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Dimensional Tolerances in Mechanical Assemblies: A Cost-Based Optimization Approach

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Abstract: There is a widely accepted consensus that component manufacturing precision is directly correlated with improved functional performance. However, this increase in precision comes at the expense of higher manufacturing costs, resulting in a trade-off between quality and affordability. In light of this opposing behavior, low-cost products typically exhibit lower quality, whereas high-quality products tend to be more expensive. This study introduces a novel approach for optimizing the dimensional tolerances of mechanical assembly components, taking into account both their manufacturing requirements and the associated costs of non-quality. Furthermore, the method considers the functional constraints imposed by interrelated tolerance chains within the product. Instead of relying on an exact mathematical solution, the proposed solution employs a heuristic approach through a simple and flexible algorithm. This enables practical implementation, as different cost-tolerance functions can be selected based on specific requirements. To provide a comprehensive evaluation of the proposed method, a concise review of the relevant literature in the field was conducted, allowing a comparison with other state-of-the-art approaches.

Keywords: dimensional tolerances; tolerance–cost optimization; interrelated dimensional chains

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1. Introduction

Manufacturing processes inherently involve uncertainties, making it impractical to achieve exact geometries and dimensions for every part. When defining the geometry and dimensions of product features, it becomes necessary to account for these uncertainties, specifically the manufacturing variation, while ensuring that the final result allows the product to fulfill its intended design function. To specify a range of acceptable variations, it is important to consider the sensitivity of the product function to geometric and dimensional uncertainties, although at a lower level of magnitude. These acceptable ranges of variations are commonly referred to as "tolerances", which are defined by maximum and minimum values. The concept of tolerance development is illustrated in Figure 1. The tolerance range, representing the specified extent of variation, is applicable to components, assemblies of components, or simply referred to as "assemblies".

In practice, the specification of dimensional tolerances becomes relevant when mechanical components interact through contact surfaces, forming a mechanical assembly. When determining the tolerances for a product, achieving a balance among the following factors is a recurring concern:

1. Functional design requirements, also known as design constraints, encompass both functional and quality considerations. These constraints include aspects such as fitting with other assemblies, alignment between shafts, lubrication and sealing requirements, flow and thermal considerations, as well as visual and aesthetic requirements, among others. Typically, these constraints are identified as functional key characteristics (FKCs) or simply key characteristics (KCs).

2. The accuracy and precision of manufacturing processes play a crucial role in specifying tolerances. The dimensional variation in manufacturing processes is influenced by multiple factors, including the capability and condition of the machinery being used, the setup of the process, the fixture holding the part, vibrations, tool temperature, and environmental conditions. These factors collectively contribute to the overall variation in dimensions during manufacturing.
3. Manufacturing costs, which depend on the chosen processes, have a direct impact on the overall cost of the product. These manufacturing costs directly influence the retail price of the product and, subsequently, its competitiveness on the market. Therefore, careful consideration of manufacturing costs is essential to ensure a competitive position in the market.

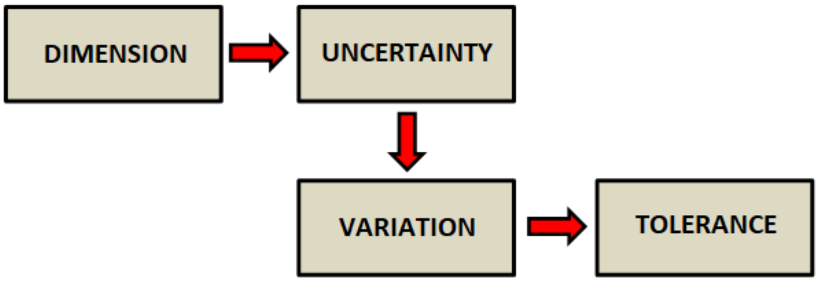

Figure 1. The geometric and dimensional tolerances.

The advantages of tolerance–cost optimization are widely recognized. However, its full potential remains untapped due to the complexity and limited comprehensibility of existing methods found in the literature [1]. This study aims to address this gap by introducing a more flexible and simplified approach to tolerance–cost optimization. The primary objective is to achieve the optimal balance between manufacturing cost and product quality. To accomplish this, a novel cost-based optimization heuristic is proposed for the process of specifying tolerances in both individual parts and assemblies.

In the optimization process, various other design factors are taken into account as variables. The algorithm developed takes into account both simplicity and flexibility, allowing for parameter adjustments that can be tailored to specific industrial facilities. This customization capability enables the algorithm to effectively incorporate specific and local cost precision data in the development of the models that it utilizes. Moreover, the proposed optimization process has the capability to handle multiple tolerance chains simultaneously, which makes it different from existing methods where this feature is often overlooked. This ability to consider multiple tolerance chains is a notable differentiating factor of the proposed approach.

The structure of this paper is organized as follows. Section 2 provides an introduction to the relevant concepts employed in this study, along with a concise review of the literature. The proposed methodology is presented in Section 3. Section 4 outlines a case study that demonstrates the execution of the first module, where the functional requirements for each tolerance chain are determined. The results of the second module are presented in Section 5, which also includes a comparative analysis of the proposed method with other approaches found in the literature. Finally, Section 6 summarizes the key findings, highlights the conclusions drawn from the study, and suggests potential avenues for future research.

2. Applicable Concepts and References Using Similar Proposal Approaches

The study of dimensional tolerances necessitates an understanding of several fundamental theoretical concepts. This section provides a comprehensive description of the key concepts used in this proposal. Additionally, it presents a survey of recent works that focus on similar tolerance optimization methods. By referencing up-to-date literature, this section enables a comparative analysis with the state-of-the-art approaches pertaining to each topic discussed.

2.1. Tolerance Analysis

The tolerance analysis process is used to accumulate the individual tolerances of components within an assembly to determine the overall tolerance range. This outcome must align with the specified design value. In the context of mechanical assemblies, this process takes into account the sequential order of component tolerances, referred to as chain dimensioning. The inclusion of plus and minus signs in the chain is dictated by the geometric positioning of the components within the assembly. In engineering design, the two most commonly utilized tolerance analysis models are the Worst-Case (WC) and Statistical Tolerance (ST) models. These models play a crucial role in assessing and managing tolerances within the design process.

The Worst-Case (WC) method, commonly referred to as tolerance stack-up, involves the summation or subtraction of component tolerances at their maximum and minimum values in a given direction. This calculation yields the extreme limits of the assembly, representing the worst-case scenario [2–4]. The WC method is typically applied in situations where:

1. information about the manufacturing processes of the components is unavailable;
2. the assembly consists of a limited number of components and is subjected to critical functional constraints.

In the absence of process information, a 100% production inspection becomes necessary, resulting in the Worst-Case (WC) method being overly conservative. On the contrary, the statistical tolerance (ST) method is suitable for component populations. The ST method is derived from observation of measurements in well-controlled processes, where most component dimensions align closely with the center of their specified tolerance zone [5]. This allows the ST model to be applied to mass production scenarios using sampling inspection. However, it should be noted that the ST model allows a fraction of the process output to deviate from the specification, and this consideration must be taken into account during design development.

