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A Wavelet Multiscale Mathematical Model for Quality of Life Index Measuring

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Abstract: The present paper is concerned with the study of the quality of life index. Such an index has become an important index for measuring the well-being of individuals. However, the quality of life index is always subjective, intangible, and often hard to quantify with precision due to the lack of quantitative models. The main goal of the present paper is thus to propose a mathematical, quantitative model for the measurement of a quality of life index. The main novelty is firstly the construction of a wavelet dynamic multiscale model to quantify and investigate the effect of time scale on the quality of life index measuring. The proposed procedure is acted empirically on a sample corresponding to Saudi Arabia as a case study during the period from 2003 to 2020 as part of the 2030 vision plan. Saudi Arabia has implemented the so-called 2030 vision plan where quality of life improvement is one of the main goals. The findings show that wavelets are capable of localizing the time-wise behavior of the index contrarily to classical studies, which estimate a global view of the index. Moreover, the study shows the link between the quality of life behavior and many other indices.

Keywords: wavelets; multiscale; quality of life; mathematical models

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1. Introduction

Quality of life is nowadays a major and important concept in the evaluation of modern well-being. The major drawback in such an index is due to its subjectivity and intangibility compared to the standard of living concepts and factors. To date, it is hard to measure such an index in a complete way that may be applied in all situations, such as healthy well-being and economic well-being, etc. [1–3]. This is due to the fact that this index is often hard to quantify, as the factors that may be involved and that affect the overall quality of life vary by people's lifestyles and their personal preferences [4–10].

In the last few years, the Kingdom of Saudi Arabia represented by its Council of Economic Affairs and Development has implemented the so-called 2030 vision, which is a block of programs and plans of strategic importance for the government of Saudi Arabia. One of the strategic objectives to be established in such a vision is the quality of life programs, which aim essentially to make the kingdom one of the most suitable destination for both its citizens and also for foreigners and residents as a top living place, and thus a top and/or high quality of life index (see [11]).

Regardless of these factors, this measure plays an essential part in the financial decisions in everyone's lives. The purpose of this paper is to understand the topics or the factors and/or indices that seek to improve the quality of life. We aim essentially to study the quality of life and its measurements in view of KSA 2030 vision and policy to conclude about the effects of life quality on the economic situation in KSA, the general situation forecasting in Saudi Arabia as the best destination as implemented and aimed in the 2030 vision plans, and the possibility to reach such a goal in view of quantitative indices.

Our basic idea reposes on the application of wavelet theory as a new mathematical tool that is not widely applied in social sciences. The application of wavelet analysis on the data results in a decomposition into many components. A first component reflects the global shape of the data, and other components containing the noise, and it describes more adequately the fluctuations hidden in the data according to different horizons known as levels of decomposition or also time scales. Wavelets indeed posses some special mathematical requirements useful in data representation. For example, for non-stationary data, wavelets are efficient descriptors for trend, volatility clustering, and variance due to their ability to take into account both frequency and time in the multi-resolution [12–19].

In the literature, there are also other models that may be applied in social science generally and involving multiscaling laws such as multiscale time series. In this context, the time-scaling concept permits assessing the dynamical complexity of the time series across multiple temporal scales. To understand and describe well the time-scaling behavior in time series, many methods have been applied such as entropy, fractals and multifractals, chaos, fluctuation analysis, and Markov chains.

In [20], an entropy concept is applied to investigate the time-scale behavior of time series. In [21], a multiscale approach is developed to localize the peaks in time series issued essentially from neuronal recordings (see also [22,23]).

Adam and Oelschlger [24] applied hidden Markov models for modeling multiscale data issued from economic variables where the time-scale behavior is always observable at different temporal resolutions. Variables are strongly related to the ones applied in the present work such as GDP, for which the time-scaling aspect is observed on yearly, quarterly, and also monthly bases. In [25], Baranowski and Fryzlewicz developed a multiscale method to model time-series autoregression, where the linear regressors include features of paths living on multiple time scales.

Multifractal modeling of the scaling law such as time-scale and multiscale time series is also well known in data analysis, especially in biosignals and financial time series, where the data hides a complex structure. See, for example, [26,27]. Further readings on multiscale time series may be found in [28–32].

Due to many reasons—such as the availability of data in both web sites designated for it or the social media texts (which are in the heart of our study) to estimate and show their influence on the different situations of the countries, economic, political, or also social, and in order to possibly compare with existing ideas in measuring quality of life index—we proposed a mathematical multiscale in time wavelet model based on the following factors or variables to be estimated. The first variable is due to the Gross Domestic Product (GDP, in Millions of Saudi Riyals), the second is the Gross National Income (percentage of GDP), and the third is estimated by the Gross Savings (percentage of GDP). The model will include also the Unemployment rate, the Energy Consumption per Capita, the Death Rate, the Life Expectancy, and the Education Index (as percentage of GDP).

The main difference with the last existing model due to [4] is the involvement of a time-scale procedure, which permits to detect the time-wise or the time-scale behavior of the quality of life index as well as other variables, instead of the one global index computed in [4]. The new procedure will show clearly the movement of the market and generally the situation of the country and to also forecast its ability to be really a suitable future destination as a top living city.

The sample of data to be applied will be based on annual values of the variables raised above traded on the period 2003 to 2020. This is a remarkable period characterized by many political, economic and financial movements such as Qatar embargo, Yemen war, NEOM project, 2030 KSA vision, the Arab spring, and lastly the COVID-19 pandemic. All these factors have surely strong effects and thus make the findings in the present work a good basis for understanding current and future situations and may be thus bases of future decisions.

One of the main variables applied in worldwide regions when estimating many indexes such as the QoL is the poverty rate. However, in many cases, such a variable is not provided, despite its existence in the real daily life in the society subject of study. In Saudi Arabia, for example, and which is the case study in our present paper, the index of poverty or the rate is nowhere provided, except for some discussions in many global and general reports which are not provided by the concerned authorities.

This may make the evaluation of many models such as the one reviewed previously, and which has been mainly simulated in Romania, unable to represent other cases. This leads us to modify the existing model by including other indexes or variables, which may involve even implicitly the lost variables from the existing models such as poverty in our case. Indeed, our model to be developed involves the Education Index instead, which is strongly related in fact to poverty. We know that one of the main goals of the worldwide authorities, especially UNESCO, is to enable all humankind, especially poor humans, to be educated at least to some level.

Recall that the estimated indices especially in social sciences are very rarely taking into account the time scale, and they are estimated instead relatively to the entire periods of the study. This means in time scale interpreting that the indices are assumed to be stable relatively to a certain scale. While in reality, the variance with the society such as the market, money, and investments varies over time, and therefore, there should be some statistical issues to be considered related to errors in the estimates of individual well-being, securities, and their instabilities.

In this context, it is worth noticing that a major problem in data processing or data analysis that is always confronted by researchers and analysts is related to the availability and accuracy of the data. These are known in data analysis as missing, fuzzy, uncertain, and also false data. To overcome these problems, many tools have been developed by researchers to adjust the data and thus to build complete samples to be used adequately.

In our present situation, this problem is present strongly, as our data are in the majority collected from media such as web sites and social media sites, where the possibility of errors is somehow large and thus, there is a necessity for adjustments. Even in many governmental sites, the authorities often did not provide complete data. This is of course generally related to many cases when the data, for example, are related to the policy and/or the secrets of the countries, and also when the data may yield unstable movements.

In our case, to build a complete sample, and thus to reconstruct the missing and the uncertain parts, we applied the wavelet method, which is able to reconstruct our short time series on an arbitrary set of backwards and/or forwards (past and/or future, prior and/or post) values. Recall that to obtain good reconstructions in the case of statistical series, it is needed to have a significantly long past interval for training. This fact may not be satisfied in general situations such as the present one. All the existing procedures considered only the reconstruction of future values by means of preceding ones, i.e., just a forecasting procedure. See, for example, [18,19,33] and the references therein.

In our case, some values that are missing are unfortunately at the beginning of the period. We re-applied a modified version of the method developed in [17,33]. Our new idea consists first of applying a backward–forward method to reconstruct missing data. The method is principally characterized by the non-necessity of testing it on the detail parts components of the series nor its wavelet coefficients. This is essentially due to the fact that we use few values of the series, leading to short sub-samples, and next act the prediction on the sub-samples. The most positive point in the method is the fact that it requires only computing the values of the source scaling function and the associated wavelet on a suitable grid, the dyadic or the integer grid in the supports of the mother and father wavelets. Such values are well estimated in [34], which motivated the application of Daubechies wavelets in the empirical part.

The present paper will proceed as follows. Section 2 is concerned with an overview on the quality of life and the related concepts. Section 3 will review briefly the last known models that proposed a quantitative measure for the quality of life. In Section 4, we will develop our idea for a quantitative mathematical model to measure a quality of life index. In a first step, an adjustment of the data based on wavelets will be provided. Next, a multiscale wavelet model will be developed involving the time scale and wavelet decomposition into the model. Section 5 is devoted to the empirical results due to the quantitative mathematical model developed in Section 4. The sample in the study is based on Saudi Arabia as a case study in the framework of its 2030 vision plan. Section 5 is devoted to the discussion and interpretations of our empirical results. Section 6 is a final conclusion, and Section 7 is an appendix concerned with the brief review of the wavelet toolkit.

