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Fast Converging Gauss–Seidel Iterative Algorithm for Massive MIMO Systems

Dong Shen *, Li Chen and Hao Liang

School of Electronics and Information Engineering, Lanzhou Jiaotong University, Lanzhou 730000, China; zitongcl111@outlook.com (L.C.); lianghao0223@163.com (H.L.)
* Correspondence: dshen@mail.lzjtu.cn; Tel.: +86-173-9318-2470

Abstract: Signal detection in massive MIMO systems faces many challenges. The minimum mean square error (MMSE) approach for massive multiple-input multiple-output (MIMO) communications offer near to optimal recognition but require inverting the high-dimensional matrix. To tackle this issue, a Gauss–Seidel (GS) detector based on conjugate gradient and Jacobi iteration (CJ) joint processing (CJGS) is presented. In order to accelerate algorithm convergence, the signal is first initialized using the optimal initialization regime among the three options. Second, the signal is processed via the CJ Joint Processor. The pre-processed result is then sent to the GS detector. According to simulation results, in channels with varying correlation values, the suggested iterative scheme's BER is less than that of the GS and the improved iterative scheme based on GS. Furthermore, it can approach the BER performance of the MMSE detection algorithm with fewer iterations. The suggested technique has a computational complexity of $O(U^2)$, whereas the MMSE detection algorithm has a computational complexity of $O(U^3)$, where U is the number of users. For the same detection performance, the computational complexity of the proposed algorithm is an order of magnitude lower than that of MMSE. With fewer iterations, the proposed algorithm achieves a better balance between detection performance and computational complexity.

Keywords: massive MIMO; conjugate gradient; Jacobi; Gauss–Seidel; Kronecker channel

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1. Introduction

Massive MIMO technology is an extension of conventional MIMO technology that allows it to service more customers concurrently on the same frequency. This improves data rate, spectrum efficiency, energy efficiency, channel capacity, and connection stability [1]. Through further evolution, it is more frequently employed in telecommunications and related industries, such as the 5G new radio system. Massive MIMO is an essential foundational technology for future wireless systems, and it can be used in concert with other technologies like intelligent reflecting surfaces, orthogonal frequency division multiplexing, artificial intelligence, etc. Navigation, transportation, healthcare, and other fields can all benefit from the integration of communication, positioning, and sensing functions made possible by massive MIMO enabling intelligent reflecting surfaces. New human–computer interactions are made possible via the integration of artificial intelligence and MIMO technology, which will also enable extremely dependable real-time industrial information transmission [2].

Massive MIMO technology has several advantages, but it also has many drawbacks. In a massive MIMO communication system, other antennas interfere with the signal during transmission, making it far more difficult to detect the intended signal [3]. At the receiver side, a detector appropriate for enormous multiple-input multiple-output must thus be constructed to strike a compromise between low complexity and excellent performance.

The maximum likelihood [4] detection algorithm finds the signal closest to the transmitted signal by rounding through all the received signals, but the high complexity of the traversal signal makes it difficult to apply the algorithm in practice. In order to mitigate the cost of computing, linear detection schemes are recommended to obtain near-optimal detection performance, commonly known as the zero-forced [5] algorithm and the minimum mean squared error (MMSE) [6] detection algorithm. A highly complex matrix inversion is used in the MMSE method, which is difficult to implement in practice.

To avoid inverse operations in high-dimensional matrices, a few iterative algorithms have emerged based on the MMSE detection algorithm. Depending on the iterative approach, they can be classified into the following three categories. The first category is the iterative algorithm based on gradient search. The convergence speed of the steepest descent [7] iterative algorithm, influenced by the initial point, becomes slower when approaching the final estimated signal. The conjugate gradient (CG) [8] detection algorithm is offered to increase the speed of convergence, but a complex pre-processing process is required. The second category is repetitive methods that rely on series expansion, like the Neumann series-expansion algorithm [9], where the complexity is acceptable if the quantity of series-expansion terms is less than three, but high complexity arises if the quantity of series-expansion levels is larger than or equal to three. The third group is those that rely on iterative schemes to solve linear equations. The common ones are the Jacobi iterative method [10], the successive over relaxation (SOR) iterative algorithm [11], the Richardson iterative algorithm [12], the Gauss–Seidel (GS) [13] iterative algorithm, etc. Such detection algorithms reverse the matrices into solving a system of linear equations, which have a strong stepwise nature and high requirements for the filter matrix.