The Statistical Tolerance (ST) model offers various methods for tolerance analysis, but two of the most commonly used approaches are the Root Sum Square (RSS) method and the Monte Carlo (MC) method. These methods are widely applied in the field of tolerance analysis.

2.2. Tolerance Synthesis

The tolerance synthesis process serves as a counterpart to tolerance analysis and is carried out under the following circumstances:

1. when the specified design constraints are not met during tolerance analysis;
2. when the design constraints are successfully met, but there is a need for optimizing the tolerance variables of the assembly components to balance cost and quality loss.

In this process, the assembly tolerance is distributed among its components based on a specific criterion.

2.3. The Cost Approach

The cost impact is a primary concern in the product tolerance design, as overall product costs directly influence the retail price. Given the global competitiveness of the market, managing costs is typically a key priority.

In addition to cost, performance and reliability are also critical requirements in product design. The cost approach should not be viewed solely in a traditional sense but rather be linked to a broader concept that encompasses the penalties incurred by the client. These penalties are associated with the non-quality costs, which are intrinsic features described in detail in this section. The motivation behind this work stems from the need for a comprehensive approach where quality requirements are integrated into the design activity.

2.3.1. Manufacturing Costs

The manufacturing cost of a product is influenced by various factors, including material, facilities, labor, and parts design specifications [6]. Dimensional tolerances play an important role in this regard, as the cost of manufacturing increases with a tighter specified precision [7]. It is widely accepted that tighter tolerances lead to higher production costs [8–10]. This cost increase is particularly evident below a certain tolerance value, as illustrated in Figure 2.

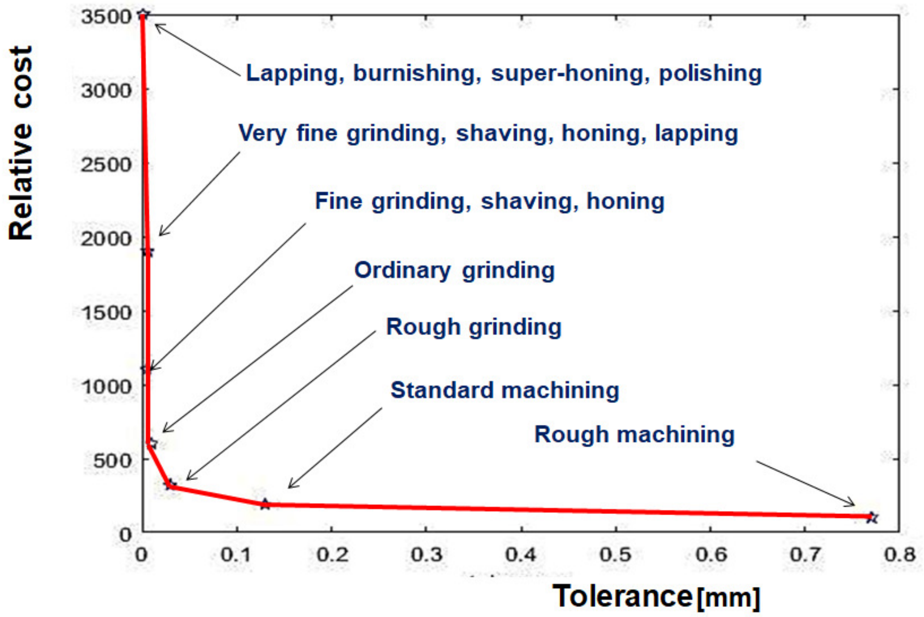

Figure 2. Relative cost vs. tolerance vs. process type.

Manufacturing costs associated with tolerances vary depending on the production site and are typically treated as proprietary information that is not publicly disclosed. In practical applications, the cost–tolerance relationship is often represented by multiple linear functions, as exemplified in Figure 2, derived from discrete tabular values.

Several references have been chosen to illustrate the various ways in which the cost of dimensional tolerances can impact a product, encompassing aspects such as design, manufacturing, and application. For example, Curran et al. proposed considering components as a function of the product's operating costs. They demonstrated the impact of widening manufacturing tolerances on the aerodynamic behavior of an aircraft nacelle to reduce costs [11]. Etienne et al. introduced the concept of activity-based tolerance allocation, which involves evaluating tolerance costs based on design and process selection activities [12]. Chiang et al. explored the use of the skew normal distribution optimization strategy (SNDOS) as an approach to address tolerance costs. They provided an example that highlights the reduction in car seat rework rates. It is worth noting that normal distributions are commonly assumed to be symmetric [13].

Armillotta conducted a comprehensive review of the tolerance cost functions employed in tolerance synthesis processes. The author emphasized the importance of appropriately selecting parameters within cost functions to avoid inconsistent results [14]. On a related note, Hallmann et al. proposed a cost optimization approach that considers interrelated key characteristics (KCs) [15].

2.3.2. Non-quality Costs

In addition to manufacturing costs, non-quality costs are associated with the use of a product and can have significant consequences. Poor-quality products can compromise functionality, reliability, and safety, leading to customer dissatisfaction and potential damage to the supplier's reputation. The severity of these non-quality costs can be measured using key characteristics (KCs), which are features with the greatest impact on the fit, performance, or service life of the finished product from the customer's perspective. The KC approach involves an interrelation between design and process [16–18].

The non-quality cost approach, also known as quality loss, was originally introduced by Taguchi [19]. It involves analyzing the cost penalties incurred by customers due to product defects or failures from the moment the product is released for shipment. The concept of a quadratic loss function has been widely studied and applied [20–23], as shown in Figure 3. For bilateral or symmetric tolerances, the economic safety factor ϕ is defined according to Taguchi's formulation [22] as

$$\phi = \frac{\triangle_0}{\Delta} \tag{1}$$

where $\triangle_0$ is the tolerance to the functional limit of the product and Δ is the tolerance to the specification with lower or higher limits.

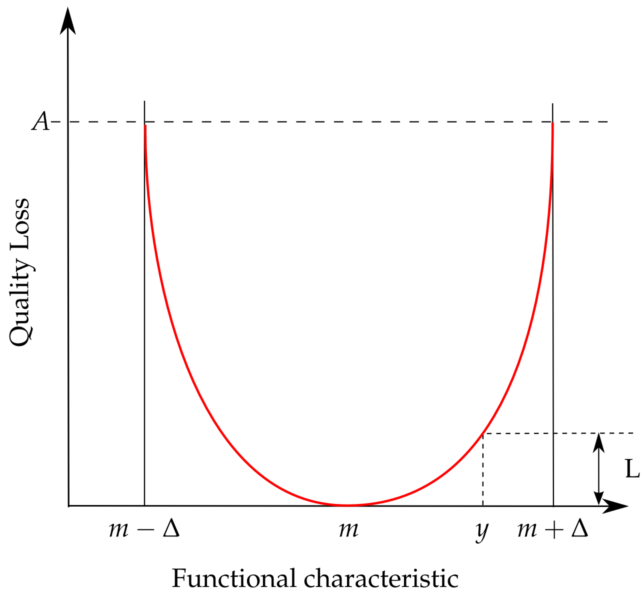

Figure 3. Relationship between quality loss and deviation Δ from an objective value m. A represents the cost of scraping or reworking; L represents the loss caused by deviation of y from m.

