


Article

One-Dimensional Consolidation Properties of Soft Clay under Multi-Stage Loading

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Abstract: The consolidation characteristics of soft clay under multi-stage loading and single-stage loading exhibit significant differences. In order to investigate the consolidation behavior of soft clay under multi-stage loading, one-dimensional oedometer tests were conducted on marine sedimentary soft clay from northern China. The results indicate that the overall time-deformation pattern of multi-stage loading is a cyclic nonlinear extension of that of single-stage loading. The final deformation between multi-stage loading and single-stage loading is approximately equal; however, the consolidation rate of single-stage loading is four times that of multi-stage loading. Furthermore, the coefficient of consolidation (C_v) decreases with increasing stress. Subsequently, the traditional Terzaghi one-dimensional consolidation equation was modified and a consolidation equation suitable for multi-stage loading is proposed in this study. The analysis of engineering applications demonstrates that the traditional theory provides more accurate predictions of consolidation rate and settlement when the load is small. However, when the load is large, the settlement predicted using the Terzaghi one-dimensional consolidation equation may have an error of 0–25% compared to that using the modified equation. The modified Terzaghi one-dimensional consolidation equation provides a more accurate representation of the actual consolidation of soft soil.

Keywords: surcharge preloading; multi-stage loading; consolidation coefficient; foundation settlement; consolidation rate



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1. Introduction

Soft clay typically has a high water content, poor permeability, high compressibility, and low strength [1–6]. When subjected to an external load, the excess pore water pressure gradually dissipates as the free water in the soil is discharged. This leads to an increase in effective stress and the development of settlement deformation until the soil reaches a stable state. While consolidation theory provides an idealized concept of soil settling, it is important to note that the actual process is highly complex in practical engineering. This complexity is a major factor contributing to errors in Terzaghi's classical consolidation theory [7]. For instance, in highway construction, preloading is often employed to mitigate post-construction settlement and reduce the construction period [8–10]. Accurately predicting embankment settlement is crucial for ensuring the stability of the foundation [11,12]. The traditional Terzaghi theory assumes a constant consolidation coefficient and a uniformly distributed and continuously applied external load. However, this is rarely the case in practical engineering conditions, leading to a significant discrepancy between the settlement estimated using the Terzaghi theory and the measured value. Therefore, it has become crucial in soil mechanics research to enhance the consolidation theory of soft soil to account for the complexities encountered in practical situations.

Davis and Raymond [13] improved the linear relationship between deformation and stress by incorporating the nonlinear stress–deformation relationship of e – $\lg P$. However, they still made the assumption that the coefficient of consolidation remains constant and

that the permeability is directly proportional to the compressibility. They successfully derived the analytical solution for one-dimensional non-linear consolidation. In a study by Abuel-Naga et al. [14], indoor and field tests were conducted to investigate the thermal conductivity of Bangkok saturated soft clay. The findings indicated that the thermal conductivity of clay increases with soil density. The study also discussed the reliability of the results obtained through different test methods and provided insights for enhancing the consolidation process of soft soil using heat treatment technology. Gibson et al. proposed that the permeability coefficient, k , and the volume compressibility coefficient, mv , are polynomial and exponential functions of depth, respectively. They utilized the finite difference method to solve the one-dimensional problem of soft soil consolidation under instantaneous loading [15]. Yuan and Wang [16] propose that the amount of soil deformation is equivalent to the volume of water discharged from the soil pores. They theoretically derive a calculation method for the soil consolidation degree based on water content. Wang et al. [17] demonstrate that the consolidation coefficient of soft soil is influenced by the pre-consolidation coefficient and varies with the consolidation stress. Chai et al. [18] investigate the compression and consolidation behavior of structured natural clay and propose formulas for settlement and consolidation with greater sensitivity. Qi et al. [19] propose a three-dimensional thaw consolidation theory that combines Biot's consolidation theory with conduction equations, taking into account the effects of the ice–water phase change through specific heat. Cao et al. [20] develop an axisymmetric large strain consolidation model with the void ratio as the variable under equal strain conditions. They provide difference schemes for the model equation, initial condition, and boundary condition, and evaluate the influence of different consolidation theories and self-weight stress on average consolidation. Xie et al. investigated the impact of pumping velocity, loading pressure, pumping and loading type on consolidation behavior. They developed analytical solutions for one-dimensional consolidation induced via time-dependent pumping and loading, taking into account the effects of different pumping types and loading conditions [21].

