alpha = 0.6$		$\alpha = 0.8$	
	Intl. GL	**Trans. Costs**	**Intl. GL**	**Trans. Costs**	**Intl. GL**	**Trans. Costs**	**Intl. GL**	**Trans. Costs**
s1	3, 12, 14	223,129,610	1, 12, 14	365,549,030	1, 12, 14	506,512,065	1, 13, 14	647,009,456
s2	3, 12, 14	267,755,532	1, 12, 14	438,658,836	1, 12, 14	607,814,479	1, 13, 14	776,411,347
s3	3, 12, 14	178,503,688	1, 12, 14	292,439,224	1, 12, 14	405,209,652	1, 13, 14	517,607,565
s4	3, 12, 14	208,191,272	1, 12, 14	325,888,407	1, 12, 14	438,658,836	1, 12, 14	551,429,264
s5	1, 12, 14	241,491,590	1, 12, 14	410,647,234	1, 12, 14	579,766,838	1, 14, 15	743,667,894
s6	3, 13, 14	260,392,325	1, 13, 14	429,436,112	1, 13, 14	596,487,252	1, 13, 14	763,538,391
s7	3, 12, 14	183,698,164	3, 12, 14	297,876,805	1, 12, 14	410,647,234	1, 12, 14	523,417,662
s8	3, 13, 14	201,692,264	1, 13, 14	318,068,686	1, 13, 14	429,436,112	1, 13, 14	540,803,538
s9	1, 12, 14	236,054,009	1, 12, 14	405,209,652	1, 14, 15	572,475,719	1, 14, 15	736,129,225
RM	3, 12, 14		1, 12, 14		1, 12, 14		1, 12, 14	

Note: s1–s9: scenario number; RM: robust optimization model; Intl. GL: optimal international gateway locations; Trans. costs: total transportation costs in the respective optimal solutions.

Note that in robust solutions, the total transportation costs are different in each scenario; hence, the values of the "Trans. Costs" are left empty. Observe in Table 4 that, for each scenario, the optimal international gateway locations are not always the same when the alpha value changes. For instance, the solution of the base case suggests locating tthe international gateways at Nodes 3, 12, and 14 for $\alpha = 0.2$, Nodes 1, 12, and 14 for $\alpha = 0.4$

and $\alpha = 0.6$, and Nodes 1, 13, and 14 for $\alpha = 0.8$. This proves that the optimal solutions are sensitive to the alpha value. In addition, the magnitude of the total transportation costs increases with the increase in the alpha value.

For each alpha value, the optimal international gateway locations may be different in each of the scenarios. For example, when $\alpha = 0.8$, although Node 1 and Node 14 were selected as international gateways in all of the scenarios, the third gateway was not always the same. Specifically, the optimal solution in Scenarios 1, 2, 3, 6, and 8 was $\{1, 13, 14\}$, while in Scenarios 4 and 7, it was $\{1, 12, 14\}$, and in Scenarios 5 and 9, it was $\{1, 14, 15\}$. If an alliance route network is constructed based on the optimal solution in the base case, the international gateways are PEK, DTW, and LAX, which will be different from the optimal gateways PEK, JFK, and LAX in Scenarios 4 and 7 and the optimal gateways PEK, LAX, and MSP in Scenarios 5 and 9. This suggests that the international gateways selected for the base case will no longer be optimal in some other scenarios. In general, when only the cost data in a certain scenario are optimal, the solutions might not be optimal in other scenarios, and may even deviate significantly from the original optimal solutions.

We can see from Table 4 that the optimal solutions of the robust model are different from the solutions obtained in some scenarios. In order to illustrate that the robust solution can adapt to many possible scenarios with different cost parameters, we took $\alpha = 0.8$ as an example and calculated the relative deviation of the robust solution in each scenario. The results are shown in Figure 2.

The calculation results show that in each scenario, the relative deviation of the robust solution was small. Even in the worst case, such as Scenario 8 $(-20\%,\ 20\%, -20\%)$, the relative deviation was 1.68%. In other words, even if the robust solution $\{1, 12, 14\}$ differed from the optimal solution in Scenario 8, the relative deviation from the total cost of the optimal solution$\{1, 13, 14\}$ did not exceed 2%. In addition, we found that even though the robust solution was the same as the optimal solutions of Scenarios 4 and 7, there was a certain deviation in the total transportation cost. This is due to the differences in transport paths and flow volume on each path.

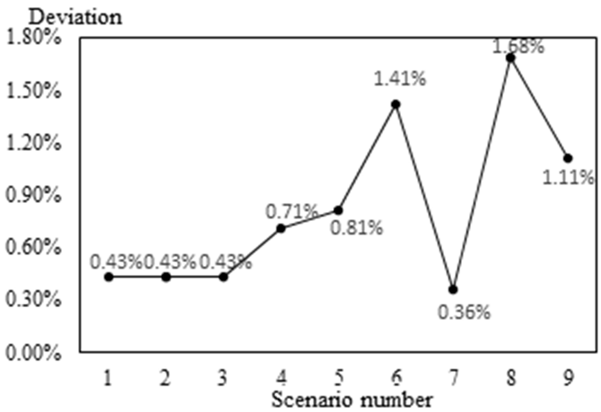

Figure 2. Relative deviations.

We conclude that, since the optimal solutions are sensitive to the unit transportation costs, with high amounts of uncertainty in the unit transportation costs, it is better to adopt the solution obtained with the robust model instead of adopting the solution of a particular scenario. This proves that the model presented in Section 3.2 can be used to realize transportation through the alliance routes and reduce the impact of cost uncertainty on the design of an alliance route network.