If A_0 represents the average financial loss incurred when the objective characteristic of the product exceeds the functional limits, which corresponds to the customer's loss caused by a defective product, and A represents the average financial loss when the objective characteristic of the product exceeds the production tolerance limit, corresponding to the manufacturer's loss caused by a defective product, the safety factor can be calculated as

$$\phi = \sqrt{\left(\frac{A_0}{A}\right)}. \tag{2}$$

References in the literature often discuss optimization proposals for both tolerance cost and quality loss. Several relevant studies have been selected for consideration. For example, Peng et al. focused on optimizing the tolerance design considering correlated characteristics and the present value of quality loss [24]. Similarly, Zhao et al. presented a tolerance optimization design based on the current value of loss in service quality [25]. Huang and Shiau proposed an optimization approach for manufacturing costs, quality loss, and the reliability index, considering statistical distributions with normal and logarithmic normal variations. They accounted for asymmetric distributions to represent mean shifts and made adjustments to the Taguchi quality loss function, resulting in a more complex solution compared to symmetrical distributions. In their study, the inclusion of the reliability index as an acceptance limit was demonstrated in an example involving the life of a bearing, with the goal of 75% of the nominal value in the quality loss model [26]. Kumar and Padmanaban introduced an optimization approach to manufacturing cost and quality loss, using a reciprocal exponential cost–tolerance function [27].

2.4. Interrelated Chains

A chain refers to a sequential arrangement of elements, where each element shares one endpoint with its preceding element, and the other endpoint with its succeeding element. Interrelated dimension chains are characterized by not encountering the same endpoint (referred to as an elementary chain) or utilizing the same dimension (known as a simple chain) only once, as mentioned earlier. The relationship between the dimensions within a chain is denoted as the "fundamental equation", which defines the response function of the assembly specified in the design [28,29].

Limited research references are available on interrelated chains within the tolerance synthesis process, and most studies resort to heuristic methods to address them [29,30]. The complexity of the problem, compounded by multiple concurrent variables, makes finding exact solutions infeasible at present.

2.5. General Approaches Used for Tolerance Optimization

Optimizing the component tolerances of an assembly becomes relevant only when at least one dimensional constraint exists; otherwise, unrestricted variation would be possible. In this context, optimization represents a tolerance synthesis process that allows for the allocation of tolerances based on one or more variables. Cost is typically considered the primary variable, given its economic importance, as discussed earlier, followed by quality considerations such as functional performance and reliability. However, in situations where dimensional constraints must adhere to KCs related to safety or critical damage, they take precedence over cost and quality requirements.

The Lagrange multiplier (LM) method is a classical approach used to solve constrained optimization problems. Its concept involves transforming the constrained optimization problem into an unconstrained one. This transformation is achieved by introducing undetermined multipliers and incorporating the constraint to create a new unconstrained objective function, known as the Lagrangian function. The transformed constrained problem can then be solved using numerical optimization methods designed for unconstrained problems, such as utilizing Taylor's expansion.

Cheng and Tsai proposed a tolerance allocation approach for the reciprocal exponential cost–tolerance function, which integrates the Lambert W function, a complex multivalue function [31]. Similarly, Kumar et al. developed a least cost–tolerance allocation method based on the Lagrange multiplier technique and the Lambert W function. To obtain the desired result, they simultaneously derived a reciprocal power cost function and a quality loss function [32]. Furthermore, Armillotta applied the Lagrange multiplier method using an extended formulation of the reciprocal power cost–tolerance function [33].

In addition to the works mentioned above, a wide range of papers can be found in the literature that explore various approaches to optimizing cost to tolerance [10,34–42]. These papers contribute to the understanding and advancement of the field by presenting different methodologies and techniques. A recent comprehensive review by [1] provides a detailed examination of tolerance versus cost optimization. The review includes up to 290 articles published since 1970 that are relevant to the topic. The authors elucidate important concepts related to tolerancing, including tolerance–cost optimization, tolerance analysis, and tolerance synthesis, among others. Despite the well-known benefits of these approaches, their full potential is often underutilized due to their inherent complexity and the limited understanding of the proposed methods.

3. A Proposal for Tolerance Synthesis

An analysis of the existing literature indicates that problems involving multiple chains are often tackled using heuristic methods because of their unique characteristics and complexity. The objective of the proposed optimization approach is to provide a practical and user-friendly tool. By adopting a heuristic approach instead of a complex mathematical treatment and by offering flexibility in selecting different cost functions, the proposed method is designed to be simple and adaptable for practical applications.

The main objective of the cost–quality optimization task is to determine a set of tolerance values, denoted as t_i, for n components. These tolerance values are synthesized in such a way that they minimize the total cost, denoted as CT, which consists of both the manufacturing cost, denoted as $C(t_i)$, and the non-quality cost, denoted as $Cq(t_i)$. The proposed approach aims to find the optimal tolerance values that result in the lowest possible total cost CT while considering both manufacturing cost and non-quality cost factors

$$CT = \sum_{i=1}^{n} C(t_i) + \sum_{i=1}^{n} Cq(t_i). \quad (3)$$

The graphical representation of (3) is depicted in Figure 4. In the synthesis process, it is essential to ensure that the assembly tolerance T satisfies one or more functional constraints denoted as R_f, as

$$T \leq R_f. \quad (4)$$

These functional constraints define the acceptable limits or requirements that the assembly must meet in order to ensure proper functionality. The proposed pragmatic approach highlights two features that are not simultaneously addressed in the referenced works. Considering both features together, the optimization framework offers a comprehensive solution that takes into account their mutual influence and provides a more holistic approach to the problem.

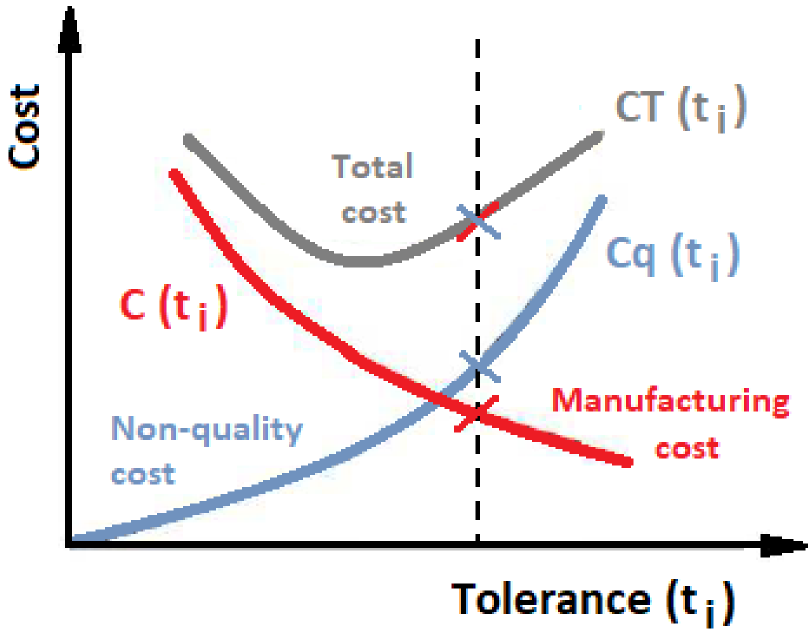

Figure 4. Graphical representation of the sum of manufacturing costs and non-quality costs. For each tolerance value (t_i), the total cost CT is the sum of the manufacturing cost $C(t_i)$ and the non-quality cost $Cq(t_i)$. The minimum value is the optimum total cost to be considered.