While there have been numerous studies on the consolidation characteristics of soft soil, the research on the consolidation characteristics of soft soil under multi-stage loading remains understudied. Further investigation is needed for settlement prediction in multi-stage preloading projects. In this study, we present a method to accurately predict the settlement of a soft soil foundation subjected to multi-stage preloading. A series of multi-stage loading consolidation tests on coastal soft soil in northern China were conducted to investigate deformation, the degree of consolidation, and variations in the consolidation coefficient. By combining the observed variations in the experiments with the traditional Terzaghi consolidation theory, we propose a new settlement calculation method specifically designed for grading surcharge preloading.

2. Multi-Stage Loading Consolidation Test

The soil samples were collected from Cangzhou City, Hebei Province, China, near the Bohai Sea. These samples consisted of coastal sedimentary soft soil and were taken at a depth of 8 m below the groundwater level. The stratigraphic profile can be seen in Figure 1. The physical and mechanical properties of the soil samples are presented in Table 1. The high water content in the soil samples made them soft and plastic, which made the structure susceptible to damage [22,23]. To mitigate the influence of soil structure, remolded soil samples were used. The initial consolidation state of the remolded soil sample was controlled with a dry density of $\rho_d = 1.532 \text{ g/cm}^3$. The preparation process involved air-drying the soil sample, passing it through a 0.5 mm sieve, weighing a subsample, mixing it in a mixer, and thoroughly incorporating airless water with a water content of 30.47%. A specimen with a diameter of 61.8 mm and a height of 20 mm was extracted from the mixed sample using a sample maker. The specimen was then placed in a vacuum saturation device (refer to Figure 1) and connected to the vacuum cylinder and extractor. The extractor was started and the vacuum pressure gauge was monitored until it reached a value close to atmospheric pressure. At this point, the pipe clamp was slightly opened to

introduce clean water into the vacuum cylinder. Once the water submerged the saturator, the pumping was stopped and the pipe clamp was opened to allow air into the vacuum cylinder. The sample was left to stand for 12 h to ensure full saturation. Subsequently, the vacuum cylinder was opened and the specimen was removed using a ring knife. The total mass of the ring knife and sample was weighed to calculate the water content, and the saturation was determined using Equation (1). If the saturation was below 98%, further pumping saturation was performed. Once the sample saturation reached or exceeded 98%, the test could proceed.

$$S_r = \frac{w_{sr}G_s}{e} \tag{1}$$

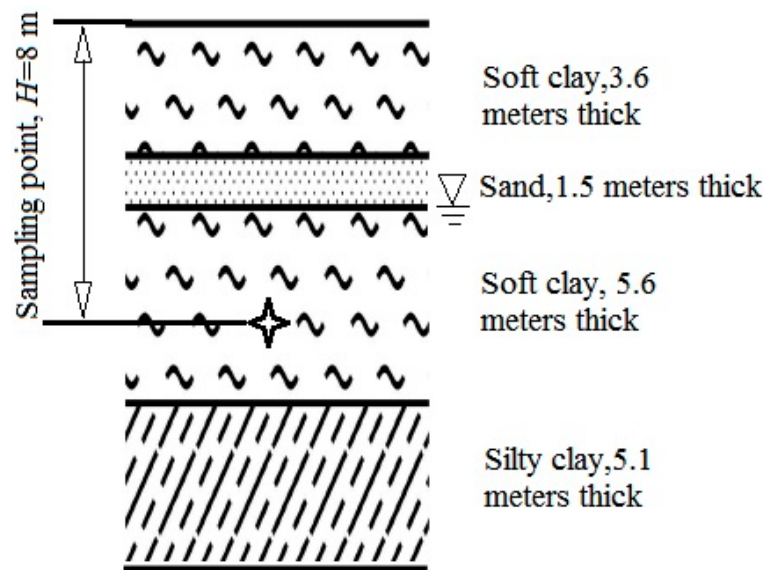


Figure 1. Strata profile.

Table 1. Physical and mechanical properties of soil specimen.

Natural unit weight (kN/m ³)	Moisture content (%)	Plastic limits (%)	Liquidity limits (%)	Plasticity index	Liquidity index	Clay content (%)	Void ratio (e ₀)
16.95	37.4	22.44	42.40	19.59	0.77	70.7	1.19
Permeability coefficient (cm/s)	Compression index (MPa ⁻¹)	Compression modulus (MPa)	Internal friction angle (°)	Cohesion (kPa)	Specific gravity (g/cm ³)	Water Saturation (%)	
4.21 × 10 ⁻⁸	0.55	3.33	11.38	11.63	2.73	97.33	

The experiment adopted the new automatic high-voltage consolidation instrument (Figure 2). It has direct electrical conversion, an ultra-high speed, and a large flow, along with a fast reaction speed and load stability time of ≤1 s. Once the consolidation pressure is stable, the pressure error is less than or equal to 1.0%. Overall, this device effectively stabilizes the loading process while minimizing its impact on the test, thereby enhancing the accuracy and precision of experiments. It has a loading range of 0–3200 kPa and an air pressure control range of 0–0.9 Mpa. The whole process of the test is controlled using a computer, and data collection is completed automatically. The data collection accuracy is 0.001 mm.

