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Column Penetration and Diffusion Mechanism of Bingham Fluid Considering Displacement Effect

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Abstract: The diffusion progress of grout is hindered by groundwater, which means the diffusion distance cannot reach the designed values required in engineering for water plugging or reinforcement. In this study, based on the generalized Darcy's law and the continuity equation of steady column penetration, a column diffusion mechanism for Bingham fluid, considering the displacement effect of grout on groundwater, is proposed. This diffusion mechanism is then validated by the penetration grouting experiments that have been previously performed. The influences of the grouting pressure, groundwater pressure, water–cement ratio and penetration coefficient of porous media on the diffusion radius are analyzed. Based on the Comsol Multiphysics platform, a three-dimensional numerical simulation program for this mechanism is developed using computer programming technology. Numerical simulations of the penetration and diffusion morphology of Bingham cement grout in porous media are then carried out. The results show that the theoretical calculation values of diffusion radius obtained from this mechanism are closer to the experimental values than those obtained from the column penetration grouting theory of Bingham fluid, without considering the displacement effect. The results of this study can provide theoretical support for practical grouting engineering.

Keywords: Bingham fluid; penetration grouting; displacement effect; column diffusion

1. Introduction

Geological environment conditions are getting more complicated and changeable with the effect of rainfall, rock weathering and changes in in situ stress in engineering construction [1]. Engineering practice shows that grouting technology, as a method to improve the physical properties of rock and soil, is often used to deal with a complex geological environment [2]. Grout is injected into the cracks and pores in rocks and soil to plug water and reinforce the intensity of the injected body using grouting technology. Therefore, it has been widely used in several application domains such as construction engineering, mining engineering, water conservancy and hydropower engineering and civil engineering [3], for example.

Several investigations have been conducted in penetration grouting mechanism and grouting experiments to study diffusion laws in porous media. For instance, Van Genuchten [4] proposed an analytical expression for the relative penetration of unsaturated soil, which could accurately predict the influence of unsaturated characteristics on penetration. Huang [5] derived a driving-water penetration grouting mechanism of Newtonian fluid in sand layers. The actual results showed that the mechanism developed by Huang was closer to the engineering practice than the Maag formula. Yang [6] proposed a penetration grouting mechanism of gravel soil reinforced by power-law fluid. This theoretical mechanism was validated by comparing results from theoretical analyses, indoor experiments and numerical simulations. Yang [7] developed a spherical penetration and diffusion mechanism of Bingham fluid. He obtained grouting pressure that satisfied the design requirements of the diffusion radius of the Bingham grout and the variation laws of the diffusion radius with a formation depth. Coskun [8] analyzed the penetration process of Bingham grout in fully saturated soil. He then proposed the penetration theory of immiscible two-phase fluid for grout–groundwater in porous media. The theoretical results were in good agreement with the experimental results, which indicated that the theory could provide support for construction design. Maghous and Saada [9,10] studied the influence of the filtration effect on penetration and diffusion in grouting. They proposed a spatial–temporal variation relationship between the penetration coefficient and porosity, which deepened the understanding of the microscopic penetration grouting mechanism in porous media. Bouchelaghem [11,12] obtained a propagation law of miscible grout in saturated deformed porous media, based on the spatial volume average statistical model of Bear. From the perspective of conservation of quality, Kim and Whittle [13] developed an iterative numerical calculation method considering the penetration effect and the corresponding grid model. Based on this model, the numerical solution of the sand column grouting model considering the percolation effect was obtained. Ye [14] studied the spherical diffusion laws for cement and sodium silicate mixed grout while considering the time-dependency and space effect of viscosity. He also analyzed the influence of C-S viscosity parameters on the grout diffusion radius. The influence of viscosity parameters on grout diffusion distance was also discussed. Zhang [15–17] established the spherical permeation and diffusion model of Bingham and power-law fluid under pulsating pressure and developed the model using the Comsol Multiphysics numerical simulation software. The model and the practical results were compared and analyzed to verify its efficiency. Zhang [18] developed the penetration grouting diffusion model for C-S grout considering the spatiotemporal variation of viscosity, and the theory was verified through one-dimensional simulation experiments. Zhou et al. [19,20] established a permeation grouting model considering the tortuous effect of the diffusion path of cement grout in porous media. Yang further considered the influence of circuitous and tortuous effects on the column penetration grouting mechanism of power-law fluid in porous media and obtained the column penetration grouting mechanism of power-law fluid while considering the circuitous and tortuous effects. The rationality of the theoretical mechanism was verified using a model test and numerical simulation [21].

