supplementary materials

Dimensionless power under inertial load

To derive the expression for dimensionless output power under inertial load, we begin by transforming Equations (1) and (7) to the frequency domain, resulting in Equations (S1) and (S2)

$$m\ddot{x} + C_M\dot{x} + K_M x + f = m\ddot{y} \tag{1}$$

$$\dot{x} = \frac{1}{\theta^2}\left(Af + B\dot{f}\right) \tag{7}$$

$$-m\omega^2\hat{x} + i\omega C\hat{x} + K_M\hat{x} + \hat{f} = m\omega^2\hat{y} \tag{S1}$$

$$i\omega\hat{x} = \frac{\hat{f}}{\theta^2}(A + i\omega B) \tag{S2}$$

From the square of the magnitude of Equation (S2), together with Equation (9), we can relate $\dot{x}$ to P_T. We use the parameters defined in Tables 1 and 2 to reduce the expression.

$$P_T = \frac{A\left|\hat{f}\right|^2}{2\theta^2} \tag{9}$$

$$|i\omega\hat{x}|^2 = \frac{A^2\hat{f}^2}{\theta^4}\left(1 + \left(\frac{\omega B}{A}\right)^2\right) = \frac{2P_T}{C_M\alpha}(1 + (\beta\gamma)^2) \tag{S3}$$

Next, we use the relationship between x and y to express P_T in terms of base excitation. From Equation (S1) and Equation (S2), we can derive:

$$\hat{y} = \hat{x}\frac{C_M}{m\omega}\left(-i\left(1 + \frac{\alpha}{(1 + (\beta\gamma)^2)}\right) + \varepsilon - \frac{\alpha\beta\gamma}{(1 + (\beta\gamma)^2)}\right) \tag{S4}$$

Taking the magnitude gives:

$$|i\omega\hat{y}| = |i\omega\hat{x}|\frac{C_M}{m\omega}\sqrt{\left(\frac{\alpha}{(1 + \ (\beta\gamma)^2)} + 1\right)^2 + \left(\frac{\alpha\beta}{(1 + \ (\beta\gamma)^2)} - \varepsilon\right)^2}$$

Next, we square both sides and use Equation (S3) to arrive at:

$$|i\omega\hat{y}|^2 = \omega^2 y_0^2 = \frac{2P_T}{C_M\alpha}(1 + \ (\beta\gamma)^2)\left(\frac{C}{m\omega}\right)^2\left(\left(\frac{\alpha}{(1 + \ (\beta\gamma)^2)} + 1\right)^2 + \left(\frac{\alpha\beta\gamma}{(1 + \ (\beta\gamma)^2)} - \varepsilon\right)^2\right)$$

Extraction of P_T and reduction of expression leads to:

$$\frac{P_T}{\frac{(\omega^2 y_0)^2 m^2}{C_M}} = \frac{\alpha}{2\left((1 + \ \alpha)^2 + (\beta\gamma)^2 - 2\varepsilon\alpha\beta\gamma + \varepsilon^2(1 + \ (\beta\gamma)^2)\right)} \tag{S5}$$

To find the dimensionless power over the load we use the resistive division terms defined by Equation (10). Using the parameters from Tables 1 and 2 we can rewrite Equation (10) in the general form of Equation (S6).

$$\begin{aligned} \text{PEH:} \quad & P = P_T \frac{R_P}{R_P + R_L} \\ \text{EMEH:} \quad & P = P_T \frac{R_L}{R_P + R_L} \end{aligned} \tag{10}$$

$$P = P_T \beta \xi_C \tag{S6}$$

The dimensionless output power under inertial load thus becomes:

$$\frac{P}{\frac{(\omega^2 y_0)^2 m^2}{C_M}} = \frac{\alpha\beta\xi_C}{2\big((1+\ \alpha)^2 + (\beta\gamma)^2 - 2\varepsilon\alpha\beta\gamma + \varepsilon^2(1+\ (\beta\gamma)^2)\big)} \tag{S7}$$

Dimensionless power under prescribed displacement

The expression for dimensionless output power under prescribed displacement can be derived using Equation (S2) and solving for f, with $x = x_0 \sin(\omega t)$. Equation (9) together with Equation (S6) then gives the power over the load. An alternative derivation is to use Equation (3) together with Ohm's law to derive the average power over the load resistance. Under prescribed displacement we can also assume $x_0 = y_0$.