There is a limit to what can be achieved using a single algorithm for detection [14]. Therefore, the combination of different detection algorithms in a complementary manner can further improve the detection capability. A hybrid iterative algorithm combining adaptive damped Jacobi and conjugate gradient was proposed in the literature [15], which requires several iterations to achieve near-optimal performance. The literature [16] indicates that combining QR with traditional techniques for detection provides excellent manifestations and effectively improves the plant throughput. In the literature [17], a hybrid iterative algorithm such as QR-MLD, has been suggested that further reduces the complexity of the ML detection algorithm by using a sphere decoding algorithm suitable for high signal-to-noise ratio and an easy-to-use DM algorithm. The literature [18] proposes JAGS detection methods, which is initialized via the Jacobi method and then estimated via GS, but the convergence speed is slow. The preprocessing Gauss–Seidel iterative algorithms [19,20] use the preprocessing matrix to transform the original linear equation into a completely new linear equation, which results in a faster convergence rate in the new framework. However, complex preprocessing processes create additional computational difficulties.

1.1. Contributions

To mitigate the high cost of the inverse operation of MMSE detection, this work employs a low-complexity iterative approach for signal identification based on solving linear equations. The following is a summary of this work's main contributions:

1. An improved Gauss–Seidel iterative algorithm based on conjugate gradient and Jacobi (CJ) joint preprocessing is proposed, which can be described as the CJGS iterative method. The proposed algorithm attempts to combine CG and Jacobi iteration to accelerate the convergence of GS. Then the GS detector is employed to converge faster and iterate less.
2. A well-chosen initialization technique can lower computing cost and increase algorithmic accuracy. The best appropriate initialization technique for the advised approach is chosen by contrasting the three initialization strategies.
3. Software simulation and data analysis are utilized to provide more detailed examples of the suggested algorithm's advantages in terms of complexity and performance. Simulation representations demonstrate that, independent of channel correlation, the CJGS iterative scheme surpasses both the enhanced GS-based iterative program and the conventional Gauss–Seidel repeated approach in terms of BER ability. Because of

its reduced complexity, the MMSE detection ability may be attained with fewer iterations. As a result, the recommended strategy performs better in terms of complexity and detection effectiveness.

1.2. Paper Outline

The remaining parts of the paper are structured as follows: The large-scale MIMO system model, the channel model, and the principles of the MMSE detection algorithm are all covered in Section 2. The suggested detection methodology is explained in the third part. In the fourth part, the BER performance of the proposed method will be evaluated in light of previous study findings. Next, we determine how difficult the recommended approach is. The final portion contains the conclusions.

1.3. Notation

Vectors and matrices are displayed in bold lowercase and bold uppercase letters, respectively. Scalars are represented in lowercase letters. The operators $(.)^{-1}$ and $(.)^{H}$ indicate matrix inverse and Hermitian transpose. The identity matrix $U \times U$ is represented by I_U, which denotes the element in row i, column j of the matrix A. The result after the i-th iteration is the vector $s^{(i)}$.

2. Massive MIMO System Model

The uplink of an uncoded massive MIMO system is studied, and consists of B antennas configured at the received side serving U single-antenna users within the same time-frequency resource [21]. Figure 1 illustrates a schematic diagram. And B is much greater than U.

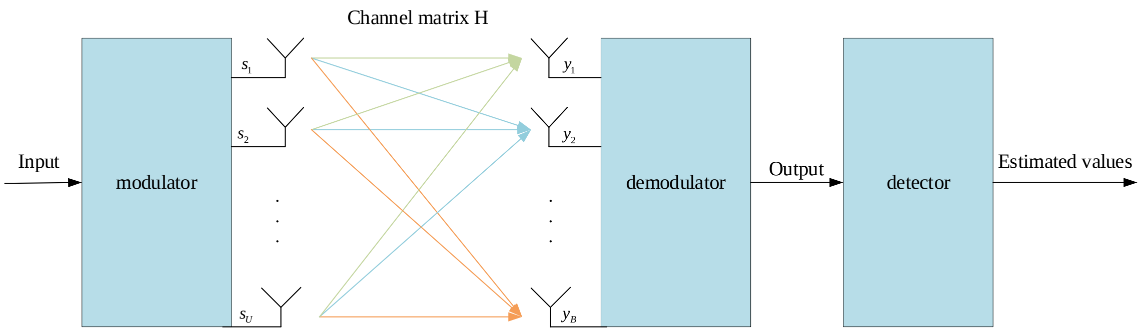


Figure 1. Signal-detection framework for massive MIMO communications.