In summary, studies on penetration grouting theory have focused on the factors of flow pattern, time-dependent behavior of viscosity, grouting pressure and penetration effect [22]. However, groundwater pressure was simplified to zero in the validation of the theories. The displacement effect of grout on groundwater in practical engineering applications was not considered, especially for penetration resistance and water interception in rock mass penetration and water-blocking curtains in dams. Therefore, here, a column penetration and diffusion model of Bingham fluid which considers the displacement effect based on the simplified Generalized Darcy's Law and the continuity equation of steady-state column penetration is deduced. The influence of the grouting pressure, groundwater pressure, water–cement ratio and penetration coefficient of porous media on the diffusion radius is analyzed. Compared with the existing experiments, the column penetration and diffusion model of Bingham fluid considering the displacement effect are more consistent with the actual situation than those that do not consider the displacement effect. This can provide theoretical support and guidance for practical grouting construction.

2. Penetration and Diffusion Equation of Bingham Fluid

Due to the difference between grout and groundwater, the flow of grout in the stratum cannot be simplified to the penetration of a liquid. Under the action of grouting pressure, free water in stratum pores is driven by grout. The groundwater is displaced by grout and the diffusion is completed, which is referred to as the displacement effect [23,24]. The displacement effect is characterized by two types of fluids with different physical and chemical properties and different motion modes in porous media.

2.1. Basic Assumptions

The penetration and diffusion of Bingham fluid during grouting are analyzed based on the following assumptions [25–27]:

① The injected stratum is homogeneous and isotropic; ② the injected grout is Bingham fluid without considering the time-dependent behavior of the viscosity of the grout, while the effect of gravity is ignored; ③ the grout is injected through a complete hole or from the bottom to the top of the capillary channel when the grouting process is not segmented. More precisely, the capillary channel passes through an aquifer to the bottom impermeable layer. Based on this grouting method, column diffusion is used.

2.2. Penetration Motion Equation

The theoretical diffusion model used in the derivation of the column penetration and diffusion mechanism of Bingham fluid, considering the displacement effects, is shown in Figure 1. The grouting disturbance range of this experiment is 5 m, and the boundary of the experimental box verified through the experiment is 0.3 m [28].

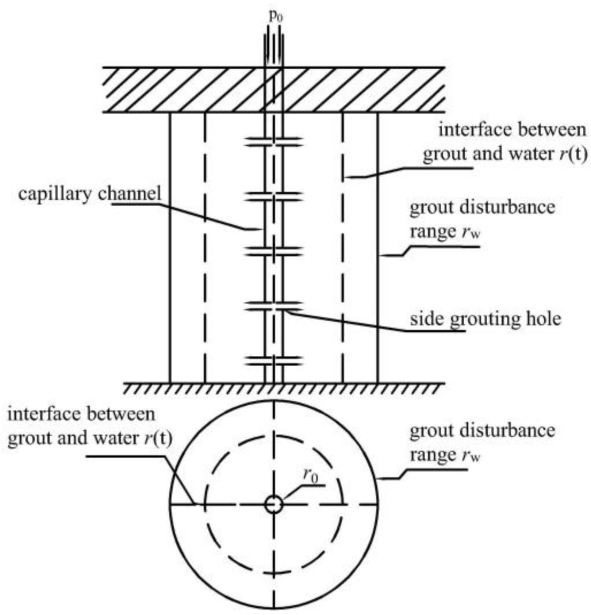

Figure 1. Schematic diagram of the column diffusion of Bingham fluid.

As mentioned in [29], the capillary group theory is used in the derivation of the mechanism. In Figure 2, the radius of a single capillary channel is assumed to be r_s. In addition, the central axis of the capillary is considered as the axis of symmetry. The force conditions of a micro unit with a length of dl and a radius of r are shown in Figure 2, where p and $p + dp$ are, respectively, the pressures on both sides of the column micro unit, r_p is the radius of the flow core of Bingham fluid, τ is the shear stress of grout when it permeates and diffuses in porous media, v_p is the flow velocity of the grout within the radius of the flow core and v is the flow velocity of the grout outside the radius of the flow core.

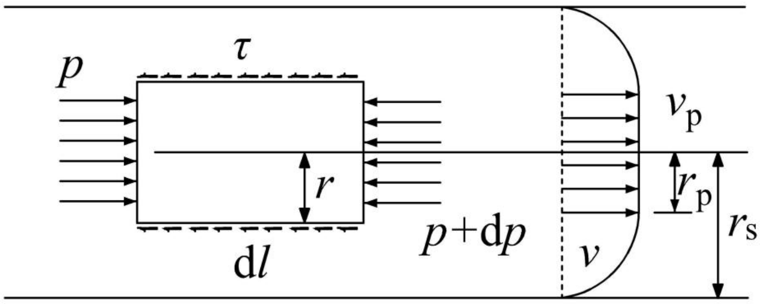

Figure 2. Force analysis of grout in the capillary channel.

