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Evolution of the Complex Supply Chain Network Based on Deviation from the Power-Law Distribution

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Abstract: The power-law distribution is an important descriptive characteristic of scale-free complex supply chain networks (SCN). The power-law distribution and deviation phenomena of SCN nodes are explored in combination with complex network theory, so it is imporant to accurately characterize the dynamic characteristics of network evolution on a time scale. Based on the analysis of the topological structure and evolutionary characteristics of the small-world network and scale-free SCN, the single and double power-law distribution and evolutionary dynamic characteristics of the complex SCN are further analyzed, and the deviation phenomenon of the power-law distribution is analyzed. On the premise of setting three parameters of new nodes, new edges, and node reconnection in the process of network evolution, the power-law distribution deviation evolution model under a complex network environment is constructed, and then the parameters of the SCN evolution model are analyzed. Combined with numerical simulation and model simulation, the evolution of SCN with two kinds of power-law deviation is analyzed. The results show that the deviation of the two-stage power-law distribution is not caused by the process of adding nodes or connecting edges, while it has a certain influence on the change of the power index, and the deviation of the power-law distribution in SCN increases with the extension of the evolution time. When $p_1 = 0$, the single power-law distribution of SCN tends to change to a δ distribution when the time step is large enough.

Keywords: complex network; evolutionary dynamics; power-law distribution deviation model; network evolution mechanism

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1. Introduction

Since the 1970s, the rapid development of industrialization has promoted the speed of product updates. Customer needs have personalized and diversified characteristics. The traditional enterprise management model needs to be optimized, and the supply chain management mechanism has quickly been applied and developed in many management fields. However, with the continuous deepening of global economic integration and trade reforms in recent years, e-commerce in particular has promoted the rise and development of emerging business models. The internal and external environment and management models of supply chain companies are also increasingly complex, which undoubtedly makes the management and development of supply chain enterprises and networks difficult. Accurately and effectively characterizing the complex supply chain network's (SCN) structure and evolution mechanism has become an urgent demand.

In the fields of super networks and complex networks, as scholars have discovered the existence of scale-free networks, complex network theory has been widely applied to various research fields, such as interpersonal relationship networks in the field of sociology and brain nerves in the field of life sciences, and networks, transportation road networks, and SCNs in management. Complex network theory has a good ability to solve nonlinear and nondeterministic problems. It can analyze the structural characteristics and evolution of complex networks from a relatively macro perspective. Moreover, with the rapid development of computer science in recent years and the in-depth study of complex networks, the theory of complex networks has been further developed.

In recent years, most SCNs gradually exhibited complex characteristics under the influence of dynamic and changing internal and external factors. Many domestic and foreign research scholars have cited the theory of complex networks in the study of SCNs to explore their development and evolution laws and have obtained certain research results. As early as 1999, Barabási et al. [1] proposed that many large networks have complex topological structures, vertex connectivity follows a scale-free power-law distribution, and the evolution of the network is accompanied by the addition of new nodes and their preferred connection mechanism. A BA model suitable for scale-free networks is developed. Since then, the achievements of using complex network theory to study the SCN based on different perspectives have continuously emerged.

In the study of power-law characteristics of the degree distribution of complex networks, Pei Weidong et al. [2] proposed a dynamic evolution algorithm based on complex network models and used the method of mathematical statistics to numerically simulate the degree distribution of the built model to obtain the complex network model generated by this evolutionary algorithm, which has the characteristics of uniform and power-law mixed distribution. Tao et al. [3] extended the BA model under the incentive of priority and uniform connection ideas and methods, established a complex network model of power-law accelerated growth and a mixture of two connection mechanisms, and discussed the degree distribution for numerical simulation. Chao et al. [4] studied the evolutionary prisoner's dilemma game based on the Barabási-Albert scale-free network and analyzed the effect of power-law-distributed human activity heterogeneity on cooperative evolution. More recently, Contreras-Reyes [5] studied the dynamics of Conway's game of life cellular automata under the power-law function perspective.

However, in recent years, the field has gradually discovered that the degree distribution of realistic complex networks does not always conform to a theoretical power-law distribution, and there is a phenomenon that deviates from the power-law distribution. Many scholars have tried different models and mechanisms to explain this power-law deviation, including additive statistical mechanics, size effects, information filtering mechanisms, aging and cost constraints, birth and death processes, and geopolitical constraints. Li Yaya et al. [6] tried to use Tsallis statistics to fit the power-law deviation, but the results were not very good. James et al. [7] proposed a fixed-scale wealth distribution model. At each time step, a unit of wealth is distributed to a randomly selected or optimally selected agent with different probability. Their research shows that the distribution of wealth undergoes a transition from a Gaussian distribution to a power-law distribution. Mossa et al. [8] constructed a scale-free network growth model under information filtering conditions. Nodes can only process the information of a part of the existing nodes in the network and find that the in-degree distribution has an exponential deviation. Guimera et al. [9] proposed a model with preference selection and geographic distance limitation. The dual power-law distribution generated by this model coincides with the data of the world aviation network. Maillart et al. [10] generated the Zipf degree distribution using the generation and deletion mechanism of consecutive edges.

In summary, most of the current research on the power-law distribution of complex SCNs is mainly based on network dynamics, cooperation, and evolution. However, there are few domestic and foreign studies on the deviation of complex SCNs. Therefore, in this paper, the complex SCN conforms to the characteristics of the small-world and the scale-free network. Based on the entire network data set, the model is constructed, and the deviation of the power-law distribution is described by the model. It is characterized by standard statistical methods. In reality, the underlying model of the power supply law distribution and deviation characteristics of complex SCNs explores the phenomenon of power supply network deviations from the perspective of complex networks.

Therefore, this paper addresses the SCN in a complex environment. Combining complex network theory, this paper analyzes the topological structure and evolution characteristics of the SCN with small-world and scale-free characteristics and explains that the current network distribution in many studies does not describe an accurate phenomenon. On the basis of the three parameter settings of new nodes, new connecting edges, and connecting edges, an SCN evolution model based on the deviation of the power-law distribution is established. Model parameters are resolved. This research is conducive to accurately characterizing the node distribution of complex SCNs, exploring the actual evolution mechanism of the network, and providing theoretical and technical support for supply chain enterprise and network layer management and optimization.

2. Evolutionary Characteristics and Model Analysis of Complex SCNs

In the real world, with the further development of economic and trade marketization and globalization, the process from early production to users within the supply chain system of various industries continues to expand into suppliers, manufacturers, distributors, consumers, and transportation companies. In the functional network system composed of "node" enterprises, the relationship between nodes, topology structure, and network evolution process are dynamic and changeable with time, gradually showing the characteristics of complex networks. The initial SCN gradually evolved from an incomplete state to a relatively ideal and balanced state, making the SCN transform from disorder to order. Therefore, with the help of relevant knowledge of complex network theory, the distribution characteristics and evolution mechanism of complex SCNs can be explored more deeply and accurately. The following will specifically analyze the evolution characteristics of different types of SCNs based on small-world networks and scale-free networks and conclude that complex SCNs conform to the characteristics of small-world networks and scale-free networks.

2.1. Analysis of SCN Based on Small-World Network Evolution

In the early days, a type of complex SCN that was studied more internationally was a random network. In an ideal state, when the scale of a random SCN gradually tends to infinity, the degree distribution of the network will obey the Poisson distribution [11], as follows:

$$P(k) = e^{-pN}\frac{(pN)^k}{k!} = e^{-\langle k\rangle}\frac{\langle k\rangle k}{k!}, \quad (1)$$

where $\langle k\rangle$ is the average value of the node degree of the SCN. Usually, the number of nodes in a random SCN will peak at a node degree value of $\langle k\rangle$, and then the number of nodes will decay exponentially.

