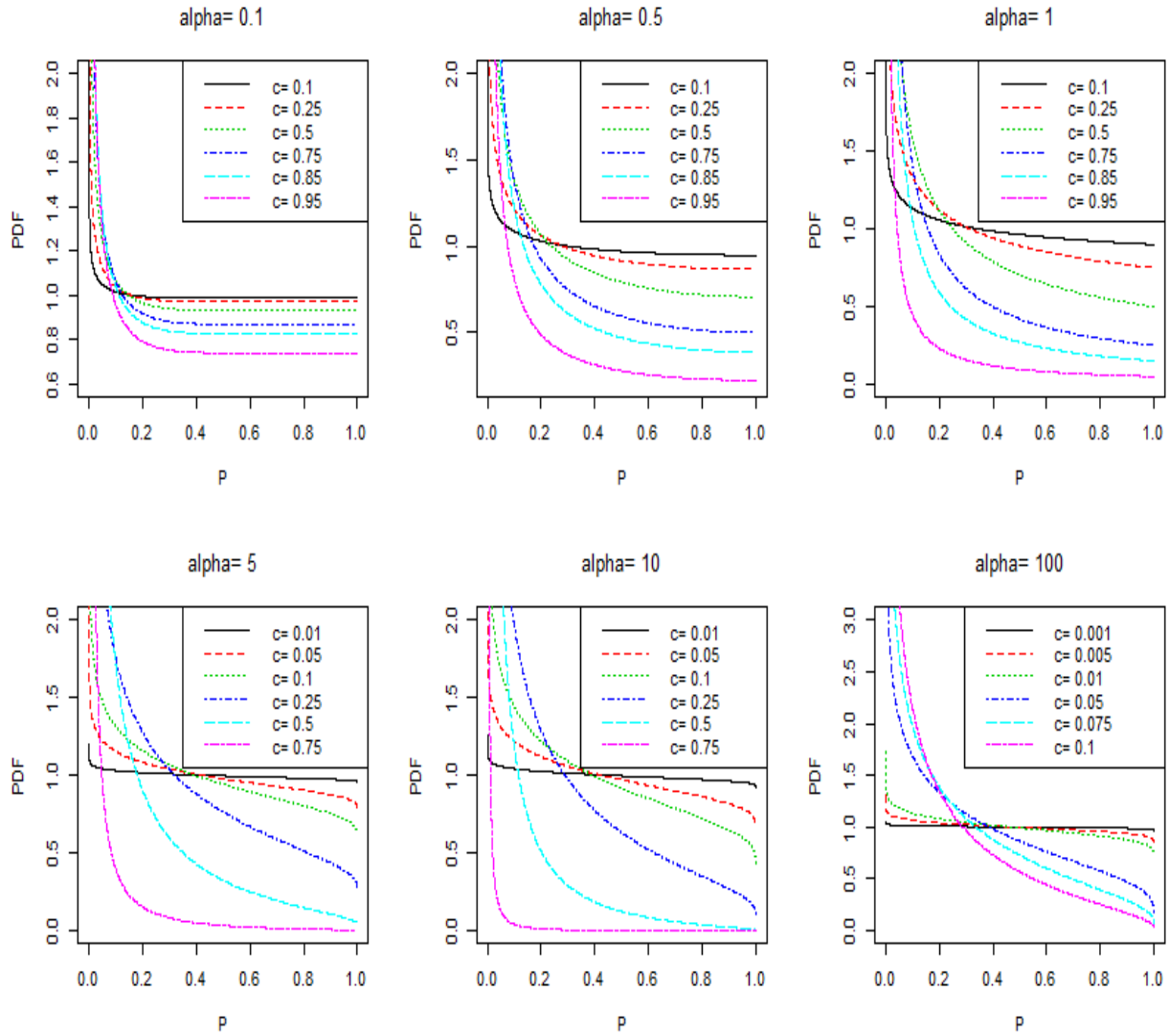


# Optimal tests for combining p-values

Zhongxue Chen

## Supplementary file

### S1. Some densities of $f_{\alpha,c}(p)$

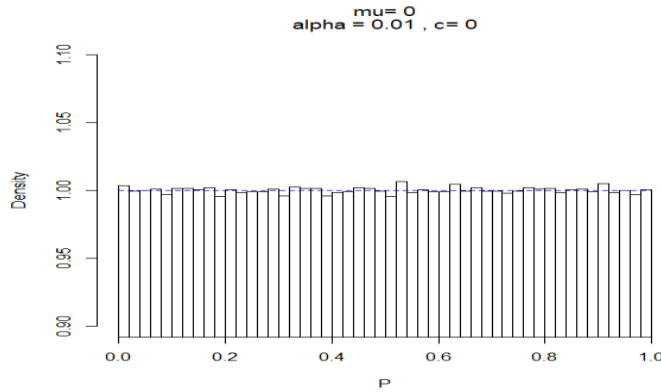


**Figure S1.** Some density functions of  $f_{\alpha,c}(p) = (1 - c)^{\alpha} e^{c F_{G(\omega)}^{-1}(1-p)}$  with different values for  $\alpha$  and  $c$ .

