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Production Optimization in a Grain Facility through Mixed-Integer Linear Programming †

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Featured Application: The methodology described in this article is applicable to production planning optimization problems with multiple products that can be processed in any of several production lines, including storage and limited shelf life constraints.

Abstract: This article introduces a Mixed-Integer Linear Programming model for cost optimization in multi-product multi-line production scheduling. This model considers discrete time windows and includes realistic constraints. The NP completeness of the problem is proven. A novel scheme based on embedding bounds is applied to speed up convergence. The model is tested on 16 input configurations of a real case study from the top Uruguayan grain production facility. The numerical results show that the model significantly improves the outcome of the current ad hoc heuristic planning, reducing on average 10% the overall production costs; and that the introduction of the embedded bounds-based scheme reduces significantly the elapsed time, on average by 22%.

Keywords: grain facility optimization; multi-product multi-line scheduling; Mixed-Integer Linear Programming; theoretical bounds analysis

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1. Introduction

The fourth industrial revolution, including the digital transformation of production and business models, is characterized by the concept of Industry 4.0 [1]. Industry 4.0 integrates innovative approaches and new digital technologies, to take advantage of robotics, automation, data analytics, and computational intelligence to assist in better decision making and create added value. The main goal is to anticipate the demands of the market and customers, providing levels of efficiency and responsiveness that were not achievable in the traditional industrial model. In turn, the digital revolution also offers many opportunities for applying sustainable production and sustainable development approaches [2].

National and regional companies are required to adapt their production systems to consider the technological shift, for example in the agri-food sector [3]. Agricultural and food companies are encouraged to introduce smart technologies in manufacturing and production processes, promoting the shift from product-oriented business models to service-oriented business models [4,5].

Artificial intelligence and optimization models are of paramount importance in the era of Industry 4.0. Although most media attention is devoted to conceive and develop new business models, there is still a large space for optimizing production operations, within the currently applied models. Specifically, in agriculture and related industries, information technologies are increasingly applied in problems such as production forecast, scheduling, planning, and distribution [6–8].

Following this line of research, this article introduces a Mixed-Integer Linear Programming (MILP) optimization model based on discrete time windows for a capacitated multi-product multi-line production scheduling problem. It allows modeling several constraints that are typical in the context of grain production, including limited storage space and shelf life of products. The model is extended by proposing a bounds-based scheme intended to accelerate convergence to the optimal solution.

The model was tested with real data, taken from the top Uruguayan grain producer and exporter company and its soybean harvest in 2020. The goal was to plan the production of 17 products along 16 weeks, using two production lines. Storage, line cleaning times and limited shelf life constraints are part of this real instance. Significant cost reductions are achieved when compared to the ad hoc planning so far applied by the company. The tests also show that significant time reductions are achieved by applying the embedded bounds-based scheme proposed in this article.

This article contributes toward a relevant research topic by proposing a MILP model for a relevant grain production problem. The problem is analyzed both theoretically and empirically, and the presented case study is one of the first reported research of applying ad hoc systematic optimization models for a real grain production company in Uruguay. The article extends our previous conference publication 'A case study of smart industry in Uruguay: grain production facility optimization' [9] presented at IV Ibero-American Congress on Smart Cities, Cancún, México, December 2021. New content and contributions in this article include: (i) an expanded review of the related literature about optimization and the application of MILP models for grain production problems; (ii) the NP-completeness of the problem is demonstrated; (iii) a novel bounds-based scheme for speeding up convergence is introduced; and (iv) the real test case is run with 16 input sets where the original variables are altered to illustrate the performance of the algorithm under different scenarios.

The article is organized as follows. Section 2 reviews relevant related works. Section 3 describes the addressed grain facility optimization problem and the proposed MILP formulation. Section 4 presents a study of the computational complexity of the optimization problem and computes useful lower bounds. A case study for a real grain production company in Uruguay is described in Section 5. The experimental evaluation of the proposed model and bounds is reported in Section 6. The computed results are validated by a comparison with a heuristic ad hoc planning method, applied by the company before the model was conceived. Finally, Section 8 presents the main conclusions of the research and formulates the main lines for future research opportunities.

2. Related Work

Several articles in the literature have proposed linear programming (LP) or MILP models for solving different optimization problems related to grain production, logistics, and/or chain supply problems.

Early works in the area applied linear and mixed-integer programming models for problems related to harvesting methods and machinery selection. Al-Soboh et al. [10] applied MILP for mixed cropping systems, computing an optimal spacing for planting navy beans. Ait Si Larbi et al. [11] proposed a mathematical model for multi-stage optimization in agri-food supply chain. A real case study in Algeria was solved. The proposed model significantly outperformed a planning heuristic. A subsequent article by the same authors [12] applied the Agent Unified Modeling Language (AUML) protocol specification to build an effective system for production and transportation. The proposed model was effective to compute accurate values of the optimized functions.

Mendéz et al. [13] introduced a very important survey (until that date) contained a review of optimization methods for short-term scheduling of batch processes. The authors introduced general optimization constraints that in particular can be perfectly instantiated, for example, for the efficient and effective production of a soybean production plant, among other grains. Allowing the minimization of waiting times, optimizing the use of machines online, the hours of the operators, and other parameters that have a strong impact on the productivity of the plant. In addition, they provide a list of available academic and commercial software, and tackle the issue of rescheduling capabilities of the various optimization approaches, and include extensions that go beyond short-term batch scheduling. Bilgen and Ozkarahan [14] proposed a multi-period MILP model for minimizing costs of blending and shipping on the wheat supply chain. A real-life case study demonstrated the approach as effective. An hybrid deterministic/non-deterministic model was proposed by Granillo et al. [15] for minimizing the building cost of a distribution network farm. Results reported for the supply chain of barley in Mexico showed that the model computed accurate solutions.

The analysis and optimization of production lines of a processing plant has an superlative economic value as well as a strategic impact for the industrial market, allowing an increase in quality and quantity of production in less time, producing more with less cost. Based on this premise, Belletini et al. [16] used the Method of Problem Analysis and Solving (MAPS), an efficient set of statistical tools that provides a methodology and an improvement in process quality, with the goal of optimizing the production line in a wheat mill, transforming unproductive into productive hours. Using data collection at stops, at random times, there was an average operations/day value of 60.5% and a best result of 83%, taken as an objective to be accomplished. The authors found that the problem of excessive stops focused on four major causes, representing 87% of all cases. Through the combination of different mathematical strategies as well as knowledge of industrial practice, it was possible to efficiently attack and solve these four problematic cases of stops. As a global result, they found that increasing the number of productive hours in 22.5%, monthly income grew to superlative levels when compared to the corresponding cost of investing in more hours of feasible dedication of the workers.

Sanches et al. [17] applied a multi-period MIP model for the optimization of production scheduling of fruit beverages. A case study in Brazil was used for validation of the proposed approach. Results demonstrated the applicability of the proposed approach under realistic assumptions. Li et al. [18] studied MILP formulations for the Wheat Blending Problem and proposed an hybrid Evolutionary Algorithm (EA) including an exact subordinate LP operator to solve a linear-relaxation of the problem. The method was evaluated on synthetic instances, outperforming previous results. Aguirre et al. [19] introduced multiple and efficient MILP-based models for the planning and scheduling of multiproduct multistage continuous plants with sequence-dependent changeovers in a supply chain network under demand uncertainty and price elasticity of demand. The authors considered multiple key parameters and phases associated with the complete production chain of the products for the different optimization approaches studied. The global problem allows multiple plants, multiple manufactured products, multiple collection and distribution centers from which the final products are distributed to the final points of sale. A hybrid discrete/continuous model was proposed for the global problem and the Hierarchical Model Predictive Control (HMPC) method was applied to solve scenarios with uncertainty. The effectiveness of the optimization method was evaluated by comparing the obtained results on a large-scale instance with the solution delivered by a classic Cutting Plane algorithm.

Hosseini et al. [20] proposed a two-stage mixed-stochastic approach to deal with uncertainty in costs, demand, and supply in the wheat supply chain network. The first stage considered the selection of sites for locating silos and waste collection centers. The second stage optimized the wheat flow in the supply the network, considering uncertainty on the demand. A real case study in Iran was solved, where the proposed model was able to outperform a deterministic model. Chakraborty et al. [21] proposed a MILP formulation to optimize cost in a real-life food transportation problem in India. A heuristic method was applied for resolution, since authors claimed that commercial optimization software failed to compute exact solutions for large problem instances in a practical time limit. The viability of the proposed model was evaluated by solving a case study involving nine sites and eight relevant costs. For this reduced problem instance, the gap of the proposed heuristic over results computed using CPLEX were between 1.57% and 11.71%. No further comparisons with other resolution methods or real results was presented. León et al. [22] proposed a MILP formulation for minimizing the total cost of the bioethanol supply chain. Accurate results were computed to satisfy the demand in a case study considering corn and barley residues in México. The proposed MILP approach was robust and applicable to other similar problems. Osaki et al. [23] introduce important theoretical contributions and suitable models with the aim of understanding the double-crop production systems that make Brazil one of the world's leading and most competitive grain-producing countries. The authors propose a decision support model focused on optimal agricultural planning that pointed to multiproduct farms under risk conditions and applies this theoretical model of farm planning that uses operations research and optimization to comprehend the different productive resource allocations in farms engaged in grain production. The designed and developed model was used in the region of Sorriso, Brazil, resulting in better financial results with lower risks for grain producers in that region. As an example, the efficient frontier curves computed in the financial analysis showed that the representative farms in this region maximized their production factors. This produced a result of very high value for investors.