The E-R model in the random SCN is a network distribution model in which the degrees of vertices obey the binomial distribution, namely $P(k) = C_{n-1}^k p^k (1-p)^{n-1-k}$ and the basic idea is to connect each pair of nodes in the N nodes with probability p. However, the two problems in the E-R model are p is a constant in the model, but in reality, p and n are inversely proportional; the mean value of the network nodes is linearly related to n, but the actual mean growth rate will be slower. A schematic diagram of the E-R stochastic SCN is shown in Figure 1a. Its network node N = 20 and node connection probability p = 0.1.

However, with the deepening of scientific research, many of the complex SCNs observed in reality do not simply meet the new edge to join with the same probability; the network's agglomeration coefficient and degree value have significant heterogeneity, rather than Obey the uniformity and homogeneity network characteristics with an agglomeration coefficient of $C_{ER} \approx \dfrac{\langle k\rangle}{N}$. To further study the network topology and evolution mechanism between a completely regular network and a random network, Watts and Strogatz of Cornell University proposed the concept and model of a WS small-world network in 1998.

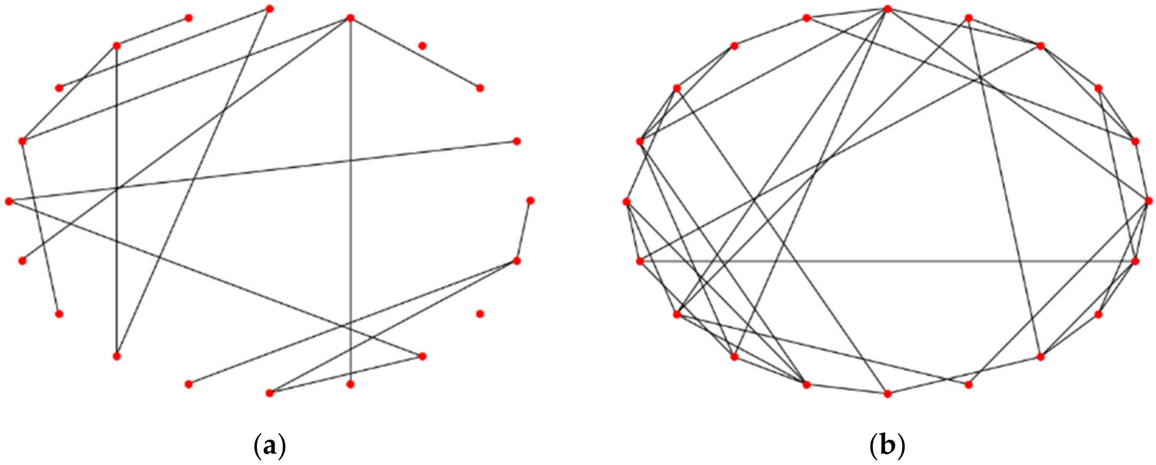


Figure 1. (**a**) E-R stochastic supply chain network; (**b**) WS small-world network.

In reality, many complex SCNs exhibit the "small-world" characteristics of clusters; that is, any SCN node can connect with nodes on any other network in a few simple steps, which is unique to complex SCNs. Figure 1b shows a schematic diagram of a WS small-world SCN with 20 nodes, each node having four neighbors, and a random edge reconnection probability of 0.3. The main idea of its construction is based on a node with N nodes, and each node has a closed regular network of k-nearest neighbors that interrupts and reconnects the connected edges in the network with a certain probability. The specific construction process of the WS small-world SCN is shown in Figure 2.

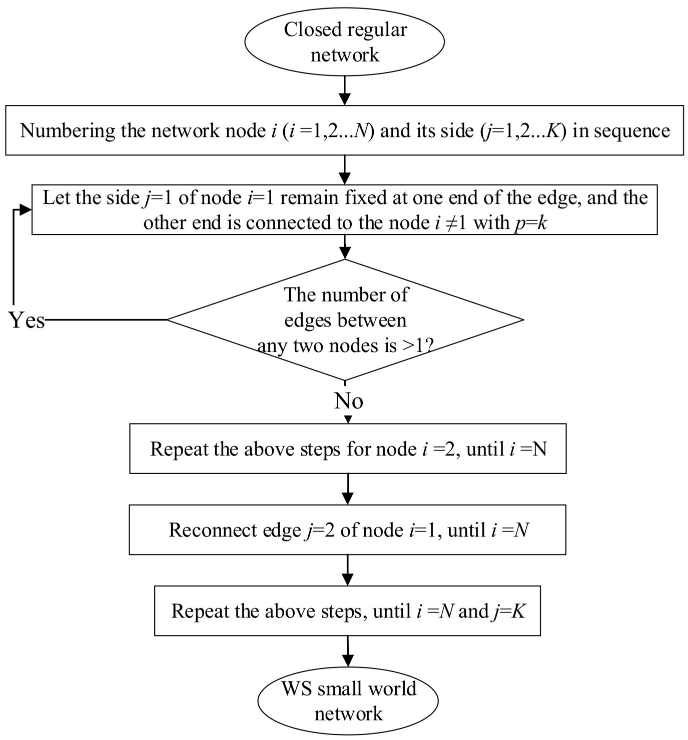


Figure 2. WS small-world network construction flow chart.

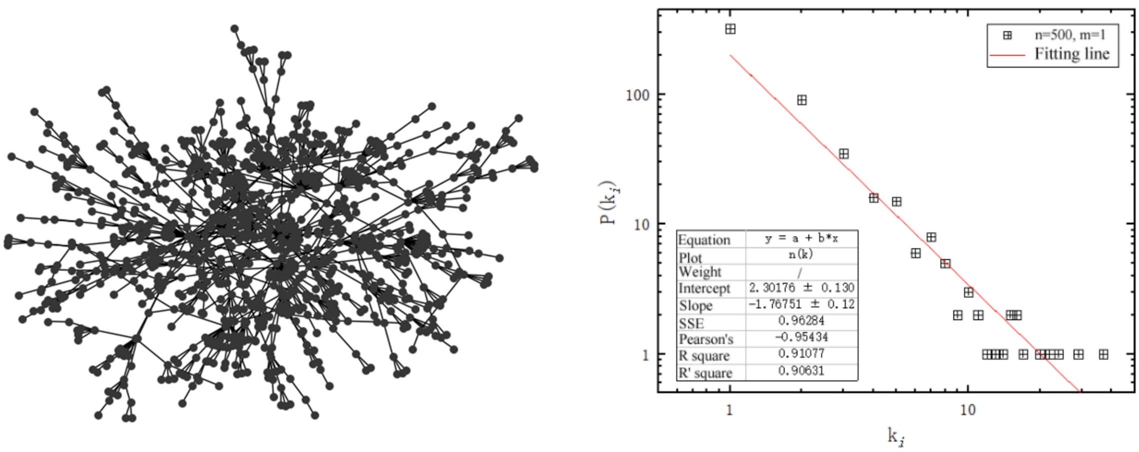
The SCN conforming to the "small-world" characteristics has a clustering coefficient of $C(0) \approx (\mathrm{k}-1)/\mathrm{k}$ similar to the regular network and a smaller average path length $dist_c \propto \log(N)$ similar to the random SCN. In the evolution of complex SCNs over time, for a small reconnection probability p, the network aggregation coefficient can be large, but the change is small, and the average network path length can be small, but the attenuation is large [12].