The flow rate in a single capillary can be computed as [29]:

$$q = \pi r_p^2 v_p + \int_{r_p}^{r_s} 2\pi r v dv = -\frac{\pi r_s^4}{8\mu}\left(\frac{dp}{dl}\right) \times \left[1 - \frac{4}{3}\left(\frac{2\tau_0/r_s}{-dp/dl}\right) + \frac{1}{3}\left(\frac{2\tau_0/r_s}{-dp/dl}\right)^4\right] \quad (1)$$

τ_0 is the yield stress in the constitutive equation of Bingham-type grout and μ is the viscosity of Bingham-type grout. The equilibrium flow velocity in the capillary channel is $\widetilde{v} = q/\pi r_s^2$, and $\widetilde{v}$ can be rewritten as:

$$\widetilde{v} = -\frac{r_s^2}{8\mu}\frac{dr}{dl} \times \left[1 - \frac{4}{3}\left(\frac{2\tau_0/r_s}{-dp/dl}\right) + \frac{1}{3}\left(\frac{2\tau_0/\tau_s}{-dp/dl}\right)^4\right] \quad (2)$$

When the flow rate in the capillary channel is zero, the starting pressure gradient can be obtained:

$$\lambda = \frac{2\tau_0}{r_s} = -\frac{dp}{dl} \quad (3)$$

Based on the Dupuit–Forchheimer relation, namely $v = \phi\widetilde{v}$, the penetration motion equation of Bingham fluid can be obtained, where v is the penetration velocity, ϕ is the porosity of the stratum and $K = \phi r_s^2/8$ is the absolute penetration of the stratum [29]:

$$V = -\frac{K}{\mu}\left(\frac{dp}{dl}\right) \times \left[1 - \frac{4}{3}\left(\frac{\lambda}{-dp/dl}\right) + \frac{1}{3}\left(\frac{\lambda}{-dp/dl}\right)\right] \quad (4)$$

By extending Equation (4) to the three-dimensional case, the generalized Darcy law of Bingham fluid can be written as:

$$V = -\frac{k}{\mu} \bigtriangledown p\left[\left(1 - \frac{\lambda}{|\bigtriangledown p|}\right)\right] - \frac{1}{3}\left[\frac{\lambda}{|\bigtriangledown p|}\left(1 - \frac{\lambda^3}{|\bigtriangledown p|^3}\right)\right] \quad (5)$$

In Equation (5), $\bigtriangledown$ is the Hamilton operator, which is $\bigtriangledown = \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right]$. To simplify the calculation, the higher-order term in brackets in Equation (5) is omitted, and therefore, the simplified generalized Darcy's law can be obtained [23,29]:

$$V = -\frac{K}{\mu} \bigtriangledown p\left[1 - \frac{\lambda}{|\bigtriangledown p|}\right] \quad (6)$$

3. Column Permeation Diffusion Model of Bingham Fluid

As mentioned in [5,29], the continuity equation of steady column penetration can be written as:

$$\frac{1}{r}\frac{\partial}{\partial r}(r \cdot V) = 0 \tag{7}$$

Substituting Equation (6) into Equation (7), and in $\frac{\partial p}{\partial r}$ rather than ∇p, the differential equation of column penetration of Bingham fluid can be expressed as:

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} - \frac{\lambda}{r} = 0, \left|\frac{\partial p}{\partial r}\right| > \lambda \tag{8}$$

It is assumed that the injected grout is Bingham fluid, while the groundwater is Newtonian fluid. The gravity and compressibility of the fluid are ignored. The penetration differential equation and boundary conditions of this model are given by:

$$\begin{array}{ll} \frac{\partial^2 p_1}{\partial r^2} + \frac{1}{r}\frac{\partial p_1}{\partial r} - \frac{\lambda}{r} = 0, & r_0 < r \le r(t) \\ \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p_2}{\partial r}\right) = 0, & r(t) \le r < r_w \\ p_1 = p_0, & r = r_0 \\ p_2 = p_w, & r = r_w \end{array} \tag{9}$$

where p_1 is the pressure of Bingham fluid in the penetration zone, p_2 is the pressure in the area of groundwater, r_0 is the radius of capillary channel, p_0 is the grouting pressure, r_w is the grouting disturbance range and p_w is the groundwater pressure.

The pressure and the velocity (flow rate) are continuous at the grout front $r = r(t)$:

$$\begin{array}{c} 2\pi rh\left[-\frac{K_1}{\mu_1}\frac{\partial P_1}{\partial r}\left(1 - \lambda\left(\frac{\partial p_1}{\partial r}\right)^{-1}\right)\right] = -2\pi rh\frac{K_2}{\mu_2}\frac{\partial P_2}{\partial r} \\ p_1 = p_2 \end{array} \tag{10}$$

where μ_1 and μ_2 are, respectively, the viscosity coefficient of grout and water, K_1 and K_2 are, respectively, the absolute penetration of grout penetration zone and groundwater penetration zone and h is the grouting height.