Griffini et al. [24] analyzed and estimated the whole-farm costs of conducting on-farm trials using a modification of the classic down-time model in an LP framework, compared to previously estimated potential benefits. From the comparison, the authors highlighted that after accounting for the whole farm costs there are still benefits to on-farm trials. Moreover, when on-farm trials cause planting and harvesting field operations to be conducted outside the optimal time, crop yields may be adversely affected. On the other hand, they affirmed that farm decision makers should take into account certain research questions that do not necessitate adversely impacting these windows until experience has been gained. In all cases, the comparative models used were based on LP to determine optimal solutions to maximize contribution margins on each considered scenario.

Gameiro et al. [25] introduced an LP mathematical model for assistance in the planning and management of agricultural production, as well as to assist in estimating potential gains from the use of integrated systems. They stand out as a strategic point, introducing diversification in production as a necessary condition for economic viability. The numerical results obtained support this point. A total cost reduction potential of about 30% was found when comparing the scenario with lower levels of diversification was contrasted with one of higher levels. In addition to financial gains, the authors note that the integrated systems bring benefits to the environment, more precisely with respect to to the reuse of resources. As example, the model can reuse nitrogen, phosphorus, and potassium present in animal waste. Another important conclusion is that the diversification model has a clear impact on the reduction of transaction costs.

Motivated by the little recent research on optimizing seed supply chains (particularly soybeans) in Canada, Shekarian [26] introduced a novel MILP formulation for a soybean supply chain network. The objective function considered was to maximize the profit. The model takes into account constraints involving multiple products, growers, potential farm company facilities, potential locations of distributers, and customers. In addition, the author extends the model by adding uncertainty to certain parameters. In order to measure the effectiveness and efficiency of the optimization model, it was applied on a soybean supply chain network in Ontario where there are several soybean companies and growers in this region. The output of the optimization model is displayed by means of maps, highlighting different characteristics of importance for soybean plantations (e.g., showing the optimal soybean supply chain network for certain products).

Susilawati et al. [27] analyzed land use in Sumbawa District (Indonesia) to produce corn and its effect in the cutting trees as well as on environment. Production and government objectives for corn production increase annually and have an indirectly impact on clearing new land as corn planting land. Using the Simplex linear optimization method, authors focused on a cutting tree optimization model to enhance the productivity of corn and optimize the benefits of corn farmers and investors. Using the proposed method, maximum production of corn in 2019 increased to 701,984 tons of corn, the paddy field area to 44,198.5 hectares and 55,292.5 hectares of land not paddy fields.

Fang et al. [28] considered that the grain processing industry often excessively pursues the objective of fine processing, resulting in increasing raw material cost and low machining efficiency, etc. The authors provided an interesting methodology, applicable to food and grain production based on a food/grain optimized processing anda data analysis system which can outperform various previously designed data analysis. The designed methodology has the following functions: grain processing database is able to store and call consumer market surveys and data of grain processing enterprises, visualizing the national diet and grain processing process. Data analysis incorporates within its functionalities tasks, such as statistical analysis of data sets extracted according to required indicators, diagnostic optimization based on Data Envelopment analysis, prediction analysis based on curve fitting, classification analysis based on clustering algorithm, etc. The implementation in industry of the designed system was shown to outperform other previous approaches.

Taşkıner et al. [29] presented a complete review of optimization models focused on harvest and production planning for food crops. Optimization models have been used extensively to provide decision makers with insights on issues related to harvest and production planning in agri-food supply chains. Taşkıner et al. studied in depth several neglected topics and provided promising research directions to stimulate research interest on agri-food supply chains, specifically the planning of harvest and production and other tasks intrinsically related, such as optimization of the production chain, minimization of downtime in the plant, maximizing the efficiency of operations within the processing plant, suitable use of available machinery, etc. Several other articles related to food supply chain and logistics have applied LP or MILP models for optimization. Soysal et al. [30] proposed a MILP model for the beef industry, extended to minimize cost and greenhouse gas emissions in transportation. Mishra et al. [31] applied a nonlinear programming model to minimize refrigeration cost in the storage process of leafy greens.

This article contributes towards this line of research, by modeling and studying, both theoretically and empirically, a relevant grain production problem. In turn, the case study presented in this article is one of the first reported studies applying ad hoc systematic optimization models for a real grain production company in Uruguay.

3. The Grain Facility Optimization Problem

The problem concerns a real-life grain processing facility, which must treat seeds using different types of resources, including workers, machinery, storage silos with a given capacity, etc. The main goal for the company that operates the facility is to meet pre-planned delivery demands for each harvest. In the grain facility, several varieties of seeds are processed and delivered. A specific feature of the problem is that some products, i.e., *inoculated products*, have a limited shelf life: they expire a certain number of days after being processed. This issue imposes a restriction concerning the production dates and the withdrawal dates. Once the products are processed, they are stored in silos, waiting for clients to withdraw their orders. Orders are based on a planned list of withdrawals that were previously communicated to the company in the planning phase.

The sales department establishes a calendar of product withdrawals and the planned production must respect several constraints. One the one hand, there must be enough stock of each product available at the time of each withdrawal. On the other hand, the processes required by the seeds are performed by employing a limited number of production lines. The total throughput (tons/h) is also limited, which imposes a limit on the restocking rate. In addition, the total storage capacity is limited, which makes it mandatory to alternate batch production and withdrawals. Some products have a limited shelf life, which imposes a limit on the time window between their production and withdrawal.

The production lines must be supervised by operators while they are operational. Machines generally produce at full capacity and operator shifts are established and assigned to each line. The number of shifts is set for each week and it does not change within the same week. Every time a line finishes the production of a certain product, it must be stopped and cleaned before continuing with the processing of a different product. If the line stops after a certain demand has been satisfied and then returns to producing the same product, cleaning is not necessary. The production process covers several weeks per year. A set of demanded products must be produced and withdrawn each week, according to the specific constraints determined by the throughput, stocking/restocking availability, storage capacity, production lines, shelf life and worker shifts.

Taking into account the presented realistic production situation, the model is defined to minimize production costs. Among other relevant factors, the cost function must consider the number of production hours, the number of operator shifts, and the number of cleanings performed on each machine. Furthermore, in order to be applicable in a real commercial environment, the problem model must guarantee that the production is sufficient for the withdrawals to be fulfilled, that the expiration periods of those products that expire are respected, and that the production that is not delivered immediately does not exceed the available storage capacities. Since inoculated products must be delivered in a limited period of time, it is important to keep track of their processing dates. The proposed problem model defines specific variables to distinguish batches of such products belonging to different dates, as described in the next subsection.

The mathematical formulation of the studied grain facility optimization problem considers the following elements:

- Sets:
  1. Set V_I denotes products that expire.
  2. Set V_T denotes products that do not expire.
  3. Set V denotes all products, i.e., $V = V_I \cup V_T$.
  4. Set L denotes the available production lines.
- Parameters:
  1. Parameter S_f denotes the number of weeks considered for production scheduling (the *scheduling horizon*).
  2. Parameter M denotes the maximum number of withdrawals for each product.
  3. Parameter C denotes the total storage capacity of the company (measured in tons).
  4. Parameter τ denotes the shelf life of products that expire (measured in weeks).
  5. Parameter N denotes the maximum number of shifts per week.
  6. Parameter H_f denotes the number of productive hours in a shift.
  7. Parameter π_l denotes the productivity of the production line $l \in L$ (measured in tons/h).
  8. Parameter L_l denotes the cleaning time of line $l \in L$ (measured in hours).
  9. Parameter B_v denotes the number of batches of product v; in case $v \in V_I$, if $v \in V_T$ then B_v=1.
  10. Parameter S_v^r denotes the week in which the r-th withdrawal of product $v \in V$ is located.
  11. Parameter δ_v^r denotes the demand in the r-th withdrawal of product $v \in V$, for $r = 1 \cdots M$.
  12. Parameter d_v^s denotes the demand to withdraw in week s for product $v \in V$.
  13. Parameter D_s denotes the working days for each week s. Two cases are considered: a five day working week (from Monday to Friday) and a five and a half working week (from Monday to Saturday noon).

14. Parameters P, Q, and R are weights for each of the three components considered in the cost function to optimize; they are defined as unit conversion coefficients that allow expressing the overall cost in a single unit.