1. The non-quality cost is an inherent aspect of the optimization process, influenced by the system configuration, as depicted in Figure 4. Therefore, the inclusion of an

economic safety factor, as indicated in (1) and (2), serves as an additional and significant control mechanism. By considering the economic safety factor, the optimization approach accounts for the potential financial implications of non-quality costs and ensures that the resulting solution aligns with cost-effectiveness considerations.

2. The pursuit of a global minimum cost, as demonstrated in several cited works, does not offer the flexibility to impose restrictions on the minimum values of component tolerances. This limitation hinders the ability to choose suitable manufacturing processes at a specific production site. It is important to note that lower tolerance values correspond to lower non-quality costs. However, such narrow tolerances may result in economically unviable processes for the manufacturer. However, the selection of a manufacturing process typically allows maximum tolerance values to be accommodated. This flexibility ensures that the chosen process remains within the realm of economic feasibility while still meeting the required quality standards.

The formulation of this optimization proposal was influenced by an in-depth analysis of the limitations present in the current available optimization methods. The primary goal of developing this method was to provide a competitive solution to the problem of dimensional tolerance specification while minimizing the limitations observed in similar approaches, particularly their inherent complexity.

Furthermore, it is important to note that the total tolerance cost is just one component among many that contribute to the overall assembly cost. To assess its significance for a specific manufacturer, its magnitude should be evaluated in relation to other cost factors. By considering the relative importance of the total tolerance cost in the context of the entire cost structure, manufacturers can make informed decisions about the optimization of tolerances and their potential impact on the overall affordability of the product.

Naturally, the proposed method has limitations. It is specifically designed to handle interrelated chains in a single dimension. The handling of three-dimensional tolerances requires alternative approaches, such as the utilization of geometric tolerances [43]. However, the proposed method remains effective in addressing a wide range of tolerable issues encountered in mechanical design. Although it may not be suitable for all scenarios, it provides a valuable tool to tackle numerous tolerable challenges in the field. The main objective of this proposal is to take advantage of the unidimensional tolerating approach for a simple application. In fact, most cases in industrial design deal with analysis and synthesis of unidimensional tolerances. For these cases, the inherent simplicity of the proposed algorithm can lead to a significant reduction in costs without affecting quality and safety.

The Proposed Method

The following guidelines were considered:

1. The definition of assembly functional constraints involves establishing quality criteria that consider both the alignment with the customer's requirements and the cost-effectiveness of the product's performance in terms of quality considerations.
2. The specification of tolerance ranges for assembly participant dimensions in the chain analysis is performed without imposing initial restrictions on maximum values. This process draws upon tables found in literature sources such as Swift and Booker and Trucks [44,45], or preferably, on a company's internal data. By adopting this approach, it becomes possible to minimize the initial cost associated with each component.
3. The chain tolerance analysis is conducted utilizing the Root Sum Square (RSS) method.
4. The tolerance synthesis involves optimizing the manufacturing costs of components based on their respective tolerance ranges. The algorithm is designed to identify the most effective distribution of tolerance ranges among the components, ensuring compliance with the assembly tolerance range specified by the adopted functional constraints. To ensure flexibility, the algorithm is capable of calculating both simple and interrelated chains. In cases where multiple chains are present, the calculation can be performed sequentially. The main algorithm starts by running the first (main)

tolerance chain. Tolerances for components shared with other chains, which have already been optimized in the first chain, are considered predetermined and "frozen" in subsequent steps. This enables the main algorithm to re-run with only the newly determinable tolerances of the additional chains. If required, a preliminary sensitivity test can be performed to select the preferred main chain (without involving tolerance calculation). This entails running a preliminary analysis by treating the common components of interrelated chains as a single chain. This allows for evaluating the sensitivity of the components and helps in determining the main chain that will be considered in the subsequent algorithm run. This is an additional distinctive feature of the method. Although a discretized linear function was used due to its practical applicability in the industry, the algorithm and the MatLab® software do not impose any restrictions on the choice of cost–tolerance function.

5. The system provides the capability to specify both maximum (for cost optimization) and minimum (for process feasibility) values for determinable tolerances. This feature plays a crucial role in the selection and/or restriction of manufacturing processes during the design phase of each component. It enables the method to be employed by various manufacturers, accommodating their specific limitations and requirements.

The optimization algorithm consists of two modules. The first module focuses on establishing functional constraints and is associated with Guideline (1). The chains are arranged in ascending order on the basis of the magnitude of their respective functional constraint limits. The second module of the algorithm is responsible for performing the optimization process. The step-by-step procedure is outlined in Algorithm 1. The algorithm assumes that an assembly consists of m parts and p chains. Initially, a set of tolerances $\{t_k\}$ is provided as input. Furthermore, the minimum tolerance value for each part is specified based on the available manufacturing process, denoted as $\{\min(t_k)\}$. These inputs correspond to Guidelines (2) and (5), respectively. Each chain within the assembly has its own set of tolerances for the parts, denoted as $ch_i = \{t_{i,j}\}$, where $t_{i,j} \in \{t_k\}$. The chain set, $\{ch_i\}$, is ordered on the basis of the ascending order of their respective functional constraint limits. The functional constraints are defined as $\{Rf_i\}$. Upon completion of the optimization algorithm, the output is a set of optimized tolerances denoted as $\{to_k\}$.

The optimization algorithm, as depicted in Algorithm 1, begins by determining the tolerances for each chain. This step is in accordance with Guideline (3), where the chain tolerance should be lower than its corresponding functional constraint limit. The process involves handling the tolerances associated with each chain individually, executed within a for loop starting at Step 9 of Algorithm 1.

The proactive optimization process is carried out within the **ProcessChainTolerance s** routine, as presented in Algorithm 2. When processing each tolerance within a chain, the **ProcessChainTolerances** routine is invoked. If the tolerance has not reached its minimum value and has not been optimized in the previous chain, the routine attempts to reduce its value. This procedure is associated with Guideline (4). The set δ consists of all the tolerances that have already been optimized. The cost $C_{i,j}$ for the current tolerance, as well as the cost $C^d_{i,j}$ for the reduced tolerance, are determined. Finally, the difference between the two costs is stored in the set γ.

In the optimization algorithm, specifically in line 13 of Algorithm 1, the complete set γ is generated, containing all the calculated cost differences. If γ is not empty, the routine **DetermineOptimizedTolerancesCosts** is run to carry out the actual optimization process. The details of this routine can be found in Algorithm 3. During optimization, the minimum cost difference is determined. Only the tolerances associated with the minimum difference cost will be reduced. As a result, the set of optimized tolerances, denoted δ, is updated accordingly.