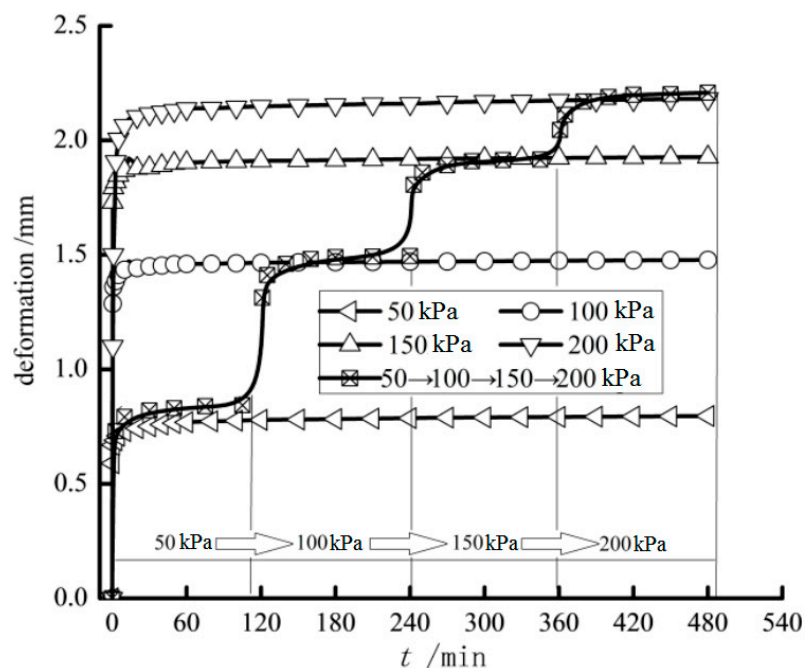


Figure 2. Variation in deformation with t for multi-stage loading and single-stage loading.

Two groups of eight soil specimens were tested in total. In group A, four soil specimens were tested using the multi-stage loading compression consolidation test with a loading sequence of 50→100→150→200 kPa. Each load level was maintained for 6 h, resulting in a total of 24 h of compression. In group B, four soil specimens were tested under single-stage loading. The consolidation pressure was applied at 50 kPa, 100 kPa, 150 kPa, and 200 kPa, respectively, with each load maintained for 24 h. Data were collected every 10 min for both test groups.

3. Test Results and Discussion

3.1. Time Variation of One-Dimensional Compression Deformation

Figure 2 illustrates the relationship between time and deformation under multi-stage loading and single-stage loading conditions. The deformation of the specimen during single-stage loading can be categorized into three stages: rapid increase, slow increase, and stabilization. In the initial stage of loading (approximately 0–5 min), the specimen exhibits significant compression deformation. The magnitude of deformation is directly proportional to the applied load, with the initial stage accounting for more than 85% of the total deformation. This behavior can be attributed to the soft marine sedimentary soil composition of the soil sample, which possesses relatively large pores and high water content, rendering it in a soft plastic state. Additionally, the soil has a high permeability coefficient, leading to the rapid discharge of pore water and the consequent compression of the samples.

The pore ratio and permeability coefficient decrease as the effective stress increases further (second stage). Consequently, the pore water pressure dissipates until the permeability coefficient rapidly stabilizes and the pore ratio enters the third stage. In this stage, the deformation of the sample enters a stable growth phase (5 min–60 min), which contributes to approximately 13.5% of the total deformation. Overall, the deformation in this stage exhibits minimal changes over time, gradually reaching a stable state.

Under multi-stage loading, the time–deformation law of each stage of the load is similar to that of single-stage loading. Multi-stage loading can be divided into three stages: rapid increase, slow increase, and stabilization. The total time–deformation law is the cyclic extension of the law of each loading step, and the total deformation is the superposition of each step loading deformation. In the test, the loads at all levels were

equal ($\Delta P = 50$ kPa), yet each load did not result in equal degrees of deformation. Overall, the deformation gradually decreased with each step, indicating the nonlinearity of the compression deformation of saturated soft soil. Figure 2 shows that the final amount of deformation during multi-stage loading and single-stage loading is approximately the same, but the deformation rate of single-stage loading is significantly greater than that of multi-stage loading. Therefore, when treating the foundation with graded stacking, it is recommended to increase the load of each stage as much as possible to accelerate the consolidation rate of the foundation and ensure its strength.