- Decision variables:
  1. Variable $x_{v,l}^{s,r} \in \mathbb{R}^+$ denotes the quantity of product $v \in V$ produced in the line $l \in L$ in the week s for the r-th withdrawal (integer variable, measured in tons).
  2. Variable $t_l^s \in \mathbb{Z}^+$, the number of shifts to consider in line $l \in L$ in week s (integer variable).
  3. Variable $y_{v,l}^s$ denotes the production of a given product on a line (binary variable), defined by

$$y_{v,l}^s = \begin{cases} 1 & \text{if and only if line } l \text{ is producing any quantity of product } v \in V \text{ in week } s \\ 0 & \text{otherwise.} \end{cases},$$

  4. Variable $p_{v,l}^s$ denotes if a product is the first to be produced in a line (binary variable), defined by

$$p_{v,l}^s = \begin{cases} 1 & \text{if and only if product } v \in V \text{ is the first product to be produced in week } s \text{ in } l \in L, \\ 0 & \text{otherwise.} \end{cases}$$

  5. Variable $u_{v,l}^s$ denotes if a product is the last to be produced in a line (binary variable), defined by

$$u_{v,l}^s = \begin{cases} 1 & \text{if and only if product } v \in V \text{ is the last product to be produced in week } s \text{ in } l \in L, \\ 0 & \text{otherwise.} \end{cases}$$

  6. Variable $w_{v,l}^s$ denotes if a product ends a week and starts the next week in the same line (binary variable), defined by

$$w_{v,l}^s = \begin{cases} 1 & \text{if product } v \in V \text{ ends the week } s \text{ and starts the week } s+1 \text{ in line } l \in L, \\ 0 & \text{otherwise.} \end{cases}$$

  7. Variable c_l^s denotes if a cleaning is avoided in a line (binary variable), defined by

$$c_l^s = \begin{cases} 1 & \text{if in the week } s\text{, one cleaning of the line } l \in L \text{ is avoided,} \\ 0 & \text{otherwise.} \end{cases}$$

Considering the previously defined sets, parameters, and variables, the mathematical formulation of the problem as a Mixed-Integer Linear Programming (MILP) problem is presented in Equations (1)–(16).

$$\min \quad P\sum_{l\in L}\sum_{s=1}^{S} t_l^s + Q\sum_{l\in L}\frac{1}{\pi_l}\sum_{v\in V}\sum_{r=1}^{B_v} x_{v,l}^{s,r} + R\sum_{l\in L} L_l(\sum_{s=1}^{S}\sum_{v\in V} y_{v,l}^s - \sum_{s=1}^{S-1} c_l^s), \tag{1}$$

$$\text{subject to} \sum_{z=S_v^r-\tau}^{S_v^r}\sum_{l\in L} x_{v,l}^{z,r} - \delta_v^r \geq 0, \forall v \in V_I,\ \ r = 1\cdots B_v, \tag{2}$$

$$\sum_{z=1}^{S_v^r}\sum_{l\in L} x_{v,l}^{z,1} - \sum_{z=1}^{S_v^r} d_v^z \geq 0, \forall v \in V_T,\ \ r = 1\cdots M, \tag{3}$$

$$\sum_{z=1}^{s}\sum_{v\in V}\sum_{l\in L}\sum_{r=1}^{B_v} x_{v,l}^{z,r} - \sum_{z=1}^{s}\sum_{v\in V} d_v^z \leq C\ \ \forall s = 1\cdots S, \tag{4}$$

$$t_l^s \leq N\ \ \forall l \in L,\ \ \forall s = 1\cdots S \tag{5}$$

$$d_v^s.y_{v,l}^s - \sum_{r=1}^{B_v} x_{v,l}^{s,r} \geq 0\ \ \forall l \in L,\ \ \forall vs. \in V,\ \ \forall s = 1\cdots S, \tag{6}$$

$$\sum_{r=1}^{B_v} x_{v,l}^{s,r} - y_{v,l}^s \geq 0\ \ \forall l \in L,\ \ \forall vs. \in V,\ \ \forall s = 1\cdots S, \tag{7}$$

$$y_{v,l}^s - p_{v,l}^s \geq 0\ \ \forall l \in L,\ \ \forall vs. \in V,\ \ \forall s = 1\cdots S, \tag{8}$$

$$\sum_{v\in V} p_{v,l}^s \leq 1\ \ \forall l \in L,\ \ \forall s = 1\cdots S, \tag{9}$$

$$y_{v,l}^s - u_{v,l}^s \geq 0\ \ \forall l \in L,\ \ \forall vs. \in V,\ \ \forall s = 1\cdots S, \tag{10}$$

$$\sum_{v\in V} u_{v,l}^s \leq 1\ \ \forall l \in L,\ \ \forall s = 1\cdots S, \tag{11}$$

$$2w_{v,l}^s - u_{v,l}^s - p_{v,l}^{s+1} \leq 0\ \ \forall l \in L,\ \ \forall vs. \in V,\ \ \forall s = 1\cdots S-1, \tag{12}$$

$$c_l^s - \sum_{v\in V} w_{v,l}^s \leq 0\ \ \forall l \in L,\ \ \forall s = 1\cdots S-1, \tag{13}$$

$$|V|p_{v,l}^s + |V|u_{v,l}^s + \sum_{v_1\in V, v_1\neq v} y_{v_1,l}^s \leq 2|V|\ \ \forall l \in L,\ \ \forall vs. \in V,\ \ \forall s = 1\cdots S \tag{14}$$

$$\pi_l(D_s H_f t_l^s - L_l\sum_{v\in V} y_{v,l}^s - c_l^s) - \sum_{v\in V}\sum_{r=1}^{B_v} x_{v,l}^{s,r} \geq 0\ \ \forall l \in L,\ \ \forall s = 1\cdots S-1 \tag{15}$$

$$\pi_l(D_S H_f t_l^S - L_l\sum_{v\in V} y_{v,l}^S) - \sum_{v\in V}\sum_{r=1}^{B_v} x_{v,l}^{S,r} \geq 0\ \ \forall l \in L, \tag{16}$$

The objective function of the problem is expressed in Equation (1). It proposes minimizing the generalized cost of production for the considered grain facility. The cost is composed of three different terms, to properly model the costs associated with labor, production, and cleaning. The generalized cost is defined as a number that summarizes these three cost sources for each production cycle. The first term of the objective function represents the labor cost, which is proportional to the total number of shifts (of operators) to be employed for the scheduled production, for all the considered machines that operate each production line. The second term of the objective function represents the costs associated with the time during which the machines produce, other than labor. Some relevant costs included in the second term are energy and amortization costs, as well as other variable costs. The third term of the objective function represents the costs incurred every time a line is cleaned, other than labor. Some relevant costs included in the third term are supplies costs and energy costs. Energy costs for cleaning have a different hourly consumption pattern than the energy costs for production (included in the second term). A linear combination approach is applied to add the three components of the cost function, considering the weights defined by parameters P, Q, and R. The parameter P accounts for the labor cost of one shift during one week serving one line. The parameter Q accounts for the variable costs (other than labor) of using one line to produce during one hour. The parameter R accounts for the variable costs per hour (other than labor) of every cleaning procedure on one line. The linear combination approach is appropriate since all considered costs are variable and vary in a directly proportional manner with respect to the decision variables, e.g., labor cost is proportional to the number of scheduled shifts; energy, supplies and amortization costs are proportional (considering different rates) to the machines operational hours and to the cleaning hours, respectively.

Equations (2)–(16) formulate the constraints of the optimization problem. The constraint in Equation (2) states that production must be greater than demand, guaranteeing that there is sufficient production of inoculated products to be delivered. Since inoculated products have an expiration date, the model must ensure that the number of weeks elapsed since inoculated products are produced until they are delivered does not exceed the shelf life for expiring products, given by parameter τ. The constraint in Equation (3) guarantees that enough treated products are produced to satisfy the demand. In the same way as for constraint in Equation (2), the production must be greater than the demand, but in this case the products can be produced at any moment of the production cycle, since shelf life is not a constraint for treated products. The constraint in Equation (4) controls the storage capacity. The first term on the left side of the inequality corresponds to the overall production of week s and the second term corresponds to the deliveries scheduled for that week. The difference between production and delivery must be positive and cannot exceed the total storage capacity, given by parameter C, to guarantee that the surplus of production can be stored. The constraint in Equation (5) specifies a limit for the number of shifts to be performed for each production line in each week.

The group of constraints in Equations (6)–(11) controls the activation of binary variables $y^s_{v,l}$ $p^s_{v,l}$, $u^s_{v,l}$ y $w^s_{v,l}$. These variables are used as auxiliary variables for the calculation of the number of cleanings avoided in the production lines. The constraints in Equations (6) and (7) operate together to activate the variable $y^s_{v,l}$ if and only if some quantity of a certain product v is produced on the line l in the week s. The constraint in Equation (8) states that the necessary condition for the variable $p^s_{v,l}$ to be activated (meaning that v is the first product to be produced on line l in week v) is that some quantity of product v has been produced in the week s. The constraint in Equation (9) assures that the first product produced in week s on line l is unique, thus guaranteeing a consistent definition for variable $p^s_{v,l}$. Inequalities in Equations (10) and (11) are analogous to the constraints in Equations (8) and (9), but in this case are applied to the last product produced in the week. To assure the existence of a last product v to be produced on the line l in week s, there must be a production of said product v. Furthermore, Equation (9) guarantees that the existing last product is unique. The constraint in Equation (12) forces the variable $w^s_{v,l}$ to take the value 1 only if the production of product v is split on two weeks, i.e., product v is the last product of week s and the first of the week $s+1$ for the production line l. The inequality in Equation (13) controls the cleaning variable c^s_l, which indicates whether a cleaning is saved on production line l for week s. The sum of variables c^s_l appears in the objective function with a negative sign, therefore, the optimization model presses for this quantity to have the maximum possible value. The set of constraints in Equation (14) guarantees that if a certain product v is the first and the last produced on week s for a production line l, no other product $v_1 \neq v$ is produced on line l in the same week.

The set of constraints in Equation (15) indicates that the maximum production capacity of line l must be greater than the quantity produced in that line for every week. Two terms are subtracted from the productivity of the production line l: the stop times needed and the cleanings saved, according to a correct sequencing of the products to be produced. Finally, constraint in Equation (16) is analogous to the previous one, but in this case formulated to cover the particular case of the last week, where there are no savings in cleaning (no more productions are to be scheduled after reaching the production horizon).

4. Problem Complexity and Lower Bounds

This section presents a study of the computational complexity of the proposed grain facility optimization problem and computes useful lower bounds to speed up the problem resolution.