## S2. Histogram of p-values and the estimated pdf for $f_{\alpha,c}(p)$

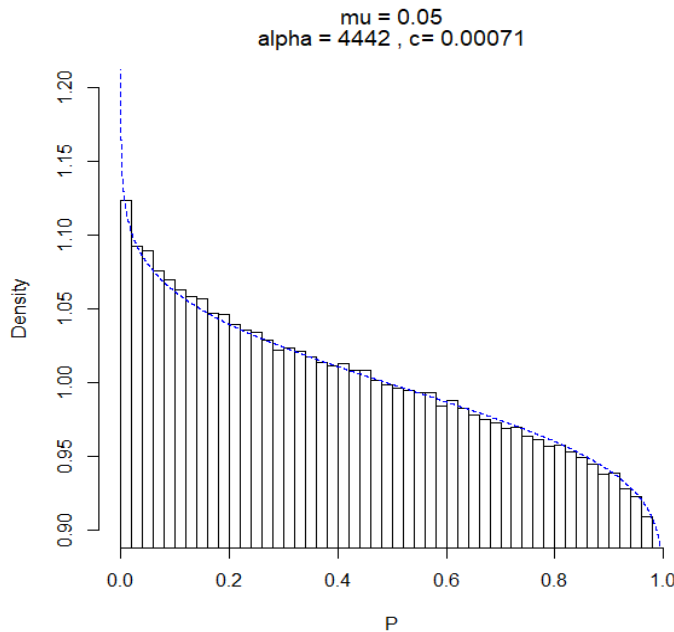
We simulate  $p_i = 1 - \Phi(Z_i)$  ( $i = 1, \dots, 10^7$ ), where  $Z \sim N(\mu_i, 1)$ . The parameters  $\alpha, c$  of  $f_{\alpha,c}(p)$  are estimated from those simulated p-values. The histogram is the plot for those simulated p-values and dash line is the pdf  $f_{\hat{\alpha},\hat{c}}(p)$ . The estimated  $\hat{\alpha}$ , and  $\hat{c}$  are also shown in the plots.

### S2. 1 Under the null ( $\mu_i = 0$ ):

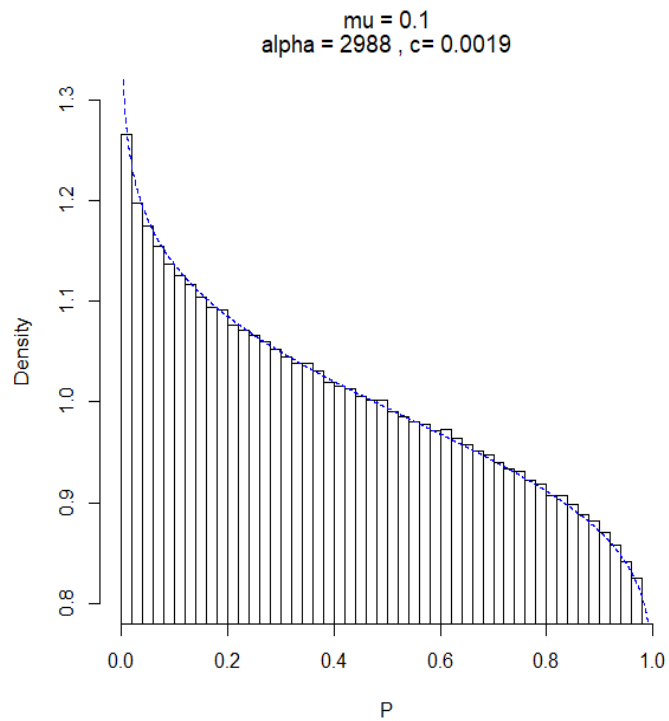


**Figure S2.** Histogram and the estimated density from the simulated data when  $\mu_i = 0$ .

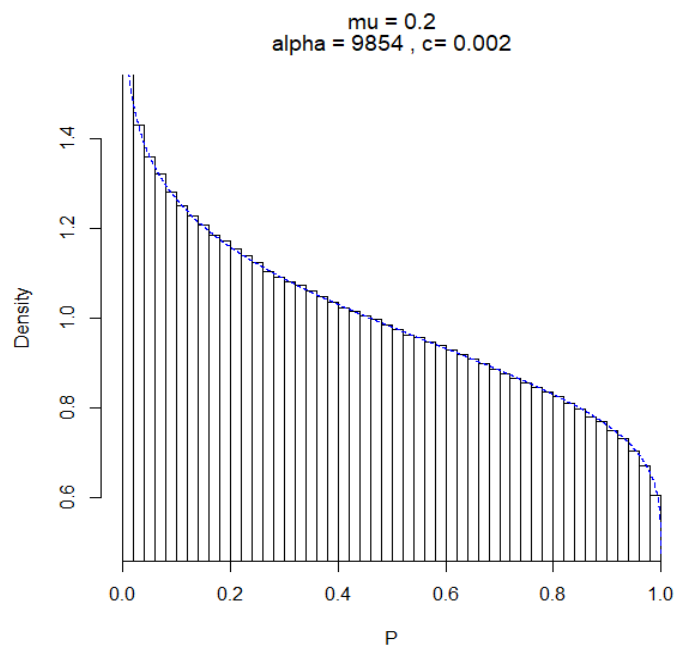
### S2. 2 Homogenous effects (i.e., $\mu_i$ 's are identical)