3.2. Variation Law of Consolidation Coefficient

Excess pore pressure is generated in soil when it is subjected to external loads. The water present in the soil pores is gradually discharged, leading to a reduction in excess pore pressure [24,25]. The rate at which this dissipation occurs is commonly quantified using the consolidation coefficient (C_v). In this study, a one-dimensional compression test was conducted to determine the relationship between the deformation of undisturbed soil specimens over time under different consolidation pressures. The consolidation coefficient was calculated using Equation (2):

$$C_v = 0.848 \frac{H^2}{t_{90}} \quad (2)$$

where C_v is the vertical consolidation coefficient; H is the maximum drainage distance; t_{90} is the time required for the degree of consolidation to reach 90%, which can be obtained by drawing the curve between the reading of the dial indicator of the consolidation test and the square root of time.

Figure 3 illustrates the variation in the consolidation coefficient of saturated soft soil under multi-stage loading. As the consolidation pressure (P) increases, the consolidation coefficient (C_v) gradually decreases. This decrease can be observed in two stages: an initial rapid decrease followed by a slower decrease. During the first stage of loading ($P = 50$ kPa), the soil undergoes a rapid discharge of pore water and the compaction of pores, transitioning from a soft plastic state to a plastic state. This stage is known as the rapid consolidation stage. Subsequently, after consolidation and stabilization from the first stage, the soil enters a stage of slow consolidation. In the second, third, and fourth stages of loading, the discharge of pore water is hindered, resulting in a slower rate of consolidation deformation. Studies conducted by Yang et al. [26] on soft soils in Shenzhen, Chu et al. [27] on soft soils in Singapore, and by Kassim et al. [28] on soft soils in Gemas and Kluang have indicated that the consolidation coefficient decreases with increasing consolidation pressure. However, Wang et al. [17] have demonstrated that the consolidation coefficient is also influenced by the stress history. Both overconsolidated and underconsolidated soft soils exhibit distinct patterns of change.

The process of consolidation deformation of soft soil can be analyzed using the microstructural model of soft soil established by Wong et al. [29]. In its natural state, the soil contains a significant amount of pore space, including overhead macropores and micropores. The soil particles mainly consist of silt and sand particles, which do not form the soil skeleton. During the rapid consolidation stage, the external load is primarily supported by the pore water in the overhead macropores. The soil contains numerous interconnected macropores, resulting in a considerable permeability coefficient (k_v) and consolidation coefficient (C_v). As pore water is discharged, the macropores gradually compress. The compression of macropores in the soil allows larger consolidation pressures to compact and block more drainage channels compared to under smaller pressures, resulting in a significant decrease in k_v . This decrease in k_v is the main reason for the sharp decrease in C_v when P increases from 0 to 50 kPa.

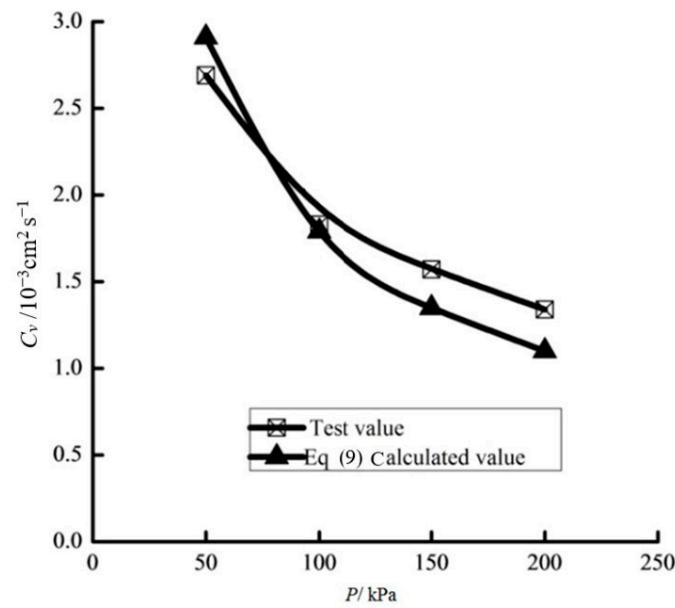


Figure 3. Variation in C_v with P for multi-stage loading.

