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Determinants of Top Speed Sprinting: Minimum Requirements for Maximum Velocity

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Abstract: Faster top sprinting speeds require shorter ground contact times, larger vertical forces, and greater thigh angular velocities and accelerations. Here, a framework using fundamental kinematic and kinetic relationships is presented that explores the effect of body dimensions on these mechanical determinants of sprinting performance. The analysis is applied to three hypothetical runners of different leg lengths to illustrate how these mechanical determinants of speed vary with body dimensions. Specific attention is focused on how the following variables scale with leg length and top speed: ground contact time, step rate, step length, ratio of step length to leg length, ratio of vertical force to body weight, total thigh range of motion, average thigh angular velocity, and maximum thigh angular acceleration. The analysis highlights the inherent biological tradeoffs that interplay to govern the optimal dimensions for sprinting speed and underscores that accounting for leg length may facilitate interpretation in future investigations examining the relationship between these mechanical variables and top speed. Furthermore, for athletes with given body dimensions and sprinting performance goals, this framework could help to establish the minimum requirements for maximum velocity.

Keywords: sprinting biomechanics; bipedal gait; kinematics

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1. Introduction

Research in locomotor biomechanics has increasingly elucidated the kinematic and kinetic mechanisms underpinning high-speed running performance. During upright running, ground contact time decreases with increasing speed [1–3], demanding greater mass-specific vertical forces to meet the necessary vertical impulse requirements [2–4]. Additionally, numerous publications have established that limb angular motion is an important factor in determining running speed [5–8]. Specifically, recent research has demonstrated that thigh angular velocity and thigh angular acceleration have a strong linear correlation to top speed [5,9].

Body dimensions (and particularly leg length and body mass) affect the kinetic and kinematic requirements for running at a given steady speed, whether examining human sprinters of different dimensions [8,10,11] or comparing various species of bipedal runners [12]. Based on limb excursion angles normally selected during ground contact [12–14], the constraints imposed by leg length are especially relevant to the factors of ground contact time, vertical impulse, and thigh angular motion. Therefore, a specific analysis of how body dimensions affect these factors is warranted.

Here, a simple framework utilizing basic kinematic and kinetic relationships is presented, in order to predict output variables observed from human runners when sprinting. The relationship of these variables represents the theoretical requirements for attaining maximum velocity across a range of body dimensions and top speeds. This analysis may provide additional insight into the mechanics underlying high-speed running, and potentially serve as a point of comparison for practitioners working with athletes.

2. Materials and Methods

The framework presented here enables the evaluation and prediction of kinematic and kinetic variables from top-speed sprinting by simply using the input parameters of running speed and leg length.

During steady-speed running, the center of mass horizontal velocity during the flight phase is constant (discounting resistance due to air friction), and so the runner's forward *Speed* (m/s) is determined by the time it takes the center of mass to traverse the contact length L_c (m) during the ground contact time t_c (s) [3,15]:

$$Speed = \frac{L_c}{t_c} \tag{1}$$

L_c is determined by leg length L_0 (m) and the total excursion angle during contact θ_c (deg):

$$L_c = 2 \cdot L_0 \cdot \sin \frac{\theta_C}{2} \tag{2}$$

For faster running speeds, prior research has demonstrated that $\theta_c \approx 60$ deg for humans and other bipedal runners [12–14]. Based on Equation (2), it can be approximated that $L_c \approx L_0$ (see Figure 6 in [5]). Equation (1) then shows that t_c is directly related to L_0 and *Speed*:

$$t_c = \frac{L_0}{Speed} \tag{3}$$

Aerial time t_a (s) is the time interval from takeoff of one foot until touchdown of the contralateral foot. Prior research has demonstrated that t_a is approximately 0.12 ± 0.02 s for runners across a broad range of top speeds (~6 to 11 m/s) [2–5]. Step time t_{step} (s) is the time interval to complete one step, and is equal to the sum of t_c and t_a:

$$t_{step} = t_c + t_a \tag{4}$$

Step Rate *SR* (steps/s) is the number of steps taken per second and is the inverse of t_{step}:

$$SR = \frac{1}{t_{step}} \tag{5}$$

Step Length *SL* (m) is the distance traveled per step and can be calculated from *Speed* and *SR*:

$$SL = \frac{Speed}{SR} \tag{6}$$

During steady-speed running where the net vertical displacement of the body is zero, the time-averaged vertical ground reaction force must equal the body's weight. Thus, the ratio of stance-averaged vertical force to body weight (F_{Zavg}/mg) can be determined if t_c and t_a are known:

$$\frac{F_{Zavg}}{mg} = \frac{t_{step}}{t_c} = \left(1 + \frac{t_a}{t_c}\right) \tag{7}$$

where m (kg) is body mass and g (9.8 m/s^2) is gravitational acceleration [16,17]. Given that t_c is generally inversely related to top speed, and t_a remains relatively constant across a range of runners and top speeds, F_{Zavg}/mg generally increases with top speed [2–4,17].

Limb angular motion is also considered within the context of this framework. Figure 1a depicts a simplified example of harmonic oscillatory thigh motion that assumes symmetrical peak thigh flexion and extension values. Because this framework is based on equations of harmonic motion [18], thigh angular kinematics as a function of time are determined by the parameters of frequency and θ_{total}, which is the total thigh range of motion from peak extension through peak flexion (Figure 1b). The frequency f (Hz) can be expressed in terms of the period T (s) for one full cycle of the leg or equivalently in terms of t_{step}:

$$f = \frac{1}{T} = \frac{1}{2 \cdot t_{step}} \tag{8}$$

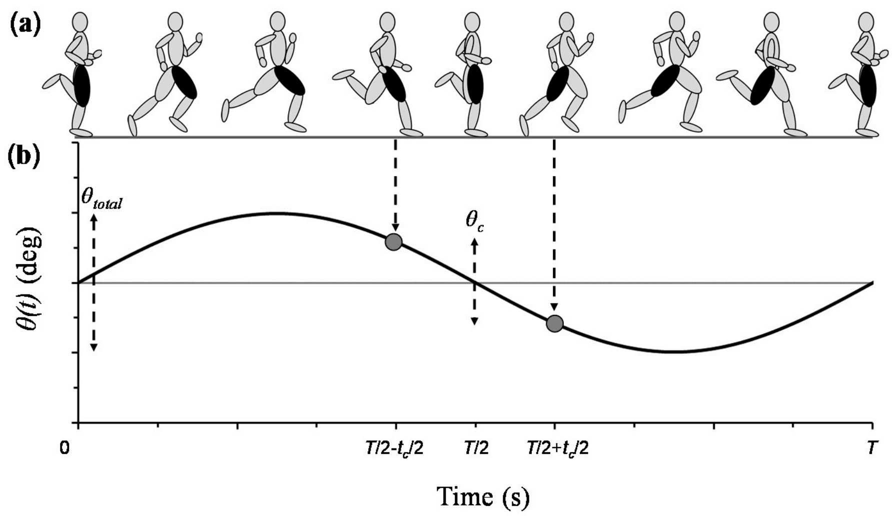

Figure 1. Leg angular motion during the gait cycle. (**a**) Simplified illustration of harmonic oscillatory thigh motion that assumes symmetrical peak thigh flexion and extension values. (**b**) Thigh angular position $\theta(t)$ during the gait cycle. Thigh angular kinematics as a function of time are determined by the parameters of frequency ($f = 1/T$, where T is the period of the cycle) and θ_{total}, which is the total thigh range of motion from peak extension through peak flexion. The total excursion angle during contact (θ_c) occurs in the contact time (t_c) between $t = T/2 - t_c/2$ and $t = T/2 + t_c/2$.