Assuming that $\partial p_1/\partial r = Y_1$, $\partial p_2/\partial r = Y_2$, Equation (9) can be reduced to the following equations:

$$\begin{array}{c} \frac{\partial Y_1}{\partial r} + \frac{1}{r}Y_1 - \frac{\lambda}{r} = 0 \\ \frac{\partial Y_2}{\partial r} + \frac{1}{r}Y_2 = 0 \end{array} \tag{11}$$

The solution can be resolved as $Y_1 = \lambda + C_1/r, Y_2 = C_2/r$, and C_1 and C_2 are the coefficients of the general solution. The expression of p_1 and p_2 is given by:

$$\begin{array}{c} p_1 = C_1 lnr + \lambda r + C_3 \\ p_2 = C_2 lnr + C_4 \end{array} \tag{12}$$

C_3 and C_4 are constants of the general solution. By considering Equations (9), (10) and (12), the following can be obtained:

$$\begin{array}{c} p_0 = \lambda r_0 + C_1 lnr_0 + C_3 \\ p_w = C_2 lnr_w + C_4 \\ \lambda r(t) + C_1 lnr(t) + C_3 = C_2 lnr(t) + C_4 \\ -\frac{K_1}{\mu_1}\left(2\lambda + \frac{C_1}{r(t)}\right) = -\frac{K_2}{\mu_2}\frac{C_2}{r(t)} \end{array} \tag{13}$$

Considering $(K_1/\mu_1)/(K_2/\mu_2) = M$, when the absolute penetration of the stratum of grout flowing zone is similar to that of the groundwater flowing zone, $M = \mu_2/\mu_1$

is the ratio of the viscosity of water to the viscosity of grout. Considering the boundary conditions in Equation (9), it can be deduced that:

$$C_1 = \frac{p_w - p_0 - \lambda r(t) + \lambda r_0 + 2M\lambda r(t)(lnr(t) - lnr_w)}{lnr(t) - lnr_0 - Mlnr(t) + Mlnr_w} \quad (14)$$

Considering $A = p_w - p_0 + \lambda r_0$, $B = 1 - M$ and $D = M(lnr_w - lnr_0)$ in Equation (14), the latter can be rewritten as:

$$C_1 = \frac{A - \lambda r(t) + 2M\lambda r(t)[lnr(t) - lnr_w]}{D + Blnr(t)} \quad (15)$$

By substituting Equation (15) into Equations (12) and (13), the distribution laws of pressure in grout diffusion area and underground flowing area along with diffusion radius are obtained:

$$p_1 = \frac{A - \lambda r(t) + 2M\lambda r(t)ln[r(t)/r_w]}{D + Blnr(t)} \times ln\frac{r}{r_0} + \lambda(r - r_0) + p_0 \quad (16)$$

In order to ensure the accuracy of the calculation results, the generalized Darcy's law without simplification is used to calculate the flow rate of grout. According to Equations (5) and (16), the flow rate can be expressed as:

$$Q(r) = -2\pi rh\frac{K_1}{\mu_1}\frac{\partial p_1}{\partial r} \times \left[1 - \frac{4}{3}\left(\frac{\lambda}{-\partial p_1/\partial r}\right) - \left(\frac{1}{3}\left(\frac{\lambda}{-\partial p_1/\partial r}\right)\right)\right] \quad (17)$$

Dividing the flow rate in Equation (17) by the effective water area ($A = 2\pi rh \times \phi$, h is the height of column diffusion), it can be deduced that

:

$$\frac{dr(t)}{dt} = -\frac{K_1}{\phi\mu_1}\left[\frac{7\lambda}{3} + \frac{A - \lambda r(t) + 2M\lambda r(t)ln[r(t)/r_w]}{D + Blnr(t)}\frac{1}{r(t)} - \frac{\lambda^4}{3}\left(-\lambda - \frac{A - \lambda r(t) + 2M\lambda r(t)ln[r(t)/r_w]}{D + Blnr(t)}\frac{1}{r(t)}\right)^{-3}\right] \quad (18)$$

Because the original function of the differential equation cannot be expressed by the basic function, the numerical method is used to find the numerical solutions.