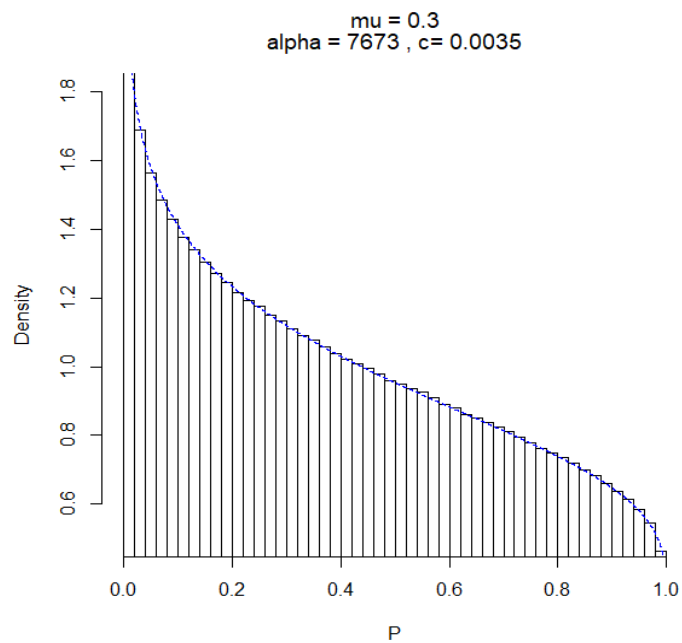
**Figure S3.** Histogram and the estimated density from the simulated data when  $\mu_i = 0.05$ .



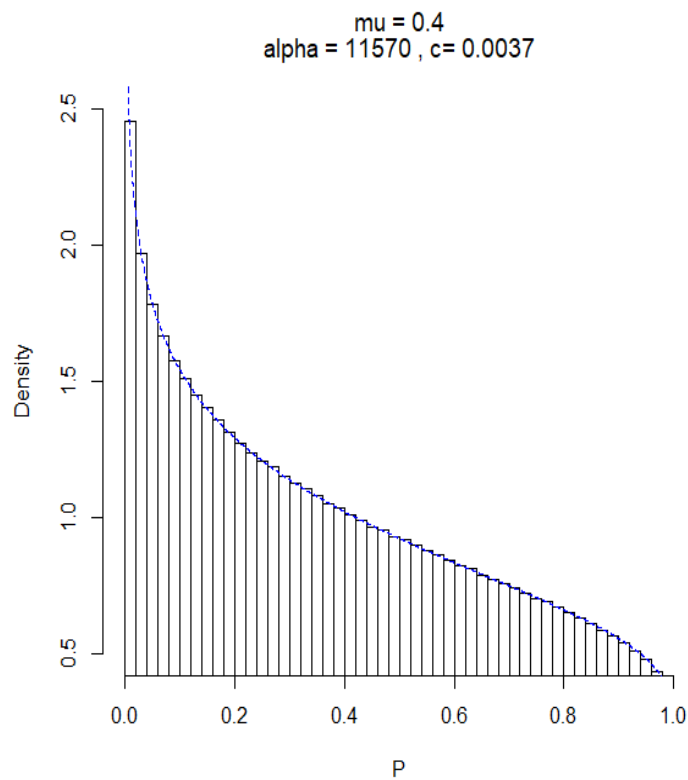
**Figure S4.** Histogram and the estimated density from the simulated data when  $\mu_i = 0.1$ .



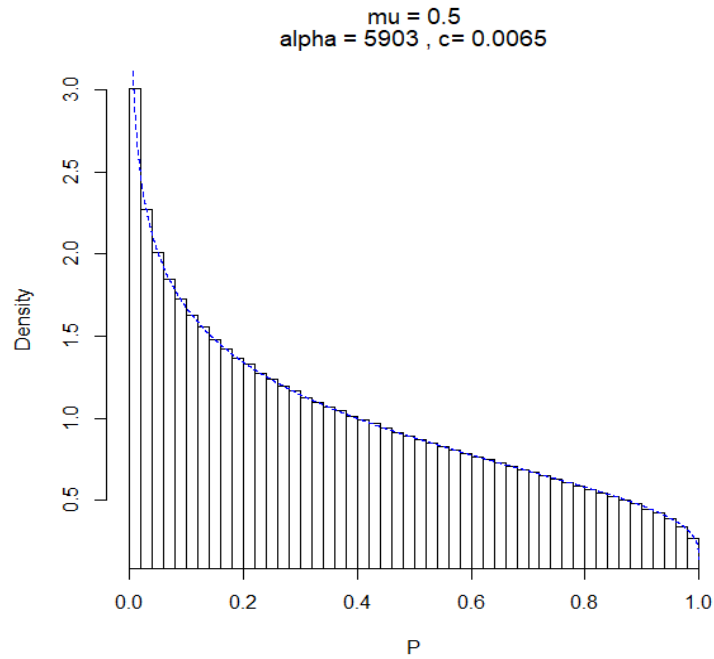
**Figure S5.** Histogram and the estimated density from the simulated data when  $\mu_i = 0.2$ .