The angular position is referenced to the condition $\theta(t) = 0$ at $t = 0$, and the equation for thigh angular position $\theta(t)$ is:

$$\theta(t) = \frac{\theta_{total}}{2} \cdot \sin(2 \cdot \pi \cdot f \cdot t) \tag{9}$$

The relationship between total thigh excursion θ_{total} (deg) and the total excursion angle during contact θ_c (deg) can then be derived from Equation (9) using the conditions $\theta(t) = \theta_c/2$ at $t = T/2 - t_c/2$, and $T = 1/f$:

$$\theta_{total} = \frac{2 \cdot \theta(t)}{\sin(2 \cdot \pi \cdot f \cdot t)} = \frac{\theta_c}{\sin(\pi \cdot f \cdot t_c)} \tag{10}$$

The thigh angular velocity averaged throughout the stride cycle ω_{avg} (deg/s) has been shown to increase linearly with top speed [5] and can be determined by:

$$\omega_{avg} = \frac{\theta_{total}}{t_{step}} = \theta_{total} \cdot SR \tag{11}$$

The modeled maximum thigh angular acceleration α_{max} (deg/s^2) during the swing phase can be derived from the second derivative of Equation (9). This variable has also been shown to increase linearly with top speed [9] and can be determined by:

$$\alpha_{max} = \frac{\theta_{total}}{2} \cdot (2 \cdot \pi \cdot f)^2 = 2 \cdot \pi^2 \cdot \theta_{total} \cdot f^2 \tag{12}$$

Collectively, the figures and equations presented above allow for predictions of key kinematic and kinetic variables across a range of top speeds. In Tables 1–3, values are calculated across a range of top speeds from 7.0 to 13.0 m/s for three hypothetical runners of different leg lengths: L_0 = 0.85 m, 0.95 m, and 1.05 m (greater trochanter to ground in standing position). Data for 13.0 m/s were calculated to explore the requirements for achieving this level of performance, with the recognition that no human runner has yet attained this speed. The theoretical calculations required only two simplifying assumptions across all conditions, θ_c = 60 deg and t_a = 0.12 s.

Table 1. Calculated values across a range of speeds for a hypothetical athlete with L_0 = 0.85 m.

Speed (m/s)	t_c (s)	*SR* (steps/s)	*SL* (m)	SL/L_0 (ratio)	F_{Zavg}/mg (ratio)	θ_{total} (deg)	ω_{avg} (deg/s)	α_{max} (deg/s^2 $\times$ 10^3)
7.00	0.121	4.14	1.69	1.99	1.99	84.5	349.8	7.15
8.00	0.106	4.42	1.81	2.13	2.13	89.2	394.3	8.60
9.00	0.094	4.66	1.93	2.27	2.27	94.1	438.6	10.09
10.00	0.085	4.88	2.05	2.41	2.41	99.0	482.8	11.62
11.00	0.077	5.07	2.17	2.55	2.55	104.0	526.9	13.18
12.00	0.071	5.24	2.29	2.69	2.69	109.0	571.1	14.77
13.00	0.065	5.39	2.41	2.84	2.84	114.0	615.2	16.38

Table 2. Calculated values across a range of speeds for a hypothetical athlete with L_0 = 0.95 m.

Speed (m/s)	t_c (s)	*SR* (steps/s)	*SL* (m)	SL/L_0 (ratio)	F_{Zavg}/mg (ratio)	θ_{total} (deg)	ω_{avg} (deg/s)	α_{max} (deg/s^2 $\times$ 10^3)
7.00	0.136	3.91	1.79	1.88	1.88	81.0	316.9	6.12
8.00	0.119	4.19	1.91	2.01	2.01	85.2	356.9	7.38
9.00	0.106	4.43	2.03	2.14	2.14	89.5	396.6	8.68
10.00	0.095	4.65	2.15	2.26	2.26	93.8	436.3	10.01
11.00	0.086	4.85	2.27	2.39	2.39	98.2	475.8	11.38
12.00	0.079	5.02	2.39	2.52	2.52	102.6	515.3	12.77
13.00	0.073	5.18	2.51	2.64	2.64	107.1	554.8	14.18

Table 3. Calculated values across a range of speeds for a hypothetical athlete with L_0 = 1.05 m.