As mentioned in [26], the column penetration and diffusion model of Bingham fluid without considering the displacement effect is expressed as:

$$\nabla P = p_0 - p_w = \frac{\phi\mu r^2}{2Kt}ln\frac{r}{r_0} + \frac{4\lambda}{3}(r - r_0) \quad (19)$$

4. Comparison between the Theoretical and Experimental Results

In order to analyze the applicability and accuracy of the mechanism, the relevant penetration grouting experimental results obtained by Yang [30] are used. The experimental device of Yang is shown in Figure 3. Experimental results and theoretical calculation values which were obtained respectively from Equations (18) and (19) are compared. Two kinds of cement grout with water–cement ratios of 0.90, 1.00 and 1.25 [30,31] are considered to perform the penetration grouting experiments. It can be seen from the literature of Yang that they are typical Bingham fluids. The experimental scheme is shown in Table 1. The rheological equations of two cement grouts are shown in Table 2. The water viscosity is μ_w = 0.001 Pa·s, and the stratum parameters are shown in Table 1 [27].

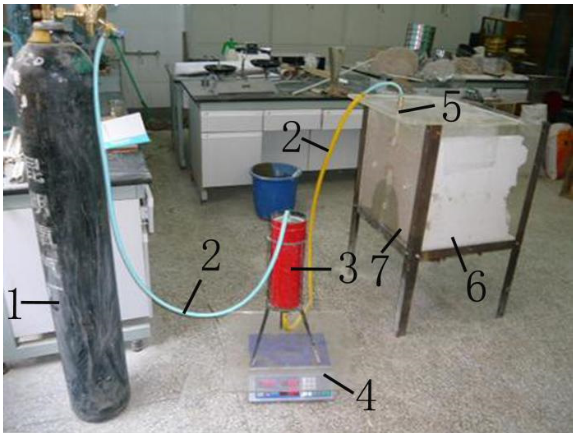

Figure 3. Grouting experimental device [30] (1—pressure supply equipment, 2—connecting pipe, 3—grout storage box, 4—electronic scale, 5—capillary channel, 6—experimental box, 7—gravel soil).

Table 1. Parameter values in the penetration grouting experiments [30].

Test Number	Water–Cement Ratio (w/c)	Grouting Pressure/MPa	Grouting Time/s	Penetration Coefficient k/cm·s^{-1}	Porosity
G1	0.90	0.12	26.50	3.45	0.4519
G2	1.00	0.10	24.90	3.09	0.4391
G3	1.25	0.08	23.50	2.89	0.4414
G4	1.00	0.05	22.50	3.90	0.4524
G5	1.00	0.06	21.70	2.11	0.4505

Table 2. Rheological equations of two kinds of cement grout [30].

Water–Cement Ratio	Rheological Equation
1.25	$\tau = 0.1136 + 0.0159\ \gamma$
1.00	$\tau = 0.8593 + 0.0169\ \gamma$
0.90	$\tau = 1.7876 + 0.0194\ \gamma$
0.75	$\tau = 3.2130 + 0.0203\ \gamma$

The theoretical calculation values of the diffusion radius can be obtained by the mechanism (cf. Equation (18)) and column penetration grouting formula (cf. Equation (19)) of Bingham fluid without considering the displacement effect. The results are compared with the experimental values, as shown in Table 3.

Table 3. Comparison between the experimental results and theoretical calculation values.

Experimental Number	Theoretical Values Obtained from Equation (18)/cm	Theoretical Values Obtained by Equation (19)/cm	Experimental Value /cm
G1	34.2	64.83	11.10
G2	29.74	58.20	11.70
G3	25.66	49.74	12.50
G4	23.52	44.63	11.80
G5	18.15	35.89	10.80

It can be deduced from Table 3 that the theoretical calculation values of diffusion radius obtained by the mechanism are smaller than those obtained by the column penetration and diffusion formula of Bingham fluid without considering the displacement effect. The former is closer to the experimental values than the latter. Therefore, it can be seen that the mechanism is better than the column penetration and diffusion formula of Bingham fluid without considering the displacement effect, in order to reflect the diffusion morphology and laws of Bingham fluid in porous media.

It can also be seen from Table 3 that there is still a certain deviation between the theoretical calculated values of the diffusion radius obtained by the mechanism and the experimental values. The main reasons for this deviation are summarized as follows: (1) the grouting theory deduced in this study only considers the influence of the displacement effect on the penetration diffusion morphology. Several studies show that different factors affect the grouting diffusion effect. For instance, Ye, Zhang, Yang and Liu et al. [14,18,25,32] deduced that the rheological parameters of cement grout have an important influence on the diffusion radius. Han [24] believes that the self-weight of grout will affect its diffusion radius in different penetration directions. In addition, Liu's study [32] showed that the cement grout in the process of grouting will continue to water, which results in a change in viscosity and the increase in the difficulty of grouting. Subsequently, Maghous and Saada [9,10,33] showed that precipitation, blockage and other penetration effects may occur when grout passes through the pore channels of porous media. (2) The gravel soil used in the penetration grouting experiments is not the ideal isotropic uniform porous medium. The porous medium is assumed to be isotropic and homogeneous in the theoretical derivation. The influence of the internal structural characteristics of the porous medium on the penetration and diffusion of grout is ignored.