The error between the theoretical calculation and experimental values is significant, as depicted in Figure 3. This discrepancy can be better understood by examining the definition and calculation formula of the soil’s consolidation coefficient. The consolidation coefficient is a crucial parameter that indicates the rate at which soil consolidates. In the context of one-dimensional seepage consolidation theory for saturated soil, the consolidation coefficient is defined as follows:

$$\frac{\partial e}{\partial t} = \frac{k(1 + e_0)}{\gamma_w} \frac{\partial^2 u}{\partial z^2} \tag{3}$$

where k is the permeability coefficient; e_0 is the initial pore ratio; γ_w is soil bulk density. From the compression curve $e \sim \log \sigma$ of the soil,

$$e = e_0 - C_c \lg \left(\frac{\sigma'}{\sigma'_0} \right) \tag{4}$$

where C_c is the compression index; σ'_0 is the initial consolidation stress; \lg means logarithm to base 10.

According to the effective stress principle,

$$\sigma' = \sigma - u \tag{5}$$

Substituting Equation (5) into Equation (4) and taking the derivative of u , we can obtain

$$\frac{\partial e}{\partial u} = \frac{C_c}{2.33(\sigma - u)} \tag{6}$$

where

$$\frac{\partial e}{\partial t} = \frac{\partial e}{\partial u} \frac{\partial u}{\partial t} \tag{7}$$

Substituting Equation (6) and Equation (7) into Equation (3), we can obtain

$$\frac{\partial e}{\partial t} = \frac{2.33k(1 + e_0)\sigma'}{\gamma_w C_c} \frac{\partial^2 u}{\partial z^2} \tag{8}$$

According to Terzaghi’s consolidation equation,

$$C_v = \frac{2.33k(1 + e_0)}{\gamma_w C_c} \sigma' \tag{9}$$

The permeability coefficient of soft soils varies widely during the consolidation process [30] and has a large impact on the consolidation process. According to the results of a large number of tests, for the variation characteristics of $k - \sigma'$, the relationship between the permeability coefficient and the stress level is as follows:

$$k = k_0 e^{-a\sigma'} \tag{10}$$

Substituting Equation (10) into Equation (9), we can obtain

$$C_v = \frac{2.33(1 + e_0)k_0}{\gamma_w C_c} \sigma' e^{-a\sigma'} \tag{11}$$

It can be seen from Equation (11) that the consolidation coefficient, C_v , is related to the consolidation stress. With an increase in the load, the permeability coefficient decreases, so the consolidation coefficient also decreases.

3.3. Variation Law of Consolidation Degree

According to the measured displacement during the test, the degree of consolidation can be calculated:

$$U_t = \frac{s_{ct}}{s_c} \tag{12}$$

where s_{ct} is the deformation value measured at a certain time, and s_c is the final deformation value measured in the consolidation test. Figure 4 illustrates the changes in the degree of consolidation over time under both single-stage loading and multi-stage loading conditions. Upon reaching a loading of 200 kPa, the specimen experiences rapid and significant deformation. Within 60 min, the degree of consolidation exceeds 95%, and the settlement rate is less than 0.01 mm per hour after 240 min. This signifies the completion of primary consolidation and the beginning of the secondary consolidation stage. In the case of specimens subjected to multi-stage loading, each stage of the load application triggers immediate large deformations, and the degree of consolidation increases non-linearly with each subsequent load. The number of stages directly impacts the duration of primary consolidation, with a higher number of stages leading to an extended consolidation period.

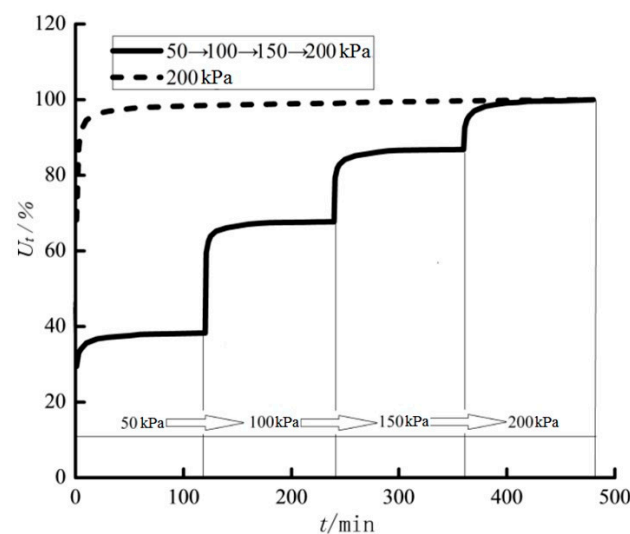


Figure 4. Variation in U_t with t for multi-stage loading.