Algorithm 1 Optimization Algorithm

Input
m: number of parts
p: number of chains
$\{t_k\}$: set of all tolerances $1 \leq k \leq m$
$\{\min(t_k)\}$: set of all minimum tolerances, considering the specific process $1 \leq k \leq m$
$c_i = \{t_{i,j}\}$: set of all tolerances j for chains i, it is known that $t_{i,j} \in \{t_k\}$
$\{R_i^f\}$: functional constraints for all chains $1 \leq i \leq p$
Output
$\{t_k^o\}$: set of all optimized tolerances
$\{t_k^o\} \leftarrow \{t_k\}$: Copy the set of tolerances
<Calculate the chains tolerance$\{t(c_i)\}$ >
$\delta \leftarrow \emptyset$: Initiates the set of optimized tolerances with empty set
for i = 1:p **do**: Process all chains
    flag $\leftarrow$ false: This flag indicates if all tolerances reached the minimum
    while $t(c_i) > R_i^f$ and **not** flag **do**: The chain tolerance is not optimized
        flag $\leftarrow$ true
        $\gamma \leftarrow \emptyset$: Initiates the set of cost differences
        for j = 1:num($\{t_{i,j}\}$) **do**: Process all tolerances in chain i
            γ, flag $\leftarrow$<ProcessChainTolerances>(i, j, $\{\min(t_k)\}$, $t_{i,j}^o$, δ, flag, γ)
        end for
        if not flag **then**: The set γ is not empty
            $\delta, \{t_{i,j}^o\}$ $\leftarrow$< DetermineOptimizedTolerancesCosts >(m, i, γ, δ, $\{t_{i,j}^o\}$)
        else
            for j = 1:num($\{t_{i,j}\}$) **do**: Process all tolerances in chain i
                γ, flag $\leftarrow$<ProcessAllChainTolerances>(i, j, $\{\min(t_k)\}$, $t_{i,j}^o$, flag, γ)
            end for
            if not flag **then**: The set γ is not empty
                $\delta, \{t_{i,j}^o\}$ $\leftarrow$< DetermineOptimizedTolerancesCosts >(m, i, γ, δ, $\{t_{i,j}^o\}$)
            else
                The chain cannot be optimized
            end if
        end if
    end while
end for

Algorithm 2 Process Chain Tolerances

Input
m: number of parts
i: current chain
j: current tolerance
$\{\min(t_k)\}$: set of all minimum tolerances, considering the specific process $1 \leq k \leq m$
$t_{i,j}$: tolerances j for chains i, it is known that $t_{i,j} \in \{t_k\}$
δ: set of optimized tolerances
γ: set of cost differences
Output
γ: set of cost differences
flag: indicates if the tolerance was optimized
ProcessChainTolerances(m, i, j, $\{\min(t_k)\}$, $t_{i,j}$, δ, γ)
if $t_{i,j}^o - t_d > \min(t_{i,j}^o)$ and $t_{i,j}^o \notin \delta$ **then**: The minimum has not been reached
    flag $\leftarrow$ false: At least one tolerance did not reach the minimum
    $t_{i,j}^d \leftarrow t_{i,j}^o - t_d$: Decrement the tolerance
    $C_{i,j}^d \leftarrow cost(t_{i,j}^d)$: Calculate the cost for the new tolerance
    $C_{i,j} \leftarrow cost(t_{i,j}^o)$: Calculate the cost for the actual tolerance
    $\Delta C_{i,j} \leftarrow C_{i,j}^d - C_{i,j}$: Calculate the difference between the two costs
    $\gamma \leftarrow \gamma \cup \Delta C_{i,j}$: Include just the calculated cost difference in the set
end if

Algorithm 3 Determine Optimized Tolerances Costs

Input

m: number of parts
i: current chain
γ: set of cost differences
δ: set of optimized tolerances
$\{t^o_{i,j}\}$: set of tolerances for chains i, it is known that $t_{i,j} \in \{t_k\}$

Output

γ: set of cost differences
$\{t^o_{i,j}\}$: set of tolerances for chains i, it is known that $t_{i,j} \in \{t_k\}$

1: **DetermineOptimizedTolerancesCosts**(m, i, $\{t^o_{i,j}\}$, γ, δ)
2: $k \leftarrow \min_i(\gamma)$: Determines the tolerance with minimum cost difference
3: **for** j = 1:num($\{t_{i,j}\}$) **do**: Process all tolerances in chain i
4: **if** $\Delta C_{i,j} == \Delta C_{i,k}$ and $\Delta C_{i,j} \in \gamma$ **then**: If the cost is equal to the minimum
5: $t^o_{i,j} \leftarrow t^o_{i,j} - t_d$: The tolerance with minimum cost is optimized
6: $\delta \leftarrow \delta \cup \{t^o_{i,j}\}$: Include the optimized tolerance in the set
7: **end if**
8: **end for**

If no tolerance within the current chain can be optimized, an alternative procedure is executed. This occurs when γ is empty or the flag is set to true. This condition is represented by the statement else on line 14 of Algorithm 1. In this scenario, the routine **ProcessAllChainTolerances** in Algorithm 4 is executed for all tolerances in the chain. This routine is similar to the previous **ProcessChainTolerances**, with the exception that it reprocesses the tolerances that have already been optimized. The only difference between the two algorithms lies in line 2, where the algorithm attempts to reprocess the already optimized tolerances and determines if further optimization is possible within the current chain.

Algorithm 4 Process All Chain Tolerances

Input

m: number of parts
i: current chain
j: current tolerance
$\{\min(t_k)\}$: set of all minimum tolerances, considering the specific process $1 \leq k \leq m$
$t_{i,j}$: tolerance j for chain i, it is known that $t_{i,j} \in \{t_k\}$
γ: set of cost differences

Output

γ: set of cost differences
flag: indicates if the tolerance was optimized

1: **ProcessAllChainTolerances**(m, i, j, $\{\min(t_k)\}$, $t_{i,j}$, γ)
2: **if** $t^o_{i,j} - t_d > \min(t^o_{i,j})$ **then**: The minimum has not been reached
3: flag $\leftarrow$ false: At least one tolerance did not reach the minimum
4: $t^d_{i,j} \leftarrow t^o_{i,j} - t_d$: Decrement the tolerance
5: $C^d_{i,j} \leftarrow cost(t^d_{i,j})$: Calculate the cost for the new tolerance
6: $C_{i,j} \leftarrow cost(t^o_{i,j})$: Calculate the cost for the actual tolerance
7: $\Delta C_{i,j} \leftarrow C^d_{i,j} - C_{i,j}$: Calculate the difference between the two costs
8: $\gamma \leftarrow \gamma \cup \Delta C_{i,j}$: Include just the calculated cost difference in the set
9: **end if**

The execution returns to Algorithm 1, and if the set γ is not empty, the optimization continues using the same routine as previously described. However, if the set γ is empty, it indicates that the current chain cannot be further optimized. In such cases, user intervention becomes necessary, such as modifying the manufacturing procedure by assigning a new minimum tolerance value to a specific part within that chain.