Speed (m/s)	t_c (s)	*SR* (steps/s)	*SL* (m)	SL/L_0 (ratio)	F_{Zavg}/mg (ratio)	θ_{total} (deg)	ω_{avg} (deg/s)	α_{max} (deg/s^2 $\times$ 10^3)
7.00	0.150	3.70	1.89	1.80	1.80	78.3	290.1	5.30
8.00	0.131	3.98	2.01	1.91	1.91	82.0	326.4	6.41
9.00	0.117	4.23	2.13	2.03	2.03	85.8	362.6	7.56
10.00	0.105	4.44	2.25	2.14	2.14	89.7	398.5	8.74
11.00	0.095	4.64	2.37	2.26	2.26	93.6	434.4	9.95
12.00	0.088	4.82	2.49	2.37	2.37	97.6	470.2	11.18
13.00	0.081	4.98	2.61	2.49	2.49	101.6	505.9	12.44

3. Results

Data across a range of top speeds for the three hypothetical runners of varying leg lengths are presented in Tables 1–3 and Figure 2. These values are calculated from the equations and simplifying assumptions presented in the Methods. The predictive lines illustrated in Figure 2 are based on the outcomes listed in Tables 1–3.

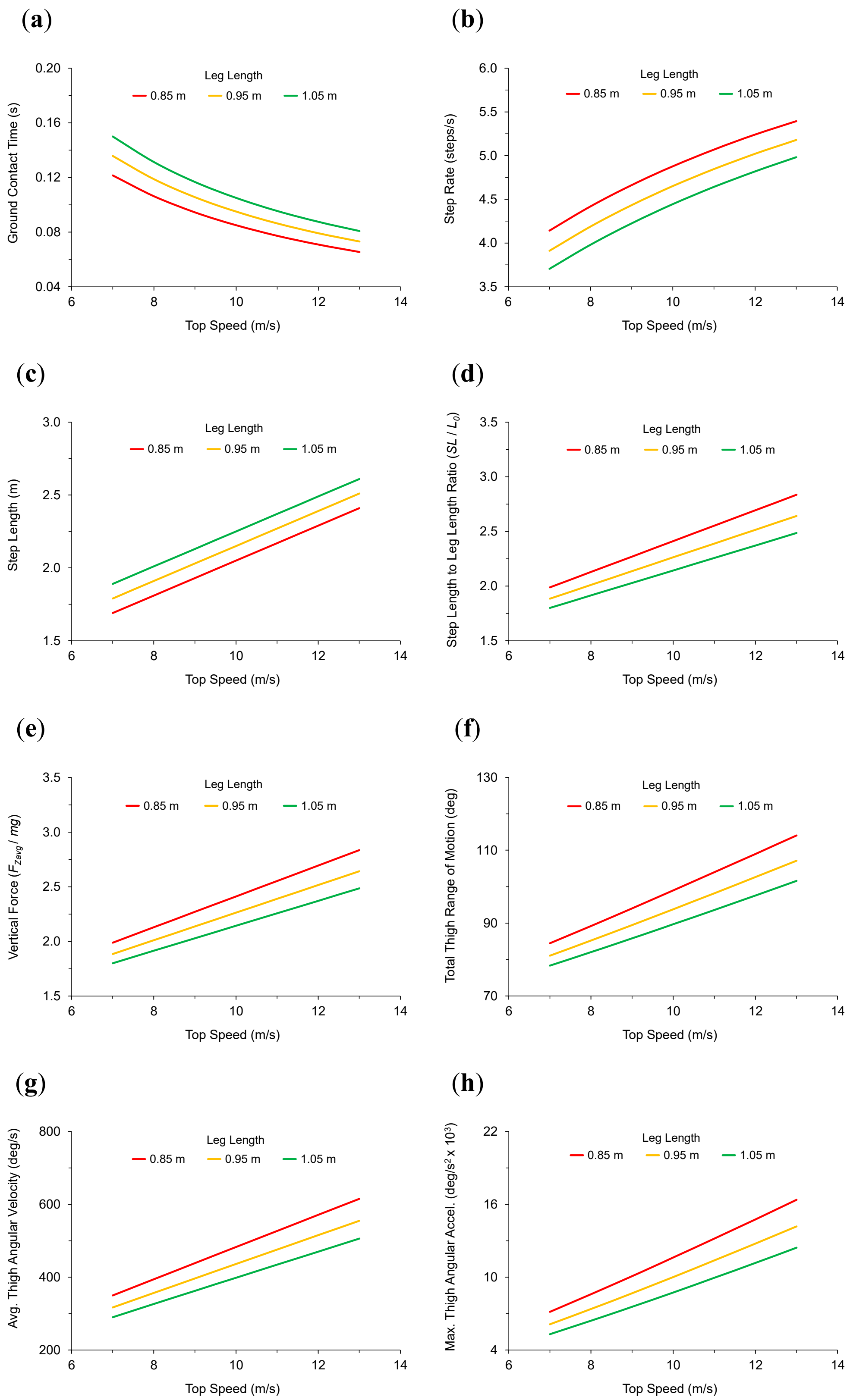

Figure 2. Calculated data across a range of top speeds for the three hypothetical athletes of different body dimensions. Illustrated lines based on values in Tables 1–3, arranged by athlete leg length (L_0). (**a**) ground contact time (t_c), (**b**) step rate (SR), (**c**) step length (SL), (**d**) ratio of step length to leg length (SL/L_0), (**e**) ratio of stance-averaged vertical force to body weight (F_{Zavg}/mg), (**f**) total thigh range of motion (θ_{total}), (**g**) average thigh angular velocity (ω_{avg}), (**h**) modeled maximum thigh angular acceleration (α_{max}).