The rate of consolidation for single-stage loading is generally faster than that for multi-stage loading. As a result, when using multi-stage preloading for foundation treatment, it is recommended to reduce the number of preloading stages and expedite the consolidation rate of the foundation soil to meet the required strength.

There is a strong correlation between the average degree of consolidation of soft soil and the consolidation stress under multi-stage loading. A regression formula has been developed to determine the design value of the vertical degree of consolidation based on the load value. After comparing several commonly used fitting formulas, it was found that U_t and P have a logarithmic relationship. The fitting formula and the correlation coefficient value, R^2 , are presented in Figure 5. The empirical relationship between the average vertical degree of consolidation and consolidation stress of soft marine soil in northern China can be expressed as follows:

$$U_t = M \ln P + N \tag{13}$$

where $M = 53.38$, $N = -181.755$, M , and N are the undetermined coefficients. The values for soft soil in different places can be calculated using the least-square method based on the test results. The empirical formula can be used to determine the degree of vertical consolidation of soft soil under various loads, which serves as the foundation for calculating the settlement of multi-stage loading.

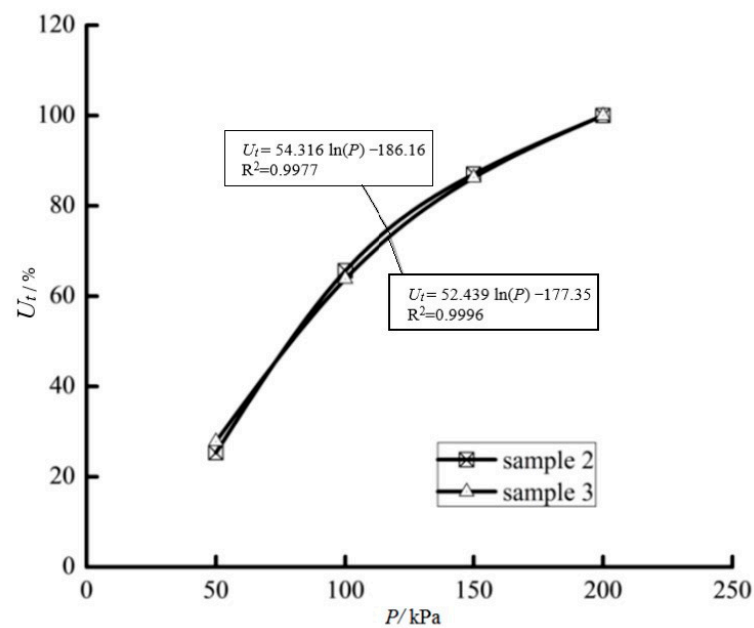


Figure 5. Variation in U_t with P for multi-stage loading.

4. Modified Terzaghi One-Dimensional Consolidation Equation

Terzaghi’s one-dimensional consolidation theory assumes a constant coefficient of consolidation (C_v). However, when the foundation is subjected to multi-stage loading surcharge preloading, the coefficient of consolidation decreases with the application of various loads. If the traditional Terzaghi’s consolidation equation is used for consolidation calculations, the calculated value will significantly deviate from the actual situation. Therefore, the traditional Terzaghi one-dimensional consolidation equation needs to be modified based on the consolidation coefficient obtained from experiments. This paper presents a new method suitable for consolidation calculations using the multi-stage preloading method. The traditional Terzaghi one-dimensional consolidation equation can be solved as follows [31]:

$$\begin{cases} U_t = 1 - \frac{8}{\pi^2} \sum_{m=1}^{\infty} \frac{1}{m^2} \exp\left(-\frac{m^2 \pi^2 C_v t}{4}\right) \\ S_{ct} = S_c U_t \end{cases} \tag{14}$$

Substituting Equation (12) into Equation (15), the solution of the Terzaghi one-dimensional consolidation equation is modified:

$$\begin{cases} U_t = 1 - \frac{8}{\pi^2} \sum_{m=1}^{\infty} \frac{1}{m^2} \exp\left(-\frac{m^2 \pi^2 C_v t}{4H^2}\right) \\ C_v = \frac{2.33(1 + e_0)k_0}{\gamma_w C_c} \sigma' e^{-a\sigma'} \\ S_t = S_c U_t \end{cases} \quad (15)$$

where H is the maximum drainage distance. U_t is the degree of soil consolidation, S_{ct} is the settlement at time t , and S_c is the final settlement. The modified Terzaghi one-dimensional consolidation equation takes into account the nonlinear effect of soil consolidation and considers the consolidation coefficient, C_v , as the variable of stress level. The compensates for the deficiency in the traditional Terzaghi one-dimensional consolidation equation C_v constant. Therefore, the modified Terzaghi one-dimensional consolidation equation better reflects the consolidation behavior of soil under multi-stage loading.