4.1. NP-Completeness

This subsection presents a demonstration of the NP-completeness of the decision problem of determining if there is a solution whose cost is smaller than a given value C. Let this decision problem be called SOJA(C). The first important remark is that SOJA(C) is in NP, since it takes a polynomial time to verify the feasibility of a solution. Therefore, it is enough to polynomially reduce SOJA(C) to any NP-Complete problem. Let the chosen problem be the PARTITION problem, i.e., the one of deciding, given positive integers $n_1, \ldots, n_k$, whether there is a subset of them that sum $h := (n_1 + \cdots + n_k)/2$, i.e, if there is a subset $S \subset \{1, \cdots, k\}$ of the indices such that Equation (17) holds.

$$\sum_{i \in S} n_i = \sum_{i \notin S} n_i = h. \tag{17}$$

We consider two production lines of equal productivity, i.e., $\pi_1 = \pi_2 = 1$ tons per week; k product types; equal week withdrawals for all the products, i.e., $r = 1$ and $S_i^r = T + h + 1$ weeks; demands $d_i^1 = d_i = n_i$ with $i = 1, \cdots, k$; equal cleaning times $L_1 = L_2 = 1/k$ and cleaning cost $R > 0$. Next it is proven that if PARTITION has a solution, then SOJA(C) has solution if and only if $C \geq 2h + R(k-2)$; and that conversely, if there exists a solution to SOJA$(2h + R(k-2))$, then PARTITION has a solution.

The following statement holds: if line 1 produces $a_0, \cdots, a_{k_1}$ tons of products of type $t_0, \cdots, t_{k_1}$ with $t_i \neq t_{i+1}$ and line 2 produces $b_0, \cdots, b_{k_2}$ tons of products of type $t'_0, \cdots, t'_{k_2}$ with $t'_i \neq t'_{i+1}$, then the total number of cleaning is $k_1 + k_2$ and the total time T' spend on production is given by Equation (18).

$$T' = \max\left\{\frac{k_1}{k} + A, \frac{k_2}{k} + B\right\} \quad \text{with} \quad A = \sum_{i=0}^{k_1} a_i, \quad \text{and} \quad B = \sum_{i=0}^{k_2} b_i. \tag{18}$$

The overall cost of production is $\hat{C} = 2h + (k_1 + k_2)R$, which is at least $2h + (k-2)R$.

For proving that if PARTITION has a solution, then SOJA(C) has a solution of cost $2h + R(k-2)$, consider the solution S to PARTITION, with parameters $k_1 = |S| - 1$, $k_2 = k - k_1 - 2$, $a_i = n_i$ for $i \in A$ and $b_i = n_i$ for $i \in S^c$. With this assignment, the total time is given by Equation (19), so the solution is feasible.

$$T' = \max\left\{\frac{k_1}{k} + h, \frac{k_2}{k} + h\right\} = h + \max\left\{\frac{k_1}{k}, \frac{k_2}{k}\right\} \leq h + \frac{k-2}{k} < h + 1. \tag{19}$$

The overall cost $\hat{C} = 2h + (k_1 + k_2)R = 2h + (k-2)R$ is optimal, so SOJA(C) has solution if and only if $\hat{C} \leq C$.

Conversely, if SOJA$(2h + (k-2)R)$ has a solution, then PARTITION has as well. Indeed, first observe that $k_1 + k_2 = k - 2$ since $k_1 + k_2 \geq k - 1$ and the cost is at least $2h + (k-1)R$. Therefore, since the number of cleanings $(k-2)$ is the minimum possible, all product types should be different, and for each i there exists one and only one j_i such that $a_i = n_{j_i}$, and the same for b_i. Finally, since the solution is feasible, then $A + k_1/k < h + 1$ and $B + k_2/k < h + 1$, thus $A < h + (1 - k_1/k) < h + 1$ and $B < h + (1 - k_2/k) < h + 1$, i.e., $A \leq h$ and $B \leq h$. However, $A + B = 2h$, then $A = B = h$, and PARTITION has solution $S = \{j_i : i = 0, \cdots, k_1\}$.

4.2. Lower Bound

Computing bounds on the exact optimal value for an optimization problem is a typical challenge to complement the design of both exact and approximation algorithms [32]. Such bounds are useful values for determining the maximum error of an approximation method or to speed up the algorithm when using exact methods. Several techniques are applied for bound calculation (e.g., witnesses/theoretical proofs, relaxation, and coarsening/restricting the original problem). Bound calculation is a traditional tool in operations research and it has been applied in many articles related to production, scheduling and planning, and other engineering problems [33–37].

A lower bound to the objective function is computed by bounding each of the three terms of that function, i.e., the total number of shift, the time of production and the total number of cleanings. First, consider the total number of cleanings. A lower bound for the objective function arises from Section 4.1 (Equation (20)).

$$\sum(1 - c_l^s) \geq |V_I| + |V_T| - |L|. \tag{20}$$

The production time is greater or equal to the total demand divided by the productivity of the quickest line (Equation (21), where $\hat{\pi} = \min_l \pi_l$).

$$\sum_{l \in L} \frac{1}{\pi_l} \sum_{v \in V} \sum_{r=1}^{B_v} x_{v,l}^{s,r} \geq \frac{1}{\hat{\pi}} \sum_{r,v} \delta_v^r. \tag{21}$$

Thus, a lower bound to the number of shift is the integer part of the total working hour divided by the number of hours per week (Equation (22), where $\hat{D} = \max_l D_l$.).

$$\sum_{s,t}^{S} t_l^s \geq \left\lceil \frac{1}{\hat{D}_l} \frac{1}{\hat{\pi}} \sum_{r,v} \delta_v^r \right\rceil. \tag{22}$$

Taking into account all previously described lower bounds, the lower bound for the objective function described in Equation (23) is obtained, where $\hat{L} = \min_l L_l$.

$$P \left\lceil \frac{1}{\hat{D}} \frac{1}{\hat{\pi}} \sum_{r,v} \delta_v^r \right\rceil + Q \frac{1}{\hat{\pi}} \sum_{r,v} \delta_v^r + R\hat{L}(|V_I| + |V_T| - |L|). \tag{23}$$

The presented lower bound is valuable from a theoretical point of view. In turn, the computed bound was useful in the real case scenario of the grain facility optimization problem (described in Section 5), improving over the bounds computed by the resolution approach using CPLEX. However, the bound was not as effective in the defined variations of the case study. For this reason, the theoretical bound was not considered/included in the resolution approach described in Section 7.

5. A Real-World Case Study: Optimization in a Grain Processing Facility in Uruguay

A specific real-world case study is considered to evaluate the proposed MILP model. The case study corresponds to the production planning of a company that processes and sells soybeans. The main details of the considered scenario are presented in this section.

5.1. Description of the Case Study

The case study corresponds to an Uruguayan agricultural company, specifically Barraca Erro. This is a very relevant real-world case study in Uruguay. Among other products and services, Barraca Erro produces grains and seeds (soybeans, wheat, barley, etc.). The company is one of the main producers and exporters in Uruguay. In 2021, Barraca Erro exported goods for a total amount of 248 million USD, which represented 15% of the total Uruguayan exports within its sector of economic activity and 2.5% of the total exports of the Uruguayan economy.

The considered case study accounts for producers who send orders for 17 different kinds of soybeans for each harvest (i.e., $|V| = 17$). The soybeans must be processed and delivered according to a schedule that extends over a period of 19 weeks (i.e., $S_f = 19$), which defines the annual soybean harvest. The main problem that the company faces when the orders are received is the accurate planning of production and the proper preparation the deliveries in a timely manner.

The company has two production lines (line 1 and line 2, i.e., $|L| = 2$) with a processing capacity of 6.165 and 6.65 tons per hour, respectively, (i.e., $\pi_1 = 6.165$ and $\pi_2 = 6.650$). Both lines are capable of producing any of the 17 products considered. Every time the machines finish processing a product, they must be stopped and cleaned in the case that a different product is to be processed next. The duration of the stoppage and cleaning times is 6 h for production line 1 and 8 h for production line 2 (i.e., $L_1 = 6$ and $L_2 = 8$). The company stops producing during the period of time that the machines are down for cleaning, and therefore reducing this downtime is of the utmost importance to improve production efficiency and profit. The adequate quantity to be produced, as well as the correct sequencing of the products that must enter each of the two production lines, help to reduce stoppages and generates important savings in time and associated costs. Two important decisions make it possible to reduce downtime and generate significant savings in production times and associated costs: (i) determining the appropriate quantity of each product to be produced and (ii) determining the correct sequencing of the products that must enter each of the two production lines.

A proper production sequencing strategy often involves producing more soybeans than withdrawals in particular weeks. In this case, surplus soybeans must be stored. For storage purposes, the company owns silos with a total storage capacity of 2700 tons of grain. Using machinery efficiently and taking advantage of the available storage silos, the company must produce and store enough grain to cover the withdrawals required by its customers.

In the defined case study, the two types of products defined in Section 3 are considered. Processed products are categorized into two classes: inoculated and treated. This categorization implies an additional control that the company must resolve. Products within the inoculated class (7 out of the 17 products considered, i.e., $|V_I| = 7$) have an expiration time. Therefore, these products cannot be manufactured and stored too far in advance, due to the proximity of their expiration date. In the case study considered, the expiration time is two weeks, which imposes an important limitation for the planning of the inoculated products. This restriction does not apply to treated products, since there is no expiration time for products within the treated class (10 out of the 17 products considered, i.e., $|V_T| = 10$).

The company has personnel (operators) who carry out the two main tasks for a correct development of the production process: the supervision of the machinery during production and the cleaning tasks of the production lines. The operators work in shifts, a total of 7.5 effective hours in each shift. The company has defined a set of management policies for its workers, which imply certain rules. Given a week w and a specific line l, the operators must work the same number of shifts (1, 2 or 3) from Monday to Friday and half of those shifts on Saturday of that week. The number of shifts can be set to different values for each combination of week and line (w, l).