$$\begin{aligned} \text{PEH:} \quad & \theta \dot{x} = i \\ \text{EMEH:} \quad & \theta \dot{x} = v \end{aligned} \tag{3}$$

$$\text{PEH:} \quad V_{Load} = \left(\theta\omega y_0/\sqrt{2}\right) \times |Z_{total}| \rightarrow P_{PD} = \left(\frac{\theta\omega y_0}{\sqrt{2}}\right)^2 \frac{|Z_{total}|^2}{R_L}$$

$$\text{EMEH:} \quad I_{Load} = (\theta\omega|\hat{y}|/\sqrt{2})/|Z_{total}| \rightarrow P_{PD} = \left(\frac{\theta\omega y_0}{\sqrt{2}}\right)^2 \times \frac{R_L}{|Z_{total}|^2}$$

We can express this in a unified manner using the same set of dimensionless parameters as before:

$$P_{PD} = \left(\frac{\theta\omega y_0}{\sqrt{2}}\right)^2 \left(\frac{\xi_C}{\sqrt{\gamma^2 + (\xi_C + \xi_E)^2}}\right)^2 \frac{1}{\xi_C \omega_N B} \tag{S8}$$

For comparability with the case of inertial load, we can extract the same reference power as mentioned previously, which gives:

$$\bar{P}_{PD} = \frac{2\zeta^2}{\gamma^2} \frac{\alpha\beta\xi_C}{(1 + (\beta\gamma)^2)}$$

Efficiency under inertial load

To derive the efficiency under inertial load we first need to define the input power. We use the definition from Yang et.al. [1], which gives:

$$P_{IL\_In} = F \times \dot{y} = \ m(\ddot{x} + \ddot{y})\dot{y}$$

$$P_{IL\_In\_Avg} = \frac{1}{2} m|\ddot{x}||\dot{y}| \times \sin(\emptyset_x)$$

$\emptyset_x$ is the phase difference between x and y. Combined with Equation (9) we can define the efficiency as:

$$\Gamma = \frac{P_{IL\_avg}}{P_{IL\_In\_Avg}} = \frac{A|\hat{f}|^2}{2\theta^2} \frac{1}{\frac{1}{2} m|\ddot{x}||\dot{y}| \times \sin(\emptyset_x)} = \frac{A|\hat{f}|^2}{\theta^2 m|\ddot{x}||\dot{y}| \times \sin(\emptyset_x)} = \frac{A|\hat{f}|^2}{\theta^2 m|-\omega^2 \hat{x}||i\omega\hat{y}| \times \sin(\emptyset_x)}$$

If we rearrange Equation (S2) as,

$$\frac{1}{i\omega(1 + i\,\beta\gamma)} = \frac{A\hat{f}}{(-\omega^2\hat{x})\theta^2} \tag{S9}$$

and factor in $\hat{f}/(im\omega\hat{y})$ and $1/\sin(\emptyset_x)$ on both sides we see that we have the expression for efficiency to the right if we just take the magnitude. From Equation (S1) and Equation (S2), we find that:

$$\frac{\hat{f}}{im\omega\hat{y}} = i\omega \frac{\alpha}{((1 + i\varepsilon)(1 + i\,\beta\gamma) + \alpha)} \tag{S10}$$

To derive $1/\sin(\emptyset_x)$ we begin by determining the phase difference, $\emptyset_x$, between x and y, which is equal to the phase angle of $\hat{y}/\hat{x}$, defined by $\tan^{-1}\left(-\frac{Im(\hat{y}/\hat{x})}{Re(\hat{y}/\hat{x})}\right)$. From Equation (S4) we can directly see that:

$$\emptyset_x = \tan^{-1}\left(\frac{\alpha + 1 + (\beta\gamma)^2}{\beta\gamma\alpha - \varepsilon(1 + (\beta\gamma)^2)}\right) \tag{S11}$$