Taking $\alpha = 0.8$ as an example, the international gateways chosen according to the robust solution are PEK, JFK, and LAX. Connections between the city airports can be seen in Figure 3.

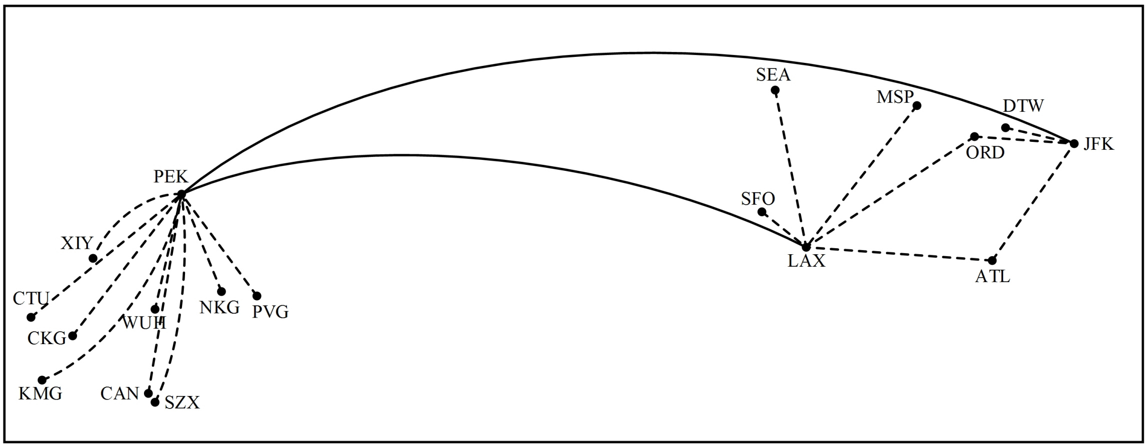

Figure 3. Alliance route network optimization design.

The dotted lines indicate the connections between "spokes" and international gateways, and the solid lines indicate the connections between the international gateways. PEK was selected as the gateway of China, and JFK and LAX were selected as the gateways of the US. Thus, passengers across the two areas can be transported through the links between the international gateways.

To evaluate the impact of the number of international gateways on the robust solutions, we calculated the optimal solutions for different p values according to different alpha values, as shown in Table 5.

As can be seen from Table 5, for different alpha values, the frequency of each gateway selection is not the same. In our case, for example, when $\alpha = 0.6$ or $\alpha = 0.8$, the frequency of gateway selection is $1, 14 > 12 > 3 > 15$. With the increase in the number of international gateways, the total transportation cost of the alliance network continues to decrease. The model proposed in this paper can select gateway locations and network configurations, and can also provide more decision-making options for airline partners.

Table 5. Optimal results for different numbers of international gateways.

	$p = 2$	$p = 3$	$p = 4$	$p = 5$
$\alpha = 0.2$	3, 14	3, 12, 14	1, 3, 12, 14	1, 3, 12, 13, 14
$\alpha = 0.4$	1, 14	1, 12, 14	1, 3, 12, 14	1, 3, 12, 13, 14
$\alpha = 0.6$	1, 14	1, 12, 14	1, 3, 12, 14	1, 3, 12, 14, 15
$\alpha = 0.8$	1, 14	1, 12, 14	1, 3, 12, 14	1, 3, 12, 14, 15

Through our simulations, we obtained more results. A large number of gateways contributes to excessive diversion and flow dispersion, leading to a discount capacity for inter-hub transportation. A small number of gateways leads to limited choices of gateways for the origin–destination (OD) flow transfer, resulting in serious amounts of bypass transportation. Consequently, the gateway number should be seriously considered for the airlines and airline partners when constructing an alliance route network. Specifically, when multiple regional hubs have better knowledge of uncertain parameters, the gateway number can be increased so as to improve the capability for coordination among airlines. Moreover, by having a robust optimization formulation, the deviation under uncertain costs can be reduced, which makes an optimized route network more resistant to risks and economic decisions.

5. Conclusions

International alliances are built on the extensions of existing bilateral relationships and are implemented to allow the largest international carriers in the world to link their routes and frequent flyer programs to make international networks. The rational construction of an alliance route network is important for the airlines to maximize the use of the alliance network's resources and to expand their network.

In this paper, we constructed an alliance route network based on an HS airline network. Uncertainties were taken into account in the design of the alliance network. We built an alliance route network optimization model that considers the uncertainty in unit transportation costs by modifying a three-subscript model of an HS network. To date, this has not been proposed in the literature. The proposed model was tested on a real-world dataset of a China-based airline and a US-based airline. The optimal alliance route network showed that the two airlines achieved inter-area transport through their international gateways. The calculation results indicated that the optimal solutions were sensitive to the unit transportation costs, and it was better to adopt the solution obtained with the robust model instead of adopting the solution of a particular scenario. By evaluating the impact of the number of gateways on the robust solutions, we concluded that the model proposed in this paper can reduce the total transportation costs compared with those of previous studies.

The proposed model and method enabled the design of a route network with the capability of being resistant to risks and economic decisions. The model was extended from a three-subscript route network model, which is promising for addressing airline alliance problems.

The methodology developed in this paper shows potential for other alliance network optimization problems with more complicated structures, such as the capacity limitations among hub airports with the consideration of passenger demands and cost uncertainties. Dynamic and flexible costs impact the optimization performance, and the real-time estimation of costs also demands further investigation.

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