2.2. SCN Analysis Based on Scale-Free Network Evolution

Based on research on the evolution of small-world SCNs, many experts have found that the topological structure characteristics of most complex SCNs are different from those of stochastic SCNs and WS small-world supply chains when they conduct further research on real networks in different fields. For the network, the most significant difference is the degree distribution of the network.

It can be seen from the previous section that the more average the degree value of the random SCN is, the more concentrated the degree distribution interval; there are also fewer nodes with larger deviations from the mean value of the degree, and it decays exponentially for the larger degree. The exponential form rises to the apex and then decays to the exponential form, approximately obeying the Poisson distribution, the degree values of most nodes are gathered around a certain value, and the number of other degree values is relatively small. However, in most realistic networks, such as complex SCNs and complex aviation networks, etc., the terminal on the right side of the degree distribution curve has the characteristic of a "fat tail", indicating that such SCNs have certain nodes with large degree values; that is, the network existing nodes have a large number of connected edges and are highly nonuniform, unlike random SCNs and small-world SCNs. At present, the observed degree distributions of most realistic and complex networks have power-law characteristics. Because power laws have the same properties on all scales, such networks are called scale-free networks.

Dynamic growth and preferential connection are two evolutionary mechanisms for forming scale-free networks. To further analyze the topology structure and network degree distribution characteristics of the scale-free SCN, Python software was used to generate a BA scale-free network with node $n = 500$ and the initial number of neighbors $m = 1$ for each node. As shown in Figure 3, it can be seen that (a) that the network structure is unevenly distributed, with a large number of nodes with small degree values, but there are also a few core nodes with large degree values, and the network node degree value is [1, 37]. The distribution of the network degree of the logarithmic transformation of the ordinate is shown in (b), where the discrete points of the degree value are fitted. The coefficient R2 of the linear fitting function is 0.91, which has a good fit, so it shows that the scale-free SCN follows the power-law distribution feature.



(**a**) (**b**)

Figure 3. BA scale-free network simulation diagram: (**a**) Network topology; (**b**) Network degree distribution fitting.

For a complex SCN under economic globalization and an open market environment, each node company's position and structural characteristics in the network are different, and this will be due to the participation and cooperation between nodes or supply chains. Eliminate network nodes, and node enterprises develop the continuous evolution of the SCN with a nonuniform growth model under the motivation of survival and maximum profit. Through research and analysis, the complex SCN has the following three characteristics:

- Growth: Assuming that the initial SCN G0 (n0, l0) has n0 nodes and l0 connected edges, from time t0, a new node with m edges is added in an exponential form $N(t) \propto e^{c_n t}$, and the new node will have a certain probability of connecting existing nodes in the network.
- Optimal connectivity: When new nodes join the SCN, they will generally prefer to select larger-scale, higher-degree node companies to connect; that is, the probability of selecting a connected node $\prod_{j \to i}$ is proportional to the node degree value k_i. The connection probability expression is as follows:

$$\prod_{j \to i} = \frac{k_i}{\sum_j k_j}, \tag{2}$$

- Dynamics of network evolution: Due to the competition and cooperation mechanism of the economic market, node companies need to constantly learn and update to maintain the policy operation of the entire SCN and not only will increase as the SCN evolves but also because they cannot adapt to the market. Elimination leads to the dynamic nature of the evolution of the network topology over time. The probability of selecting node i as the disconnected end is as follows:

$$\prod_{j \to i}{}' = \frac{1}{N(t) - 1}(1 - \frac{k_i}{\sum_j k_j}), \tag{3}$$

In Formula (3), $N(t)$ is the total number of network nodes when disconnecting the edges.

- Emergence of network evolution: SCN node companies may be affiliated with multiple supply chains simultaneously in the network, and nodes have multimodal competitive relationships and nonlinear interactions, and there are intersections between multiple supply chains or multiple networks. The phenomenon of nesting promotes the evolution of the network. The SCN will emerge with new structures and functional characteristics under the joint action of internal factors and the external environment and other systems, such as the small-world nature of network functions, cascading failures and agglomeration, robustness, community structure, etc., on the network structure.

2.3. Complex SCN Evolution and Degree Distribution Model

The first two sections briefly analyze the evolution mechanism and topology structure of the SCN with small-world and scale-free characteristics. Through an analysis of the previous material, it can be concluded that the complex SCN is a scale-free network that conforms to the characteristics of the "small-world" and follows the nonuniform dynamic evolution mechanism, such as difference, preference, growth, maintenance, and emergence. From a macro perspective, the SCN is a network of business operation models derived from node enterprises to adapt to the dynamic economic market environment. It is also a product of competition and cooperation between node enterprises on the network under different time and space conditions. The formation process of the SCN is a dynamic self-organization evolution process. This process is based on the competition and cooperation mechanism of the enterprises in the market, and carries out the growth of the enterprise itself, the dynamic competition, and cooperation between the enterprises in the international market environment. It optimizes with the survival of the fittest in this process, i.e., the joining and exiting of network nodes.

With the continuous transformation of the value chain in the contemporary social economy, the continuous increase in the scale of the enterprise's operational planning, the in-depth refinement of the division of labor in the production process, and the continuous transformation of the production and sales model, the complexity and dynamics of SCN have increased, while the evolution mechanism of the network in time can be explored from multiple dimensions. First, from a microlevel analysis, the evolution of complex SCNs is manifested by the node joining and elimination by node companies to adapt to the market environment and the pursuit of maximizing benefits, as well as the disconnection and reconnection of node companies. Behavior mode is also a product of competition and cooperation between enterprises. Second, from a macro perspective, the evolution of the SCN is reflected in the dynamic change in the overall topology, the time-varying network robust performance, and the scale of the network. Continuous expansion occurs, and the network distributed structure gradually tends to the ideal state. In general, the evolution mechanism of complex SCNs is based on internal and external environments and interactions, and the evolutionary dynamics of "node enterprises-single supply chain-SCN" gradually change from the underlying individual behavior changes to overall network structure changes. Subtle changes in any link may cause huge changes in the SCN system. These changes are closely related to the topology and macro nature of the SCN itself.

The SCN has gradually evolved from a very small initial stage to a network system with complex characteristics. At this time, the degree distribution of the SCN follows the power-law characteristic and satisfies the following formula:

$$P(k) \propto k^{-\gamma}, \tag{4}$$

where k is the node degree value of the network and γ is the exponent of the power-law distribution. The power index γ will be different in different SCNs and evolution periods. It can be seen from Formula (4) that the probability of network size distribution is inversely proportional to the power index γ. When γ becomes larger, it indicates that the number of node enterprises with a larger degree value in the network is reduced, and the scale of the node enterprises tends to be homogeneous. When γ decreases, it indicates that the number of node enterprises with a larger degree in the network increases, and the development scale of node enterprises is more heterogeneous. However, due to the influence of internal and external factors in the SCN, the value of the power index γ will not decrease or expand indefinitely, and γ will eventually stabilize at a constant greater than 0, achieving a dynamic and stable distribution state in the structure. For example, the Tokyo Industrial Park Ohta-Ward includes 8311 companies, including TOSHIBA and other companies. The agglomerated SCN composed of these companies has a self-similar structure without a scale network degree distribution and a high degree of dispersion. The distribution of the degree of network nodes conforms to the power-law distribution of the power exponent γ = 2.3 [13], as shown in Figure 4.