We can use the definition $\sin(\arctan(\vartheta)) = \frac{\vartheta}{\sqrt{1+\vartheta^2}}$, to rewrite $1/\sin(\emptyset_x)$ as:

$$1/\sin(\emptyset_x) = \frac{\sqrt{\left(\beta\gamma\alpha - \varepsilon(1 + (\beta\gamma)^2)\right)^2 + (\alpha + 1 + (\beta\gamma)^2)^2}}{\alpha + 1 + \beta^2} \tag{S12}$$

Applying Equation (S10) to Equation (S9) and taking the magnitude results in:

$$\frac{A|\hat{f}|^2}{\theta^2 m|-\omega^2\hat{x}||i\omega\hat{y}|} = \left|\frac{\alpha}{(1 + i\,\beta\gamma)((1 + i\varepsilon)(1 + i\,\beta\gamma) + \alpha)}\right| = \frac{\alpha}{\sqrt{(1 + (\beta\gamma)^2)((1 + \alpha - \beta\gamma\varepsilon)^2 + (\varepsilon + \beta\gamma)^2)}}$$

By applying Equation (S12) we arrive at the expression:

$$\frac{P_{IL\_avg}}{P_{IL\_In\_Avg}} = \frac{\alpha\beta\xi_C}{1 + (\beta\gamma)^2 + \alpha}$$

Efficiency under prescribed displacement

In the same way as for the inertial load case, we use the definition of input power as $P_{In} = F \times \dot{y}$, which for the case of prescribed displacement is equivalent to $F \times \dot{x}$. Using Equation (1) we can the define the input power as:

$$P_{PD\_In} = (m\ddot{x} + C_M\dot{x} + K_M x + f)\dot{x} \tag{S13}$$

As x is known we assume it is a simple harmonic function $x = x_0 \sin(\omega t)$. To derive P_{PD_In} we must first derive the instantaneous value of f. In the frequency domain, using Equation (S2), we can express $\hat{f}$ as Equation (S14).

$$\hat{f} = x_0 \frac{\frac{\sqrt{2\pi}}{2}\big(\delta(\omega - a) - \delta(\omega + a)\big)\omega\theta_E\theta_F(A - i\omega B)}{(A^2 + \omega^2 B^2)} \tag{S14}$$

From which the inverse Fourier transform gives:

$$f = x_0 \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{\frac{\sqrt{2\pi}}{2}\big(\delta(\omega - a) - \delta(\omega + a)\big)\omega\theta_E\theta_F(A - i\omega B)}{(A^2 + \omega^2 B^2)} e^{i\omega t}\ d\omega$$

$$f = \frac{\theta^2 x_0 \omega}{A^2 + (\omega B)^2}(A\cos(\omega t) + B\omega \sin(\omega t)) \tag{S15}$$

Applying Equation (S15) to Equation (S13), and using $x = x_0 \sin(\omega t)$, we can write:

$$P_{PD\_In} = x_0\omega\cos(\omega t)\,[-m x_0\omega^2 \sin(\omega t) + C_M x_0\omega\cos(\omega t) + k_M x_0 \sin(\omega t) + \frac{\theta^2 x_0\omega}{A^2+(\omega B)^2}(A\cos(\omega t) + B\omega\sin(\omega t))]$$

and simplify this to:

$$\frac{P_{PD_{In}}}{\frac{(y_0\omega^2 m)^2}{C_M}} = \frac{2\zeta^2}{\gamma^2}\left[\left(1 + \frac{\alpha}{1 + (\beta\gamma)^2}\right) + \left(1 + \frac{\alpha}{1 + (\beta\gamma)^2}\right)\sin\left(2\omega t + \frac{\pi}{2}\right) + \left(\frac{\alpha}{1 + (\beta\gamma)^2} - A\varepsilon\right)\beta\gamma\sin(2\omega t)\right]$$

From which the average becomes:

$$\frac{P_{PD\_In\_Avg}}{\frac{\left(y_0\omega^2 m\right)^2}{C_M}} = \frac{2\zeta^2}{\gamma^2}\left(1 + \frac{\alpha}{(1 + (\beta\gamma)^2)}\right)$$

Together with Equation (S8), the input to output power efficiency, under prescribed displacement, is then given by:

$$\frac{P_{PD\_avg}}{P_{PD\_In\_Avg}} = \frac{\alpha\beta\xi_C}{1 + (\beta\gamma)^2 + \alpha}$$