2. Quality of Life Brief Overview

2.1. Different Measurements of the Quality of Life

To understand and/or conclude whether a person or society has reached some level of life quality, several measurements have been defined by researchers. Some of them are subjective; others are qualitative. However, few of them are quantitative. Some researchers considered personal well-being aspects such as emotional experience, joy, stress, sadness, anger, and affection as factors to measure the quality of life. Others suggest to measure or compare to some scale or threshold to conclude that the person is living well. The scale may be related to income, fertility, productivity, and also to the education level. Health care, wealth, and materialistic goods are also factors that are taken into consideration in many cases. In developing countries, for example, and/or poor countries, we know that health care and social securities are not well supported by the states (see [1–3,35,36]).

However, in these countries and poor societies, a great part of the community relates the quality of life to other even simple actors such as global family happiness and living in a family, especially in a safe situation. We know that in many countries such as India, south African countries, South America, and even in the USA, the overage of crime is high. This makes life unsafe. Sociologists considered the so-called physical quality of life index based on basic literacy, infant mortality, and life expectancy and sometimes a country's ecological footprint as an indicator [8,9,37–42].

In political economics to measure the quality of life, researchers considered liveability, which takes into account the place of birth, the place of childhood, family situation, and stability such as the rate of divorce in the society or satisfaction with the infrastructure. Other factors may be also included such as government taxation, government aid, sponsorship programs, freedom and political rights, the rate of employment, and racism. Readers may refer here to [43–46].

2.2. Quality of Life and Marketing

One of the important concepts that may be related or regarded simultaneously with quality of life is marketing. Most of the relevant links between marketing and quality of life appear in theory only from the last two decades. Indeed, nowadays, governments, organizations, and individuals are paying more interest in how their activities impact on quality of life at the regional, national, and global levels. Factors such as global resources, environmental circumstances, politics, competition, technology, and education may have a significant impact on populations regardless of their level of development. See for instance [1,4–7,10,36,37,47,48].

A conceptualization of quality of life based on the involvement of citizens in exchanging resources to new resources or to ultimate satisfaction in a set of arenas of action may be of interest. The so-called objective and subjective approaches to measuring quality of life address different aspects of the exchange relationships and their context. Marketing at least indirectly may contribute to a higher quality of life by rendering work life possible and by providing the goods and services entering the consuming life arena. In addition, marketing may induce a negative influence by giving priority to short-term, materialistic needs (see [47,48]).

Moreover, quality of life defined as the individuals' subjective perceptions on objective conditions related to welfare and standard of living may be a main cause of decisions on consumption and purchase; therefore, marketing specialists should take it into consideration when developing the product, establishing the price, and designing the distribution system and the communicational strategy [38,39].

Another link may be due to the influence on social marketing, given that this is an area of marketing that is concerned especially with society's problems, trying to promote a responsible behavior for firms and citizens in order to increase the level of satisfaction with every aspect of life [40–42,49].

The present project aims to build a research study into quality of life and marketing by reviewing the research literature dealing with this concept and develop a set of antecedents and consequences of that relationship. The task of marketing in quality of life may be regarded as marketing practices designed to enhance the well-being of customers while preserving the well-being of the firm's other stakeholders. We think that the consequences of marketing beneficence and normal efficiency are high levels of customer well-being, customer trust and commitment, and positive corporate image and company goodwill. These factors may also be influenced by environmental factors, organizational factors, and individual factors.

2.3. The Impact on the Whole Community

- Investments will participate in decreasing the rate of unemployment.
- Investments permit the country be transferred to an industrial country that will export technology, foods, drugs, and energy more than importing.
- High-level educated individuals will improve the scientific level such as research in national universities. This will improve the rank of these universities and will give them an international aspect. It will be possible to interchange students and researchers worldwide.
- These facts will have a positive effect on the growth, the development, and thus the economic situation in the country.

More discussion and background may be found in [11,43–46] and the references therein.

3. Recent Quality of Life Mathematical Model Review

In the literature review on quality of life mathematical models, unfortunately, there are few ideas that apply really exact mathematical models to provide an exact powerful measure for such an index. This is due to the fact that quality of life may be viewed in different ways according to the field of examination. Health quality is different from economic quality. This later is in turn different from social well-being. Regarding the political situation, human rights are also factors that may be included in the evaluation of the satisfaction of people against the citizenship [3–7,10,50–53].

One of the recent models that seems to be a step ahead is due to [4] where the author has introduced a mathematical model to estimate some real-valued measurement of the QoL. The model in its original form is based on economic variables issued from knowledge and introduced in [4]. It applies to the following variables,

- x_1^t Health;
- x_2^t Family life;
- x_3^t Community life;
- x_4^t Financial situation (GDP per capita, in \$);
- x_5^t Political stability and security;
- x_6^t Climate and geography;
- x_7^t Job safety (unemployment rate);
- x_8^t Political freedom;
- x_9^t Gender equality;
- LE Life expectancy at birth;
- LEI Life expectancy index;
- EI Degree of access to education;
- $MYSI$ Education period;

- II Revenue indicator;
- $GNIpc$ Gross national income at purchasing power parity per capita.

As explained in [4], the variables $x_1^t, \ldots, x_9^t$ are assumed to be independent and reflect the mathematical objective characteristics of the quality of life, giving individuals in a society the subjective satisfaction at a given time t. Denote for the next $\overline{X}$ the arithmetic mean of $x_1^t, \ldots, x_9^t$. The mathematical model provides an index HDI relative to a period of time $[t_0, t_1]$, as an average

$$HDI_m = \frac{1}{(t_1 - t_0)^2} \int_{t_0}^{t_1} HDI(t)dt \, . \int_{t_0}^{t_1} \overline{X}(t)dt. \tag{1}$$

At an instant of time t, we have

$$HDI = \sqrt[3]{LEI \times EI \times II},$$

where

$$LEI = \frac{LE - 20}{82.3 - 2.0}, \quad EI = \frac{\sqrt{MYSI \times EYSI}}{0.951}, \quad MYSI = \frac{MYS}{13.2}, \quad EYSI = \frac{EYS}{20.6},$$

and

$$II = \frac{\ln(GNIpc) - \ln 100}{\ln(107.721) - \ln 100}.$$

More details and related discussion may be found in [5–7,52,53].

4. Development of a Multiscale Wavelet Model

In the present section, we provide details concerning the mathematical model to be investigated in the present paper. In fact, such a model is not completely new, but the new task resides in the involvement of the time-scale behavior and the use of wavelets in the construction or the adjustment of the data sample. For convenience, and for a non-mathematics community, we proposed a brief review of wavelet theory in Appendix A as the main tool in our work.

4.1. Wavelet Adjustment of Data Samples

Let (t_i, X_i), $i = 1, 2, \ldots, N$ be the statistical series, and its wavelet decomposition at a level J,

$$X^J = \sum_k a_{0,k}\varphi_{0,k} + \sum_{j=1}^{J}\sum_k d_{j,k}\psi_{j,k}. \tag{2}$$

The missing values will be categorized into two categories.

- A missing value, X_k, is situated between two segments of known values, after and before it. Thus, we chose the one with greater length for the prediction of the missing value X_k. Denote, for example, L_k the length of the greater segment $I_k = \{i_1^k, i_2^k, \ldots, i_{L_k}^k\}$.
- Consider next the truncated time series $\widetilde{X}_i$ corresponding to (t_i, X_i) $i \in I_k$, and its wavelet decomposition as in (2).
- If I_k is an after-interval to X_k, we estimate the value $X_{i_1^k - 1}$ by

$$X_{i_1^k - 1} = \sum_l a_{0,l}\varphi(i_1^k) + \sum_{j=1}^{J}\sum_l d_{j,l}\psi(2^j(i_1^k) - l). \tag{3}$$

- If I_k is a before-interval to the missing value, we estimate the value $X_{i^k_{L_k}+1}$ by

$$X_{i^k_{L_k}+1} = \sum_l a_{0,l}\varphi(i^k_{L_k} + 1 - l) + \sum_{j=1}^{J}\sum_l d_{j,l}\psi(2^j(i^k_{L_k} + 1) - l). \tag{4}$$

- Whenever the after and before intervals have the same length, we take the mean value of the two predicted ones.
- In the case where many successive values are missing, we take the extremities of the missing segments as starting points to be reconstructed.
- Finally, each predicted value is added to the series, and the new series is reconsidered for the next step.

We recall again that the process necessitates knowing the values of ψ on the dyadic grid $\{2^j(N+1) - l, l\}$ and those of φ on the integer grid $\{N+1-l, l\}$ in the supports.

4.2. The Multiscale Mathematical Model

Notice that in model (1), we get a global index relative to a whole period, which may not reflect the real situation during localized times, as it did not take into consideration the time scale. To improve the model and exploit the time scale's role and influence, we proposed to implement a multiscale QoL index by considering multiple time periods. We estimate a relative HDI according to a time scale $j \in \mathbb{N}$, for which

- The scale 0 corresponds to a period of 1 year dynamics;
- The scale 1 corresponds to a period of 2 years dynamics;
- The scale 2 corresponds to a period of 4 years dynamics;
- The scale $j \geq 0$ corresponds to a period of $2^j = 2^{j+1} - 2^j$ years dynamics.

The maximal value of the scale j will be J_{max} depending on the size of the data sample or equivalently to the period of the empirical study.