The power-law distribution is a sign of the transition of the self-organized critical system from the steady state to the chaotic state. The literature [9] found that the power-law distribution of the complex SCN of agricultural products will turn at kc, which is described by the single power-law distribution. This network degree distribution feature is not accurate, as shown in Figure 5. To accurately describe the power-law distribution of a complex SCN, a complementary distribution of degrees is proposed, which describes the power-law distribution from a turning point into two parts. The complementary distribution of the power-law distribution follows the following formula:

$$F(k) = P(K > k) = \frac{\lambda}{\gamma - 1} k^{-(\gamma-1)}, \tag{5}$$

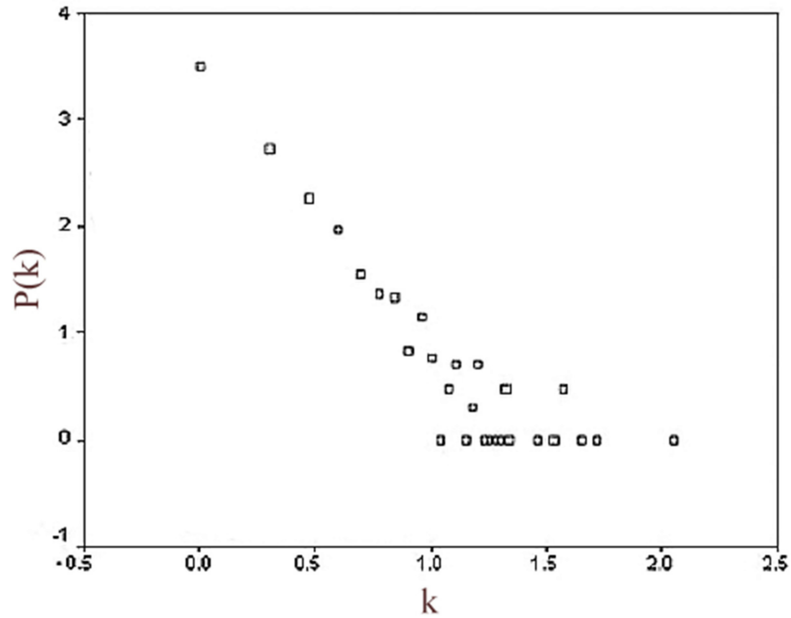


Figure 4. The Tokyo Industrial Park Ohta—Ward SCN degree distribution.

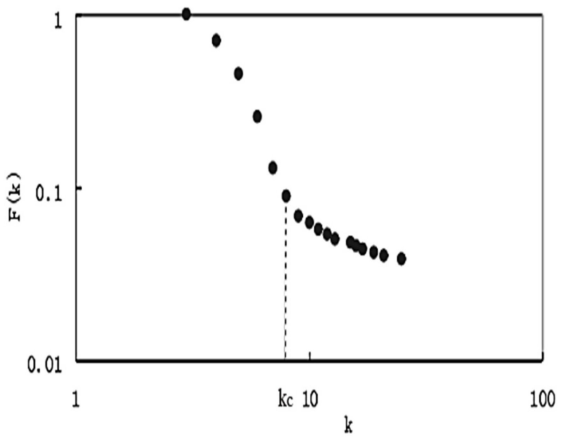


Figure 5. The agglomerated SCN degree distribution.

The formula for calculating the power index γ using maximum likelihood estimation is as follows:

$$\hat{\gamma} = 1 + n\left[\sum_{i}^{n} \ln\left(\frac{k_i}{k_{\min}}\right)\right]^{-1}, \tag{6}$$

Among them, $k_{\min}$ is the minimum degree value of the degree distribution conforming to the power rate characteristic part, and $k_{\min} > 0$.

In addition, an increasing number of studies have found that a single power-law distribution does not fully describe the distribution law of real-world systems. The degree distribution of many complex SCNs will change during the evolution process and obey two power laws with different exponents. Distribution, such as China’s airline network, social network, and SCNs such as automobiles and transportation ports.

Therefore, the distribution of the complex network power-law distribution containing two sections of different power-law regions is called a two-stage power-law distribution, which has different power exponents. The function expression [14] is as follows:

$$P(k) = \begin{cases} k^{-\gamma_1}, k < k_c \\ k^{-\gamma_2}, k > k_c \end{cases}, \tag{7}$$

Among them, γ_1 and γ_2 are the indexes of the two-stage power-law distribution, and k_c is the turning point in the two-stage power-law distribution.

2.4. Analysis of the Deviation of the Power-Law Distribution

An important prerequisite for previous studies on the evolution model of complex SCNs is that the total number of connected edges and the total number of nodes in the network where nodes and connected edges that grow over time are always dynamically linearly related, and the average value of the network will not be changed during the evolution degree value. However, the nonlinear accelerated growth of complex SCNs in reality is a common evolutionary phenomenon, so the following model construction and analysis are based on the scale-free accelerated growth SCN as the research object, which is more in line with the real complex supply chain evolution mechanism and form of the network.

Most of the complex SCNs widely existing in reality are self-organized critical systems on the edge of chaos. An important feature of this type of network is that the degree distribution meets the power-law characteristic, that is $p(k) \sim k^{-\gamma}$, and the value of the power index γ is usually less than 3. In the field of complex network research, many related models and mechanisms have been proposed to explain the important topology property of power-law distribution, such as the priority connection mechanism, adaptive model, accelerated growth model, and logarithm proposed by Barabasi and Albert. They present a growth model, random preference model, etc. [1].

However, according to a large number of empirical studies, it is found that there are few power-law distributions that completely obey factor $k^{-\gamma}$ in the real SCN. Most of the complex SCN's degree value data only appear as a power-law distribution within the main range of the degree distribution, and the phenomenon of deviation from the power-law distribution often occurs in the range of small or large degree values. Many scholars have also discovered this kind of power-law deviation in empirical research. Some scholars proposed different models and mechanisms to explain the phenomenon of power-law distribution deviation, such as the size effect, information filtering, nonadditivity, statistics, and other mechanisms. While Barrat [15] and Matteo [16] studied the world aviation network and the business management network, respectively, they found that the cumulative degree distribution would have an exponential deviation and constructed a model describing the power-law deviation, as shown in Formula (8):

$$p(k) = k^{-\gamma} \exp\left(\frac{k}{k_x}\right), \tag{8}$$

There is a similar exponential deviation in the citation network of scientist papers, and this phenomenon has been described since the construction of the model, as shown in Formula (9):

$$p(x) = \frac{c_1}{c_2 + (|x - x_m|)^{1+\alpha}} \exp(x), \tag{9}$$

Among them, x is the number of times a paper is cited, x_m is the maximum probability value of x, and c_1 and c_2 are constants. In the case where the aviation network with a two-stage power-law distribution exhibits data deviation, a mathematical equation $p(k) = \frac{a}{k^{\gamma_1}(1 + bk^{\gamma_2})}$ can be established to represent the deviation characteristics of the power-law degree distribution of the network. In addition, there are similar deviations in the cumulative distribution of road networks in the US and Denmark.

Similarly, the deviation of the power-law distribution is found in most scale-free complex SCNs, and these networks usually use truncated data sets to fit the power-law distribution model. For example, Clauset et al. [17] divided the degree value of the network into two parts by $x_{\min}$; the network only obeyed the power-law distribution when it was $x \geq x_{\min}$ and estimated $x_{\min}$ based on the distance test method, using the standard maximum likelihood method to estimate the value of alpha. However, this method has three obvious deficiencies: (1) the data before all the truncation points of $x \leq x_{\min}$ are lost; (2) different types or periods of networks have different $x_{\min}$, which makes the degree distribution of the comparison network difficult; and (3) because the data set of $x \leq x_{\min}$ is not modeled, it is not feasible to predict the evolution of the network on the entire data scale.