**Figure S6.** Histogram and the estimated density from the simulated data when  $\mu_i = 0.3$ .

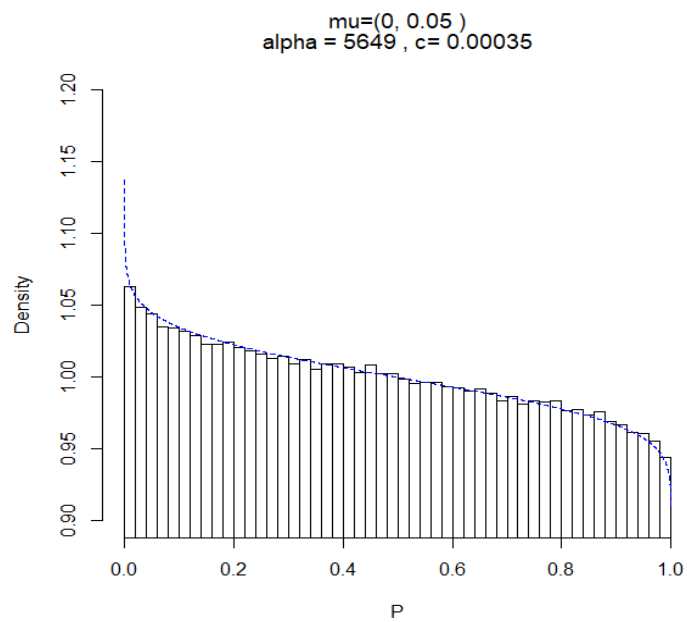


**Figure S7.** Histogram and the estimated density from the simulated data when  $\mu_i = 0.4$ .

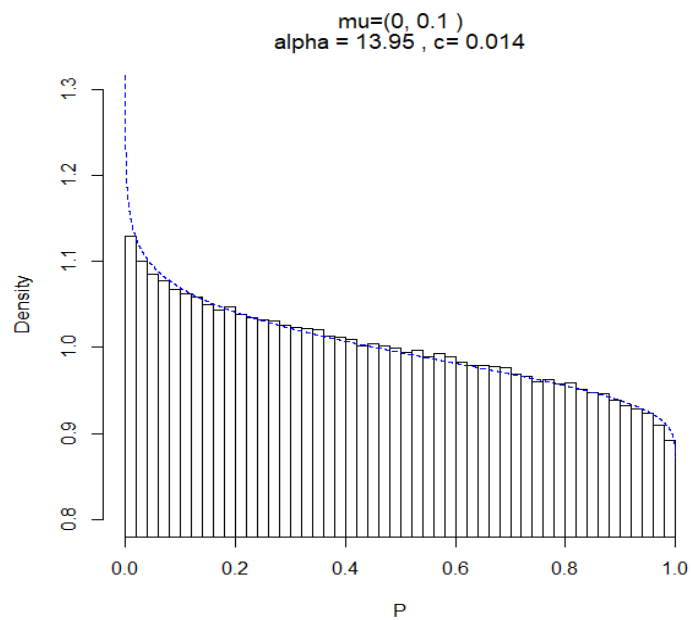


**Figure S8.** Histogram and the estimated density from the simulated data when  $\mu_i = 0.5$ .

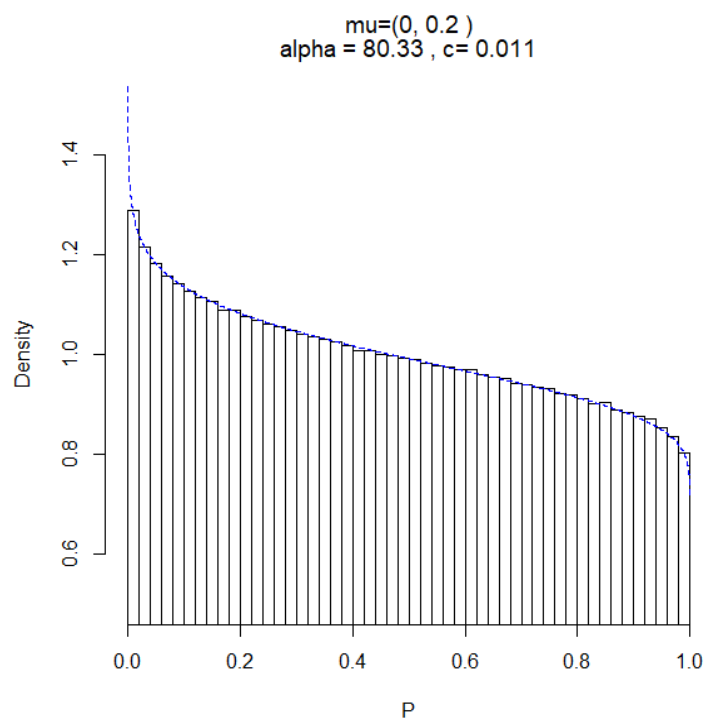
### S2. 3 Heterogeneous effects (two different $\mu$ 's, each for half alternatives)