5. Analysis of Displacement Effect

This section analyzes the change laws of the influencing factors, such as the diffusion radius, grouting pressure, groundwater pressure, water–cement ratio of Bingham cement grout and the coefficient of penetration of porous medium, under two theoretical models of the Bingham fluid penetration grouting mechanism: considering the displacement effect and without considering the displacement effect. The basic parameters are shown in the G5 experiment in Table 1.

5.1. Pressure Distribution in Grout Diffusion Zone

The penetration and diffusion model of grout considering the displacement effect is developed based on the continuity conditions of the pressure and flow rate pressure at the interface between grout and groundwater. The distribution of fluid pressure in the penetration and diffusion zone of grout is analyzed. The relationship curve between the grout pressure and diffusion distance is shown in Figure 4.

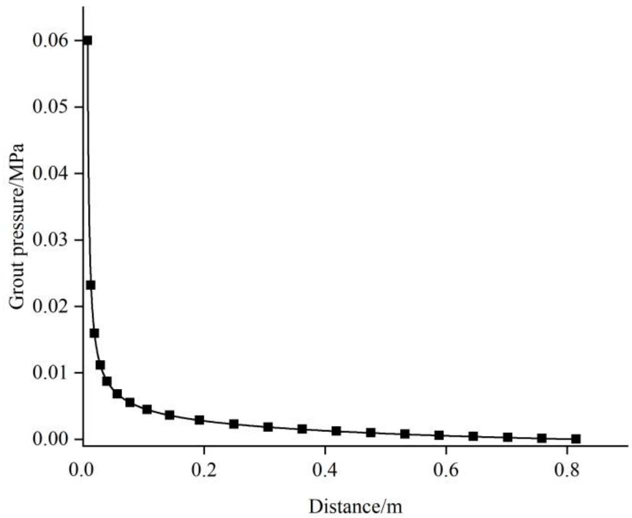

Figure 4. Pressure distribution in grout diffusion zone.

Figure 4 shows that the grout pressure rapidly decreases with the increase in the diffusion distance of grout. The pressure attenuation mode considering the displacement effect is concave. Under the influence of groundwater, the pressure balance between grout and groundwater rapidly balances. That is, the pressure and flow rate at the interface between grout and groundwater are continuous. The resistance in the diffusion of grout increases, and therefore, the grout pressure rapidly decreases. The pore water in the stratum is driven away after the grout injection. Its penetration movement is actually a deceleration movement, which is consistent with the results obtained by Yang [34]. The results show that the column penetration and diffusion mechanism of Bingham fluid considering the displacement effect are consistent with the practice.

5.2. Analysis of the Influence Factors

Under two theoretical models, the influence laws of the grouting pressure, groundwater pressure, grout water–cement ratio and penetration coefficient of porous media on the diffusion radius are shown in Figures 5–7, respectively.

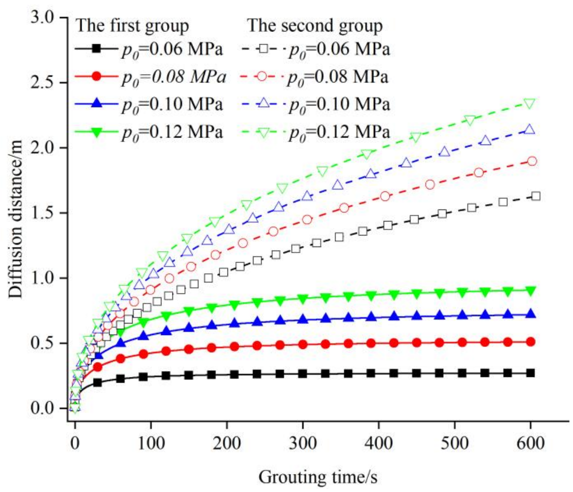

Figure 5. Relationship between diffusion distance and time under different pressures. Two groups of data exist in the figure: the data considering the displacement effect and the data without considering the displacement effect. The two groups of data in Figures 5–7 have the same meaning as Figure 5.

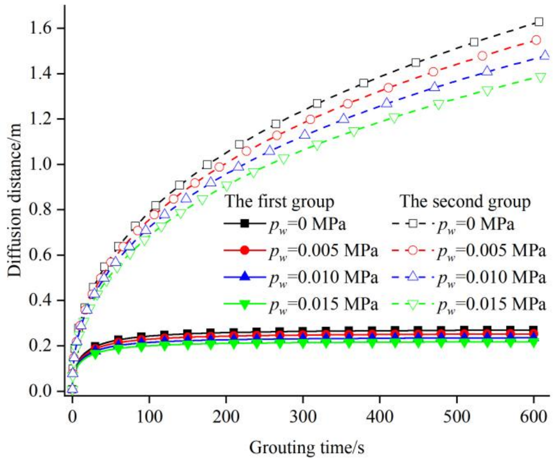

Figure 6. Schematic diagram of the diffusion radius with different groundwater pressures.

