

Viscoelasticity: Mathematical Modelling, Numerical Simulations, and Experimental Work

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Viscoelastic materials are abundant in nature and present in our daily lives. Examples include paints, blood, polymers, biomaterials or food products. It is thus important to study and understand the viscoelastic behaviour of these different materials.

In this Special Issue, a total of eleven contributions (ten research papers and one review paper) from different areas of viscoelasticity (mathematical modelling, numerical simulations) are presented.

McDermott et al. [1] proposed an improved viscoelastic turbulence model in a fully developed drag reducing channel flow, where turbulent eddies are modelled by a $k-\epsilon$ representation, together with polymeric solutions described by the finitely extensible nonlinear elastic Peterlin (FENE-P) constitutive model. The performance of the model was evaluated using a variety of direct numerical simulation data described by different combinations of rheological parameters and was able to predict all regimes of drag reduction (low, intermediate, and high) with good accuracy. Ingelsten et al. [2] developed a new Lagrangian–Eulerian method for the simulation of viscoelastic free surface flows. The approach was developed from a method in which the constitutive equation for viscoelastic stresses was solved at Lagrangian nodes connected by flow and interpolated onto a Eulerian grid using radial basis functions. In the new method, a backwards-tracking methodology was used to allow fixed locations for the Lagrangian nodes to be chosen a priori. The proposed method was also extended to the simulation of viscoelastic free surface flows with the volume of fluid method. Bertoco et al. [3] presented the HiGTree–HiGFlow solver for numerical simulations of the KBKZ integral constitutive equation. The numerical method used finite differences and tree-based grids, which leads to greater accuracy in local mesh refinement. Wojcik et al. [4] performed fluid dynamic simulations using the FENE-P model and an incompressible Newtonian fluid to understand the role of elasticity in the formation of vortices in a narrow channel with a 90° curvature. The analysis bridged the flow behaviour of a purely elastic fluid and that of a Newtonian fluid. Their predictions were in good agreement with previous experimental and numerical works. Liu et al. [5] investigated singularities in the stress field of the flow of a viscoelastic fluid at the stagnation point for various viscoelastic constitutive models. Exact analytical solutions of two-dimensional steady wall-free stagnation point flows for the generic Oldroyd 8-constant model were obtained for the stress field using different material parameter relationships. Compatibility with the conservation of momentum was considered for all solutions.

Aabid et al. [6] studied and summarised the active control of high-speed aerodynamic flows. Vishalakshi et al. [7] studied 3D MHD fluid flows under the influence of a magnetic field with an inclined angle. Their results have been used in many real-world applications, e.g., automotive cooling systems, microelectronics, heat exchangers, etc. Anusha et al. [8] studied the two-dimensional magnetohydrodynamic problem for a steady incompressible



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flow over a porous medium. They concluded that the porosity and radiation parameters enhance the temperature distribution, while the suction/injection parameter suppresses the temperature distribution. Skinner et al. [9] developed a computational algorithm based on an accepted analytical model to investigate the viscoelastic behaviour of carbon fibre-reinforced polymer composite flywheel rotors with an aluminium hub mounted by press-fit. The simulations showed that over time the viscoelastic effects are likely to reduce the peak stresses in the composite rim. However, viscoelasticity also affects the stresses in the hub and at the hub–rim interface, leading to rotor failure. It was also found that the charge/discharge cycles of the flywheel energy accumulator can lead to significant fatigue loads.

Furlan et al. [10] derived different formulations to obtain a solution for Giesekus' constitutive model for a flow between two parallel plates. Bertoco et al. [11] presented a numerical study of the development length (the length from channel entry required for the velocity to reach 99% of its fully developed value) of a pressure-driven viscoelastic fluid flow (between parallel plates) modelled by the generalised constitutive Phan–Thien–Tanner equation (gPTT). They concluded that at low values of the Weissenberg number (Wi), the highest value of the development length was achieved for $\alpha = \beta = 0.5$; at high values of Wi , the highest value of the development length was achieved for $\alpha = \beta = 1.5$.

Although submissions for this Special Issue have now closed, research into the field of viscoelasticity continues to address various challenges we face today: medicine (e.g., drug delivery, foods that consider their rheology, and complex blood flow), development of new and smart materials (e.g., paints, biomaterials, and clothing), new industrial developments.

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References

1. McDermott, M.; Resende, P.; Charpentier, T.; Wilson, M.; Afonso, A.; Harbottle, D.; de Boer, G. A FENE-P k – ϵ Viscoelastic Turbulence Model Valid up to High Drag Reduction without Friction Velocity Dependence. *Appl. Sci.* **2020**, *10*, 8140. [\[CrossRef\]](#)
2. Ingelsten, S.; Mark, A.; Kádár, R.; Edelvik, F. A Backwards-Tracking Lagrangian-Eulerian Method for Viscoelastic Two-Fluid Flows. *Appl. Sci.* **2021**, *11*, 439. [\[CrossRef\]](#)
3. Bertoco, J.; de Araújo, M.; Leiva, R.; Sánchez, H.; Castelo, A. Numerical Simulation of KBKZ Integral Constitutive Equations in Hierarchical Grids. *Appl. Sci.* **2021**, *11*, 4875. [\[CrossRef\]](#)
4. Wojcik, B.; LaRuez, J.; Cromer, M.; Villasmil Urdaneta, L. The Role of Elasticity in the Vortex Formation in Polymeric Flow around a Sharp Bend. *Appl. Sci.* **2021**, *11*, 6588. [\[CrossRef\]](#)
5. Liu, J.; Oberlack, M.; Wang, Y. Analytical Investigation of Viscoelastic Stagnation-Point Flows with Regard to Their Singularity. *Appl. Sci.* **2021**, *11*, 6931. [\[CrossRef\]](#)
6. Aabid, A.; Khan, S.; Baig, M. A Critical Review of Supersonic Flow Control for High-Speed Applications. *Appl. Sci.* **2021**, *11*, 6899. [\[CrossRef\]](#)
7. Vishalakshi, A.; Maranna, T.; Mahabaleshwar, U.; Laroze, D. An Effect of MHD on Non-Newtonian Fluid Flow over a Porous Stretching/Shrinking Sheet with Heat Transfer. *Appl. Sci.* **2022**, *12*, 4937. [\[CrossRef\]](#)
8. Anusha, T.; Mahesh, R.; Mahabaleshwar, U.; Laroze, D. An MHD Marangoni Boundary Layer Flow and Heat Transfer with Mass Transpiration and Radiation: An Analytical Study. *Appl. Sci.* **2022**, *12*, 7527. [\[CrossRef\]](#)
9. Skinner, M.; Mertiny, P. Effects of Viscoelasticity on the Stress Evolution over the Lifetime of Filament-Wound Composite Flywheel Rotors for Energy Storage. *Appl. Sci.* **2021**, *11*, 9544. [\[CrossRef\]](#)
10. da Silva Furlan, L.; de Araujo, M.; Brandi, A.; de Almeida Cruz, D.; de Souza, L. Different Formulations to Solve the Giesekus Model for Flow between Two Parallel Plates. *Appl. Sci.* **2021**, *11*, 10115. [\[CrossRef\]](#)
11. Bertoco, J.; Leiva, R.; Ferrás, L.; Afonso, A.; Castelo, A. Development Length of Fluids Modelled by the gPTT Constitutive Differential Equation. *Appl. Sci.* **2021**, *11*, 10352. [\[CrossRef\]](#)

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