4. Application Example

The application example used a real-world problem that involves unidirectional interrelated chains, specifically the alignment of a belt power drive system with the ancillary equipment of an internal combustion engine. Figure 5 provides a visual representation of a generic internal combustion engine belt power system. Ensuring proper alignment between the drive and the driven pulleys is crucial to optimize the life of the belt and mitigate issues such as noise. Although not severe, these problems can be challenging to address using alternative solutions.

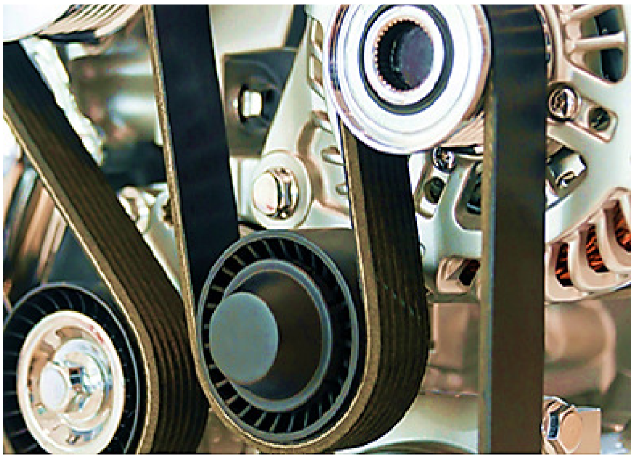

Figure 5. An example of an automotive belt power drive system. Credits.

Figure 6 provides a front view of the proposed system. The lower pulley, connected to the engine crankshaft (CRS), drives both the alternator pulley (ALT) and the hydraulic pump of the power steering system (HP). Two additional idle pulleys (IPs) are placed to ensure the desired contact angle and prevent excessive span length on the back side of the belt. One of these idle pulleys also serves as a belt tensioner. These specifications adhere to standardized values [46]. The lengths $L1$, $L2$, $L4$, and the pitch arc length $L3$ are essential in the calculation of the functional constraint.

Figure 7 provides a longitudinal view of the system, showcasing the dimensions and components involved in the construction of the chains, as well as the resulting tolerance tables. In Figure 8, a schematic representation of the assembly chains is presented. It is important to note that standardized items with predetermined tolerances do not participate in the optimization process, but are included in the calculation of assembly tolerances. The axial clearance value $d4$ is also considered predetermined, as it is limited by other functional requirements and has a fixed value. In the results tables, predetermined tolerances are highlighted in italics to ensure proper identification.

The functional constraint values were determined using the concept of economic safety factor, which takes into account the loss of quality function introduced by [20]. For the design of multi-V belts, an upper limit value of $\alpha_0 = 4.5°$ was adopted for the extension angle to prevent the belts from disengaging from the pulley grooves [47]. The relationship between factors is shown in Figure 9, while Figure 6 provides the definitions of the lengths $L1$, $L2$, $L3$, and $L4$. By utilizing the design data and referring to Figure 6, the following relationships can be established for the primary chain ($CRS \Leftrightarrow ALT$):

$$L = L1 = 250 \text{ mm: } \Delta_{01} = 250 \times \tan(4.5°) = 19.7 \quad \text{mm}$$

Regarding the secondary chain ($ALT \Leftrightarrow HP$),

$$L = L2 + L3 + L4 = 200 \text{ mm: } \Delta_{02} = 200 \times \tan(4.5°) = 15.7 \quad \text{mm}$$

The estimation of the mean financial loss is based on the functional limits. If these limits are exceeded, it would compromise the functioning of both the power steering system and the alternator, potentially resulting in accidents that could cause material and human health damage. The estimated cost of this damage is $A_0 = 500{,}000.00$. Furthermore, the cost of repairing a noise complaint during the warranty period of the vehicle includes expenses such as parts replacement, labor, administrative costs, and other associated expenses, totaling $A = 800.00$.

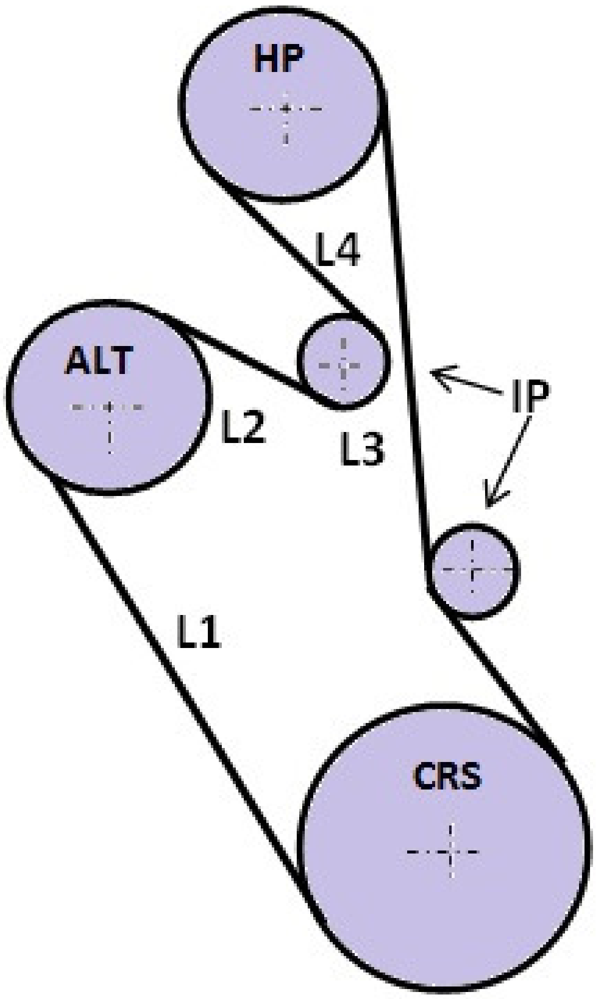

Figure 6. The belt power transmission system considered in the proposal example.

The economic safety factor of the data from the former items is

$$\phi = \sqrt{\frac{A_0}{A}} = \sqrt{\frac{500{,}000}{800}} = 25.$$

The functional constraint limits for the main and secondary chains are determined as follows:

$$\Delta_1 = \frac{\Delta_{01}}{\phi} = \frac{19.7}{25} = 0.79 \quad \text{mm} \quad \text{and} \quad \Delta_2 = \frac{\Delta_{02}}{\phi} = \frac{15.7}{25} = 0.63 \quad \text{mm}.$$

The value Δ_1 can be increased to account for the wear caused by the product. However, in the current example, this possibility will not be considered. This is because the difference between the specified value and the maximum recommended value for misalignment [46] is 0.58 mm per 100 mm, resulting in a total of $0.58 \times 2.5 = 1.45$ mm for this case. It should be noted that 1.45 mm is significantly greater than 0.58 mm, indicating that wear effects are not a significant concern in this scenario. The second functional constraint, on the other hand, is not influenced by wear effects.