5. Engineering Application Analysis

The expressway is constructed on soft soil subgrade, with a thickness of 6 m. To improve the foundation, multi-staged surcharge consolidation is performed. The loading path consists of four stages: 50→100→150→200 kPa. Each load is applied after the foundation appears consolidated and stable. The simplified model, shown in Figure 6, does not consider the drainage plate. According to the traditional Terzaghi theory, engineers believed that the consolidation coefficient remains constant in engineering design. They assumed that the consolidation coefficient under the initial load is the same as the consolidation coefficient throughout the entire consolidation process. In practical engineering, the consolidation coefficient can vary significantly due to factors such as consolidation stress and permeability coefficient. This paper investigates the variations in the degree of consolidation and settlement of the roadbed, calculated using Terzaghi’s one-dimensional consolidation equation, before and after applying the proposed correction. The study is based on Terzaghi’s theory of one-dimensional consolidation.

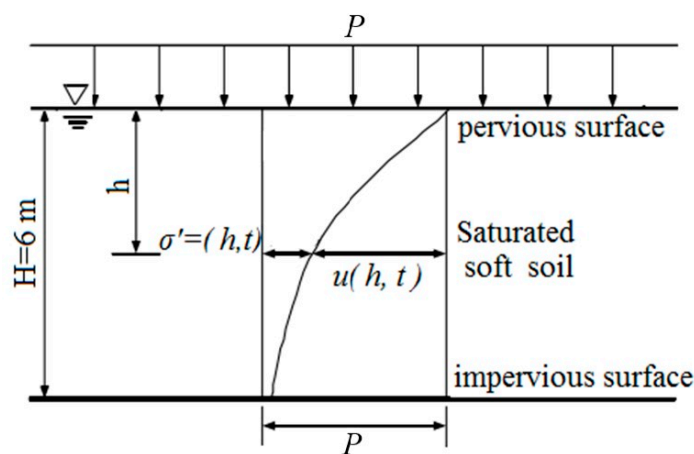


Figure 6. Simplified diagram of model.

Figure 7 illustrates the initial degree of consolidation calculated using the modified Terzaghi one-dimensional consolidation equation as 0. As consolidation progresses, the degree of consolidation gradually increases and eventually stabilizes when $U_t = 1$. However, the time taken for stabilization is significantly shorter before the modification compared to that after the modification. This is because the modified Terzaghi equation considers the influence of overburden load on consolidation stability, taking into account the decrease in pore ratio and permeability coefficient. In contrast, the traditional Terzaghi theory assumes a constant consolidation coefficient, resulting in a degree of consolidation prediction that

is independent of the load. It is important to note that the U_t-t curve corresponding to different overlying loads ($P = 50, 100, 150,$ and 200 kPa) is the same, which contradicts actual engineering observations.

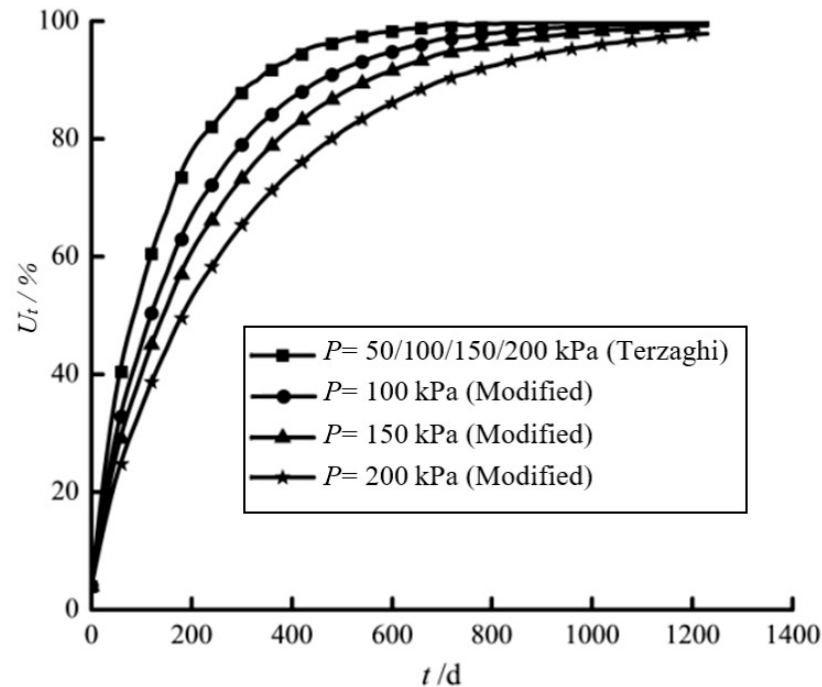


Figure 7. Relationships between the consolidation degree and time under multi-stage loadings.