The idea consists of computing for each scale j a corresponding HDI index, which will be called the j-level HDI index as

$$HDI_j = \frac{1}{2^{2j}} \int_{2^j}^{2^{j+1}} HDI(t)dt \,.\, \int_{2^j}^{2^{j+1}} \overline{X}(t)dt. \tag{5}$$

5. Results

In the present section, we aim to apply the multiscale mathematical model developed above to discuss the situation of the QoL measure in Saudi Arabia, based on special data extracted from the social media, which reflects somehow the opinion of both nationals, non-national residents, as well other interested peoples to such a region such as investors from the economic and energy sectors, for example. We essentially focused on the period 2003–2020, resulting in 18 years. As a consequence, the maximum level will be fixed to $J_{max} = 4$. Our model evaluation will be based on the following variables,

- V_1: Gross Domestic Product (GDP, in Millions of Saudi Riyals SARs);
- V_2: Gross National Income (percentage of GDP);
- V_3: Gross Savings (percentage of GDP);
- V_4: Unemployment Rate;
- V_5: Energy Consumption per Capita;
- V_6: Death Rate;
- V_7: Education Index (as percentage of GDP);
- V_8: Life Expectancy.

In Table 1 below, we provide the values of the variables gathered. Some of them are compared to existing values already provided in authorized web sites specialized in such data statistics. The red values in Table 1 are the missing ones estimated by means of our preceding procedure developed in Section 4.1 using Daubechies wavelet *Db*8 as a mother wavelet [34].

Table 1. Variables and their values: (black) real values created by the authors based on data provided on different websites, (red) missing values reconstructed by wavelet method as in Section 4.1.

Year	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8
2003	809,279	11.242	34.816	5.56	6.456	3.59	0.580	73.01
2004	970,283	7.958	42.139	5.82	6.398	3.57	0.594	73.10
2005	1,230,771	5.574	48.653	6.05	6.801	3.56	0.604	73.19
2006	1,411,491	2.788	48.505	6.25	6.995	3.54	0.619	73.28
2007	1,558,827	1.847	48.927	5.73	7.022	3.53	0.634	73.37
2008	1,949,238	6.250	52.752	5.08	7.270	3.51	0.649	73.46
2009	1,609,117	−2.060	36.598	5.38	7.506	3.49	0.664	73.64
2010	1,980,777	5.040	43.563	5.55	7.962	3.46	0.691	73.83
2011	2,517,146	9.997	50.590	5.77	8.004	3.44	0.719	74.01
2012	2,759,906	5.411	48.928	5.52	8.534	3.42	0.747	74.20
2013	2,799,927	2.699	44.608	5.57	8.871	3.41	0.770	74.38
2014	2,836,314	3.652	38.924	5.72	9.267	3.41	0.787	74.48
2015	2,453,512	4.106	25.502	5.59	9.485	3.41	0.802	74.59
2016	2,418,509	1.670	27.235	5.65	9.333	3.43	0.803	74.69
2017	2,582,198	−0.741	30.371	5.89	9.151	3.45	0.784	74.80
2018	3,062,170	2.434	33.227	6.04	8.954	3.47	0.789	74.90
2019	3,013,561	0.331	33.597	6.13	8.434	3.51	0.789	75.06
2020	2,637,629	−4.106	25.255	8.22	8.263	3.5	0.854	75.22

Next, we provide the wavelet multiscale analysis of each variable at the level 4, using herewith the well-known Daubechies wavelet Db8 (See [34]). Tables 2–9 represent the four components due to the four-level wavelet decomposition. The first column in each table represents the approximation component A_4 relative to each variable, and the second, third, fourth, and fifth columns represent, respectively, the detail components D_1, D_2, D_3, and D_4 already for each variable.

Table 2. Four-level wavelet decomposition of V_1.

A_4	D_1	D_2	D_3	D_4
13.6398	−0.0359	−0.1138	−0.0672	−0.3505
13.7696	0.0158	−0.0194	−0.1024	−0.2980
13.9706	0.0526	0.0865	−0.1073	−0.2245
14.1841	−0.0240	0.1539	−0.0883	−0.1321
14.3258	−0.0664	0.1181	−0.0540	−0.0309
14.3779	0.1050	−0.0025	−0.0151	0.0561
14.3789	−0.0877	−0.1416	0.0164	0.1147
14.4705	0.0285	−0.1568	0.0446	0.1411
14.6995	0.0391	−0.0022	0.0693	0.1368
14.8639	−0.0332	0.1117	0.0815	0.1195
14.8761	−0.0310	0.0862	0.0788	0.1028
14.8139	0.0441	0.0099	0.0591	0.0803
14.7116	0.0015	−0.0846	0.0244	0.0535
14.6949	0.0037	−0.0927	−0.0091	0.0292
14.8253	−0.0611	0.0372	−0.0310	0.0085
14.9016	0.0330	0.0993	−0.0437	−0.0011
14.8549	0.0638	0.0241	−0.0467	0.0009
14.8311	−0.0457	−0.0238	−0.0424	−0.0005

Table 3. Four-level wavelet decomposition of V_2.

A_4	D_1	D_2	D_3	D_4
11.0367	0.2053	2.0380	2.3487	0.3666
8.6928	−0.7348	−0.1612	2.3803	0.3879
4.7299	0.8441	−2.7730	1.2976	0.3878
2.4961	0.2919	−2.8414	−0.5104	0.3655
3.4032	−1.5562	0.3882	−2.4143	0.3400
3.3979	2.8521	1.7395	−3.3599	0.3624
1.8750	−3.9350	−0.1305	−2.6500	0.4590
3.3736	1.6664	−0.0957	−0.8784	0.6298
7.3882	2.6088	2.0807	1.2229	0.8588
7.8662	−2.4552	1.4130	2.6468	1.0865
4.0935	−1.3945	−1.9316	2.5602	1.2615
1.9360	1.7160	−2.7348	1.5977	1.3796
2.7195	1.3865	−0.2637	0.3630	1.4143
2.7536	−1.0836	1.3615	−0.6843	1.3232
1.6384	−2.3794	1.1667	−0.9664	1.0890
0.3848	2.0492	0.6175	−0.9308	0.7000
−1.3631	1.6941	−0.3359	−0.9063	0.1747
−2.5174	−1.5886	−0.8588	−0.7252	−0.4095

Table 4. Four-level wavelet decomposition of V_3.

A_4	D_1	D_2	D_3	D_4
37.0951	−2.2791	−4.7882	−3.3139	−0.9225
40.8820	1.2570	−1.2043	−3.9412	0.3596
45.8691	2.7839	3.0641	−3.9847	1.7042
50.3946	−1.8896	6.2395	−3.2464	3.0227
51.3069	−2.3799	5.3467	−1.7768	4.1736
47.6891	5.0629	0.1665	−0.0998	4.9549
41.8344	−5.2364	−6.5255	1.4147	5.2298
41.5055	2.0575	−6.8480	2.6718	4.9631
48.5453	2.0447	1.2520	3.4202	4.1864
51.2447	−2.3167	6.1430	3.4182	3.0632
45.0606	−0.4526	3.1558	2.6198	1.7588
36.3746	2.5494	−1.4576	1.0723	0.3260
28.0662	−2.5642	−5.3802	−0.8362	−1.0869
25.5446	1.6904	−4.4002	−2.1723	−2.2382
31.4484	−1.0774	3.3774	−2.3684	−2.9759
34.3101	−1.0831	6.6704	−1.6996	−3.2190
29.7973	3.7997	1.5103	−0.5234	−2.9828
27.1184	−1.8634	−1.9759	0.4517	−2.4919

Table 5. Four-level wavelet decomposition of V_4.

A_4	D_1	D_2	D_3	D_4
5.5492	0.0108	−0.3690	0.1172	0.3401
5.8074	0.0126	−0.0491	0.0873	0.2660
6.1869	−0.1369	0.4061	0.0523	0.1718
6.1765	0.0735	0.4915	0.0045	0.0595
5.5848	0.1452	0.0031	−0.0473	−0.0655
5.2344	−0.1544	−0.2899	−0.0586	−0.1921
5.4054	−0.0254	−0.1292	−0.0141	−0.3117
5.5504	−0.0004	−0.0428	0.0625	−0.4191
5.5898	0.1802	−0.0772	0.1350	−0.5100
5.6454	−0.1254	−0.0351	0.1282	−0.5845
5.6183	−0.0483	0.0229	0.0020	−0.6401
5.6269	0.0931	0.1455	−0.1822	−0.6659
5.7274	−0.1374	0.3050	−0.3481	−0.6519
5.6632	−0.0132	0.1288	−0.3870	−0.5890
5.5373	0.3527	−0.3559	−0.2270	−0.4706
5.9918	0.0482	−0.4165	0.0472	−0.3021
7.0382	−0.9082	0.0746	0.3288	−0.0918
7.6892	0.5308	0.2793	0.4812	0.1488

Table 6. Four-level wavelet decomposition of V_5.

A_4	D_1	D_2	D_3	D_4
6.3741	0.0819	−0.0981	0.1548	−0.9344
6.5154	−0.1174	0.0106	0.0725	−0.8514
6.7721	0.0289	0.1355	0.0173	−0.7137
6.9714	0.0236	0.1405	−0.0470	−0.5213
7.0470	−0.0250	−0.0114	−0.1315	−0.2902
7.2317	0.0383	−0.0849	−0.1913	−0.0633
7.5764	−0.0704	−0.0120	−0.2127	0.1325
7.8657	0.0963	−0.0106	−0.1813	0.2869
8.0967	−0.0927	−0.0913	−0.0862	0.3980
8.4591	0.0749	−0.0577	0.0388	0.4924
8.9288	−0.0578	0.0790	0.1655	0.5856
9.2730	−0.0060	0.1262	0.2832	0.6528
9.4043	0.0807	0.0523	0.3541	0.6796
9.3832	−0.0502	−0.0135	0.3221	0.6571
9.2003	−0.0493	−0.0451	0.1676	0.5726
8.8634	0.0906	−0.0825	−0.0746	0.4401
8.4889	−0.0549	−0.0892	−0.3328	0.2687
8.2506	0.0124	−0.0177	−0.4704	0.0502

Table 7. Four-level wavelet decomposition of V_6.