The parameters that define the case study are summarized in Table 1.

Table 1. Parameters for the considered case study.

Parameter	*Variable*	*Value*
number of production lines	L	2
processing capacity line 1	π_1	6.175 tons/h
processing capacity line 2	π_2	6.650 tons/h
cleaning time line 1	L_1	6 h
cleaning time line 2	L_2	8 h
total storage capacity	C	2700 tons
number of inoculated products	$\|V_I\|$	7
shell life of inoculated products	τ	2 weeks
number of treated products	$\|V_T\|$	10
operators shift (Monday to Friday)	H_f	7.5 h
operators shift (Saturday)		3.75 h
number of shifts		1 to 3

5.2. Conception of the Model

This subsection describes the process followed to conceive the model introduced in this article.

The primary need of the company was to improve the efficiency of the batch production scheduling process, in terms of minimizing the related costs while meeting all constraints. The company had been using a non-formalized production planning procedure, based on common sense and intuitive criteria, referred to hereafter as ad hoc.

With the purpose of better understanding the problem and providing the company with a primary quick tool to improve the planning, a simple model of the problem was created using a Microsoft Excel spreadsheet. Several cells played a role equivalent to that of decision variables x and t of the mathematical formulation in Section 3. The spreadsheet was organized with rows representing the weeks and columns representing (tons of) products to produce on each line. The spreadsheet allowed for manually changing values on each cell, while controlling the corresponding use of lines and storage capacities, and showing eventual constraint violations using conditional formats. A heuristic procedure (described in Section 6.3) was conceived on top of the spreadsheet and used to create the real 2020 production plan followed by the company. The heuristic method conveyed a reduction of 20% on the number of cleaning cycles needed when compared to the ad hoc planning that the company had already devised. The spreadsheet was of paramount importance to help the company managers and the consultant team to agree on all details of the problem and its further modeling as a MILP optimization problem. Heuristic production planning is used as a reference baseline to compare the results computed by the proposed MILP model, as described in the next section.

6. Experimental Evaluation

This section presents the experimental evaluation of the proposed model for the considered case study.

6.1. Evaluation Methodology

The proposed formulation was implemented using the AMPL modeling program language and solved with IBM CPLEX Solver ver. 20.1.0.0. The model was executed on an Intel Core i9-9900K CPU @ 3.60GHz, 16 processors with 64 Gb. of RAM. The operating system was CentOS Linux release 7.7.1908.

All executions were run with a preset amount of time, chosen as one of the stopping criteria in the experiments. The other criterion for stopping the execution is the gap between the best integer solution found by the solver and the best value of the objective function computed so far (`CPX_PARAM_EPGAP` parameter), set to 1×10^{-5}.

The performance of the proposed method is evaluated using two metrics: an execution time threshold and the gap of the solution found with respect to a certain lower bound computed by the solver. The gap is related to the execution time; an inversely proportional relationship is expected, as increasing the total execution time of the model allows computing more accurate results. For the considered case study, several executions of the proposed model are performed considering as stopping criterion 1, 5, 30, 60, 120, 180, and 240 min, respectively.

6.2. Description of the Real-World Problem Instance Solved

The experimental evaluation of the proposed MILP model for grain production optimization was performed on the specific instance of the case study described in Section 5.

According to the values presented in Table 1, the parameters values that define the considered problem instance are as follows:

- $V_I = \{1, \cdots, 7\}$; $V_T = \{8, \cdots, 17\}$; $V = V_I \cup V_T$; $\tau = 2$.
  The seven inoculated products products of the real case are coded with an index $1 \dots 7$. The treated products are coded as $8 \dots 17$. The τ parameter specifies the shelf life of the inoculated products and it is set to 2 weeks.
- $L = \{1, 2\}$; $\pi_l = \{6.165, 6.5\}$; $L_l = \{6, 8\}$, with $l \in L$.
  These parameters correspond to the set of machines available, their production capacity, and their cleaning times in hours, respectively.
- $S_f = 19$; $C = 2700$; $N = 3$; $H_f = 7.5$; $D_s = 5.5 \forall s$.
  The real case harvest spans 19 weeks, therefore, the parameter S_f is set to 19. The storage capacity of processed grains (parameter C) is set to 2700 tons, the maximum number of operator shifts (parameter N) is set to 3, and the number of productive hours in a shift (parameter H_f) is set to 7.5. The number of working days for each week (parameter D_s) is set to 5 days (i.e., from Monday to Friday and a half day at Saturday).
- The maximum number of withdrawals for each product (parameter M) is set to 10.
- The number of batches corresponding to the inoculated products is B_v={6, 7, 8, 4, 5, 2, 2} with $v \in V_I$ and the treated products belonging to a single batch is $B_v = 1 \ \forall vs. \in V_T$.
- The parameters P, Q, R of the objective function are, respectively, set to be 1000, 10 and 15, so that their linear combination properly accounts for the real incurred costs expressed in USD.

The demands of each product in every week are set based on the clients orders and estimates of the company's sales department. They are summarized in Table 2. The algorithm is fed with this table, where every value corresponds to an amount of a certain product to be handed over to the clients in a certain week. The inoculated products are labeled 1 to 7. The remaining are treated products, i.e., no shelf life limit applies to them. Table 2 allows also to generate the final pair of input parameters: (S_v^r) that represent the week in which the r-th withdrawal of product v is located, and (δ_v^r) that represents the demand for product v in its r-th withdrawal.

Several variations of the input parameters are combined building a set of 16 test cases.

Table 2. Description of the real-world problem instance solved: demand (d_v^s, in tons) for 17 products (v) in each week (s).

	Products																
Week	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**
8										20	600	60				200	
9				80	140	100				60	600	120				200	
10			40	120	400	100			80	120	600	140	60	24		200	
11		160	200	120	600				80	120	400	40	140	24	60		40
12		240	400	120	360		120	200	120	40			200	60	80		120
13	36	240	440		300		120	200	40				140	60	60		80
14	60	160	440					400					20	48			40
15	80	160	280					400						24			
16	40	80	120											40			
17	100	40	80											80			
18	40																

6.3. Heuristic Algorithm Currently Used by the Firm

In this subsection, the heuristic algorithm carried on by the planning engineers to craft the production plan is presented. The resulting plan for 2020 is then compared with the output yielded by the optimization algorithm here proposed.

The engineers work on an Excel workbook where they input the number of shifts per week and production line and the amount of each product to be produced in every combination of week, line, and product. Several cells are computed that show the capacity usage, overflows and eventual stock breaks, so that they can play until an acceptable plan is devised. The ad hoc steps so far followed by the engineers are as follows:

1. All cells that account for the number of shifts are set equal to 3.
2. All cells that account for the amount of tons of every product and line are zeroed, and this is done for every week.
3. Next, the demand of every combination of product and week p and w are copied into the cells that account for the amounts to produce. They are not split between lines, they are put in the line with the highest productivity.
4. As a result of the above, the computed cells of the worksheet will show that the plan exceeds both production capacity and space in several weeks and lines. This is solved by the next iterations.
5. Go to any week with production capacity overflow. Choose any product that is programmed that week and take it to another line, completely or partially trying to not lead to overflow the latter. Perform this step with other weeks until no further such movements solve any remaining overflow problem.
6. Look at any week w and a "treated" product p with exceeded capacity (regarding space or productivity). Take the amount produced to previous weeks, in order to lower down the excess. Perform this step for other combinations of p and w trying to eliminate as many overflows as possible.
7. Perform the previous step, now considering the "inoculated" instead of treated products. Be careful not to move the production more than the allowed shelf life of the product, since this step deals with the inoculated products.
8. Take every cell that accounts for shift numbers to its minimum possible integer value (3, 2, 1 or 0), taking care not to introduce new production capacity overflows.

Once the previous steps are performed, the engineers go through the worksheet, looking for further swaps or splittings that might help to improve the production plan while respecting all problem constraints. The planning engineer spent 20 h performing the ad hoc procedure here described (4 to create the worksheet and the remaining to iterate through all steps). The solution finally found had 319 shifts and implied 34 cleaning procedures.

6.4. Numerical Results

The presented instance of the real case study was solved using the CPLEX implementation of the proposed MILP solver, considering the parameter values described in Section 6.1.

6.4.1. Analysis of the Computed Solution

The computed solution is described in Tables 3–5. Table 3 reports the production schedule (in tons), i.e., the quantity of products to be processed each week, in each line, and for each product, in order to meet the formulated demand, subject to the defined constraints for production capacity of the machinery and the storage volume of the facilities. In addition, additional information is presented describing the hours each production requires, if a cleaning must be performed after the product has been processed, if a clean was avoided, and the stored stock of the product at the time of finalizing the corresponding production, after subtracting the withdrawals for the week (if any).

Table 3. Production schedule (in tons) by week for the considered case study.

