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Robustness Evaluation of Aerodynamic Flutter Stability and Aerostatic Torsional Stability of Long-Span Suspension Bridges

Qing Xia and Yaojun Ge *

State Key Lab of Disaster Reduction in Civil Engineering, Tongji University, Shanghai 200092, China; 1510175@tongji.edu.cn
* Correspondence: yaojunge@tongji.edu.cn; Tel.: +86-21-65983451

Abstract: Structural robustness is defined by the international engineering community as the capabilities of structural systems that enable them to survive unforeseen or unusual circumstances. In order to highlight the unforeseen and unusual characteristics of wind hazards, this study introduces the concept of structural robustness into the wind-resistant design and evaluation of bridges and proposes the robustness evaluation of aerodynamic flutter and aerostatic torsional stability of long-span bridges. Furthermore, the return period of the design wind speed that a bridge can resist is used to represent wind-resistant robustness. Aiming at the problem of aerodynamic and aerostatic stability, the analysis methods of aerodynamic flutter stability robustness and aerostatic torsional stability robustness of long-span suspension bridges are respectively established. Based on the established methods of aerodynamic flutter stability and aerostatic torsional stability robustness evaluation, robustness analysis is carried out on eight completed long-span suspension bridges and two long-span suspension bridges to be built. The evaluation method proposed in this study makes it possible to measure the ability of bridge structures to resist multiple disasters using the same index.

Keywords: long-span suspension bridges; robustness evaluation; aerodynamic flutter; aerostatic torsional stability

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1. Introduction

The spans of long-span suspension bridges have been extended to new limits nowadays. Ge and Xiang performed a feasibility study on the aerodynamic performance of a super long-span suspension bridge with a 5000 m span. Furthermore, full-bridge aeroelastic model wind tunnel tests were conducted by Ge et al. to observe the wind-induced vibrations of this 5000 m spanned suspension bridge. With ever-growing bridge span length, modern suspension bridge structures are becoming more lightweight and flexible, which means that special concerns during aerodynamic analysis are significantly necessary. Aerodynamic and aerostatic instability of bridges take place when the wind speed exceeds a certain critical value, possibly predicted by physical wind tunnel tests or numerical calculations with aerodynamic or aerostatic parameters identified through wind tunnel experiments. Due to the randomness of the wind environment and bridge aerodynamic parameters, in the assessment of bridge failure against aerodynamic and aerostatic instability, it makes more sense to determine a failure probability for a specified target safety level (e.g., a specified return period) instead of only the critical wind speed.

A literature review indicated that research on the probabilistic assessment of flutter limit prediction of bridges consists of three main aspects, i.e., sensitivity analysis, uncertainty analysis, and reliability analysis [1–10]. Sensitivity analyses of structural parameters and flutter derivatives conducted by Nieto et al. [2], Caracoglia et al. [3], and Abbas and Morgenthal [4] provided great support to quantify the effect of these key variables on the probabilistic assessment model. Mannini and Bartoli [5], Argentini et al. [6], and Abbas and Morgenthal [4] conducted uncertainty analyses of geometric parameters, material properties, and aerodynamic derivatives to perform more complete and reasonable probabilistic assessments. Several frameworks for reliability analysis were proposed by Ge et al. [7], Pourzeyanali and Datta [8], and Baldomir et al. [9] to predict the probability of bridge failure against flutter. All of these studies on probabilistic assessment models provided a better understanding of the influence of both the uncertainties and physical randomness of the wind environment and bridge dynamic parameters. Furthermore, the applications of reliability analysis models determined the clear failure probability of a bridge due to flutter for a specified return period rather than providing an intuitive safety factor.

In addition to the probabilistic-based approach, the ability of bridges to resist unforeseen wind hazards may be better evaluated by further considering the robustness of the structures. The concept of structural robustness was proposed by the international engineering community [10–13] to deal with devastating damage and collapse of structures caused by unforeseen disasters (e.g., earthquake, typhoon, and terrorist attack). According to Biondini et al. [10], structural robustness is the ability of a structural system to resist damage that is disproportionate to the initial damage. The British Standing Committee on Structural Safety (SCOSS) [11] defines structural robustness as the ability to resist disproportionate damage. Ellingwood [12] defines structural robustness as the basic characteristic of a structural system to prevent damage transmission and mitigate the risk of disproportionate failure and progressive collapse. Knoll and Vogel [13] give a more accurate definition of robustness: robustness is the ability of a system to resist unforeseen or unusual circumstances. These definitions include two parts: the first is the action that has not been previously encountered or is beyond normal, and the second is the ability of the system to resist this action, which is especially suitable for evaluating the resistance to natural hazards or man-made disasters.