The $U \times 1$ dimensional symbol vector transmitted by total users in the meantime is $s_c = [s_1, s_2, \cdots, s_U]$, where $s_U \in \Omega$ is the symbol vector sent by the U_{th} user and Ω is the set of modulated symbols for Quadrature Amplitude Modulation (QAM). The channel gain between the antennas at the receiving and transmitting apparatuses form the channel matrix $\widetilde{H}_c \in \mathbb{C}^{B\times U}$, where $\widetilde{h}_{c_{B.U}}$ is the channel coefficient between the U_{th} transmit and the B_{th} receive antenna. Then, the reception signal vector has the form [21]

$$y_c = \widetilde{H}_c s_c + n_c \tag{1}$$

where n_c denotes a $B \times 1$ dimensional complex Additive White Gaussian Noise (AWGN) vector with zero mean, and the covariance matrix is $\sigma^2 I$. To simplify the operation, Equation (1) is transformed into an analogue real-valued model as [21]

$$y = Hs + n \tag{2}$$

The assumption is that there is perfect channel state information available at the receiving end.

2.1. Channel Model

In practical large-scale MIMO communications, channels are generally correlated. The detection becomes complicated when channels are correlated. Considering the correlation between the transceiver and receiver antennas in massive MIMO communications, Kronecker's channel model [22] is described by

$$H = R_r^{\frac{1}{2}} H R_t^{\frac{1}{2}} \tag{3}$$

where H denotes the independent identically distributed complex channel fading matrix, and R_r and R_t denote the degree of channel correlation between the receiving and transmitting antennas, respectively.

$$R_r(n,m) = \begin{cases} \left(\zeta_r e^{j\phi_{n,m}}\right)^{m-n}, n \leq m \\ R_r^*(m,n), n > m \end{cases} \tag{4}$$

$$R_t(n,m) = \begin{cases} \left(\zeta_r e^{j\theta_{n,m}}\right)^{m-n}, n \leq m \\ R_t^*(m,n), n > m \end{cases} \tag{5}$$

where θ and ϕ represent the random phases of the transmit and receive antennas that are uniformly distributed, respectively. Assuming that ζ_r and ζ_t are known at the base station side, the channel correlation scenarios are classified into four types according to the different values, which are as follows [23].

(1) $\zeta_r = 0$ and $\zeta_t = 0$, the channel is independent and identically distributed.
(2) $\zeta_r = 0$ and $\zeta_t \neq 0$, the channel is user-side relevant.
(3) $\zeta_r \neq 0$ and $\zeta_t = 0$, the channel is base-station-side related.
(4) $\zeta_r \neq 0$ and $\zeta_t \neq 0$, the channel is fully correlated.

2.2. MMSE Detection

Applied at the recipient side of a massive MIMO communication, the MMSE iterative scheme offers near-optimal detection performance by degrading inaccuracies of the transmitted and received signals [24]. Estimation of the transmitted signal using the MMSE detection method is expressed in terms of [25]

$$\hat{s} = (H^H H + \sigma^2 I_U) H^H y = (G + \sigma^2 I_U) H^H y = A^{-1} y_{MF} \tag{6}$$

where $G = H^H H$ denotes the Gram matrix, and $y_{MF} = H^H y$ represents the matched filter. The MMSE detection method has $O(U^3)$ computational complexity, so it is difficult to apply in practice.

3. Proposed Algorithm

The MMSE detection algorithm has near sub-optimal detection performance when applied to large-scale MIMO systems, which generates a high exponential level of complexity. The algorithm is difficult to re-engineer in hardware devices. This paper applies the CJGS iterative algorithm to signal detection in massive MIMO communication. The scheme can be broken down into three parts, initialization, CJ joint processing, and GS iterative estimation, and the block method of CJGS detection is shown in Figure 2. During the initial stage, the optimal initial value is selected to speed up convergence. In the CJ joint processing phase, the CG iteration is used to select the optimal search direction and is combined with Jacobi iteration to reduce the number of subsequent iteration cycles. In the GS iteration estimation phase, Gauss–Seidel iteration is used to converge quickly, thus improving the overall detection performance of the massive MIMO system.

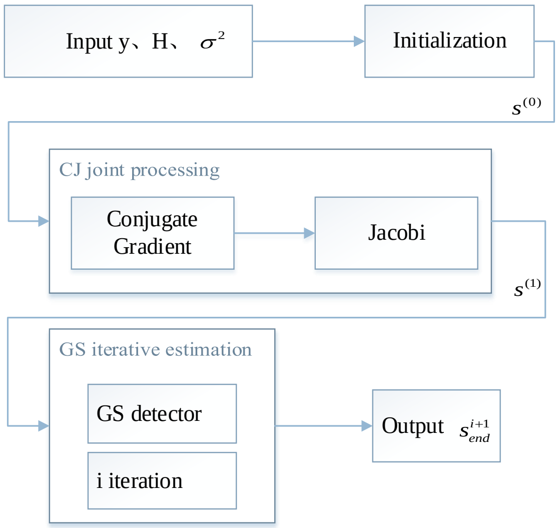


Figure 2. Block diagram of the CJGS detector.