A_4	D_1	D_2	D_3	D_4
3.5860	0.0040	0.0065	−0.0023	0.0700
3.5754	−0.0054	0.0013	0.0038	0.0589
3.5577	0.0023	−0.0063	0.0101	0.0436
3.5407	−0.0007	−0.0086	0.0162	0.0246
3.5288	0.0012	−0.0017	0.0209	0.0035
3.5121	−0.0021	0.0039	0.0207	−0.0158
3.4874	0.0026	0.0038	0.0141	−0.0307
3.4623	−0.0023	0.0031	0.0026	−0.0406
3.4390	0.0010	0.0017	−0.0113	−0.0452
3.4203	−0.0003	−0.0006	−0.0224	−0.0473
3.4102	−0.0002	−0.0016	−0.0267	−0.0485
3.4078	0.0022	−0.0031	−0.0253	−0.0474
3.4128	−0.0028	−0.0049	−0.0191	−0.0435
3.4281	0.0019	−0.0027	−0.0095	−0.0372
3.4513	−0.0013	0.0032	0.0012	−0.0282
3.4756	−0.0056	0.0071	0.0131	−0.0179
3.4955	0.0145	0.0061	0.0241	−0.0068
3.5077	−0.0077	0.0012	0.0284	0.0066

Table 8. Four-level wavelet decomposition of V_7.

A_4	D_1	D_2	D_3	D_4
0.5827	−0.0027	−0.0051	0.0128	−0.0564
0.5908	0.0032	0.0004	0.0064	−0.0523
0.6045	−0.0005	0.0056	0.0004	−0.0450
0.6199	−0.0009	0.0086	−0.0068	−0.0343
0.6336	0.0004	0.0066	−0.0144	−0.0216
0.6481	0.0009	0.0006	−0.0174	−0.0100
0.6658	−0.0018	−0.0066	−0.0138	−0.0014
0.6895	0.0015	−0.0101	−0.0049	0.0035
0.7196	−0.0006	−0.0072	0.0071	0.0051
0.7472	−0.0002	−0.0027	0.0154	0.0058
0.7682	0.0018	0.0021	0.0155	0.0073
0.7888	−0.0018	0.0120	0.0105	0.0086
0.8050	−0.0030	0.0211	0.0031	0.0093
0.8002	0.0028	0.0092	−0.0032	0.0098
0.7798	0.0042	−0.0209	−0.0048	0.0097
0.7826	0.0064	−0.0286	−0.0046	0.0100
0.8167	−0.0277	−0.0039	−0.0044	0.0108
0.8391	0.0149	0.0119	−0.0039	0.0103

Table 9. Four-level wavelet decomposition of V_8.

A_4	D_1	D_2	D_3	D_4
73.0235	−0.0135	−0.0375	0.0975	−0.3610
73.0854	0.0146	0.0010	0.0649	−0.3520
73.1909	−0.0009	0.0442	0.0323	−0.3217
73.2898	−0.0098	0.0527	−0.0116	−0.2692
73.3579	0.0121	0.0091	−0.0639	−0.2016
73.4667	−0.0067	−0.0234	−0.0914	−0.1395
73.6397	0.0003	−0.0189	−0.0813	−0.0951
73.8266	0.0034	−0.0166	−0.0399	−0.0710
74.0156	−0.0056	−0.0171	0.0210	−0.0656
74.2021	−0.0021	−0.0032	0.0628	−0.0616
74.3624	0.0176	0.0149	0.0602	−0.0472
74.4935	−0.0135	0.0268	0.0330	−0.0300
74.5991	−0.0091	0.0261	−0.0029	−0.0118
74.6850	0.0050	−0.0000	−0.0278	0.0092
74.7759	0.0241	−0.0399	−0.0223	0.0306
74.9135	−0.0135	−0.0373	−0.0056	0.0578
75.0896	−0.0296	0.0159	0.0072	0.0908
75.2003	0.0197	0.0346	0.0133	0.1159

As we know, it is generally not easy to see accurately the variations (fluctuations, increase, decrease) of these variables from the tables solely. So, for further understanding the behavior of these variables according to the time scale, we reproduced the variables V_i of Table 1 to illustrate more their variability of their behaviors. This will yield in turn and among other interpretations a good and easy reading and description of these variables according to the time scale. The following graphs illustrate the variables with their trends and dynamics or fluctuations represented by the four-level wavelet decomposition (see Figures 1–8). The strong fitting between each variable and its approximation is clearly noticed, which is always and already confirmed in wavelet theory.

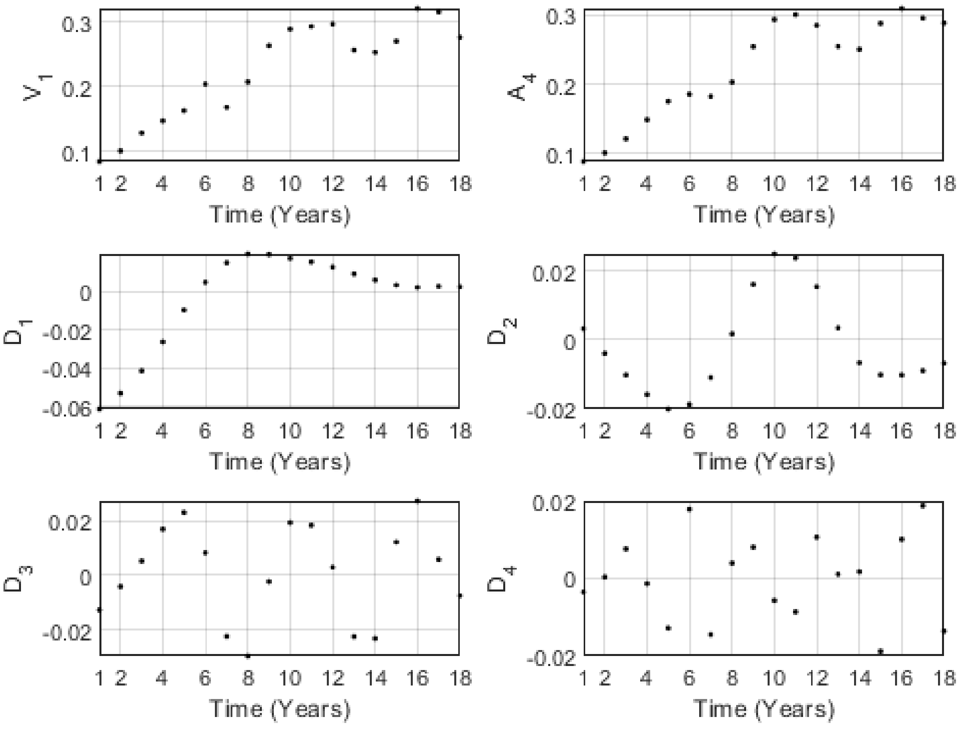

Figure 1. The wavelet decomposition of the variable V_1 at the level 4.

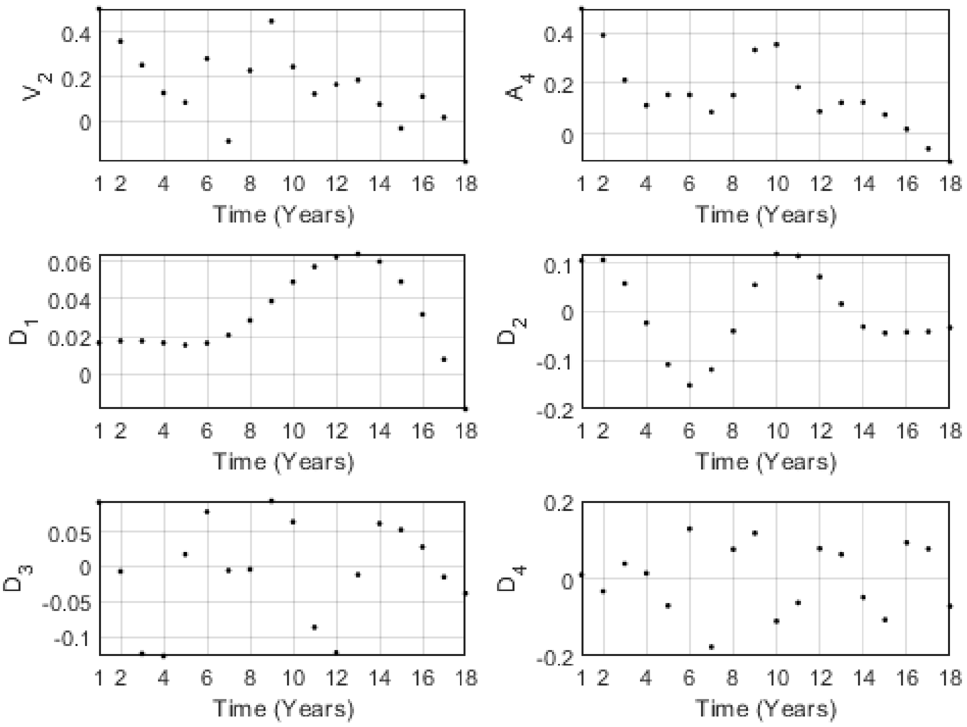

Figure 2. The wavelet decomposition of the variable V_2 at the level 4.