The calculated outcomes in Tables 1–3 include values for the following variables during top speed sprinting: running *Speed*, ground contact time (t_c), step rate (*SR*), step length (*SL*), ratio of step length to leg length (SL/L_0), ratio of stance-averaged vertical force to body weight (F_{Zavg}/mg), total thigh range of motion from peak extension to peak flexion (θ_{total}), thigh angular velocity averaged throughout stride cycle (ω_{avg}), and modeled maximum thigh angular acceleration during the swing phase (α_{max}).

4. Discussion

As demonstrated in Tables 1–3 and Figure 2, several basic relationships were observed across the range of top speeds for the hypothetical runners of different body dimensions. There was a negative relationship between *Speed* and t_c (Figure 2a), aligning with values from several experimental data sets [1–3,5]. Calculations of *SR* and *SL* (Figure 2b,c) increased with *Speed*, and values were generally in agreement with prior investigations into top-speed sprinting [2–5]. The relationship between *Speed* and F_{Zavg}/mg (Figure 2e) was positive and linear across all top speeds and corresponded to similar increases in the experimental data of Weyand et al. [2,3] across a range of top speeds in a heterogenous pool of subjects. A positive and linear relationship was also demonstrated between *Speed* and the thigh angular variables, with theoretical values of θ_{total}, ω_{avg}, and α_{max} (Figure 2f–h) aligning with recent experimental data on thigh angular motion at top speed [5,9].