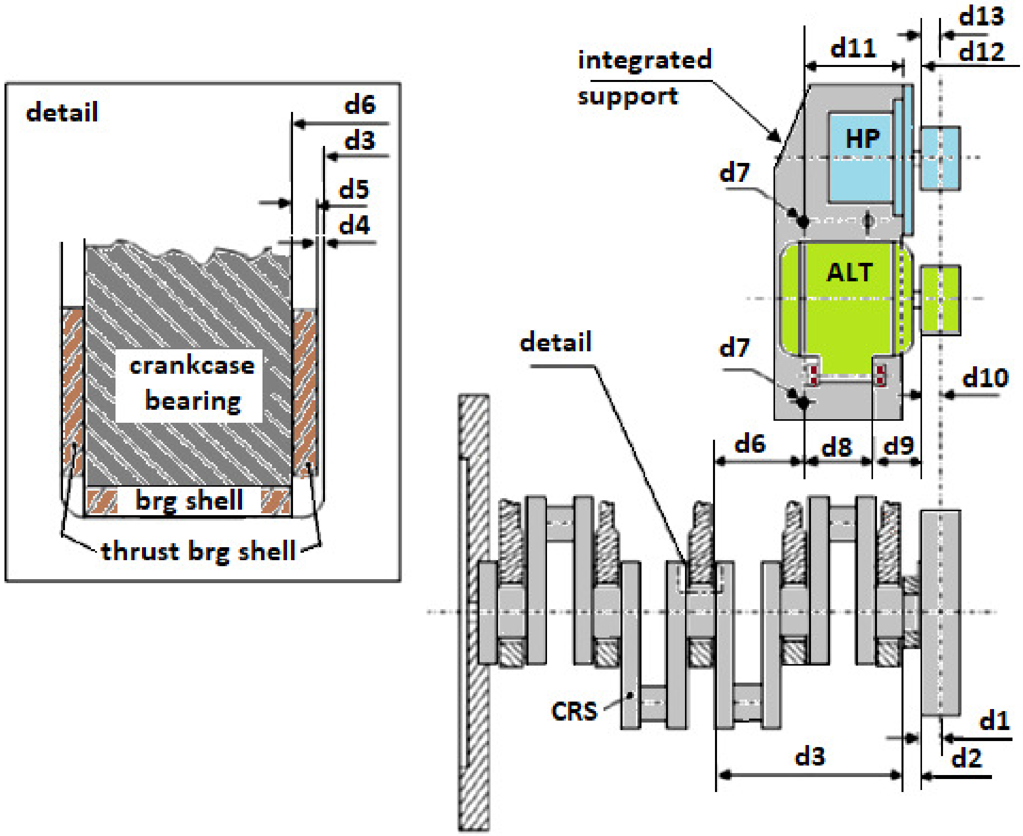

Figure 7. Side view of the system configuration.

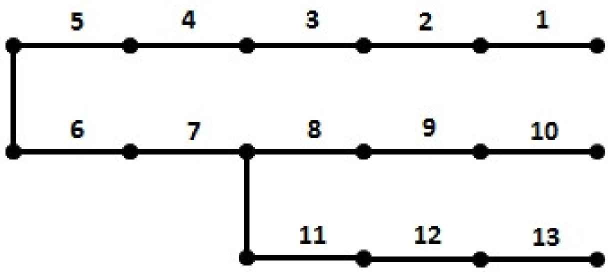

Figure 8. Schematic representation of the interdependent chains. Primary chain: items 1-2-3-4-5-6-7-8-9-10; secondary chain: items 8-9-10-11-12-13.

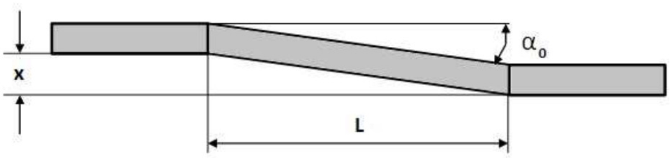

Figure 9. Relationship between angle, pulleys' axial displacement, and belt span length.

5. Results

In this section, the results of the tolerance analysis and synthesis processes are presented. These results are obtained through the application of Algorithm 1. Due to the similarity in the machining characteristics and cost within the case, the sensitivity test mentioned above was not conducted for the sake of simplification.

5.1. Tolerance Analysis Process

To simulate a real situation in the application of the proposed method, the tolerance classes specified in the international standard [48], which provide tolerance values based on component dimensions, were adopted. These classes are presented in Table 1. Additionally, the column $Spec$ provides higher values compared to class IT14. The results of the calculation for the tolerance analysis process, including the tolerances of the primary chain ($T1$) and the secondary chain ($T2$), their respective costs ($C1$ and $C2$), and the total assembly cost (CT), are summarized in the lower lines of the table for each column. Different processes present different accuracy ranges, but the same process can present a wide range of possible tolerances—several IT values—for the same size; classes for turning can vary from IT6 to IT13 [49]. For the primary chain,

$$T1 = \sqrt{t^2(1) + t^2(2) + t^2(3) + t^2(4) + t^2(5) + t^2(6) + t^2(7) + t^2(8) + t^2(9) + t^2(10)} \quad (5)$$

$$C1 = C(1) + C(2) + C(3) + C(6) + C(8) + C(10). \quad (6)$$

For the secondary chain,

$$T2 = \sqrt{t^2(8) + t^2(9) + t^2(10) + t^2(11) + t^2(12) + t^2(13)} \quad (7)$$

$$C2 = C(8) + C(10) + C(11) + C(13). \quad (8)$$

In the optimization calculation of the algorithm, the secondary chain takes priority due to its lower value functional constraint. Consequently, the tolerances of the common items shared by both chains are defined to comply with the more restrictive chain, which is the secondary chain. These tolerances are then used to calculate the tolerances of the primary chain. It is important to note that the total cost is not simply the sum of costs $C1$ and $C2$, since common items are considered only once to avoid duplication.

$$CT = C(1) + C(2) + C(3) + C(6) + C(8) + C(10) + C(11) + C(13). \quad (9)$$

The mandatory functional constraint values for the two chains, $Rf_1 = 0.79$ mm and $Rf_2 = 0.63$ mm, based on the cost of quality loss, indicate that the input data $Spec$ do not meet the following requirements:

1. class IT14 at chain #1;
2. both classes IT13 and IT14 at chain #2.

The values corresponding to these functional constraints are highlighted in bold in Table 1. The columns containing these values are duplicated in Table 2, which is used for the tolerance synthesis process.

In each column, the subsequent rows in the table present the following results:

- The tolerances T1 and T2 of the primary and secondary chains, respectively, were calculated using the RSS method. The program's data entry routine incorporated (5) and (7).
- The relative costs C1 and C2 of the chains were calculated using the program's cost routine, applying (6) and (8).
- The relative total cost (CT) of the assembly, composed of the two chains, was calculated using (9).

Table 1. Optional values for specification—values of classes IT [48] refer to their respective semi-tolerances; values in italic refer to items with predetermined tolerances.