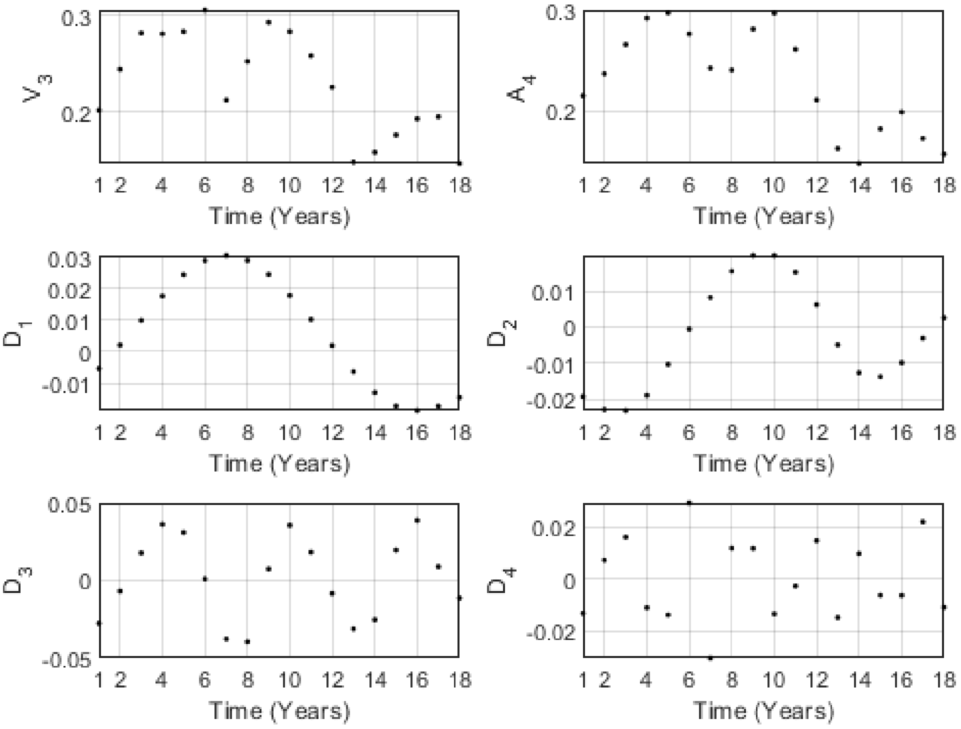

Figure 3. The wavelet decomposition of the variable V_3 at the level 4.

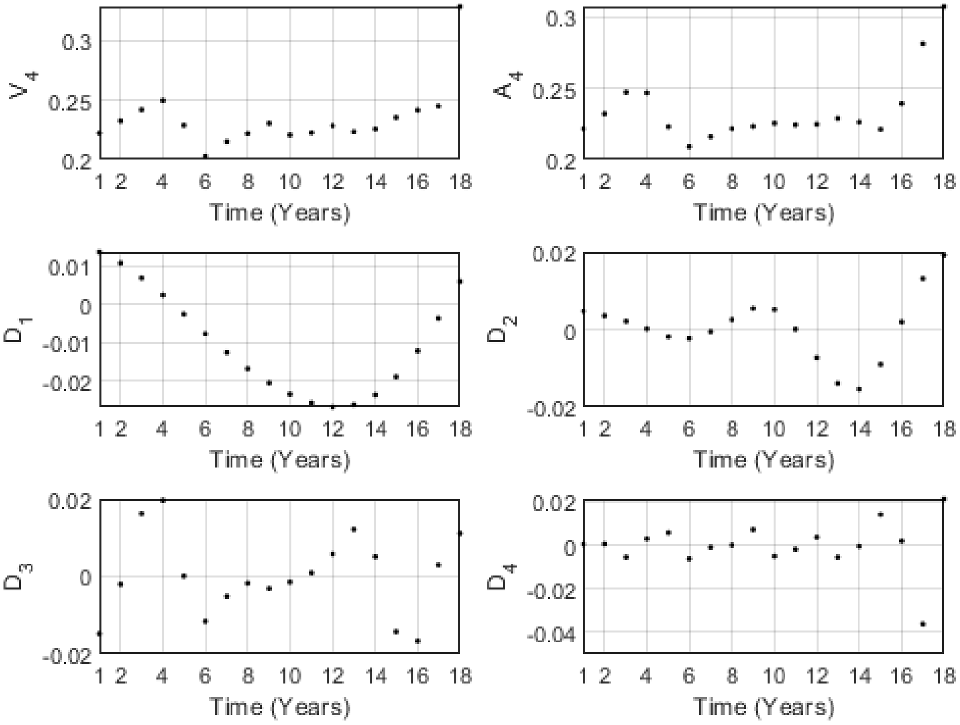

Figure 4. The wavelet decomposition of the variable V_4 at the level 4.

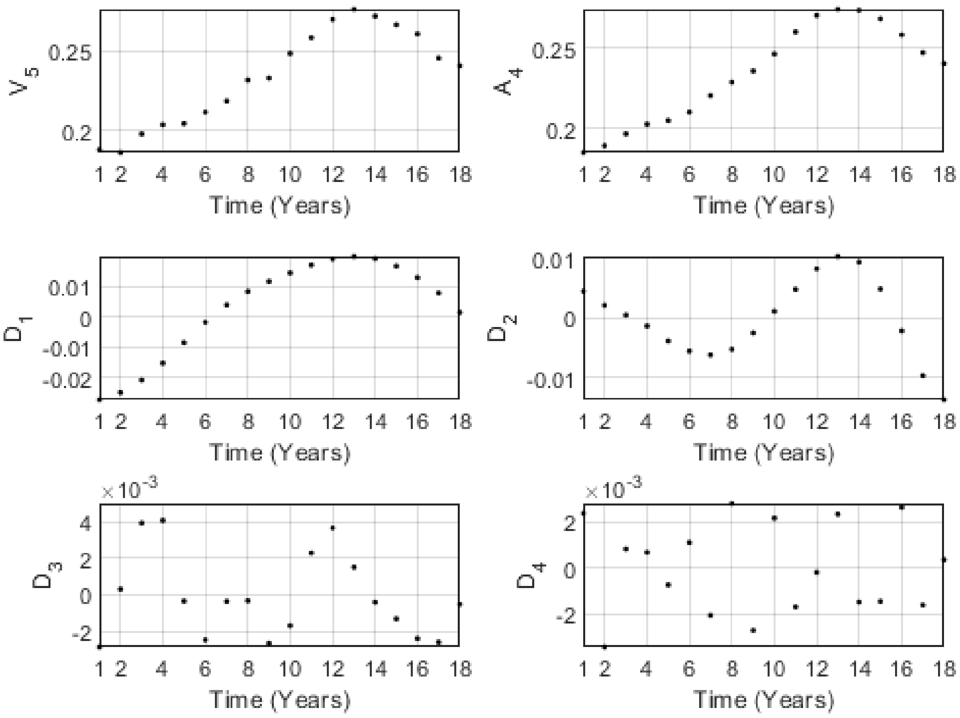

Figure 5. The wavelet decomposition of the variable V_5 at the level 4.

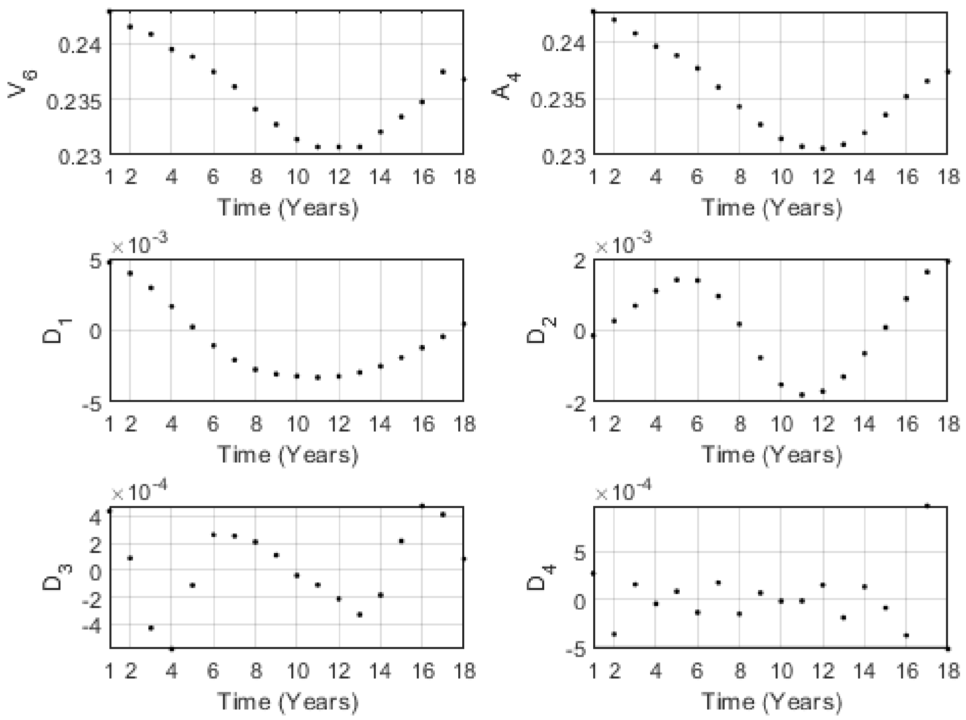

Figure 6. The wavelet decomposition of the variable V_6 at the level 4.