Therefore, this article uses a processing method different from the above literature but builds a model on the basis of the entire complex SCN data set and establishes and analyzes the power-law distribution deviation phenomenon using standard statistical methods to describe it. In reality, the underlying model of complex SCN power-law distribution and power-law deviation features overcome the model limitations caused by incomplete data.

3. Evolution Model of a Complex SCN Based on the Deviation of the Power Law Distribution

In this section, the complex SCN is taken as the research object. It is proposed that the evolution of the network includes the addition of new nodes and new edges with a certain probability and that the edges are removed and reconnected with a certain probability. To further analyze the evolution mechanism and process of complex SCN, this study uses the control variable method, i.e., by analyzing the key model parameters, such as the probability of joining new nodes p_1 and the probability of forming new edges p_2, etc., to analyze the trend of network evolution.

3.1. Model Condition Setting

Assuming that the initial SCN contains m0 nodes and n0 edges, three conditional assumptions are set for the process of network evolution based on literature research and actual network analysis and are executed with a certain probability at each time step. The specific contents of the model assumption are as follows:

- Add new nodes with probability p_1. Each newly added node establishes an edge with another node through the preferred connection mode shown in Formula (2), and m new connected edges are generated.
- Add ct new continuous edges with probability p_2, where parameter c is a constant that is much smaller than 1 and used to characterize different network features.
- n connected edges were removed and connected. Interruption and establishment of cooperation between enterprises in the network are common. To characterize this phenomenon, randomly select a node i and one of its connected edges l_{ij} with probability q, remove this edge with probability $q\prime$, and then use Formula (2) to select node $j\prime$ to form a new connected edge $l_{ij}\prime$. When the node degree value is larger, the value of $q\prime$ is smaller, and the removal of edges between nodes follows the anti-optimal disconnection rule.

In this model, all probability parameters p_1, p_2, q and $q\prime$ are in the range of [0, 1], and when q = 0. $q\prime = 0$. It is assumed that the degree value ki is continuously changing, and the initial network structure will not have a great impact on the final evolution of the network. Therefore, the above three operations will have a greater impact on the SCN degree value k_i.

3.2. Establishment of an Evolution Model of Power-Law Distribution Deviation in SCN

Based on the analysis of the above model assumptions, each step of the evolution of the SCN can be expressed using a continuous theoretical model. The specific steps are as follows:

- Add a new node to the network with probability p_1; then, the degree change rate of the node is expressed as follows:

$$\left(\frac{\partial k_i}{\partial t_1}\right) = \frac{\lambda p_1}{\sum\limits_j k_j}, \tag{10}$$

We can know $\lambda = m$.

- Adding ct new connected edges in the network with probability p_2, the rate of change in the degree of the node is:

$$\left(\frac{\partial k_i}{\partial t_2}\right) = \frac{\mu p_2 k_i}{\sum\limits_j k_j}, \tag{11}$$

Among them, after each time step, the overall change in SCN connectivity is $\Delta k = 2ct$, so $\mu = 2ct$ is available.

- When n consecutive edges in the network are reconnected, the degree change rate of the node can be expressed as:

$$\left(\frac{\partial k_i}{\partial t_3}\right) = -\frac{\omega q}{p_1 t_3} + \frac{\omega q\prime k_i}{\sum\limits_j k_j}, \tag{12}$$

Among them, $p_1 t_3$ represents the scale of the SCN at time t_3. In Formula (12), the first term on the right side of the equal sign is the reduction of the network node degree value caused by the connected edge with the probability q removed, and the second term expresses that the connected edges are reconnected with the optimal choice. The increase is in the network degree value. However, the total connectivity of the SCN did not change during the process of continuous reorganization $\omega = n$.

Since the above three changes will occur synchronously in the overall process of SCN evolution, the situation of the network node degree value change rate caused by the three evolution processes can be considered at the same time. The formula is as follows:

$$\left(\frac{\partial k_i}{\partial t}\right) = \frac{(mp_1 + 2cp_2 t + nq\prime)k_i}{\sum\limits_j k_j} - \frac{nq}{p_1 t}, \tag{13}$$

Among them, the total value of the network is $\sum\limits_j k_j = t(2mp_1 - nq + nq\prime) + cp_2 t^2$. For the node that joined at time t_i, $k_i(t_i) = m$. By iteratively solving Equation (13), the expression of the solution is as follows:

$$\begin{aligned} k_i(t) = {} & m(\tfrac{t}{t_i})^{\alpha}\left(\frac{f + cp_2 t}{f + cp_2 t_i}\right)^{\beta} - t^{\alpha}(f + cp_2 t)\beta \\ & \times \frac{nqt^{-\alpha}(f + cp_2 t)^{\alpha-1}(-f + mp_1 - cp_2 t + nq\prime) - nqt_i{}^{-\alpha}(f + cp_2 t_i)^{\alpha-1}(-f + mp_1 - cp_2 t_i + nq\prime)}{(f - mp_1 - nq\prime)(nq\prime + mp_1)} \end{aligned} \tag{14}$$

Among them

$$\begin{cases} \alpha = \dfrac{mp_1 + nq\prime}{f} \\ \beta = \dfrac{2f - (mp_1 + nq\prime)}{f} \\ f = 2mp_1 - nq + nq\prime \end{cases} \quad (15)$$

Equation (14) expresses the situation where the power-law distribution deviates in two different regions. At time t, the degree $k_i(t)$ of the node is less than the probability $p(k_i(t) < k) = P(t_i > F(m, n, p_1, q, q\prime, c, t)t)$ of k. When $t_i \ll t$, the expression of the degree $k_i(t)$ of the network can be obtained according to continuity theory as follows:

$$k_i(t) \approx m(\frac{t}{t_i})^{\alpha} \left(\frac{cp_2t}{f}\right)^{\beta} + \frac{nq(cp_2t)^2}{(f - mp_1 - nq\prime)(nq\prime + mp_1)} \quad (16)$$

$$W(m, n, p_1, q, q\prime, c, t) = m^{\frac{1}{\alpha}}(\frac{cp_2t}{f})^{\frac{2f - mp_1 - nq\prime}{mp_1 + nq\prime}}(k - \frac{nq(cp_2t)^2}{(f - mp_1 - nq\prime)(mp_1 + nq\prime)})^{\frac{-1}{\alpha}} \quad (17)$$

Assuming that the addition of nodes and edges and the reconnection of edges are completed in the same time period, the probability density of t_i is $p_i(t_i) = \frac{1}{t}$, and there is $p(k_i < k) = 1 - W(m, n, p_1, q, q\prime, c, t)$. The probability density $p(k)$ at time t can be obtained by the following formula:

$$\begin{aligned} p(k, t) &= \frac{\partial P(k_i(t) < t)}{\partial k} \\ &= \frac{fm^{\frac{f}{mp_1 + nq\prime}}}{mp_1 + nq\prime}(\frac{cp_2t}{f})^{\frac{2f - mp_1 - nq\prime}{mp_1 + nq\prime}}(k - \frac{nq(cp_2t)^2}{(f - mp_1 - nq\prime)(nq\prime + mp_1)})^{-\gamma_2} \end{aligned} \quad (18)$$

Among them, $\gamma_2 = 3 - \dfrac{n(q + q\prime)}{mp_1 + nq\prime}$.

In addition, when $t_i \sim t$ is obtained, $k_i(t) = m(\frac{t}{t_i})^2$ can be obtained. Similarly, the expression of the probability density $p(k)$ of the SCN at this time can be obtained as shown in Equation (19).

$$p(k) = \frac{\partial P(k_i(t) < k)}{\partial k} = \frac{1}{2}m^{\frac{1}{2}}k^{-\gamma_1}, \quad (19)$$

Among them, $\gamma_1 = 1.5$.