3.1. GS Iterative Estimation

When GS is used for signal detection at the receiving end of a massive MIMO communication, the GS scheme can be executed according to the following steps.

1. Matrix decomposition. If there are fewer antennas at the user site than at the base station site for massive MIMO communication, the channels tend to be orthogonal to each other. The MMSE filter matrix is a positive definite matrix with diagonal dominance [26]. Matrix A can be divided into [26]

$$A = E + F + D \tag{7}$$

where D, E, and F denote the diagonal matrix, the strict upper triangular matrix, and the strict lower triangular matrix of matrix A, respectively.

2. Calculate the initial value $s^{(0)}$.
3. GS iterative estimation. The signal-estimation formula of the GS iterative algorithm is [26]

$$s^{(i)} = (D - F)^{-1}(Es^{(i-1)} + y_{MF}) \tag{8}$$

3.2. Initialization

The initial estimate has an impact on the computational complexity, the convergence speed, and the number of subsequent iterations required in massive MIMO signal detection. Traditional detection algorithms generally have zero vectors as initial values, which are far away from the final estimate, so convergence is slower. When the matrix A is a Hermitian positive definite matrix, the GS repetitive scheme converges for any initial solution [27]. For this reason, two initialization schemes are proposed below. Using the initial estimate of the zero vector as the reference object, we simulate the performance of the two initialization strategies discussed in Section 4.1. Scheme 3 refers to repetitive computation where the beginning value is zero.

1. In a given massive MIMO, the quantity of transmitting and receiving antennas is set. Thus, the initial solution is estimated by means of a linear transformation of the number estimating the initial solution, which can avoid matrix inversion operations and further limit computing power.

$$s^{(0)} = \frac{y_{MF}}{B + U} \tag{9}$$

2. The diagonal elements of the matrix A are dominant in massive MIMO communication. Therefore, the elements of matrix A can be grouped into diagonal elements that are not negligible and non-diagonal elements that are negligible, i.e., $A \approx D$. The computational complexity of inverting the matrix could be decreased by replacing A with D.

$$s^{(0)} = D^{-1} y_{MF} \tag{10}$$

3.3. CJ Joint Processing

CJ joint processing combines CG and Jacobi to pre-iterate the initialized signal, taking advantage of the good convergence of the CG initial iteration. It is CJ joint processing that reduces the number of cycles required for subsequent iterative estimation.

The CG [8] detection algorithm searches in the direction conjugate to the gradient until the search causes the residuals to converge to zero. The CG iteration algorithm gives a good initial detection direction, but the pre-processing step generates high complexity over many iterations of the loop. Therefore, we choose one conjugate gradient iteration to avoid the increase in complexity.

In the CG iterative algorithm, it is known that the initial residuals and the starting conjugate search direction are assumed to be $r^{(0)} = y_{MF} - As^{(0)}$ and $r^{(0)} = p^{(0)}$, respectively. Then, the result for the CG iterative algorithm after one iteration is [28]

$$s_{CG}^{(1)} = s^{(0)} + u^{(0)} p^{(0)} \tag{11}$$

where $u^{(0)} = \frac{(r^{(0)}, r^{(0)})}{(r^{(0)}, Ap^{(0)})}$ is the search step.

The expression for the signal estimate of the Jacobi iterative algorithm [10] applied to massive MIMO communication is [28]

$$s^{(i+1)} = D^{-1}(y_{MF} + (E+F)s^{(i)}) \tag{12}$$

Using the CG iterative algorithm as the starting value of the Jacobi iterative scheme, the result after one repetition of Jacobi can be given,

$$s^{(1)} = D^{-1}(y_{MF} + (E+F)s_{CG}^{(1)}) \tag{13}$$

Combined with the description of the above, the CJGS iterative algorithm is shown in Algorithm 1.