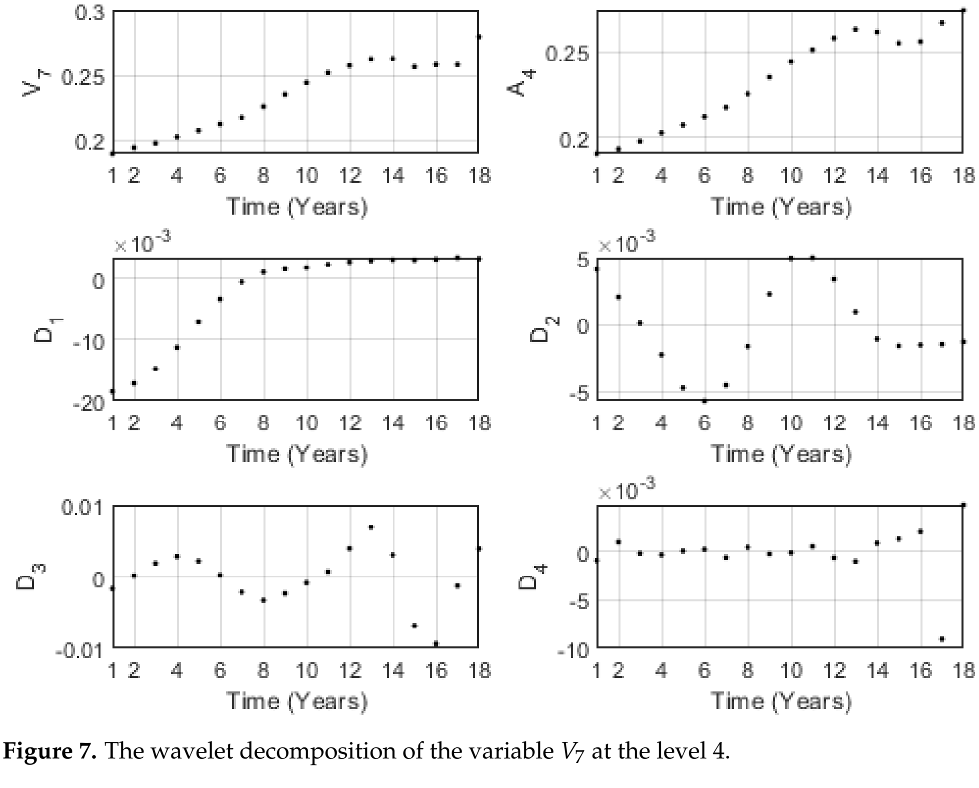

Figure 7. The wavelet decomposition of the variable V_7 at the level 4.

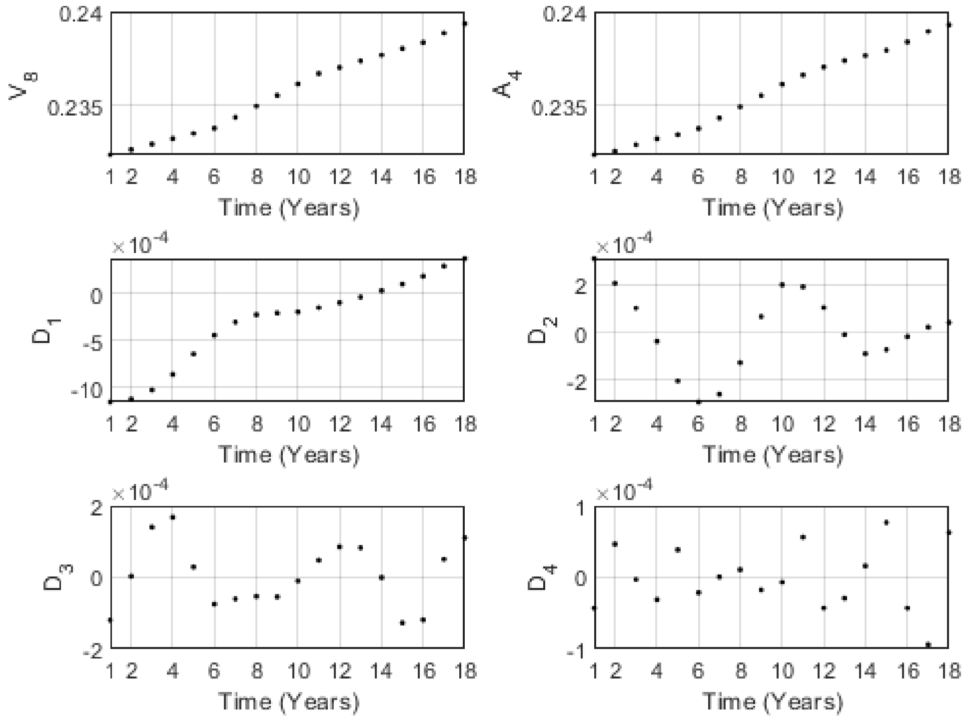

Figure 8. The wavelet decomposition of the variable V_8 at the level 4.

Next, as the components of our sample are completed and the sample has now no missing values, we provide the resulting quality of life index HDI issued from the model (5). The values are provided in Table 10 below.

Table 10. The HDI index, real and time-scale model (5).

HDI_0	HDI_1	HDI_2	HDI_3	HDI_4
0.7841	−0.0021	−0.0033	0.0046	−0.0284
0.7859	0.0021	0.0001	0.0019	−0.0260
0.7886	0.0004	0.0033	−0.0003	−0.0220
0.7920	−0.0007	0.0048	−0.0028	−0.0164
0.7962	−0.0010	0.0030	−0.0055	−0.0096
0.8008	0.0019	−0.0007	−0.0067	−0.0033
0.8057	−0.0016	−0.0039	−0.0059	0.0017
0.8108	0.0007	−0.0044	−0.0032	0.0050
0.8161	0.0002	−0.0012	0.0011	0.0068
0.8216	−0.0007	0.0013	0.0047	0.0080
0.8273	0.0004	0.0014	0.0063	0.0095
0.8329	−0.0004	0.0024	0.0065	0.0106
0.8383	0.0007	0.0037	0.0054	0.0110
0.8432	0.0004	0.0014	0.0033	0.0107
0.8477	−0.0024	−0.0036	0.0008	0.0096
0.8515	0.0020	−0.0052	−0.0023	0.0081
0.8547	0.0005	−0.0018	−0.0056	0.0062
0.8572	−0.0008	0.0013	−0.0071	0.0034

Figure 9 illustrates graphically the behavior of the quality of life index HDI and provides thus a graphical comparison between the real values and the one due to our wavelet multiscale model (5).

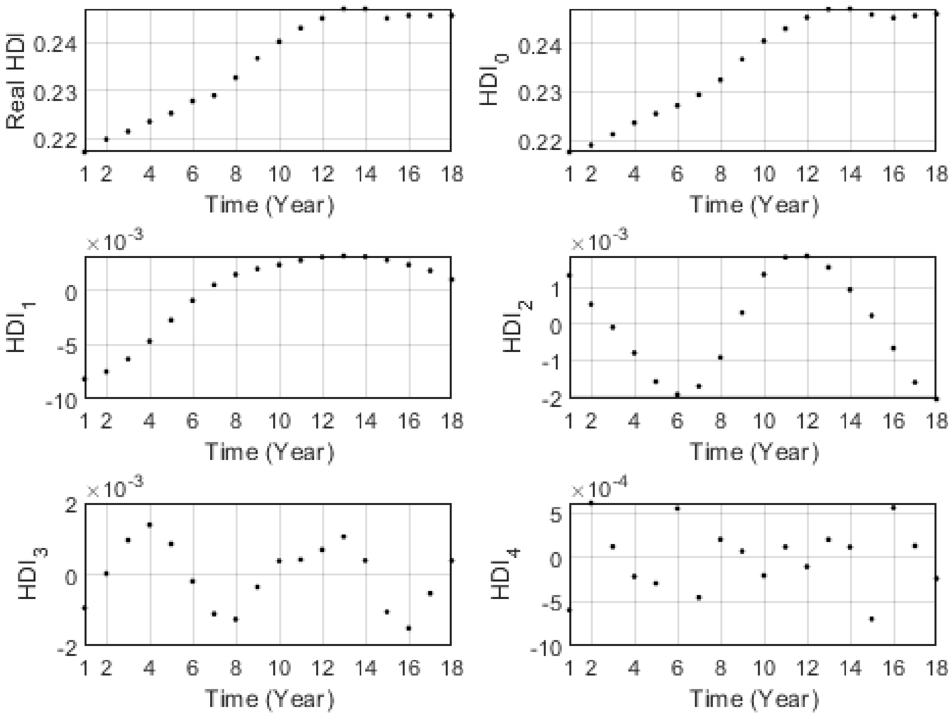

Figure 9. The plots of the HDI index, real and time scale model (5).

6. Discussion of Results

Before going on commenting and discussing our results in the framework of quality of life sense, we quickly recall some facts about the accuracy and convergence of our wavelet decomposition. This may be in fact a theoretical task which is already guaranteed in wavelet theory. We know that in wavelet theory, the wavelet approximation of the statistical series at a level J converges in the L^2-sense (variance or standard deviation measures in statistics). Mathematically speaking, we know that the projection on the multi-resolution space V_J guarantees an approximation of the order 2^{-sJ}, and thus, as we increase the order J, we keep a best convergence and accuracy. $s = NLogN$ is the regularity of the wavelet used, where N is the length of the wavelet filter. In our case, Db8 is regular of order $s = 8Log8 = 16.63$. So, the order of convergence is approximately $2^{-32Log8}$, which is an accurate order.