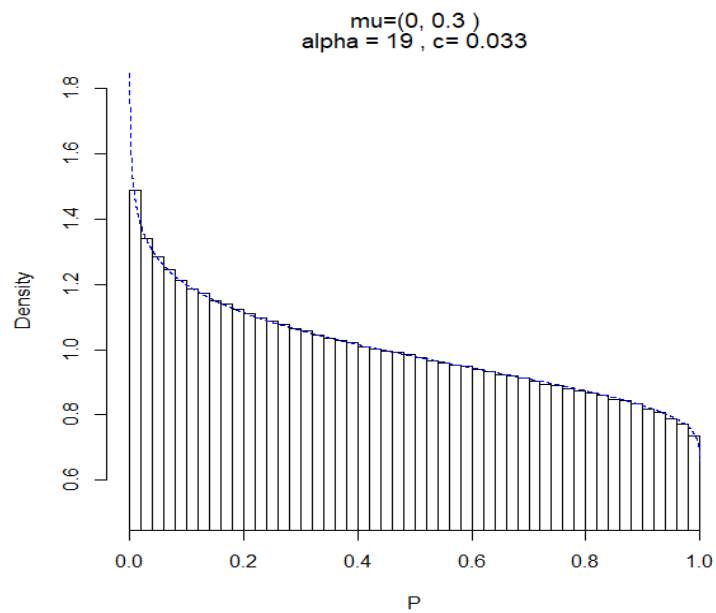
**Figure S9.** Histogram and the estimated density from the simulated data when  $\mu = (0, 0.05)$ .



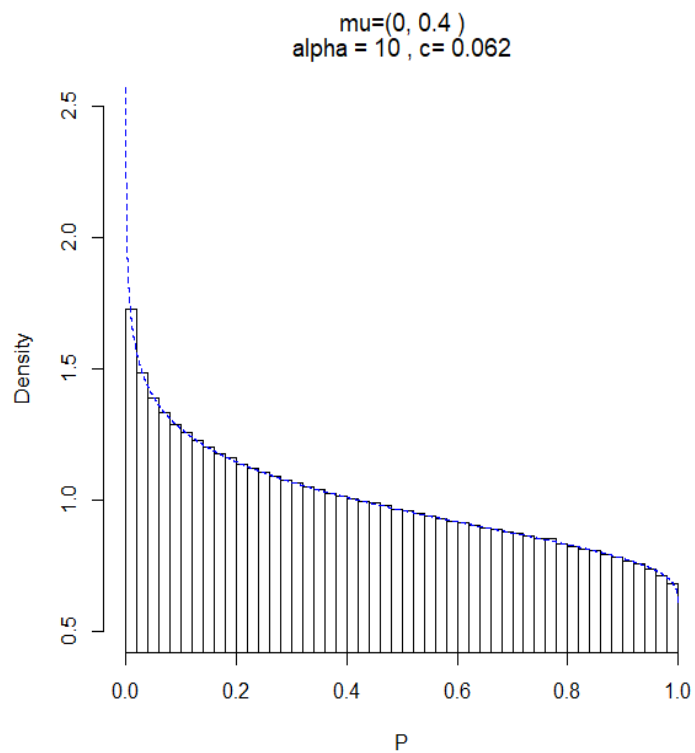
**Figure S10.** Histogram and the estimated density from the simulated data when  $\mu = (0, 0.1)$ .



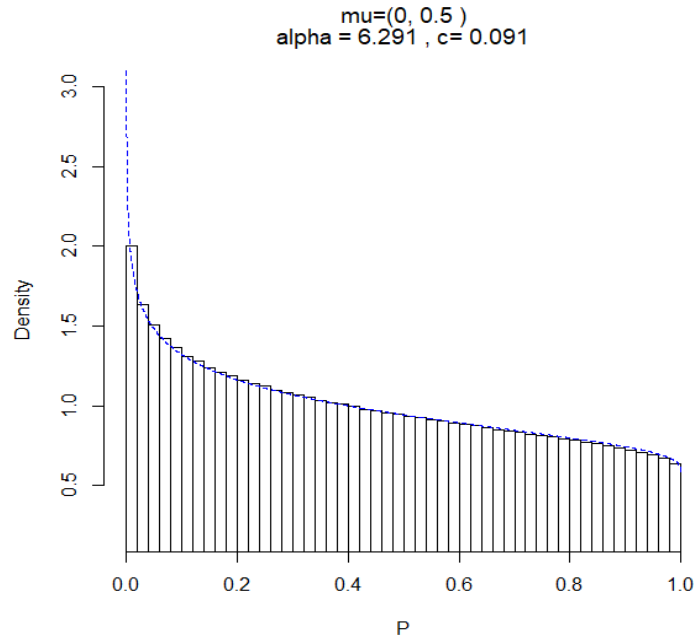
**Figure S11.** Histogram and the estimated density from the simulated data when  $\mu = (0, 0.2)$ .