Algorithm 1 CJGS iterative algorithm

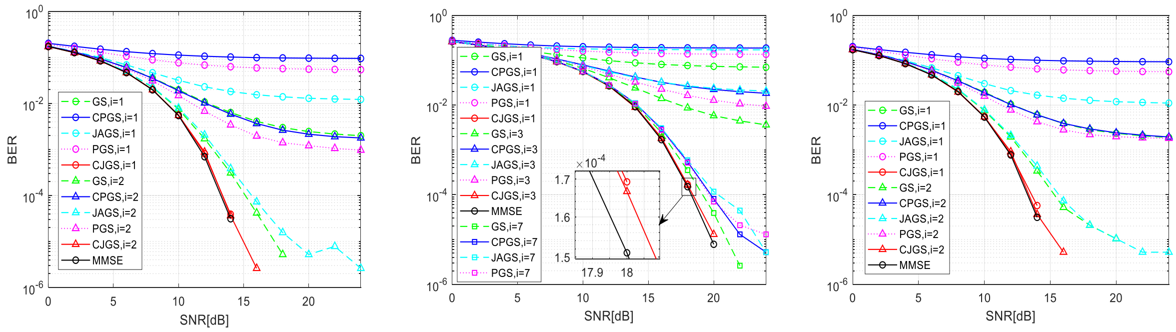
Input: y Hσ^2B U
Initialization:
1. A = $H^H H + \sigma^2 I$
2. $y_{MF} = H^H y$
3. $D = diag(diag(A))$
4. $E = -triu(A, 1)$
5. $F = -tril(A, -1)$
CJ joint processing:
6. $r^{(0)} = y_{MF} - As^{(0)}$
7. $r^{(0)} = p^{(0)}$
8. $u^{(0)} = \frac{(r^{(0)}, r^{(0)})}{(r^{(0)}, Ap^{(0)})}$
9. $s^{(1)} = D^{-1}(y_{MF} + (E+F)(s^{(0)} + u^{(0)} p^{(0)}))$
GS iterative estimation:
For i = 2 do
10. $s^{(i)} = (D-F)^{-1}(Es^{(i-1)} + y_{MF})$
End
Output: s_{end}^{i}

4. Simulation Results and Analysis

As a benchmark, MMSE detection algorithms are used to approximate optimal detection [29]. First, five iterative algorithms, GS, CPGS, PGS, and JAGS, have their bit error ratios (BERs) assessed in correlated and uncorrelated channels, respectively. The reference object used is the MMSE scheme. It functions very much the same as the greatest detecting system. Next, the CJGS iterative algorithm's starting parameters are simulated and investigated. In conclusion, an analysis was conducted to compare the computational complexity of the aforementioned techniques, where i denotes the number of iterations in the scheme.

4.1. BER Performance

A comparison and analysis of the BER performance of different iterative algorithms with different antenna configurations is shown in Figure 3. The channel matrix is independent and identically distributed, that is $\zeta_r = 0$, $\zeta_t = 0$. Signal modulation is 16–QAM.



(a) B×U=256×64 (b) B×U=128×64 (c) B×U=128×32

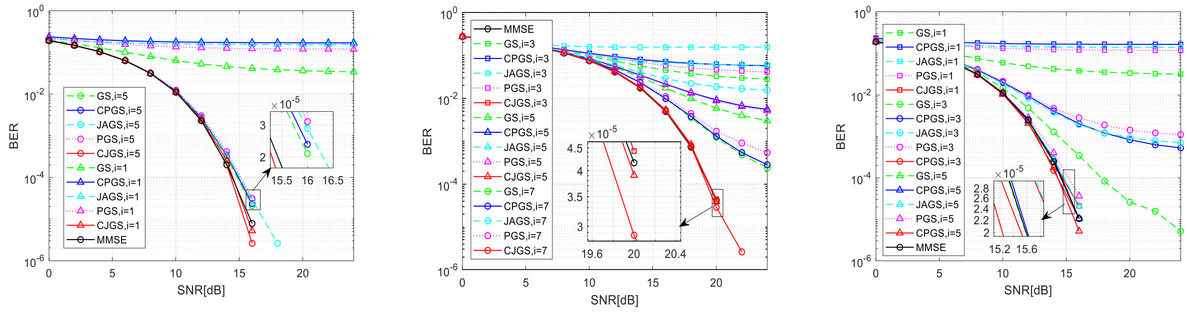
Figure 3. BER performance for independent and identically distributed channels.