Based on the above analysis, it can be found that the SCN generated according to the previously set evolution rule is a scale-free network, and the node degree distribution in the network has power-law characteristics and two different power exponents in the power-law distribution area, which is subject to the two-stage power-law distribution. At the evolution time t, for the nodes with larger degree values generated in the complex SCN, the network node degree distribution will evolve with the power exponent of γ_2 with time. Different SCNs are in second place due to different parameter values. The node enterprises on the segment power law distribution curve have different evolution speeds. The power exponent γ_2 in Formula (18) has a relatively dynamic form of change, which is more convenient for characterizing different real-world network evolution dynamics. For the node enterprises generated by the network with a small degree value, the evolution power exponent $\gamma_1 = 1.5$ has a more stable evolution mechanism.

3.3. Analysis of SCN Evolution Model Parameters

Based on the above analysis of the complex SCN evolution model, it can be seen that the evolution of the network on the time scale is subject to multiple model parameters. Next, we further analyze the influence of the parameters on the evolution of the SCN by controlling some key parameters of the model, such as the probability of adding a new node p_1, the probability of adding a new connected edge p_2, and the number of connected edges n.

- Probability of joining new edges p_2

To further analyze the dynamic evolution of the complex SCN when there is no increase in the connected edges in the network, this section sets the probability of joining the new connected edge p_2 to 0. Based on continuity theory, Formula (13) can be transformed as follows:

$$\frac{\partial k_i}{\partial t} = \frac{mp_1 k_i}{\sum\limits_j k_j} + \frac{nq\prime k_i}{\sum\limits_j k_j} - \frac{nq}{p_1 t}, \tag{20}$$

Among them, the total connection value $\sum\limits_j k_j = (2mp_1 - nq + nq\prime)t$ in the network. The first item on the right side of the medium number in the above formula represents the process of adding nodes during the evolution of the network, and the last two items are the process of reconnecting the edges in the network. When the time step is t_i, combined with the initial condition $k_i(t_i) = m$, the expression of the solution to Formula (20) is:

$$k_i(t) = \left(\frac{t}{t_i}\right)^{\alpha}\left(m - \frac{qnf}{p_1(mp_1 + nq)}\right) + \frac{qnf}{p_1(mp_1 + nq\prime)}, \tag{21}$$

The above formula is the scale expression of the SCN at time t_i. Therefore, the probability density expression $P(k,t)$ of the network node degree distribution is:

$$P(k) = \frac{\partial p(k_i(t) < k)}{\partial k} = \frac{f}{mp_1 + nq\prime}\left(m - \frac{qnf}{p_1(mp_1 + nq\prime)}\right)^{\frac{1}{\alpha}}\left(k - \frac{qnf}{p_1(mp_1 + nq\prime)}\right)^{-\gamma}, \tag{22}$$

Among them, $\gamma = 3 - \frac{n(q + q\prime)}{mp_1 + nq\prime}$. It can be obtained from the above formula analysis that this is a single power law distribution form. This result also shows that the process of adding edges in the network is a necessary condition for the deviation of the power law distribution.

In addition, through Formulas (14), (16) and (17), the expression of the curve turning point of the two-segment power law distribution is as follows:

$$k_c = (cp_2 t)^{\frac{mp_1 + nq\prime}{f}}(f + cp_2 t)^{\frac{2f - mp_1 - nq\prime}{f}}, \tag{23}$$

As seen from the above formula, the value of the turning point is directly proportional to time. In areas where the degree value is smaller than this turning point, the network degree distribution is a stable distribution with a power exponent of $\gamma_1 = 1.5$; and in areas greater than this turning point, the degree distribution follows the unsteady distribution characteristic of $\gamma_2 = 3 - \frac{n(q + q\prime)}{mp_1 + nq\prime}$, which also illustrates the long tail of the SCN. Some evolutionary situations have dynamic characteristics.

- Probability of joining new edges p_1.

However, when there are no new companies to join the SCN, i.e., when $p_1 = 0$, only the random growth and selective growth of the continuous side and the continuous reconnection process will be considered to affect the evolution of the SCN. Among them, $p_1 t$ in Formula (13) represents the scale of the SCN after time t evolves. Because $p_1 = 0$, the network scale is the initial SCN scale m0. Based on continuity theory, the formula for the rate of change in the available network degree value with time can be expressed as:

$$\frac{\partial k_i}{\partial t} = \frac{2cp_2 t k_i}{\sum\limits_j k_j} + \frac{nq\prime k_i}{\sum\limits_j k_j} - \frac{nq}{m_0}, \tag{24}$$

The first item on the right side of the medium number in the above formula represents the process of joining the edges in the evolution of the network on the time scale, and the remaining items are the process of reconnecting the edges in the network. Similarly, when the time step is ti, the expression that can be solved for the above formula is:

$$k_i(t) = t^{\alpha}(m_0 - \frac{qn^2(q\prime - q)}{m_0 t_i{}^{\alpha-2}(m_0{}^{\alpha} + nqt_i)}) + \frac{qn^2(q\prime - q)}{m_0 t_i{}^{\alpha-2}(m_0{}^{\alpha} + nq\prime t_i)}, \tag{25}$$

When a new node is added at each time step t, a node is randomly selected as an end node of the newly connected edge, and the other end node is selected according to Formula (2). During the evolution of the network, the total number of nodes remains unchanged, and the nodes with larger degrees in the network gradually increase, while the nodes with smaller degrees gradually decrease. After $t \approx$ m02/4 time steps, the entire network will generate a fully connected graph, resulting in the final degree distribution of the SCN tending to δ.

At this time, due to $p_1 = 0$, the formula $\gamma_2 = 3 - \frac{n(q + q\prime)}{mp_1 + nq\prime}$ can be expressed as $\gamma_2 = 2 - \frac{q}{q\prime}$, the turning point of the power law distribution $k_c \neq 0$, so the SCN is still a two-stage power law distribution, and the network nodes in the region with a larger degree still have a dynamic growth power index. At the same time, it also reflects that the joining process of new nodes will not change the distribution form of the node value of the entire SCN, but it has a certain effect on the deviation of the power law distribution of the network.

- Link reconnection parameter n

Similarly, analyze the evolution of the entire complex SCN when the process of reconnecting the edges of the SCN does not occur, i.e., $n = 0$. According to continuity theory:

$$\frac{\partial k_i}{\partial t} = \frac{(mp_1 + 2cp_2 t)k_i}{\sum_j k_j}, \tag{26}$$

The above formula shows the effect of the joining mechanism of nodes and edges on the rate of change of the node degree during the network evolution.

Similarly, when the time step is t_i, $k_i(t) = m(\frac{t}{t_i})^2$ can be obtained. The probability density of the SCN is the same as that in Formula (19) and the power index $\gamma_1 = 1.5$. It can be seen that the process of continuous reconnection will not affect the distribution of the nodes with a smaller value in the network, which is also in line with the characteristics of the optimal connection of the SCN, and at the same time verifies the effectiveness of the model in this paper.

In addition, according to the formula γ_2, when $n = 0$, $\gamma_2 = 3$ is obtained. Therefore, when the reconnection process does not occur, the entire SCN follows a two-stage power law distribution and has a relatively stable evolution process. At this time, the two fixed power indexes of the node degree distribution are 1.5 and 3 [11] also pointed out that the two-segment power exponents of the node degree distribution of the agricultural product SCN are 2.78 and 1.78, respectively. Similar results can also be found in the human language network [18], whose two fixed power indexes are γ_1= 1.5 and γ_2= 2.7.