**Figure S12.** Histogram and the estimated density from the simulated data when  $\mu = (0, 0.3)$ .

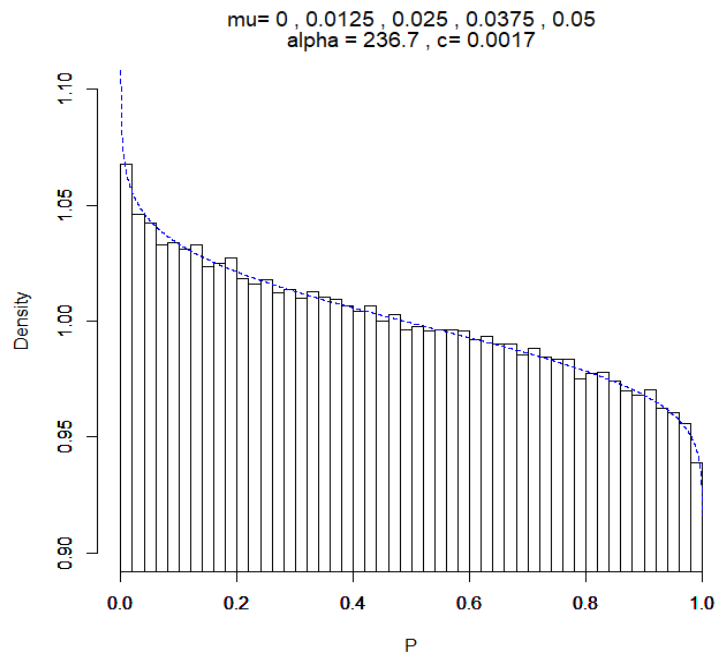


**Figure S13.** Histogram and the estimated density from the simulated data when  $\mu = (0, 0.4)$ .



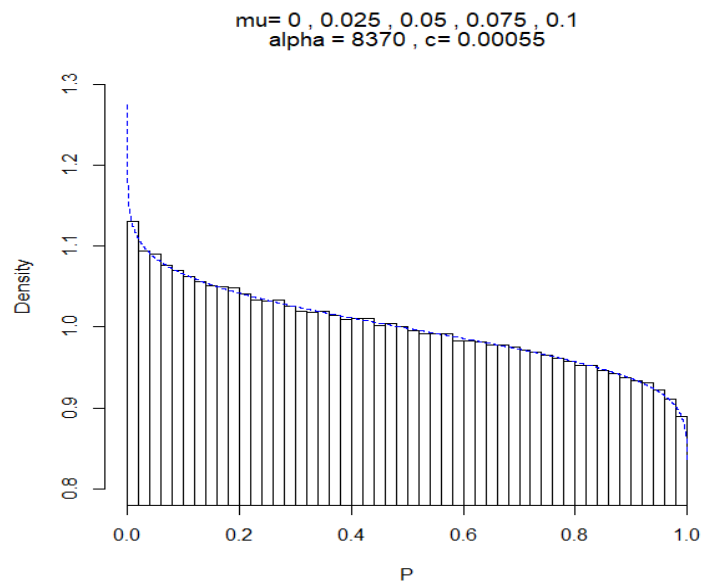
**Figure S14.** Histogram and the estimated density from the simulated data when  $\mu = (0, 0.5)$ .

#### **S2. 4 Heterogeneous effects (five different $\mu$ 's, each for one fifth alternatives)**

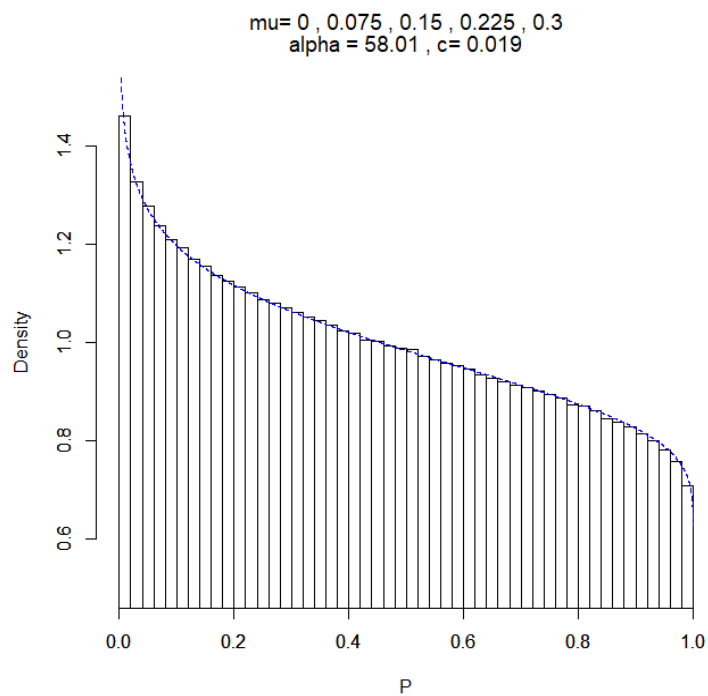


**Figure S15.** Histogram and the estimated density from the simulated data when  $\mu = (0, 1, 2, 3, 4) \times 0.0125$ .