Week	*Line*	*Product*	*Production (Tons)*	*Hours*	*Cleans*	*Cleans Saved*	*Stockpiled*
3	2	9	320	48.12	1	0	320
3	2	11	175.43	26.38	0	1	175.43
4	2	11	548.62	82.5	0	1	724.05
5	2	11	274.31	41.25	0	1	998.36
6	2	11	822.94	123.75	0	1	1821.3
7	2	11	274.31	41.25	0	1	2095.61
8	1	12	254.72	41.25	0	1	254.72
8	2	10	360	54.14	1	0	360
8	2	11	104.39	15.7	1	0	2200
8	2	16	252.15	37.92	0	1	252.15
9	1	4	155.59	25.2	0	1	155.59
9	1	6	200	32.39	1	0	200
9	1	12	105.28	17.05	1	0	300
9	1	14	168	27.21	1	0	168
9	2	5	421.89	63.44	0	1	421.89
9	2	16	347.85	52.31	1	0	400
10	1	3	295.7	47.89	1	0	295.7
10	1	4	284.41	46.06	1	0	360
10	1	13	109.94	17.8	0	1	109.94
10	2	5	822.94	123.75	0	1	1104.83
11	1	13	450.06	72.88	1	0	500
11	1	15	200	32.39	1	0	200
11	1	17	40	6.48	0	1	40
11	2	2	214.56	32.27	0	1	214.56
11	2	5	555.17	83.48	1	0	1260
12	1	7	240	38.87	1	0	240
12	1	8	200	32.39	0	1	200
12	1	17	240	38.87	1	0	240
12	2	2	425.44	63.98	1	0	480
12	2	3	344.3	51.77	0	1	400
13	1	1	127.11	20.58	0	1	127.11
13	1	8	600	97.17	1	0	600
13	2	3	769.15	115.66	0	1	769.15
14	1	1	73.56	11.91	1	0	164.66
14	1	2	160	25.91	0	1	160
14	1	14	192	31.09	1	0	192
14	2	3	274.31	41.25	0	1	603.46
15	1	2	254.72	41.25	0	1	254.72
15	2	3	316.54	47.6	1	0	480
15	2	8	400	60.15	1	0	400
16	1	1	155.34	25.16	1	0	180
16	1	2	25.28	4.09	1	0	120

A relevant feature of the computed solution is the proposed sequencing for each production line of every product to be processed within the same week. For every week and line, if the value reported in the column *cleans saved* takes the value 1, it means that the last product processed in the week in that line is also the first product to be processed in the following week, therefore, the machine does not need to be cleaned (and one cleaning is avoided).

Table 4 reports relevant indicators for each of the two considered production lines: the total number of operator shifts used, the number of products produced, the total production hours per line, the number of cleanings required for machines, the number of cleanings avoided in the computed solution, and the total hours spent on effective cleanings. Results show that a significant number of cleanings are avoided when applying the proposed MILP model for solution optimization. The cleanings saved have an important impact on the overall production costs.

Table 4. Production and cleanings for the considered case study.

Line	*Shifts*	*Production (Tons)*	*Hours per Line*	*Cleanings*	*Saved Cleanings*	*Cleaning Hours*
1	20	4531.70	733.88	14	8	84
2	31	8024.30	1206.66	8	12	64
total	51	12556	1940.54	22	20	148

Table 5 describes each week of the harvest, reporting the quantity of products stored (computed as the total production minus the withdrawn products), and the storage capacity available in the facility. According to the weekly production plan obtained in Table 3 and the demand established in Table 2, it can be seen that the occupation of the maximum storage capacity occurs in week 9, with 96.48% occupancy (2,605 tons) and the average space occupied during the entire period is 38.53%.

Table 5. Stored production and available capacity by week for the case study.

Week	1	2	3	4	5	6	7	8	9	10
stored production (tons)	0	0	495	1044	1318	2141	2415	2506	2605	2234
available capacity	2700	2700	2205	1656	1382	559	285	194	95	466
week	11	12	13	14	15	16	17	18	19	
stored production (tons)	1710	1100	880	412	439	339	39	0	0	
available capacity	990	1600	1820	2288	2261	2361	2661	2700	2700	

6.4.2. GAP Evolution and Cost Improvements

Table 6 presents an analysis of the GAP and objective function value evolution of the solution computed using the proposed MILP model for the case study, when using different fixed-effort time stopping criteria. The table reports the values of the objective function and the (percentage) GAP computed in seven executions of the proposed model with different time limits, from 60 to 14,400 s. It is useful to determine how much execution time is worth to spend in order to obtain improved solutions.

During the first hour of execution, the best solutions found are significantly improved. After one hour, the reduction in the objective function value and gap are close to 1200 units and 2.3% with respect to the solution found in few seconds (60). With higher execution times, both the gap and the objective function value almost stagnate. The variable costs involved in a typical soy harvest, which correspond to the objective function in the model, amount to approximately USD 200,000; 66% to labor, 30% to energy and consumables for production and 4% to energy and consumables for line cleaning. Therefore, a reduction of 1200 units represents a cost saving of USD 3300. The fact that most of the improvements are reached in few hours has very relevant practical consequences. Once the harvest has begun and some weeks have passed, there might be minor changes in the withdrawal dates. Storms and strikes are among the most frequent reasons. Should this happen, the algorithm might be quickly run again to re-optimize the remaining part of the harvest in light of the modified expected withdrawals.

Table 6. Analysis of the GAP evolution of the solution computed for the case study.

Execution	*Objective Function Value*	*GAP*	*Time (s)*
1	73,839	3.46%	60
2	73,700	3.02%	300
3	72,740	1.31%	1800
4	72,683	1.11%	3600
5	72,625	0.96%	7200
6	72,625	0.92%	10,800
7	72,625	0.88%	14,400

The graph in Figure 1 graphically presents the evolution of the GAP metric for the real problem instance solved, for a range of 0 to 14,000 s (4 h). The shape of the curve indicates that the convergence speed towards the global optimum is faster in the first 30 min, then it decreases significantly up to two hours, and finally it stagnates for execution times greater than two hours (the percentage of improvement in the last two hours was just 0.1%). A logarithmic regression analysis confirmed that the GAP evolution closely matches a logarithmic function $f(x) = a + b \cdot \ln(x)$, where GAP$= f(x)$ and x is the solver execution time, considering parameters $a = 5.63$ and $b = -0.52$. The correlation coefficient (defined by Equation (27)) is $r = 0.97$, suggesting a high similitude between the empirical results and the considered logarithmic function.

$$r = \left| \frac{S_{xy}}{\sqrt{S_{xy}}\sqrt{S_{yy}}} \right|, \tag{24}$$

$$S_{xx} = \sum_i (\ln x_i - \overline{\ln x})^2 = \sum_i (\ln x_i)^2 - n \cdot \overline{\ln x}^2, \tag{25}$$

$$S_{yy} = \sum_i (y_i - \overline{y})^2 = \sum_i y_i^2 - n \cdot \overline{y}^2, \tag{26}$$

$$S_{xy} = \sum_i (\ln x_i - \overline{\ln x}) \cdot (y_i - \overline{y}) = \sum_i \ln x_i \cdot y_i - n \cdot \overline{\ln x} \cdot \overline{y}. \tag{27}$$

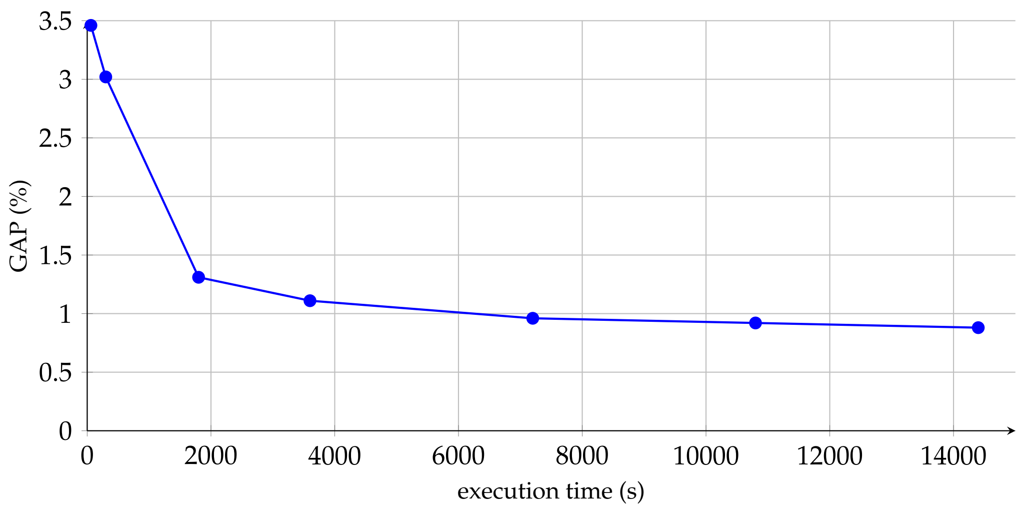

Figure 1. GAP evolution for the considered case study.

Overall, the GAP value obtained after four hours of execution of the proposed model (0.88%) is considered to be an excellent approximation to the global optimum for practical purposes.

6.4.3. Comparison with the Ad Hoc Heuristic Results

Table 7 reports the number of the number of operator shifts, number of cleanings needed, maximum storage used, and overall cost for the solutions computed by the proposed MILP model implemented in CPLEX, and the comparison with the ad hoc heuristic method considered to be a reference baseline. The Δ metric reports the reductions computed by the proposed method in each relevant indicator (in absolute value) and Δ% indicates the percent value of improvement over the heuristic result,

Table 7. Relevant metrics of the plans computed by the proposed MILP method and the comparison with the heuristic method considered to be a reference baseline.

	Heuristic	***Proposed MILP***	**Δ**	**Δ%**
shifts	58	51	7	12.1%
cleanings	34	22	12	35.3%
maximum storage used (tons)	3666	2605	1061	29.4%
overall cost (monetary units)	81,070	72,625	8445	10.4%

The overall cost (objective function) is expressed in monetary units for all minimized and duly weighted elements that define the considered objective function. This unit is in accordance with the criteria applied by the company and in order to maintain confidentiality, because the reported optimization corresponds to a real harvest.