4. Simulation of SCN Evolution

Based on the previous analysis of the distribution characteristics and evolution characteristics of complex SCNs, it is known that an important phenomenon in the evolution of SCNs is the power law deviation of the degree distribution of network nodes. Under this analysis, it is of great significance to study the evolutionary mechanism of SCNs. First, this paper uses the scale-free SCN that is more common in reality as the research object, such as the SCN of agricultural production and processing enterprises, the supply network of the automobile manufacturing industry, and the port coal transportation SCN. Second, the single SCN in the form of a power law distribution can be regarded as a special case of a double power law distribution, such as in the case where the second power exponent is the same as the first power exponent. Therefore, the SCN of energy companies is used to analyze the evolution mechanism in reality. The evolution of the single power law SCN.

4.1. Simulation Analysis of Network Evolution Based on Deviation of Two-Stage Power Law Distribution

In real life, most complex SCNs will exhibit different dual-segment power exponential distributions over time, mainly due to the inherent driving force of the growth of complex SCNs in various fields and different external environments, which in turn leads to significant differences in the evolution parameters of the network, mainly reflected in the probability parameter p_1, p_2, q, $q\prime$ and the power index γ_1, γ_2 and other parameter values. This type of SCN is generally composed of upstream raw material suppliers, component suppliers and other auxiliary product production node companies. Midstream companies are manufacturing products and processing node companies, while downstream companies mainly include transportation node companies and product demand nodal enterprises and individuals. Upstream, midstream, and downstream nodal enterprises in the network constitute an interconnected and codeveloped topology system, and the system is always dynamic over time. In this section, based on the previous model construction analysis, combined with numerical experiments and model simulation methods, and then based on the contrast analysis of the two-stage power law distribution deviation phenomenon, the network evolution and the distribution degree and characteristics of node degree values under different second-stage power index conditions are analyzed.

In the process of setting up numerical experiments, the scale of the SCN is set to a medium-sized network, the number of nodes is $N = 1 \times 10^3$ and other parameters of the evolution model are: $t = 5 \times 10^3$, $m = 2$, $p_1 = 1$, $n = 3$, $p_2 = 1$, $c = 2 \times 10^{-3}$. In addition, different types of line shapes are used to characterize the simulation results of models with different power indexes. The second-stage power index corresponding to the solid line is $\gamma_2 = 1.5$, the power index corresponding to the dotted line is $\gamma_2 = 2.5$, and the power index corresponding to the dotted line is $\gamma_2 = 3$. In addition, the parameter values corresponding to the three types of legends are different: the parameter corresponding to the dot is $q = 1$, $q\prime = 0$, the parameter corresponding to the triangle is $q = 0.5$, $q\prime = 0.5$, and the parameter corresponding to the square is $q = 0$, $q\prime = 0$.

In the case where the three power exponents are different, the numerical experiment and model simulation process are performed separately with the help of Python software, and the two calculation results are integrated into one graph. The final result is shown in Figure 6. It can be found from the figure that the two results can be fitted well and have significant two-stage power law distribution characteristics. At this time, the average degree values of the network are 1.45, 1.76, and 1.99, respectively, and the value of the curve turning point k_c of the two-segment power law distribution is 11, 13.8, and 18.6, respectively.

Among them, P(k) is the probability density function of the SCN node degree value k. It can be seen from the above figure that when the power exponent of the second segment of the distribution curve is $\gamma_2 = 1.5$, the second segment of the network node degree value distribution curve has the same slope as the first segment. A straight line is a single power law distribution. As the power exponent γ_2 of the second distribution curve continues to increase, its slope also gradually increases, which indicates that the number of nodes with larger degrees in the SCN changes faster, i.e., the number of nodes with larger degrees will accelerate. At the same time, the turning point k_c of the corresponding two-segment power law distribution curve of the SCN also gradually becomes larger, which is reflected in the figure as gradually moving to the right. In addition, according to Formula (23) and [14], it can be seen that the turning point is proportional to time, and it will become larger as the evolution time t increases.

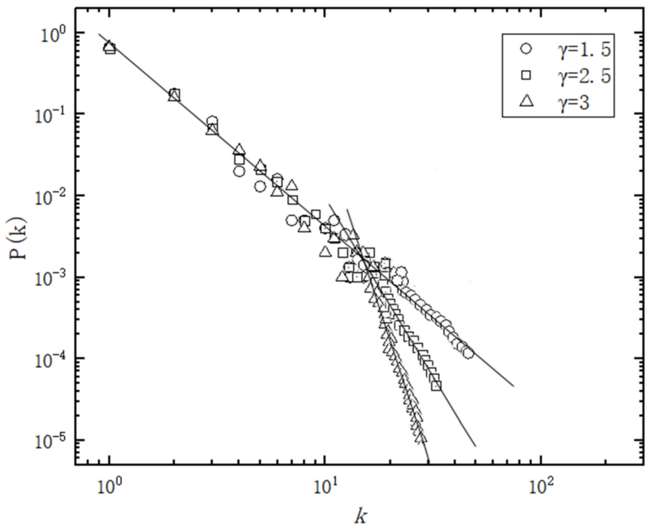


Figure 6. Network node degree distribution of the numerical experiment and model simulation under different parameters.

4.2. Simulation Analysis of Network Evolution Based on Deviation of Single Power Law Distribution

To further analyze the deviation of the power law distribution of SCN evolution, this section selects a state-owned energy technology company with a special evolution mechanism as the research object and uses the relevant theoretical knowledge and model methods of complex networks to simulate the growth evolution of the network. The phenomenon of distribution deviation was studied. In addition, the business networks of energy production companies conform to the structural characteristics of complex SCNs, and the network node degree values follow a single power law distribution.

Energy production enterprises, such as coal and petroleum-based state-controlled energy enterprises, can divide their internal organizations into procurement departments, assembly and distribution departments, production departments, and management departments according to business content. Each department can be regarded as an SCN node. The energy company has three large-scale production centers nationwide, 11 first-level sales centers, 58 second-level sales offices, and 28 raw material suppliers. Eventually, the energy supply company's business SCN can be extracted as a scale-free SCN with 100 nodes. Due to the steady development strategy of state-controlled enterprises, this SCN will have fewer new energy companies to join in a certain period of time, which can be approximately regarded as $p_1 = 0$. The evolution process and mechanism of the network on the time scale.

In the numerical simulation, to more clearly show the evolution characteristics of the energy supply enterprise's business SCN, the number of network nodes is expanded tenfold, i.e., the total network node number $m = 1 \times 10^3$ is set, and different evolution time steps t_i are set. The time step corresponding to the square is $t_1 = 5 \times 10^3$, the time step corresponding to the triangle is $t_2 = 7 \times 10^3$, and the time step corresponding to the circle is $t_3 = 1.0 \times 10^4$. Figure 7 below shows the transformation process of the degree distribution $P(k)$ of the SCN from the original single power law distribution to a more complicated distribution form under different logarithmic coordinates.

As seen from the above figure, in the initial state at t_1, the degree distribution of the energy supply network of the energy enterprise is fitted with a straight line, which is represented by a single power law distribution with a power index of 3. As the evolution time t_1 increases, a small deviation at t_i begins to appear in the distribution area of the smaller node value, the probability value corresponding to this type of node gradually decreases, and the probability value corresponding to the node with a larger degree gradually increases. When the evolution time reaches t_3, the "head" deviation phenomenon is very obvious, but the degree distribution form and Gaussian distribution are very different at this time.