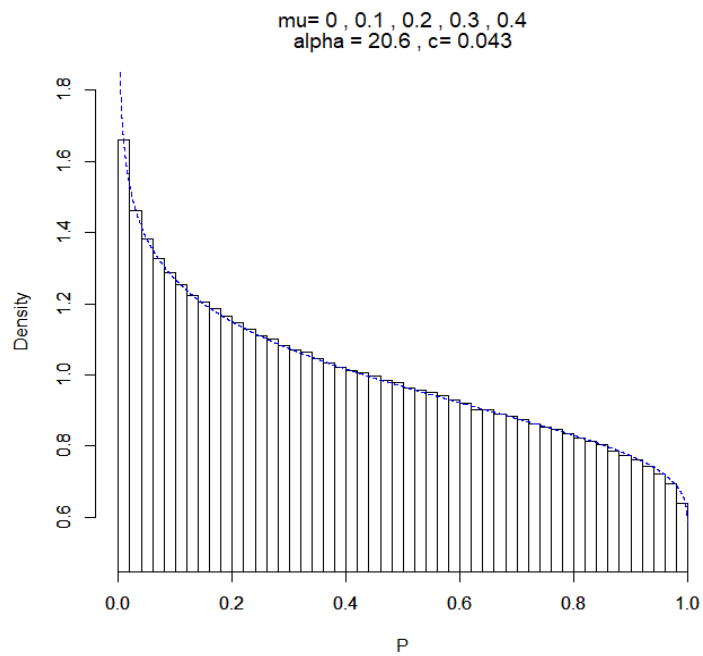




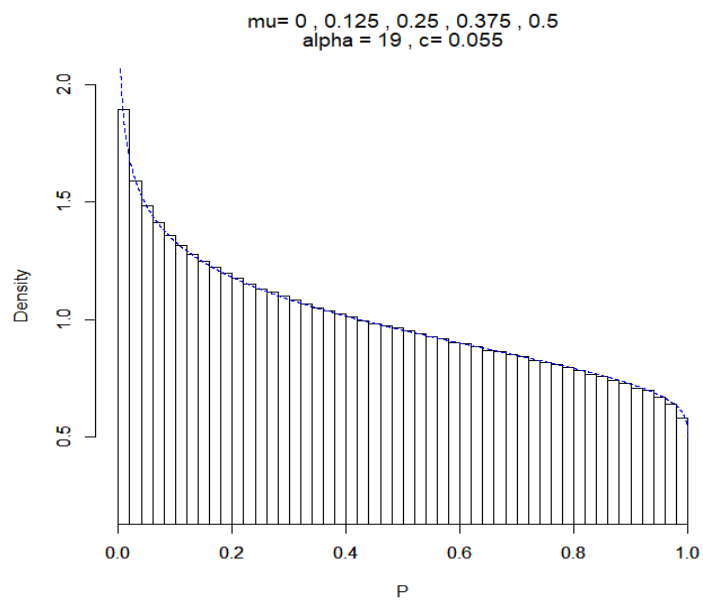
**Figure S16.** Histogram and the estimated density from the simulated data when  $\mu = (0, 1, 2, 3, 4) \times 0.025$ .