In Figure 3a, it is clear that all schemes have excellent BER performance after a modest quantity of rounds, and that the CJGS scheme has improved BER performance over the other repetition methods for the identical number of rounds. The BER of the CJGS and JAGS iterative algorithms is 2.604×10^{-6} after two iterations, but the gap between them is 8 dB. Therefore, the CJGS iterative method is better. From Figure 3b, it can be noticed that the CJGS iterative scheme requires three iterations, while the other four iterative schemes require more than seven iteration cycles to achieve the ability of the MMSE scheme when the MIMO antenna configuration is 128×32. If the signal-to-noise ratio (S/N) is 20 dB, the BER achievable via the CJGS iterative method is 1.302×10^{-5} after three iterations, and the BERs achievable via GS, CPGS, JAGS, and PGS after seven iterations are 3.906×10^{-5}, 8.073×10^{-5}, 0.0001172, and 7.031×10^{-5}, respectively. In Figure 3c, it can be seen that with the MIMO antenna configuration of 128×32, CJGS, GS, and JAGS can achieve better BER performance after two iterations, but the BER performance of CPGS and PGS is not very good.

Furthermore, by keeping U at a fixed value and increasing B, all algorithms can gain an identical BER in a fraction of the repetitions from Figure 3a,b. Conversely, if B remains fixed and U increases, all algorithms need a higher signal-to-noise ratio to obtain identical results, and the quantity of receptions required increases from Figure 3b,c.

Figure 4 is a comparison and analysis of the BER behavior of different iterative algorithms. The antenna configurations are different. The channel matrix satisfies $\zeta_r = 0.3$, $\zeta_t = 0$. Signal modulation is 16–QAM.

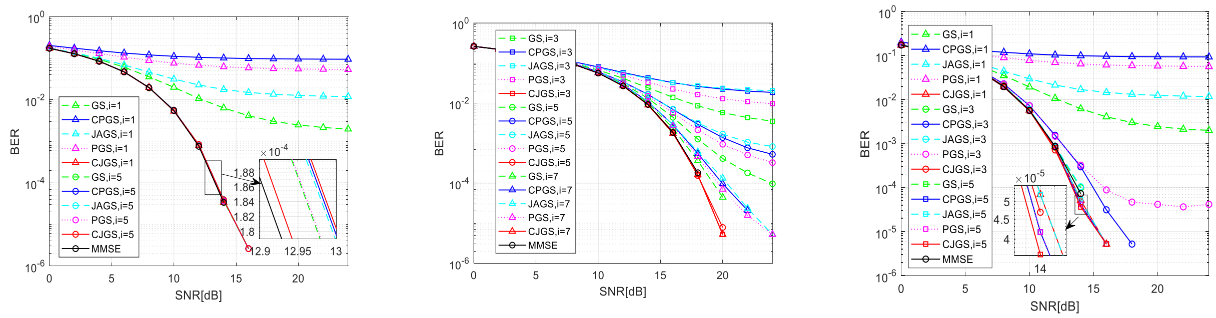
In Figure 4a, with the MIMO antenna configuration of 256×64, the performance of the other four iterative algorithms after running five iterations is still worse than that of the CJGS iterative algorithm in one iteration. The minimum BER achieved via MMSE detection is 7.813×10^{-6}, while the BER of the CJGS iteration is 5.208×10^{-6} after one iteration. From Figure 4b, the CJGS iterative scheme provides a much more superior performance when the MIMO antenna configuration is 128×64, which converges faster than the remaining iterative methods. In Figure 4c, it can be seen that the CJGS iterative algorithm after three iterations can achieve effects identical to those of the MMSE detection scheme if the S/N is 16 dB.



(a) B×U=256×64 (b) B×U=128×64 (c) B×U=128×32

Figure 4. BER performance for related channel in base station side.

A comparison and analysis of BER performance for different algorithms with different antenna configurations is shown in Figure 5. The channel matrix satisfies $\zeta_r = 0$, $\zeta_t = 0.3$. Signal modulation is 16–QAM.



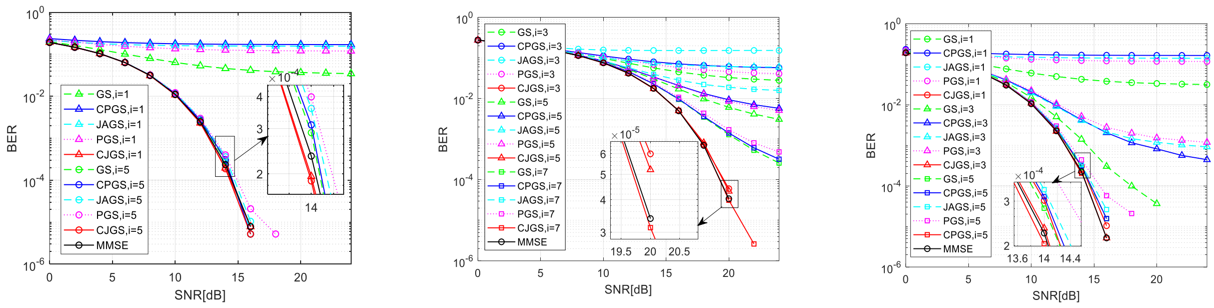
(a) B× U=256× 64 (b) B×U=128×64 (c) B×U=128×32

Figure 5. BER performance for related channels on the user side.