The results in Table 7 indicate that the proposed MILP formulation implemented in CPLEX was able to compute significant improvements over the solution computed by the ad hoc heuristic. The MILP method computed plans that reduced the number of operator shifts by more than 12%, the number of cleanings by more than 35%, and the maximum storage needed for production by almost 30%. The computed results demonstrate the accuracy of the computed plans. Indicators directly imply a better management of resources, which is crucial for improving competitiveness and minimizing operation costs. The plans computed by the proposed MILP model improved the overall values of production cost by 10.4% (approximately USD 20,000) over the heuristic planning and by more than 30% (USD 60,000) over the original manual planning.

7. Overall Improvement of Solution Time and Solution Quality Using Pre-Computed Bounds

This section describes a method for speeding up the resolution time, even improving the quality of the computed solutions, for optimization problems such as the one addressed in this article, using pre-computed bounds.

The proposed method is called the "bounds scheme" and its rationale is to compute bounds for the decision variables of the problem and using the bounds as constraints in the original problem. Depending on the variables bounded, the method is able to improve the two metrics described in Section 6.1, since adding constraints reduces the solution space to explore. On the one hand, the bounds scheme allows speeding up convergence, considering the total execution time (including the time for computing the bounds and the time for solving the problem with the added constraints). On the other hand, the bounds scheme allows reducing the gap between the objective function value of the best solution found and a lower bound computed by the solver.

The proposed method is described next. We consider the optimization problem P defined in Equation (28).

$$\text{problem } P) \quad \min \quad c_1x_1 + \cdots + c_nx_n, \text{ with } c_i > 0, \qquad \text{subject to} \quad h(x_1, \ldots, x_n) \geq 0. \tag{28}$$

The bounds scheme consists of two steps. The first step considers the set of ancillary problems P_i, with index i in some subset $S \subset \{1, \ldots, n\}$, defined in Equation (29). Problems P_i are used to find lower bounds for the values of x_i

$$\text{problem } P_i) \quad \min \quad x_i, \qquad \text{subject to} \quad h(x_1, \ldots, x_n) \geq 0. \tag{29}$$

Problems P_i are solved to find solutions x_i^*, for $i \in S$. Clearly $x_i^* = \min\{x_i : h(x_1, \ldots, x_n) \geq 0\}$, therefore, $h(x_1, \ldots, x_n) \geq 0$ implies $x_i \geq x_i^*$ Then, the second step considers the original problem P, but including the new set of $|S|$ constraints defined by $C_i)\ x_i \geq x_i^*$, with $i \in S$. In case a given problem P_i is not solved to optimality, and just a lower bound $\hat{x}_i$, i.e., $x_i^* \geq \hat{x}_i$ is found, then the new constrains C_i turns to be $C_i')\ x_i \geq \hat{x}_i$. Since $h(x_1, \ldots, x_n) \geq 0$ implies $x_i \geq \hat{x}_i$, then, including the new constraints C_i' does not modify the problem P. Summarizing, the new formulation of problem P is in Equation (30).

$$\begin{aligned} \text{problem } P) \quad \min \quad & c_1x_1 + \cdots + c_nx_n, \\ \text{subject to} \quad & h(x_1, \ldots, x_n) \geq 0, \\ & x_i \geq \hat{x}_i,\ \forall i \in S. \end{aligned} \tag{30}$$

Further improvements are obtained by relaxing some constrains in problem P. When the bounds are computed, the time can be lowered by relaxing the constraints $h(x_1, \ldots, x_n) \geq 0$. This can be done, for instance, setting an unlimited storage capacity, though at the expense of a trade-off regarding the quality of the computed bound.

The proposed method is instantiated for the grain production optimization problem addressed in this work. The considered objective function is linear with positive coefficients; thus, it verifies the hypothesis of problem P). When the proposed MILP model is executed using CPLEX, either the method finds the exact solution x_i^* or it provides a feasible solution x_i and a gap g_i, from which a lower bound is computed as $\hat{x}_i = x_i(1 - g_i)$. In the considered optimization problem, there are two families of variables, namely the number of production hours and the number of operator shifts. Thus, four possible scenarios S are defined: (i) the empty set, where no bound scheme is applied (the "NoBounds" scenario); (ii) a scenario that considers bounds just for the production hours (the "HoursBound" scenario); (iii) a scenario that considers bounds just for the numbert of shifts (the "ShiftsBound" scenario); and (iv) a scenario that combines bounds for both families of variables (the "AllBound" scenario).

Starting from the real instance (the soy harvest by the company in 2020), 16 test cases were generated, by combining two possible values for each of four parameters. The values were given by the production manager as realistic variations taking into account alternative products and production lines. These cases, numbered 1 to 16, are shown in Table 8, were the four parameters and their set values are as follows:

- *spd*: speeds in tons/h; 6.175 and 6.65 tons/h for lines 1 and 2, respectively, in half of the tests, 8.075 and 4.75 in the other half
- τ: expiration time in weeks for inoculated products; 2 and 3 weeks for each half of tests, respectively
- *cleans*: hours needed for cleaning; 6 and 8 h for lines 1 and 2, respectively, in half of the tests, 3 and 10 h in the other half

- *mix, cap*: mix of inoculated and treated products and storage capacity; 7 inoculated, 10 treated products and 2700 tons of storage capacity in half of the tests; 4 inoculated, 5 treated and 1700 tons in the other half.

Case number 1 corresponds to the real instance while cases 2 to 16 are obtained changing one or more parameters to their alternative values.

Table 8. Instance numbers as a function of parameters *spd*, τ, *mix*, *cap* and *clean*.

		Cleans **(*Mix*, *Cap*)**			
		(7i10t, 2700)		**(4i5t, 1700)**	
spd	τ	**6–8**	**3–10**	**6–8**	**3–10**
6.175–6.65	2	1	2	3	4
	3	5	6	7	8
8.075–4.75	2	9	10	11	12
	3	13	14	15	16

As Table 9 reports, for each problem instance #*I* and bounds scenario, the precomputed hours LBH and shifts lower bounds LBS, as well as the computational times T_{LBH} and T_{LBS} is spent to do it, respectively. The table also reports the actual number of hours (H) and shifts (Sh) in the best feasible solution x to P, the value *obj* of the objective function in x, the corresponding gap for this solution (*gap*), the lower bound for the objective derived from them ($best_{LB} = obj - gap$), as well as the computational time (T_{BF}) required to obtain x. Finally, the total execution time is reported in the last column (T). All execution times are reported in seconds. For example, considering the results reported for instance 3 and scenario AllBounds, the lower bound obtained for hours was 64 and for shifts was 33, whereas the best feasible solution computed has 82 h and 33 shifts. In this case, the optimal number of shifts was computed, since the gap with its lower bound is zero. The lower bound for the number of hours was computed in just one second. In turn, the lower bound for the number of shift was computed in 33 s, whereas 225 s were required to obtain the feasible solution with objective function value of 46,923. The overall execution time was 249 s.

Table 9. Solution quality and execution time for the 16 considered problem instances using the four proposed bound scheme scenarios.

#*I*	**Scenario**	**LBH**	**LBS**	***H***	***Sh***	**obj**	**Gap**	$\boldsymbol{best_{LB}}$	$\boldsymbol{T_{LBH}}$	$\boldsymbol{T_{LBS}}$	$\boldsymbol{T_{BF}}$	$\boldsymbol{T}$
1	NoBounds			154	51	**72,683**	**1.114%**	71883			3600	**3600**
	HoursBound	136		166	51	72,844	1.920%	71,445	271		3600	3871
	ShiftsBound		50	154	51	72,709	1.136%	71,842		3600	3600	7200
	AllBounds	136	50	156	52	73,680	3.040%	71,440	271	3600	3600	7471
2	NoBounds			94	50	**71,012**	0.320%	70,785			3600	**3600**
	HoursBound	87		94	50	**71,012**	0.494%	70,977	29		3600	3629
	ShiftsBound		50	94	50	**71,012**	0.095%	70,945		256	3600	3856
	AllBounds	87	50	94	50	**71,012**	**0.080%**	70,955	29	256	3600	3885
3	NoBounds			82	33	**46,921**	0.010%	46,916			2076	**2076**
	HoursBound	64		82	33	**46,921**	0.024%	46,910	1		3600	3601
	ShiftsBound		33	82	33	**46,921**	**0.007%**	46,918		23	87	110
	AllBounds	64	33	82	33	46,923	0.009%	46,919	1	23	225	249
4	NoBounds			47	33	**46,686**	0.769%	46,327			3600	**3600**
	HoursBound	40		47	33	**46,686**	0.073%	46,652	2		3600	3602
	ShiftsBound		33	47	33	**46,686**	**0.001%**	46,686		12	50	62
	AllBounds	40	33	47	33	**46686**	**0.001%**	46,686	2	12	83	97