**Figure S17.** Histogram and the estimated density from the simulated data when  $\mu = (0, 1, 2, 3, 4) \times 0.075$ .

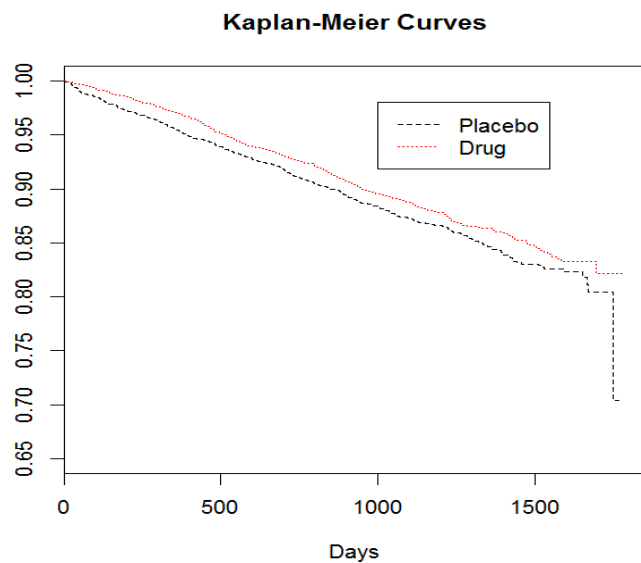


**Figure S18.** Histogram and the estimated density from the simulated data when  $\mu = (0, 1, 2, 3, 4) \times 0.1$ .

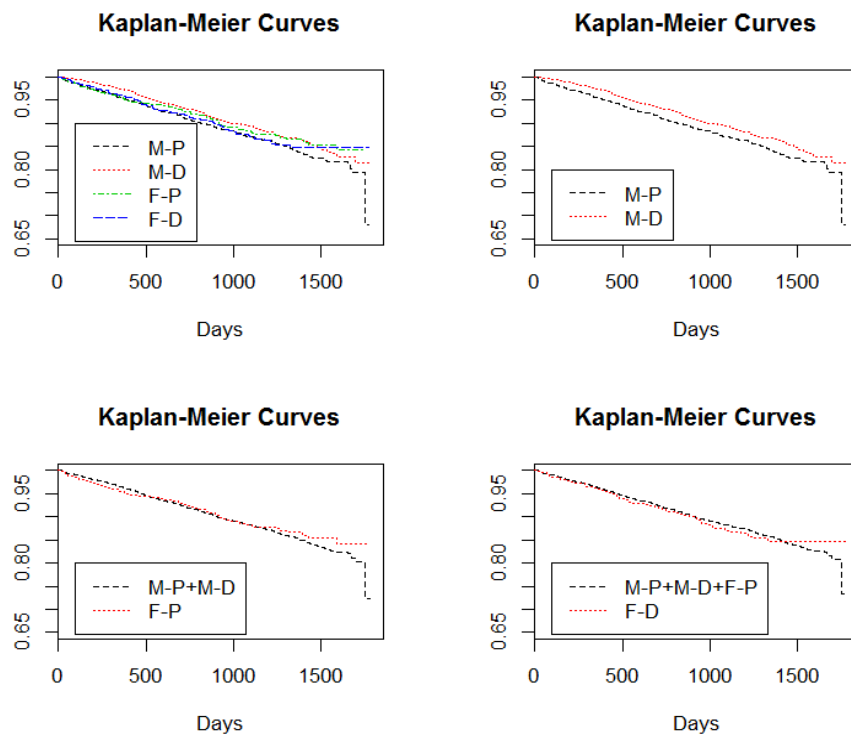


**Figure S19.** Histogram and the estimated density from the simulated data when  $\mu = (0, 1, 2, 3, 4) \times 0.125$ .

### S3. Plots for real data set #2:



**Figure S20.** Estimated survival curves from two treatment groups (drug and placebo).



**Figure S21.** Estimated survival curves from pairs of groups based on sex and treatment combination.

## S4. Simulation with small number of p-values (n=10) and 1000 replicates

**Table S1.** Empirical power from simulation under scenario 1 using  $\alpha = 0.05$ .

m, $\mu$	# concordant alternatives	Min	Fisher	Z	New
2, 1.5	1	0.44	0.34	0.18	0.36
	2	0.44	0.33	0.18	0.37
4, 1.3	2	0.76	0.69	0.40	0.73
	3	0.77	0.71	0.39	0.77
	4	0.79	0.69	0.37	0.77
6, 1.1	3	0.81	0.81	0.51	0.83
	4	0.83	0.82	0.51	0.84
	5	0.85	0.81	0.50	0.84
	6	0.82	0.82	0.53	0.85
8, 0.9	4	0.82	0.83	0.57	0.84
	5	0.80	0.81	0.56	0.83
	6	0.80	0.80	0.54	0.82
	7	0.78	0.80	0.54	0.82
	8	0.77	0.77	0.54	0.78
10, 0.8	5	0.79	0.83	0.59	0.82
	6	0.80	0.82	0.61	0.84
	7	0.78	0.80	0.57	0.81
	8	0.79	0.82	0.61	0.82
	9	0.80	0.83	0.58	0.84
	10	0.78	0.80	0.56	0.81

**Table S2.** Empirical power from simulation under scenario 2 using  $\alpha = 0.05$ .

m, $\mu$	# concordant alternatives	Min	Fisher	Z	New
2, 1.9	1	0.54	0.43	0.26	0.47
	2	0.49	0.45	0.27	0.46
4, 1.7	2	0.67	0.72	0.50	0.73
	3	0.68	0.72	0.52	0.73
	4	0.67	0.71	0.50	0.71
6, 1.5	3	0.70	0.80	0.66	0.80
	4	0.67	0.80	0.67	0.79
	5	0.70	0.82	0.66	0.81
	6	0.67	0.82	0.67	0.81
8, 1.1	4	0.45	0.65	0.54	0.63
	5	0.50	0.68	0.57	0.67
	6	0.48	0.65	0.55	0.64
	7	0.48	0.66	0.54	0.63
	8	0.49	0.66	0.55	0.63
10, 1.0	5	0.48	0.70	0.61	0.68
	6	0.44	0.69	0.60	0.68
	7	0.46	0.68	0.61	0.66
	8	0.45	0.69	0.62	0.68
	9	0.45	0.71	0.62	0.69
	10	0.44	0.67	0.59	0.66

**Table S3.** Empirical power from simulation under scenario 3 using  $\alpha = 0.05$ .

m, $\mu$	# concordant alternatives	Min	Fisher	Z	New
2, 2.2	1	0.48	0.47	0.32	0.48
	2	0.48	0.48	0.31	0.48
4, 2.0	2	0.62	0.79	0.67	0.77
	3	0.64	0.80	0.66	0.79
	4	0.64	0.78	0.63	0.77
6, 1.7	3	0.58	0.83	0.72	0.80
	4	0.56	0.83	0.75	0.80
	5	0.59	0.82	0.75	0.80
	6	0.57	0.84	0.76	0.81
8, 1.3	4	0.42	0.67	0.62	0.66
	5	0.39	0.68	0.62	0.67
	6	0.43	0.70	0.66	0.69
	7	0.446	0.70	0.65	0.70
	8	0.43	0.69	0.64	0.67
10, 1.1	5	0.34	0.64	0.59	0.62
	6	0.36	0.64	0.61	0.63
	7	0.33	0.63	0.59	0.61
	8	0.34	0.66	0.63	0.66
	9	0.38	0.65	0.63	0.65
	10	0.36	0.66	0.62	0.65

**Table S4.** Empirical type I error rates from simulation study with n=50 and 10,000 replicates using different significant levels.

$\alpha$	Min	Fisher	Z	New
0.1	0.0897	0.1033	0.0949	0.0985
0.05	0.0467	0.0505	0.0494	0.0470
0.025	0.0250	0.0238	0.0268	0.0261
0.01	0.0123	0.0086	0.0092	0.0108
0.001	0.0014	0.0009	0.0012	0.0009