Let us now comment on, interpret, and discuss our empirical results. Figure 1 illustrates the behavior of the variable V_1, which represents the Gross Domestic Product. Notice that such a variable represents a global shape showing an increase in time. However, its global shape does not reflect the real and hidden behavior of the variable according to the time scale. Indeed, at low scales, $j = 1$, the component D_1 shows quietly the same global behavior, an increasing variation according to time. However, by increasing the level or the scale, the real hidden behavior starts to appear, explaining some pseudo-periodicity and/or a fluctuation in the variation of the factor V_1. By means of quality of life vocabulary, this means that the situation is not stable, and thus, the future quality of life situation should be carefully linked to the Gross Domestic Product. However, this is unstable according to the time scale. Related to the case study, especially the period chosen, this is a natural consequence of many critical phenomena that happened in this period. The first is the starting steps in implementing the 2030 vision plans. As usual, with the absence of a real future view, precise forecasting, and prevision of this plan, the statistics as well as the real behavior of investors and citizens is always careful. The COVID-19 pandemic has been also dispersed in all the word, leading to a perturbation and sometimes a permanent stop of many important activities such as oil exportation, foreign investments, etc. These facts affect the local or the national market, especially local or domestic production: for example, by increasing prices of consummations, taxes, etc, which negatively affect the view of people regarding their lives, especially their economic situation.

The variable V_2 representing the Gross National Income is tending more toward a decreasing shape, with more stability at medium horizons. As the time scale increases, this income shows more instability with always a decreasing magnitude. This may be due as evoked previously to many causes, especially COVID-19, as well as the low movement of exportation. Recall also that the embargo against Qatar country where the exportation of a big amount of especially food industry products has been stopped for a long time, as estimated by the level $j = 3$. Another cause is the restoring legitimacy of the war in Yemen, which has largely affected the GCC continent in general. These phenomena have led to a large portion of money being directed to and reserved for the army activities and needs rather than being allocated to investments that bring more interest to the treasury.

The variable V_3 relative to Gross Savings is somehow increasing in low horizons, with a small perturbation in medium levels, and more instabilities at higher horizons. This behavior may be explained by the fact that the present or precisely the index is high at the beginning of the period, and then it starts to be unstable, decreasing as the time scale increases. A main cause is firstly the increase in prices mainly of consumed products, for which COVID-19 stopped many, even small, activities. There was an increase of taxes as well as fuel prices and cars. An important movement of immigration of citizens and thus an outside flow of money investments has taken place.

The employment rate denoted here by V_4 according to Figure 4 is somehow remaining stable and quietly constant with little variation from its average, especially at short horizons. At the end of the period, there is an extreme increase or what we call jump, which may be explained by the movement of nationalization of jobs in the kingdom as part of the politics of the government already in the application and/or the execution of the 2030 vision plan to reduce the national unemployment rate. We know that Saudi Arabia, as well as the whole continent of GCC countries, rely on a large percentage on foreign incoming labor from abroad. GCC countries have planned some 2030 to 2035 vision plans, which target among many goals reducing the non-national labor. However, in the last period, many crises have appeared such as the COVID-19 pandemic as well as the Yemen war, which severely affected the economy. Many industrial as well as small firms have been closed due to the crisis. This explains well the increase of the unemployment rate suddenly. However, we stress that this instability is not large enough to affect strongly the variation of the unemployment rate. It is worthy recalling that Saudi nationals are not widely attracting the private sector, as they are on one side seeking high salaries, and on the other side, they have more employment protections than expatriate workers, The government has made some policy to encourage the private sector for Saudi nationals recruitment such as the Nitaqat, which allocates some funds to private firms that recruit Saudi nationals. This policy has effectively increased the employment rate in some short time horizons. However, with the not strong qualifications of this national labor, the private sector often replaces these workers with expatriates, which caused in turn many problems.

The variable V_5 designates the Energy Consumption per Capita. Concerning this index, Saudi Arabia is one of the most important countries and forces that may provide the entire population with self-produced energy. Moreover, it may produce more energy, which is then exported. This makes the index of Energy Consumption globally increasing in short horizons, as it is shown in Figure 5. This may be naturally explained by the availability of energy to be delivered to the population time-wise. However, at long horizons, we notice some perturbation shown clearly in the detail component D_4 relative to this factor. This explains the influence of the global crisis due to COVID-19 and the Yemen war, which has led to a lack of technology importation as a main cause (see also [54]). We may also here relate this perturbation to the act of nationalization of jobs when including non-qualified jobs. One of the main plans in the 2030 vision is the NEOM city that the kingdom started to implement in the northwest region. This project has a great influence as a great part of energy as well as a huge amount of money is now allocated to such a project. In other words, a type of austerity policy is carried out in order to save both energy and money for the NEOM project.

Figure 6 illustrates the behavior of the variable V_6 due to the Death Rate in the kingdom. It shows a decreasing rate due to the development taking place in the health sector, in particular, and the services sector. It is also worth noting the infrastructure development, which contributed to reducing transport accidents. These factors explain well the decreasing behavior at short to medium horizons except for some perturbation at higher levels, where a jump to the top in the Death Rate has been illustrated in the last period, which is mainly due to the COVID-19 victims. This factor is strongly linked to the Life Expectancy explained by the variable V_8. Figure 8 shows a globally and strictly increasing behavior, especially in short horizons. The governmental statistics estimated that effectively, the Life Expectancy increased from approximately 70 years at 2003 to reach 75.5 at the present. This is clearly illustrated by the variable V_8. At long or higher time scales, we notice some fluctuations that are even weak around the mean value, which may be explained effectively by the previous variable where the death rate has somehow increased due to the COVID-19 pandemic. Recall that such a pandemic affects and still affects aged persons over forty years old.

The last index or factor is due to the Education Index designated here by the variable V_7. This factor or index is the most important one in the Saudi Arabia plans made many years ago as well as in its 2030 vision projects. Indeed, many years ago, the authorities in the kingdom made education and its improvement and development a primary goal. Serious revisions of programs including early primary school, secondary as well as higher education have been implemented. Moreover, since the past decade, the state has begun to establish educational institutions throughout the kingdom, including higher education, as universities have been established in every governorate center. As for the areas that are relatively far from the governorate centers, colleges have been established under the name of university colleges, which is a kind of university institution that gathers under its umbrella many literary, scientific, social, and human sciences disciplines at the same time. Within some flexible laws, the Ministry of Higher Education enables the transfer, integration, and mutation of students between these university colleges and other independent colleges, which enables the students to continue their studies smoothly.

The state has also allocated large amounts of money to reform education and infrastructure, upgrade scholarships, and establish a scholarship system abroad within the framework of master's and doctoral degrees and language studies. The scholarship system has made it possible to significantly improve the level of teachers and to develop scientific research in national universities and the administrative system as well.

The kingdom already has a platform allowing its universities to collaborate all over the world, especially with Arab countries, by recruiting qualified staff members. This mixture of different educational systems and cultures allowed the education level to be improved as well.

These facts are clearly reflected by the variable V_7 where an increasing behavior is clearly detected especially at short horizons or low time scales. However, in the few last years, there has been a small decrease in this index due to many reasons. One of the reasons may be explained by the state's austerity policy followed by the state on all governmental and national private institutions within the framework of the policy of guiding consumption and preserving public money. It should also be noted that spending on other issues such as defense and/or the army in the face of the Yemen war and Iranian expansion may have effects on reducing local internal budgets to provide them to the defense. Among the important projects, we also mention the NEOM Global Village project, which began to be established, where the Ministry of Education in general linked all its projects and outputs to the goals and projects of NEOM plans within the framework of the 2030 vision. This has made a large part of the educational institutions' allocations fall under a joint administration with NEOM (see Table 7 and Figure 7).

Now, as the components of our sample are illustrated, described, analyzed, and interpreted, we provide the resulting quality of life index HDI issued from the model (5). The values are provided in Table 10 below.

Notice from Table 10 and Figure 9 that the wavelet multiscale model succeeded to reflect or to localize the real behavior of the HDI relative to time scales. Contrarily to existing methods that yield a global value on each a priory fixed period independent from each other, and to the scale, we here notice that the HDI may possess different time-scale behaviors. Related to the case study, the HDI is increasing at short to medium horizons, reflecting good well-being and/or satisfaction versus the quality of life in the kingdom, especially at the beginning of the period of study.

Overall, the present study concludes clearly and strongly that time-scale decomposition is very important for analyzing quality of life index as well as its factors. In particular, we conclude easily that the case study of Saudi Arabia during the period 2003–2020 is an appropriate period to model the quality of life index at the time scale, which is clearly dominated by a trend. At higher scales (i.e., low frequencies), the wavelet details permit localizing the instabilities in the index behavior and thus allow the researchers to analyze the eventual causes. In low horizons, the fit is generally well.

Existing models dealing with the quality of life are in the majority independently based on the health situation or economic one. So, only the influence of the health situation of individuals or the influence of the economical variables are estimated in these models. However, there may be many dimensions and factors that influence the population life and their satisfaction such as education, security, employment, etc. The present model, even it has not considered all the facts related to quality of life, tried to include many of them to estimate a more adequate index.

7. Conclusions

The present paper concerns the estimation of a quality of life index by using wavelet theory as a mathematical tool to discover the influence of the time scale on such an index. The wavelet multiscale then provided allowed the estimation of the quality of life index in Saudi Arabia.

In the present work, we developed a quantitative mathematical model for estimating the quality of life index. The first step in building the model is a wavelet by adjusting the database. Such a base is characterized by its short size, which represents a major drawback in the prediction of statistical series. We showed that on the contrary, wavelets are able to remedy such a problem. The model is then applied to Saudi Arabia as a case study in the framework of its 2030 vision plan, and it is based on data gathered from social media.