Table 9. *Cont.*

#I	Scenario	LBH	LBS	H	Sh	obj	Gap	$best_{LB}$	T_{LBH}	T_{LBS}	T_{BF}	T
5	NoBounds			162	51	72,775	1.667%	71,562			3600	**3600**
	HoursBound	134		150	51	72,601	1.740%	71,338	348		3600	3948
	ShiftsBound		50	150	51	**72,598**	**1.250%**	71,691		3411	3600	7011
	AllBounds	134	50	156	51	72,713	1.885%	71,342	348	3411	3600	7359
6	NoBounds			94	50	70,986	0.429%	70,681			3600	**3600**
	HoursBound	87		91	50	**70,967**	0.630%	70,520	429		3600	4029
	ShiftsBound		50	91	50	**70,967**	**0.010%**	70,960		3600	2326	5926
	AllBounds	87	50	91	50	**70,967**	0.111%	70,888	429	3600	3600	7629
7	NoBounds			76	33	**46,807**	0.441%	46,600			3600	3600
	HoursBound	62		76	33	**46,807**	0.740%	46,461	2		3600	3602
	ShiftsBound		33	76	33	**46,807**	**0.009%**	46,803		18	18	**36**
	AllBounds	62	33	76	33	**46,807**	0.010%	46,802	2	18	24	44
8	NoBounds			44	33	**46,641**	1.036%	46,156			3600	3600
	HoursBound	37		44	33	**46,641**	0.099%	46,595	2		3600	3602
	ShiftsBound		33	44	33	**46,641**	**0.010%**	46,636		3	19	**22**
	AllBounds	37	33	44	33	**46,641**	**0.010%**	46,636	2	3	21	26
9	NoBounds			154	44	62,542	2.756%	60,818			3600	**3600**
	HoursBound	124		170	44	62,545	3.130%	60,587	27		3600	3627
	ShiftsBound		43	170	44	62,430	1.887%	61,252		3600	3600	7200
	AllBounds	124	43	166	43	**61,412**	**0.216%**	61,279	27	3600	3600	7227
10	NoBounds			82	41	**58,088**	1.074%	57,464			3600	3600
	HoursBound	74		82	41	**58,088**	1.159%	57,415	15		3600	3615
	ShiftsBound		41	82	41	**58,088**	**0.010%**	58,082		86	1517	**1603**
	AllBounds	74	41	82	41	**58,088**	0.010%	58,082	15	86	2118	2219
11	NoBounds			72	27	**38,378**	0.010%	38,374			68	68
	HoursBound	62		72	27	**38,378**	0.010%	38,374	2		217	219
	ShiftsBound		27	72	27	**38,378**	**0.000%**	38,378		7	5	**12**
	AllBounds	62	27	72	27	**38,378**	0.010%	38,374	2	7	13	22
12	NoBounds			36	26	**36,838**	0.010%	36,834			70	70
	HoursBound	36		36	26	**36,838**	0.010%	36,834	1		1141	1142
	ShiftsBound		26	36	26	**36,838**	0.004%	36,836		2	4	**6**
	AllBounds	36	26	36	26	**36,838**	**0.000%**	36,838	1	2	4	7
13	NoBounds			158	44	62,350	2.600%	60,729			3600	**3600**
	HoursBound	124		158	44	62,350	2.981%	60,491	62		3600	3662
	ShiftsBound		43	156	44	62,442	2.007%	61,189		3600	3600	7200
	AllBounds	124	43	162	43	**61,495**	**0.509%**	61,182	62	3600	3600	7262
14	NoBounds			91	41	**58,033**	1.036%	57,432			3600	**3600**
	HoursBound	71		91	41	**58,033**	1.214%	57,328	14		3600	3614
	ShiftsBound		41	91	41	**58,033**	**0.010%**	58,027		24	3600	3624
	AllBounds	71	41	91	41	**58,033**	0.017%	58,023	14	24	3600	3638
15	NoBounds			72	27	**38,378**	**0.010%**	38,374			1137	1137
	HoursBound	62		72	27	**38,378**	**0.010%**	38,374	2		1211	1213
	ShiftsBound		27	72	27	**38,378**	**0.010%**	38,374		11	9	**20**
	AllBounds	62	27	72	27	**38,378**	**0.010%**	38,374	2	11	81	94
16	NoBounds			36	26	**36,838**	0.100%	36,801			780	780
	HoursBound	36		36	26	**36,838**	0.438%	36,677	2		3600	3602
	ShiftsBound		26	36	26	**36,838**	0.009%	36,835		1	5	**6**
	AllBounds	36	26	36	26	**36,838**	**0.000%**	36,838	2	1	28	31

The results reported in Table 9 demonstrate that the bound schemes were able to improve the best objective value computed in all instances except one (instance 1). Regarding gaps, the bound schemes reduced the gap metric in all cases, except for instances 1 and 5. The gap was even zero in three cases: instance 11 with ShiftsBound scenario, instance 12 with AllBounds scenario, and instance 16 with AllBounds scenario. In instances 3, 4, 7, 12, and 16 the gap was below the considered value of `CPX_PARAM_EPGAP` parameter (1×10^{-5}), with scenarios ShiftsBound, AllBounds or both, depending on the instance. For the three instances where a gap of zero was achieved (instances 11, 12, and 16), the computed objective value is the same for all bound scenarios. In these cases, the bound schemes allow guaranteeing optimality, i.e., that the optimal value for the optimization problem is computed.

Efficiency results also demonstrate that the largest reductions in the execution time are computed when using the ShiftsBound scenario (e.g., in problem instances 4, 7, 11, 12, 15, and 16). These results suggest that the number of shifts has a higher impact on solution quality than the number of hours. Execution times results confirm that using the proposed bounds schemes improved the global computation times in half the cases. For some instances, significantly large improvements were achieved (e.g., for instances 3, 4, 7, 8, 11, 12, 15, and 16). In those three instances where a gap zero was achieved, the bounds scheme also helped to reduce the overall computational time by 82%, 90% and 96%, respectively. A similar situation occurred for those cases where the computed gap is lower than the considered value of `CPX_PARAM_EPGAP` parameter, achieving time improvements between 88% and 99%. The NoBounds scenario was able to achieve the CPLEX halting gap in just three instances (instances 3, 11, and 12), without spending the one hour time limit. Even in this instances, the ShitfBound and AllBounds scenarios were able to reduce the computational time significantly.

In summary, the proposed bounds scheme improved the quality of the solutions attained, while reducing the computational time in 15 out of the 16 instances solved. The average and best improvements to the execution time for those instances, as well as the average and best objective value improvements respect to the original formulation without pre-computed bounds are reported in Table 10.

Table 10. Improvements of bound schemes over the NoBounds scenario.

Scenario	*Time*		*Objective*	
	Average	**Best**	**Average**	**Best**
HoursBound	−139%	0%	0.040%	0.000%
ShiftBound	22%	99%	−0.949%	0.000%
AllBounds	14%	99%	1.914%	1.807%

The results in Table 10 indicate that the best time reductions were obtained when pre-computing bound for shifts, with an average time improvement of 22% and a best improvement of 99%. Pre-computing bounds for production hours did not yield to improvements in the execution time. However, when combined with bounds on shifts, large execution time improvements were required. Hence, results indicate that it is worth evaluating different scenarios of the bounds scheme to determine the best variable (or combination of variables) that is suitable to be prebounded. This task can be performed by applying parallel computing techniques using over high performance infrastructures [38], in order to further speed up the computation. The proposed bounds scheme is also combinable with relaxation techniques to be applied in the constrains to further reduce the execution time.

8. Conclusions and Future Work

This article addressed a relevant problem for production in the agriculture field. In this context, efficient and accurate models are needed for resource management, in order to minimize operation costs and enhance competitiveness.

A MILP optimization model based on discrete time windows was proposed for a capacitated multi-product multi-line production scheduling problem. The model incorporates relevant constraints in the context of grain production, considering both treated and inoculated products. A theoretical analysis of the NP-completeness of the optimization problem was developed. In turn, a bounds-based scheme was proposed to accelerate the convergence to the optimal solution of the optimization problem.

The proposed MILP model was implemented in AMPL and solved using CPLEX. The model was evaluated on a real-life case study, considering real data from the 2020 soybean harvest of the top Uruguayan grain producer and exporter. Many real-life situations were taken into account, including multiple products, multiple production lines, limited shelf life, storage facilities, operator shifts, and cleaning of the machinery.

The main results of the experimental evaluation indicated that the computed solutions achieved an efficient storage capacity, significantly reducing the number of cleaning stops for machinery and the number of operator shifts required. The plan computed by the proposed model achieved significant cost reductions (more than 10%, USD 20,000) over the ad hoc planning heuristic applied by the company. Furthermore, the computed plans also improving storage occupancy and costs of machinery cleaning supplies. The proposed bounds-based scheme was useful to speed up the resolution of the optimization problem in two scenarios: in a factor of 1.28× using the ShiftsBound type and in a factor of 1.17× using the AllBounds type. Conversely, execution times were worse using the HoursBound type (a factor of 0.41×).

The proposed bound scheme was useful to accelerate the resolution of the considered optimization problem. The approach based on embedding bounds in the optimization problem is suitable to be applied in any problem where the objective function is an aggregation of monotone increasing functions. In that case, the proposed method is directly applicable as we did in our research, even if the functions are not linear. Overall, the approach is also a useful idea that can be applied to other optimization problems. In this regard, the validation of the proposed model should be extended by considering new problem instances and different grain production problems, subject to the availability of data, by working with other companies in the agri-food sector.

The main lines of future work are related to improve the optimization model, and expand the case studies to properly reflect the current reality of grain production companies. The optimization model can be improved by including additional realistic features in the optimization model, such as silo farms for raw grains, shared to store the processed grains and uncertainty on demands. Regarding the discretization step considered to determine production, a problem model to plan days will certainly allow for more accurate production planning, further optimization of machinery cleanings, operator shift costs, and improved storage management. The proposed model should be adapted to take into consideration these new features and related new constraints. For executing the exact model including additional features and larger case studies, the application of parallel computing over high performance computing infrastructures is also an interesting line for future work. Finally, the design of suitable problem instances that represent realistic situations and have diverse features to perform a correct evaluation must be addressed. Both theoretical and empirical knowledge are needed to build meaningful problem instances for the evaluation of the proposed and related models.

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