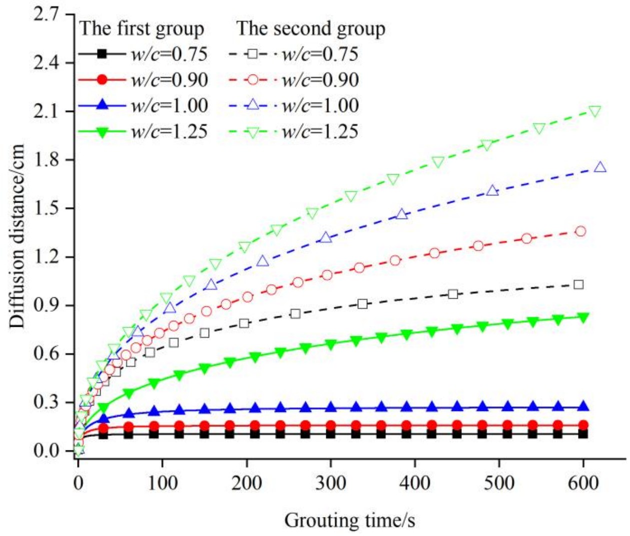

Figure 7. Influence of the water–cement ratio on the diffusion radius of Bingham cement grout.

It can be deduced from Figures 4–8 that:

(1) With the increase in grouting time, the diffusion radius obtained by the two theoretical models increases nonlinearly with time. The increment speed of diffusion radius gradually decreases with time. The increment of diffusion radius in the early period is much larger than that in the late period.

(2) The grouting pressure, groundwater pressure, water-cement ratio of Bingham cement grout and penetration coefficient of porous media significantly affect the diffusion radius under the two theoretical models. The relationship between groundwater pressure and diffusion radius is reverse. The greater the groundwater pressure, the smaller the diffusion radius. However, the grouting pressure, penetration coefficient of porous media and water–cement ratio are positively associated with the diffusion radius of Bingham cement grout. The greater the grouting pressure, porous media penetration coefficient and water–cement ratio of Bingham cement grout, the greater the diffusion radius.

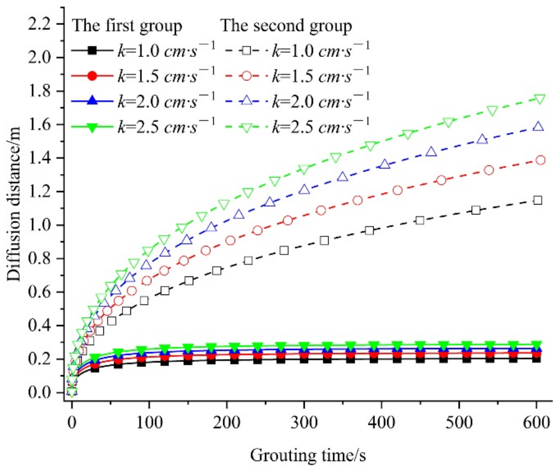

Figure 8. Influence of the penetration coefficient of porous media on the diffusion radius.

6. Numerical Simulation

6.1. Development of the Numerical Model

We make a comprehensive description of the simulation method as follows. This simulation relies on a PED module, which connects the scenarios considering the displacement effect (Equation (18)) and ignoring the displacement effect (Equation (19)). Two theoretical applications considering the displacement effect (Equation (18)) and ignoring the displacement effect (Equation (19)) are obtained by secondary development. The accurate diffusion mode of grout in porous media can be obtained with a numerical simulation program. For groundwater, given an initial groundwater pressure, the influence of groundwater on grout diffusion can be established. In order to directly compare the penetration and diffusion effects of two grouting theoretical models in porous media, the numerical model is developed in the section. Based on the Comsol Multiphysics platform, a three-dimensional numerical simulation program for this mechanism is developed using computer programming technology. Based on the experiment G5 in Section 4, the penetration and diffusion morphology effects of two grouting theoretical models in porous media are numerically simulated. The numerical model is a cube of 0.8 m × 0.8 m × 0.8 m size, and the capillary channel radius is 7.5 mm [30].

6.2. Numerical Simulation Results

The numerical simulation results are shown in Figures 9 and 10.