The results show firstly the effectiveness of wavelets in the development of multiscale-in-time models, and secondly, the effect of the time scale in the description of the behavior of the index of quality of life and its variation over time. The present model permits the possibility to overcome the subjectivity and intangibility of the major existing models, which are majorly non-quantitative, and allows thus a quantification of the quality of life index.

The present work confirms the idea stating that generally, to estimate quality of life or population satisfaction and well-being, it is necessary to define the areas where the individuals can realize their possibilities at different levels, such as the availability of services, health care, education, security and safety (economic and physical), dignity, communication, participation in decision making, etc.

The present model may be improved to include more dimensions such as material living standards, political voice and governance, social connections and relationships, environment, etc. We join here the models discussed theoretically in [52,53]. We think that a hybrid idea may lead to better estimates and understanding of the quality of life index. A future eventual extension of the present study may be also the consideration of factor analysis as a classical tool, especially in social sciences.

The present model may be also improved in the step of wavelet adjustment of the data, where missing values have been reconstructed. An extension of the datasets may be applied by considering long series (with large size) to be used for forecasting the missing values and next extract the adequate sub-series relative to the period of study. This permits long time series in the forecasting. Recall that effectively, the use of long time series yields the best accuracy and convergence of the wavelet approximation.

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Appendix A. Wavelet Processing Toolkit

Wavelets have been introduced few decades ago in mathematical theory, although their discovery was related to applications in petroleum extraction [12–16,34].

A wavelet is simply a short wave function oscillating such as the Fourier modes, but with high frequency and small support, which we call in wavelet theory localization in time-frequency and/or time-space.

To analyze a statistical (time, financial, etc.) series, we have to compute the so-called wavelet transform, a two-parameter quantity evaluated by the correlation type (a convolution) of the analyzed series with translated-dilated copies of one fixed wavelet known as the mother wavelet, which is a special function that should satisfy at least an admissibility assumption as

$$\mathcal{A}_\psi = \int_{\mathbb{R}} \frac{|\widehat{\psi}(\xi)|^2}{|\xi|} d\xi < \infty. \tag{A1}$$

The wavelet analysis appeared originally in the theoretical form as a refinement and also terminology to Fourier and harmonic analysis is general. Therefore, it associates to the analyzed function a type of transform known as the wavelet transform or exactly the continuous wavelet transform obtained by a convolution product of the analyzed function with special copies of a source function known as the mother wavelet. More precisely, denote for $s > 0$ and $u \in \mathbb{R}$ fixed

$$\psi_{s,u}(x) = \frac{1}{\sqrt{s}} \psi(\frac{x-u}{s}). \tag{A2}$$

For a function $F \in L^2(\mathbb{R})$, its continuous wavelet transform at the scale s and the position u is defined by

$$\mathcal{WT}_{s,u}(F) = \int_{-\infty}^{\infty} F(t)\psi_{s,u}(t)dt. \tag{A3}$$

The parameters s and u have many nominations according to the context of use of the wavelet transform. s is known as a frequency, scale, or a dilation or compression parameter. u is known also as the translation parameter. When covering all the real values of s and u, we obtain the so-called time-frequency or time-space domain. This transform is called continuous because of the nature of the parameters s and u that may operate on all the space $(0,\infty)\times\mathbb{R}$. It holds in wavelet theory that the function F may be reproduced by means of its continuous wavelet transform in an L^2-sense and analogously as in Fourier analysis via the L^2-equality

$$F(t)=\frac{1}{\mathcal{A}_\psi}\int\int_{\mathbb{R}}\mathcal{WT}_{u,s}(F)\psi\left(\frac{x-u}{s}\right)\frac{dsdu}{s^2}. \tag{A4}$$

We conclude that the wavelet transform operates according two parameters: the parameter s permitting to compress or to dilate the graph of ψ, which allows in turn to reach the high/low magnitude fluctuations in the signal, and the parameter u, which permits translating the graph of ψ to localize or to approach the local fluctuations.

The discrete wavelet transform is a variant of the continuous one evaluated on a discrete grid for the parameters s and u. The most known one is the dyadic, while there is no essential difference between the discrete grids. In the dyadic case, we restrict on the set $\{(s,u)=(2^{-j},k2^{-j})\,;\ j,k\in\mathbb{Z}\}$. In this case, the translated–dilated copies are defined by

$$\psi_{j,k}(t)=2^{j/2}\psi(2^jt-k). \tag{A5}$$

The continuous wavelet transform $\mathcal{WT}_{s,u}$ will be called the discrete wavelet transform, which is denoted usually by $d_{j,k}$. The recosntruction formula (A6) will be evaluated via a discrete representation as

$$F(t)=\sum_{j,k}d_{j,k}(F)\psi_{j,k}(t), \tag{A6}$$

known as the wavelet series of the function F.

In the discrete case such as statistical series and discrete signals, the wavelet transform of a series $X(t)$ (known also as the discrete wavelet transform (DWT)) is obtained by correlation-type (discrete convolution)

$$d_{j,k}(X)=\sum_n X(n)\psi_{j,k}(n), \tag{A7}$$

known also as the wavelet coefficient or detail coefficient at the level or the scale j and the position k. In wavelet theory, it is proved that any series $X(t)$ may be decomposed in a series form as

$$X(t)=\sum_{j,k}d_{j,k}(X)\psi_{j,k}(t), \tag{A8}$$

known as the wavelet series or the wavelet decomposition of $X(t)$, and it guarantees a complete reconstruction formula of the original series $X(t)$ [12–19,33,34].

The greatest advantage of this decomposition is the fact that it allows splitting the data into different horizons known as levels. Each level is associated to a component of the series, and it makes itself a refinement of the preceding one, which we call in wavelet theory the concept of multi-resolution. Denote for $j\in\mathbb{Z}$, $W_j=spann(\psi_{j,k}\,;\ k\in\mathbb{Z})$ (known as the detail space at the level j), and $V_j=\overset{\perp}{\bigoplus_{l\leq j}}W_l$ (known as the approximation space at the level j). We get an orthogonal decomposition $V_j=V_{j-1}\overset{\perp}{\bigoplus}W_{j-1}$. This permits splitting the wavelet decomposition above as

$$X(t)=\sum_{j\leq J_{\min},k}d_{j,k}(X)\psi_{j,k}(t)+\sum_{j>J_{\min},k}d_{j,k}(X)\psi_{j,k}(t), \tag{A9}$$

relative to a fixed integer $J_{\min}\in\mathbb{Z}$. For $j\in\mathbb{Z}$, the component

$$A_j(X(t)) = \sum_{l \leq j,k} d_{l,k}(X)\psi_{l,k}(t) \tag{A10}$$

belongs to V_j, and it is called the approximation of $X(t)$ at the level j. It describes the global behavior, the trend, or the shape of $X(t)$. The component

$$D_j(X(t)) = \sum_k d_{j,k}(X)\psi_{j,k}(t) \tag{A11}$$

belongs to the space W_j, and it is called the detail component of $X(t)$ at the level j. It reflects the higher frequency oscillations or the fine-scale deviations of the series near its trend. As a consequence, the wavelet decomposition of $X(t)$ in (A9) is a superposition as

$$X(t) = A_{J_{\min}}(X(t)) + D_{J_{\min}+1}(X(t)) + D_{J_{\min}+2}(X(t)) + \dots \tag{A12}$$

A second main advantage of wavelet theory is the reduction in computing the coefficients needed in the decomposition. Indeed, there exists a function φ (known as the scaling function or the father wavelet) characterized by the so-called two-scale relation

$$\varphi = \sum_{k \in \mathbb{Z}} h_k \varphi_{1,k}, \tag{A13}$$

and which is related to the function ψ by

$$\psi = \sum_{k \in \mathbb{Z}} g_k \varphi_{1,k}, \tag{A14}$$

where

$$h_k = \int_{-\infty}^{+\infty} \varphi(t)\varphi_{1,k}(t)dt, \text{ and } g_k = (-1)^k h_{1-k}. \tag{A15}$$

It holds that $V_j = spann(\varphi_{j,k}\ ;\ k \in \mathbb{Z})$, where the $\varphi_{j,k}$ values are defined similarly to the $\psi_{j,k}$ in (A5). The component $A_j(X(t))$ is therefore written as

$$A_j(X(t)) = \sum_k a_{j,k}(X)\varphi_{j,k}(t), \tag{A16}$$

where the coefficients $a_{j,k}(X)$ (known as the approximation or scaling coefficients of $X(t)$) are evaluated as the $d_{j,k}(X)$ by replacing the function ψ by φ. The relation (A13) permits computing the level decomposition from each other as

$$a_{j,k}(X) = \sum_{l \in \mathbb{Z}} h_l a_{j+1,l+2k}(X), \tag{A17}$$

$$d_{j,k}(X) = \sum_{l \in \mathbb{Z}} g_l a_{j+1,l+2k}(X), \tag{A18}$$

and

$$a_{j+1,k}(X) = \sum_l h_{l-2k} a_{j,l}(X) + \sum_l g_{l-2k} d_{j,l}(X). \tag{A19}$$

The sequence $H = (h_k)_k$ is called the discrete wavelet low-pass filter, and the sequence $G = (g_k)_k$ is the discrete wavelet high-pass filter.

The truncation of the last decomposition in (A12) in a practical finite level $J > J_{\min} \in \mathbb{Z}$ gives the so-called J-level finite wavelet decomposition of $X(t)$ as

$$S_J = A_{J_0}(S) + \sum_{J_0 < j \leq J} D_j(S). \tag{A20}$$

The lower index $J_{\min}$ is in fact more flexible, and it is usually chosen to be 0. The choice of J is always critical, and it is related to the eventual error estimates requested. See [12–16,34] for more details.

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