The detection performance of the GS, CPGS, JAGS, and PGS iterative algorithms is less satisfactory for an iteration number of one in Figure 5, while the CJGS iterative algorithm achieves a desirable performance close to MMSE detection. All iterative algorithms become better in detection performance as the number of iterations increases. As shown in Figures 3 and 5, all algorithms require more iterations to achieve optimal performance when the transmit-side antenna is correlated. If the MIMO antenna configured is set to 128×64 and the BER reaches 5.208×10^{-6}, the required S/N ratio for CJGS is 20 dB, while the S/N ratio requirement for PGS and JAGS iterations is 24 dB. Therefore, the proposed algorithms are more suitable for channel scenarios where correlation exists on the transmit side of large-scale MIMO systems.

Figure 6 concerns the fully correlated channel, i.e., $\zeta_r = 0.3$, $\zeta_t = 0.3$. The BER performances of the GS iterative algorithm, CPGS iterative method, JA-GS detection algorithm, PGS detection method, and CJGS iterative algorithm are compared and analyzed for different antenna configurations with 16−QAM modulation.

As shown in Figure 6, the detection capability of all the algorithms becomes better with more repetitions. From Figure 6a, the designed algorithm is shown to perform as well as the performance of MMSE detection at one iteration when the MIMO antenna configuration is 256×64, while all other iterations require more than five iterations to achieve the same results. If the S/N ratio is 16 dB, the BER achievable via the MMSE detection algorithm is 7.813×10^{-6}, while the BER of the CJGS iterative algorithm is 5.208×10^{-6} after five iterations. Thus, the CJGS iterative method can achieve a lower BER. From Figure 6b, it can be observed that to achieve the performance of MMSE detection when the MIMO antenna configuration is 128 × 64, the CJGS iterative algorithm needs to perform three iterations, while the other four iterative algorithms need more than seven iteration cycles. Figure 6c shows evidence that all algorithms achieve lower BER after five iterations when the MIMO antenna configuration is 128 × 32, but the suggested method has superior performance for equal iterations.



（a）B×U=256×64 （b）B×U=128×64 （c）B×U=128×32

Figure 6. BER performance for fully correlated channel.

According to the previous description, near MMSE detection performance is achieved with the proposed method with only a few iterations. The novel scheme outperforms the other four iterative algorithms. With a reduction in the quantity of the received side and an increase in the sending side, the ratio between them decreases. The proposed algorithm still shows a reliable performance, which can be well adapted to both channel-uncorrelated and channel-correlated scenarios.

As shown in Figure 7, baseline1, baseline2, and baseline3 are the initial values of schemes 1 and 2 and the zero vector mentioned in Section 3.2, part B, respectively. When the Massive MIMO antenna configuration is 128 × 32, the number of iterations is two, and the baseband modulation method is 16–QAM, the CJGS iterative method has superior BER performance and faster convergence when the initial values proposed in scheme 1 are used.

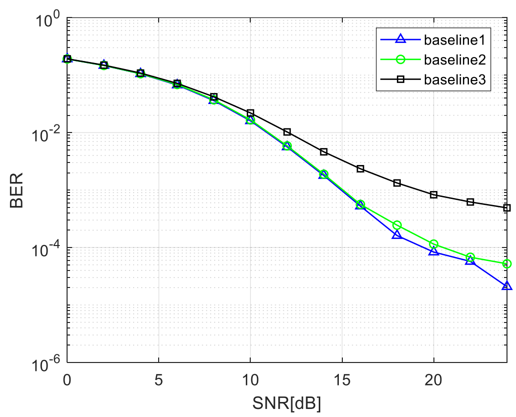


Figure 7. BER performance comparison of the CJGS iterative algorithm with different initial values.

4.2. Complexity Analysis

In massive MIMO communications, computational cost is a critical metric for signal-detection algorithms [30]. The sophistication of the CJGS iterative scheme consists of three parts, initialization, conjugate gradient and Jacobi co-processing, and GS iteration estimation, and only the number of true multiplicative operations of the method is considered in the analysis. The computation of A and y_{MF} is only once required, which is the same as the

MMSE detection algorithm [31]. Therefore, this part of the calculation is ignored and only the complexity of the subsequent steps is analyzed. One real multiplication is noted as one real multiplication time, the multiplication of a complex number with a constant is noted as two real multiplication times, and the multiplication of two complex numbers is noted as four real multiplication times.