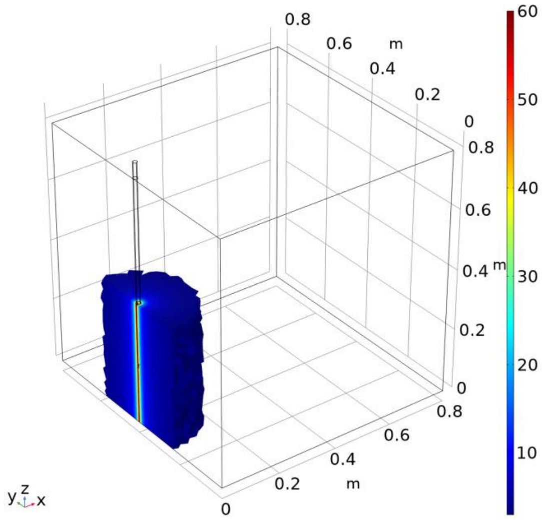

Figure 9. Simulation results of the grouting theoretical model considering the displacement effect.

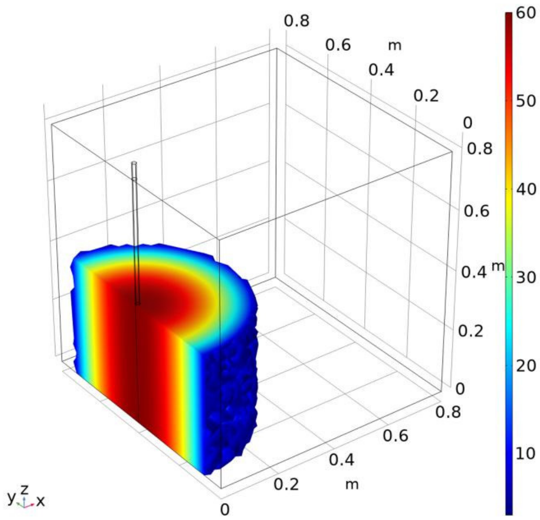

Figure 10. Simulation results of the grouting theoretical model without considering the displacement effect.

It can be seen from Figure 11 that the simulation results of the two grouting theoretical models have the same shape of column penetration diffusion, which conforms to the theoretical grouting model of column penetration. The three-dimensional penetration and diffusion morphology of Bingham cement grout in porous media simulated by the grouting theory model considering the diffusion path are smaller than those without considering the diffusion path, which is consistent with the conclusion in Section 4.

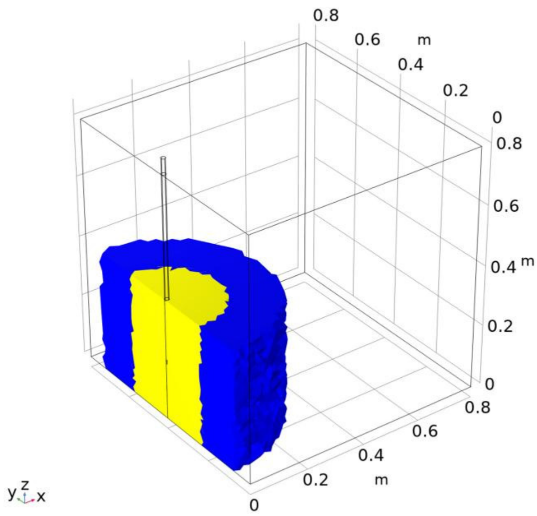

Figure 11. Comparison between the simulation results of the two grouting theoretical models; the blue and yellow colors, respectively, represent the simulation results of the grouting theoretical model without considering the displacement effect and while considering the displacement effect.

7. Conclusions

Based on the theoretical analysis, experimental comparison and other methods, the displacement effect of grout on water in grouting is analyzed in this paper. The column penetration and diffusion mechanism, as well as the pressure attenuation calculation formula of the Bingham fluid considering the displacement effect, are obtained and discussed. The main conclusions are summarized as follows:

(1) Based on the rheological equation and the steady-state column penetration continuity equation of Bingham fluid, the column penetration diffusion mechanism of Bingham fluid considering the displacement effect is proposed. Compared with the existing penetration grouting experiments, the grout diffusion radius obtained by the penetration grouting mechanism of Bingham fluid considering the displacement is closer to the experimental value than that of Bingham fluid without considering the displacement effect.

(2) The influence of the groundwater pressure on the grout diffusion radius considering the displacement effect in the grout diffusion process is more obvious than that without considering the influence of the displacement effect. The larger the grouting pressure, penetration coefficient of porous media and water–cement ratio of grout, the larger the diffusion radius of grout.

(3) Using computer programming technology and the Comsol Multi-physics platform, a three-dimensional numerical simulation program for the penetration grouting mechanism of Bingham fluid, considering and without considering the displacement, is obtained. The rationality of the simulation program is verified with model experiments, which can provide support for grouting construction design.

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