		Classes IT for Bi-Directional Tolerances According to Means									
Item	**Mean**	**IT6**	**IT7**	**IT8**	**IT9**	**IT10**	**IT11**	**IT12**	**IT13**	**IT14**	**Spec.**
1	35.0	0.008	0.013	0.020	0.031	0.050	0.080	0.125	0.195	0.310	0.50
2	5.0	0.004	0.006	0.009	0.015	0.024	0.038	0.060	0.090	0.150	0.30
3	255.0	0.016	0.026	0.041	0.065	0.105	0.160	0.260	0.405	0.650	0.650
4	*0.00*	*0.08*	*0.08*	*0.08*	*0.08*	*0.08*	*0.08*	*0.08*	*0.08*	*0.08*	*0.08*
5	*4.00*	*0.03*	*0.03*	*0.03*	*0.03*	*0.03*	*0.03*	*0.03*	*0.03*	*0.03*	*0.03*
6	190.0	0.015	0.023	0.036	0.058	0.093	0.145	0.230	0.360	0.575	0.650
7	*0.00*	*0.05*	*0.05*	*0.05*	*0.05*	*0.05*	*0.05*	*0.05*	*0.05*	*0.05*	*0.05*
8	55.0	0.010	0.015	0.023	0.037	0.060	0.095	0.150	0.230	0.370	0.500
9	*29.0*	*0.4*	*0.4*	*0.4*	*0.4*	*0.4*	*0.4*	*0.4*	*0.4*	*0.4*	*0.4*
10	25.0	0.007	0.011	0.017	0.026	0.042	0.065	0.105	0.165	0.260	0.400
11	65.0	0.010	0.015	0.023	0.037	0.060	0.095	0.150	0.230	0.370	0.500
12	*14.0*	*0.4*	*0.4*	*0.4*	*0.4*	*0.4*	*0.4*	*0.4*	*0.4*	*0.4*	*0.4*
13	30.0	0.007	0.011	0.017	0.026	0.042	0.065	0.105	0.165	0.260	0.500
Results											
T1 = Tol. chain 1		0.41	0.41	0.42	0.43	0.45	0.49	0.59	0.77	**1.12**	**1.33**
C1 = Cost chain 1		17.02	11.03	8.31	5.65	4.93	4.00	3.00	2.38	1.57	1.09
T2 = Tol. chain 2		0.57	0.57	0.57	0.57	0.58	0.59	0.62	**0.69**	**0.85**	**1.11**
C2 = Cost chain 2		14.00	8.03	4.03	3.89	3.59	2.98	1.94	1.56	1.29	0.79
CT = Total cost		24.02	15.04	10.33	7.60	6.73	5.49	3.97	3.16	2.22	1.45

Table 2. Results of tolerance synthesis process: Optimiz. Class IT14 = Optimization with respect to class IT14; Spec.: Additional specification; Optimiz. Spec. = Optimization with respect to the additional specification.

Item	**Mean**	**Class IT14**	**Optimiz. Class IT14**	**Spec.**	**Optimiz. Spec.**
1	35.0	0.31	0.31	0.50	0.31
2	5.0	0.15	0.13	0.30	0.18
3	255.0	0.65	0.39	0.65	0.38
4	*0.00*	*0.08*	*0.08*	*0.08*	*0.08*
5	*4.00*	*0.03*	*0.03*	*0.03*	*0.03*
6	190.0	0.575	0.38	0.65	0.38
7	*0.00*	*0.05*	*0.05*	*0.05*	*0.05*
8	55.0	0.37	0.13	0.50	0.13
9	*29.0*	*0.40*	*0.40*	*0.40*	*0.40*
10	25.0	0.26	0.13	0.40	0.13
11	65.0	0.37	0.16	0.50	0.13
12	*14.0*	*0.40*	*0.40*	*0.40*	*0.40*
13	30.0	0.26	0.13	0.50	0.16
Results					
T1 = Tol. chain 1		**1.12**	0.79	**1.33**	0.79
C1 = Cost chain 1		1.57	1.57	1.09	2.04
T2 = Tol. chain 2		**0.85**	0.63	**1.11**	0.63
C2 = Cost chain 2		1.29	1.59	0.79	1.59
CT = Total cost		2.22	2.83	1.45	2.83

5.2. Tolerance Synthesis Process

In Table 2, the results of the optimization calculation for the IT14 class values and the additional specification of the example are presented in the fourth and sixth columns, respectively. IT14 is the roughest ISO class selected for the case. Consequently, any tolerance above it is preferable to the lowest cost attainable when starting the optimization process. Optimization will increase accuracy (increase in manufacturing cost) depending on the other requirements (variables) involved. Processes are defined according to the specified tolerances. Different machines used in the same process (for example, lathes for turning) can present different accuracy capabilities, mainly due to the robustness and precision of the machine.

5.3. Discussion

The analysis of the data summarized in Tables 1 and 2 reveals the following findings:

1. The tolerances of the primary and secondary chains in the IT14 class and Spec. columns in Table 1 have the lowest relative costs compared to other classes. However, they do not satisfy the functional constraints due to the specified tolerances of the components.
2. The relative costs of the chains and the total cost of the non-optimized allocation processes, shown in the last columns of Table 2, are higher than those of the optimized processes. This demonstrates the effectiveness of the proposed method.
3. It was observed that the conventional allocation method, which uses equal tolerances (not shown), yields cost results that are close to the optimized values. However, upon analyzing the allocated values for the components, it was found that they may not comply with the process capability, especially when considering larger dimensions. In addition to optimizing manufacturing costs, the proposed method allows for a restriction that ensures flexibility in dealing with manufacturing feasibility.
4. The proposed method is characterized by its simplicity and quick response time (less than one minute of program run with an Intel CORE i5® processor). This allows for iterative attempts at optimized values, with validation by process experts if necessary, until an optimal solution is adopted.

Therefore, the advantages of applying the proposed method can be summarized as follows:

- the consideration and prioritization of non-quality costs through the determination of functional constraints;
- the ability to individually consider the feasibility of each manufacturing process;
- optimization of manufacturing costs related to dimensional tolerances using a low-complexity and time-efficient processing algorithm, which can be implemented using commercially available computer software.

The proposed method has certain limitations and cannot be directly applied to 3D tolerances.

6. Conclusions and Future Work

A method is proposed to optimize manufacturing costs by considering the concepts of quality loss in the design of dimensional tolerances. The methodology is detailed in the algorithms provided and has been shown to be highly effective compared to conventional tolerance allocation methods found in the literature. The proposed method offers distinct advantages over similar existing methods, particularly in its simultaneous optimization of manufacturing costs and costs associated with quality loss. This includes a simplified manufacturing cost calculation function and precise control over non-quality costs. The calculation function utilizes discrete intervals connected by linear functions, resulting in efficient processing times. Control over the non-quality cost is achieved by specifying an economic safety factor. Implementation of the proposed method has shown significant cost savings compared to conventional approaches, which can be particularly impactful in mass production volumes prevalent in today's globalized market. The authors acknowledge that the current proposal only addresses the synthesis of unidimensional tolerances and are actively working on extending the methodology to include three-dimensional approaches in future work. In addition, the application of stochastic simulation using the Monte Carlo method is being considered.

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