The relationship between load and consolidation can be observed from Figure 7. It shows that a smaller load leads to a greater degree of consolidation per unit time and a faster consolidation rate. For instance, when the degree of consolidation (U_t) reaches 0.95, the corresponding time required for different overlying loads (P) of 50, 100, 150, and 200 kPa is 1.23, 1.73, 2.05, and 2.63 years, respectively. Therefore, the loading time increases linearly for each level of load, and the consolidation time can be appropriately extended based on the foundation's strength to minimize post-construction settlement.

In Figure 8, during the initial stage of soft foundation consolidation, the initial settlement (S_{ct}) is equal to 0. As the consolidation process progresses, S_{ct} gradually increases and eventually stabilizes at S_c . Additionally, the settlement stability time predicted via the modified Terzaghi one-dimensional consolidation equation is greater than that predicted via the traditional Terzaghi one-dimensional consolidation equation. When the load is small ($p = 50$ kPa), the impact on the void ratio and permeability coefficient of a soft foundation is not significant, and the consolidation coefficient undergoes minimal change. The S_t-t relationship curves before and after the modification are relatively consistent. The void ratio and permeability coefficient of a soft foundation are significantly affected by a large load ($p = 200$ kPa). There is a large error of 0%–25% in the settlement–time relationship curve before and after correction. Hence, when the load is small, the influence of load on consolidation rate and settlement rate during the compression of a soft foundation can be disregarded. However, if the load is large, the settlement predicted by using the traditional Terzaghi one-dimensional consolidation equation will be inaccurate. It is recommended to use the modified Terzaghi one-dimensional solid solution equation, which can reduce the error and better reflect the actual characteristics during the preloading process of a soft foundation.

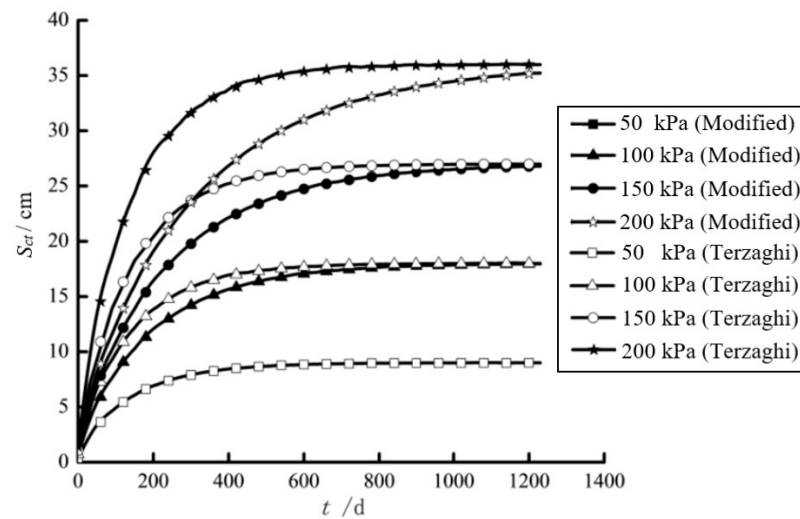


Figure 8. Relationship between settlement and time under multi-stage loading.

6. Conclusions

1. The total time–deformation law of multi-stage loading is the cyclic, nonlinear extension of that of single-stage loading. The final deformation of multi-stage loading and single-stage loading is approximately equal. However, it is important to note that the consolidation rate of single-stage loading is four times higher than that of multi-stage loading.
2. The consolidation coefficient of soft marine soil from northern China gradually decreases with an increase in consolidation stress. Follow the relationship $C_v = \frac{2.33(1 + e_0)k_0}{\gamma_w C_c} \sigma' e^{-a\sigma'}$.
3. The average degree of consolidation increases non-linearly with multi-stage loading. An increased number of multi-stage loading leads to a longer time of primary consolidation. Furthermore, the average degree of consolidation exhibits a logarithmic relationship with the load: $(U_t = M \ln P + N)$.
4. In the calculation of consolidation for a soft soil foundation under multi-stage loading, the settlement predicted via the Terzaghi one-dimensional consolidation equation may have a 25% error compared to that predicted via the modified equation. The modified Terzaghi one-dimensional consolidation equation provides a more accurate representation of the actual consolidation behavior of soft soil.

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