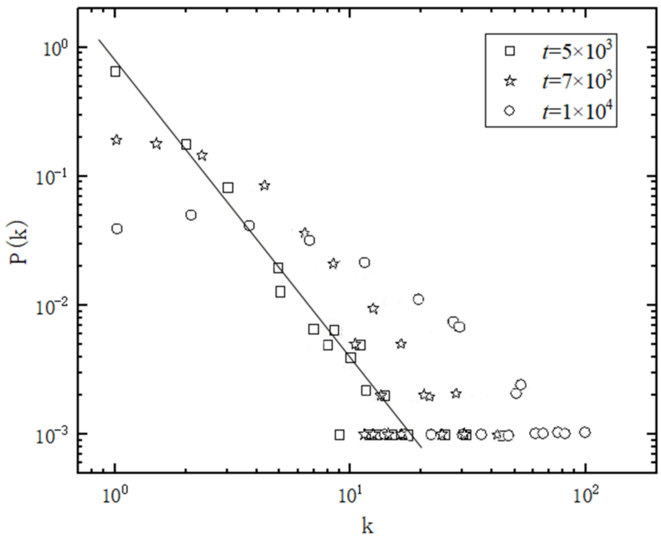


Figure 7. Evolution diagram of the network degree distribution under different time steps.

During the evolution of the entire SCN, it can be predicted that the degree distribution of the network will have a tendency to change to the δ distribution when the time step is large enough, and the form of the single power law distribution has not changed. This further validated the effectiveness of the model in this paper. At this time, the energy enterprise SCN can guarantee the connection of goods and information between any two enterprises, but the robustness of the network and the ability to resist risks are poor. Once some node companies have some problems, it will cause more problems. The division of the entire relationship network is broken, blocking the normal flow of information and products between enterprises. Therefore, after a long period of development, the SCN of energy enterprises should pay attention to strengthening the management and maintenance of the enterprise SCN, preventing the problem of network cascade failure, and improving the network topology and performance.

5. Conclusions

This paper briefly analyzes the topological structure and characteristics of SCNs with small-world and scale-free characteristics, explains the evolution of complex SCNs and node degree value distribution models, and analyzes the power law distribution deviation of the network node degree values phenomenon. Based on the above research, three mechanism conditions were set up, including a new node and new connection and connection reconnection, and then an SCN evolution model considering deviation from the power law distribution was established. Combined with the above, a simulation analysis of the evolution of the SCN based on the deviation of single and double power law distributions is carried out, and the following conclusions can be drawn: (1) The power index of the region with a large degree in the SCN is generally less than 3 and the power of the index can change with the change in parameters; (2) The deviation of the two-stage power law distribution of the network nodes is not caused by the addition of nodes or the reconnection process of the edges, because when $n = 0$ or $p_1 = 0$, the nature of the law distribution still exists, but the increase of nodes and the reconnection process of the edges have a certain effect on the change in the power index of the network distribution; (3) The deviation of the power law distribution of the SCN is proportional to the evolution time; (4) The growth speed of the chain network in the early stage of evolution is relatively fast, while in the later stage the growth rate gradually slows down and stabilizes, and the enterprise relationship structure on the network also gradually stabilizes from the chaotic state. It is an evolution process that follows the nonuniform growth mechanism. Therefore, the model constructed in this paper can be used to describe the complex network SCN in the evolution process in different periods to more accurately characterize the distribution of network node degrees and analyze the dynamic mechanism of network evolution. In future work, we will further extend the analysis to more general distributions, such as Lerch distribution.

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References

1. Barabási, A.L.; Albert, R. Emergence of scaling in random networks. *Science* **1999**, *286*, 509–512. [CrossRef] [PubMed]
2. Pei, W.; Wei, X.; Wang, Q.; Zhao, Z.; Ma, X. Study on the dynamic evolution model of complex networks with uniform and power-law mixed distribution. *J. Tianjin Norm. Univ.* **2010**, *3*, 20–23.
3. Chen, T.; Shao, Z.G. Power-law accelerating growth complex networks with mixed attachment mechanisms. *Phys. A Stat. Mech. Its Appl.* **2012**, *391*, 2778–2787. [CrossRef]
4. Liu, C.; Li, R. Power-law distributed temporal heterogeneity of human activities promotes cooperation on complex networks. *Phys. A Stat. Mech. Its Appl.* **2016**, *457*, 93–100, Erratum in *Phys. A Stat. Mech. Its Appl.* **2017**, *475*, 169. [CrossRef]
5. Contreras-Reyes, J.E. Lerch distribution based on maximum nonsymmetric entropy principle: Application to Conway's Game of Life cellular automaton. *Chaos Solitons Fractals* **2021**, *151*, 111272. [CrossRef]
6. Li, W.; Wang, Q.; Nivanen, L.; Le Méhauté, A. How to fit the degree distribution of the air network? *Phys. A Stat. Mech. Its Appl.* **2006**, *368*, 262–272. [CrossRef]
7. Bagrow, J.P.; Rozenfeld, H.D.; Bollt, E.M.; ben-Avraham, D. How Famous is a Scientist?—Famous to Those Who Know Us. *Europhys. Lett.* **2004**, *67*, 511–516. [CrossRef]
8. Mossa, S.; Barthelemy, M.; Stanley, H.E.; Amaral, L.A.N. Truncation of power law behavior in "scale-free" network models due to information filtering. *Phys. Rev. Lett.* **2002**, *88*, 138701. [CrossRef] [PubMed]
9. Guimerà, R.; Amaral, L.A.N. Cartography of complex networks: Modules and universal roles. *J. Stat. Mech.* **2005**, *2005*, nihpa35573. [CrossRef] [PubMed]
10. Maillart, T.; Sornette, D.; Spaeth, S.; von Krogh, G. Empirical tests of Zipf's law mechanism in open source Linux distribution. *Phys. Rev. Lett.* **2008**, *101*, 218701. [CrossRef] [PubMed]
11. Zhao, G. *Research on Dynamic Behavior and Controllability of Complex Supply Chain Network*; Nanjing University of Aeronautics and Astronautics: Nanjing, China, 2015.
12. Ma, J.; Fan, J.; Liu, F. Complex network and software metric analysis. *J. Beijing Jiaotong Univ.* **2016**, *40*, 23. [CrossRef]
13. Fu, P.; Li, J.; Liu, Y. Network evolution model of agglomeration supply chain based on degree and path priority connection. *Oper. Manag.* **2013**, *22*, 120–125.
14. Qian, X.; Yang, B. Research on cooperative evolution of supply chain enterprises based on complex network model. *Complex Syst. Complex. Sci.* **2018**, *15*, 4–13.
15. Barthélemy, M.; Barrat, A.; Pastor-Satorras, R.; Vespignani, A. Dynamical patterns of epidemic outbreaks in complex heterogeneous networks. *J. Theor. Biol.* **2005**, *235*, 275–288. [CrossRef] [PubMed]
16. Matteo, T.D.; Aste, T.; Gallegati, M. Innovation flow through social networks: Productivity distribution in France and Italy. *Eur. Phys. J. B* **2005**, *47*, 459–466. [CrossRef]
17. Clauset, A.; Shalizi, C.R.; Newman, M.E.J. Power-law distributions in empirical data. *SIAM Rev.* **2009**, *51*, 661–703. [CrossRef]
18. Markosova, M.; Nather, P. Language as a Small World Network. In Proceedings of the International Conference on Hybrid Intelligent Systems, Auckland, New Zealand, 13–15 December 2006.