As illustrated in Figure 2a–h, the value of L_0 had a noticeable effect on the calculated outcome variables. Across the three hypothetical athletes, increased leg length allowed a given *Speed* to be attained with: (A) longer t_c; (B) decreased *SR*; (C) longer *SL*; (D) decreased SL/L_0 ratio; (E) reduced F_{Zavg}/mg; (F) decreased θ_{total}; (G) slower ω_{avg}; and (H) decreased α_{max}. Thus, from a purely theoretical standpoint, it is clear that longer legs may allow for fast speeds to be attained with reduced mechanical requirements (i.e., prolonged ground contact times, decreased vertical forces, reduced thigh angular velocities and accelerations). This may in part explain the record-breaking performances achieved by Usain Bolt, whose unusually tall stature for a male sprinter likely provided specific advantages over his shorter competitors [10]. Of course, inherent biological tradeoffs clearly exist that interplay to govern the optimal dimensions for sprinting speed. As it relates to thigh angular motion, since torque is the product of moment of inertia and angular acceleration, and the moment of inertia is proportional to the mass and length of the leg ($mL_0{}^2$), the hip torque required to generate a given magnitude of thigh angular acceleration will escalate with increases in leg length. Therefore, while longer legs may allow for higher speeds to be attained with decreased requirements for the variables analyzed here (Figure 2a–h), longer legs may also require increases in other physical parameters such as torque generating capacity.

To enable insight into the above variables, this framework included two simplifying kinematic assumptions based on prior research: θ_c = 60 deg and t_a = 0.12 s. If runners exhibit large deviations from these simplifying assumptions, there is the possibility for experimental data to not align with the values in Tables 1–3 and Figure 2. For the limb excursion angles selected during ground contact, prior research has indicated that these angles are likely constrained by leg extensor muscle effective mechanical advantage [3,19]. However, deviations from the assumed value of θ_c = 60 deg will affect the other calculated variables, with increased θ_c allowing for longer t_c and decreased F_{Zavg}/mg to achieve a given speed, and vice versa. As it relates to aerial time, prior investigations have found that even for runners of different body dimensions and top speeds that $t_a = 0.12 \pm 0.02$ s [2–5], and thus employing a standard t_a here for hypothetical runners with varying L_0 likely did not introduce major errors. However, differences in individual running styles may exist [20,21] that could result in t_a outside the standard range presented here, subsequently affecting the other calculated variables.

In spite of the simplifying assumptions included in this theoretical approach, several important insights are highlighted by the calculated outcome variables. First, future investigations examining the determinants of top speed sprinting may aim to statistically account for leg length in order to properly analyze the relationship of mechanical variables to performance. Although this has been done in some recent investigations into human sprinting performance [8], as well as in research on bipedal locomotion across species [12], the results here indicate just how substantial the effect of leg length is on speed-specific requirements. Furthermore, the data presented here could provide normative ranges for the mechanical variables needed to attain a specific top speed for runners of different body dimensions. While further direct experimental investigation is needed to validate these kinematic variables in relation to L_0 and *Speed*, this framework may serve as a blueprint for coaches and athletes to specifically evaluate mechanics based on their body dimensions and performance goals.

5. Conclusions

During top-speed sprinting, body dimensions interplay with the mechanical variables of ground contact time, step rate and length, vertical force application, and leg angular motion to determine performance. Here, a simple framework was presented to analyze these variables within the context of running speed and leg length. The results of this investigation demonstrate that accounting for leg length may facilitate interpretation when analyzing the relationship of mechanical variables to top-speed sprinting performance. Furthermore, when evaluating athletes with given body dimensions and sprinting performance goals, this framework may help to establish the minimum requirements for maximum velocity.

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List of Symbols and Abbreviations

$\theta(t)$	thigh angle as a function of time (deg)
θ_c	total thigh excursion during the ground contact phase (deg)
θ_{total}	total thigh range of motion from peak extension to peak flexion (deg)
ω_{avg}	thigh angular velocity averaged throughout stride cycle (deg/s)
α_{max}	modeled maximum thigh angular acceleration during swing (deg/s^2)
F_{Zavg}/mg	ratio of stance-averaged vertical force to body weight mg
f	frequency of thigh angular motion (Hz)
g	gravitational acceleration (9.8 m/s^2)
L_c	contact length (m)
L_0	leg length (m)
m	body mass (kg)
SL	step length (m)
SL/L_0	ratio of step length to leg length
Speed	runner's forward speed
SR	step rate (steps/s)
T	time period of thigh angular motion (s)
t	time (s)
t_a	aerial time (s)
t_c	ground contact time (s)
t_{step}	step time (s)

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