The complexity of $s^{(0)}$: this involves multiplying a constant $\frac{1}{B+U}$ and a vector y_{MF} of $B \times 1$ with a complexity of $2U$.

The complexity of conjugate gradient and Jacobi co-processing:

The complexity of $r^{(0)}$: this involves multiplying a matrix A of $U \times U$ and a vector y_{MF} of $U \times 1$, with a complexity of $4U^2$.

The complexity of $u^{(0)}$: this concerns multiplying a vector $\left(r^{(0)}\right)^H$ of $1 \times U$ and a vector $r^{(0)}$ of $U \times 1$, with a complexity of $2U$. And it involves multiplying a matrix A of $U \times U$ and a vector $p^{(0)}$ of $U \times 1$, with a complexity of $4U^2$. It concerns multiplying a vector $\left(r^{(0)}\right)^H$ of $1 \times U$ and a vector $p^{(0)}$ of $U \times 1$, with a complexity of $2U$.

The complexity of $s^{(1)}$: this involves multiplying a constant $u^{(0)}$ and a vector $p^{(0)}$ of $U \times 1$, with a complexity of $2U$. The complexity of $D^{-1}(y_{MF} + (E+F)(s^{(0)} + u^{(0)}p^{(0)}))$ is $8U^2$.

The complexity of the GS iteration: the calculation of $s^{(i+1)}$ involves multiplying twice a matrix of U $\times$ U and a vector of $U \times 1$ with a complexity of $8U^2$.

Hence, the total complexity of the CJGS algorithm is $16U^2 + 8U + 8iU^2$. Taken together, the computational complexity of the various algorithms is shown in Table 1.

Table 1. Complex comparison of different algorithms.

Algorithm	Complexity
MMSE [6]	$8U^3 + 8BU^2 + 8BU + 2U$
GS [13]	$4U^2 + 8iU^2$
CPGS [19]	$20U^2 + 2U + 8iU^2$
JAGS [18]	$12U^2 + 8iU^2$
PGS [20]	$20U^2 + 2U + 8iU^2$
CJGS	$16U^2 + 8U + 8iU^2$

In Figure 8, the complexity of the different algorithms for MIMO systems are compared. The base station antenna count is 128 and the user-side antenna count ranges from 0 to 30 with three iterations. The suggested algorithm is much less complex than the MMSE scheme, whose complexity is $O(U^2)$.

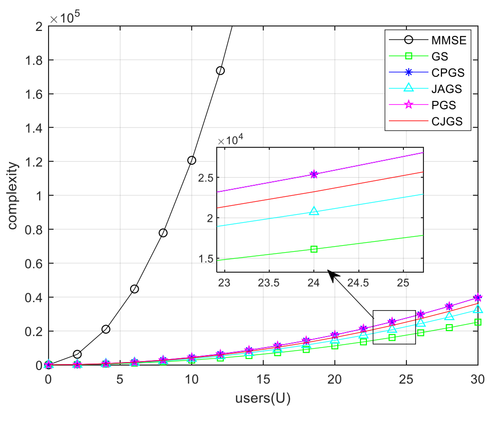


Figure 8. Comparison of algorithmic complexity.

5. Conclusions

Massive MIMO systems may be able to achieve better spectral and energy efficiency, but signal detection is still an issue that needs to be investigated. This paper introduced the Gauss–Seidel iterative technique for large MIMO communication, which is based on Jacobi iteration and shared processing conjugate gradient. To increase convergence and boost BER performance, CG and Jacobi iterations are used with GS iterations. Theoretical and simulation studies show that the CJGS algorithm performs substantially better than existing detection systems for various antenna designs, requiring only a finite number of cycles to obtain the desired results. The proposed algorithm requires one iteration to achieve the desired BER performance when the receiving and transmitting ends have 256 and 64 antennas, respectively, if the correlation coefficient at the transceiver end of the channel is 0, and the other iterative algorithms require two iterations. Similarly, if the correlation coefficient at the transceiver end of the channel is 0.3, the proposed algorithm requires one iteration to achieve the desired BER performance, while the other iterative algorithms require more than five iterations. For this reason, the suggested algorithm in this study is not considerably impacted by the channel correlation. The CJGS iterative method achieves a reduction in computational complexity by maintaining its computational complexity at $O(U^2)$. Even if the research in this study unfortunately only takes into account the influence of channel correlation and antenna setup, precoding and soft decision techniques can be introduced in later studies to further increase the reliability of detection.

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