In order to highlight the unforeseen and multi-disaster characteristics of wind hazards, it is necessary to introduce the concept of structural robustness into the wind-resistant design and evaluation of long-span suspension bridges. On the one hand, the wind-resistant robustness design and evaluation of bridges can extend the existing safety coefficient based on the allowable stress method and the partial coefficient based on the limit state method to the failure probability or return period based on the probabilistic limit state, which makes wind-resistant design and evaluation more scientific and reasonable. On the other hand, it is also possible to unify the design and evaluation methods of bridge structures under the action of multiple disasters, such as earthquake, wind, fire, collision, and so on, which means that the ability of bridge structures to resist multiple disasters can be measured using the same index. This is more convenient for identifying and analyzing the most unfavorable disasters or the most unfavorable combination of disasters in the service life of bridge structures with the return period.

As such, the objective and scope of this paper are to propose the robustness evaluation of aerodynamic flutter and aerostatic torsional stability of long-span bridges by introducing the concept of structural robustness into the wind-resistant design and evaluation of bridges and highlight the unforeseen and multi-disaster characteristics of wind hazards. The wind-resistant stability of long-span suspension bridges is taken as an example, and the evaluation methods of aerodynamic flutter stability robustness and aerostatic torsional stability robustness of long-span suspension bridges are respectively established. Aiming at the problem of aerodynamic flutter stability robustness evaluation, a stochastic model of the flutter safety domain represented by four random variables is established, and flutter robustness analysis of ten long-span suspension bridges is carried out. Aiming at the problem of aerostatic torsional stability robustness evaluation, a stochastic model of the aerostatic torsional stability safety domain represented by two random variables is proposed. Considering the different values of variation coefficients, the aerostatic torsional stability robustness of three completed long-span suspension bridges is analyzed.

2. Evaluation Methods of Aerodynamic and Aerostatic Instability

The wind-resistant performance of bridges includes strength, stiffness, and stability. Unforeseen or unusual wind speeds may cause extreme aerodynamic and aerostatic instability problems in bridges, which is seriously fatal. Therefore, aiming at the problem of aerodynamic and aerostatic instability, the present paper focused on the robustness evaluation of aerodynamic flutter stability and aerostatic torsional stability.

2.1. Deterministic Method

Two methods and indices are often adopted to evaluate the wind-resistant performance of long-span bridges, including the safety coefficient based on the allowable stress method and the partial coefficient based on the ultimate limit state method.

The safety coefficient based on the allowable stress method can be defined as follows:

$$K = \frac{U_{re}}{U_{ac}} \tag{1}$$

where U_{re} is the wind speed that the bridge can resist, and U_{ac} is the maximum design wind speed. The bridge is safe when the coefficient is larger than 1. The larger the coefficient is, the safer the bridge will be. On the contrary, the bridge will fail when the coefficient is smaller than 1.

For the ultimate limit state method, the safety factor is considered as the partial coefficient and directly expressed in the equation of the ultimate limit state. Taking the evaluation of flutter stability as an example, the safety of wind resistance is expressed as:

$$K_1 U_{re} \geq K_2 U_{ac} \tag{2}$$

where K_1 is the partial coefficient for the wind speed that the bridge can resist and K_2 is the partial coefficient for the maximum design wind speed.

2.2. Probabilistic Method

Several probabilistic evaluation methods can be used in probabilistic aerodynamic flutter analysis. The first-order reliability method (FORM) is widely used due to its simplicity [7–9,14,15]. Another estimation method is the sampling method, in which Monte Carlo simulation (MCS) is included [4–6,16]. Meanwhile, other stochastic algorithms for flutter probability analysis were developed, such as assessing the Moment Lyapunov exponent of system stability with a Euler–Monte Carlo algorithm and random eigenvalue analysis using numerical tools [17,18].

2.3. Robustness of Aerodynamic and Aerostatic Stability

Based on the definition by Knoll and Vogel [13], and considering the maximum wind speed that is unforeseen during the design stage, the definition of wind-resistant robustness of bridges can be given as the ability of the bridge to resist the maximum wind speed that is beyond the common situation. It is represented by the return period of the design wind speed and is expressed as:

$$T_m = \frac{1}{P_F} \tag{3}$$

where P_F is the failure probability for the wind resistance of the bridge structure, which can be calculated as:

$$P_F = P\{Z \leq 0\} \tag{4}$$

where Z is the random function of the safety domain, based on the fundamental variables U_{re} and U_{ac}. When T_m is shorter than the design life of the bridge (e.g., 100 years), the wind-resistant ability of the bridge cannot meet the requirement. On the contrary, the wind-resistant ability of the bridge is sufficient when T_m is longer than the design life of the bridge. The return period is not only an intuitive index for evaluating the wind-resistant performance, but also an index that can be easily compared with other disaster factors (e.g., earthquake, fire, and collision).

2.4. Formulation of Robustness Evaluation of Aerodynamic and Aerostatic Stability

The random function of safety domain Z in Equation (4) is used to describe the limit state of the bridge structure when it resists the maximum wind speed that is unforeseen or unusual. Assuming that there are n random variables affecting the robustness of bridge structures, then the random function of the structural safety domain can be written as:

$$Z = g(X_1,\ X_2,\ \cdots,\ X_n) \tag{5}$$

where $X_i(i = 1, \cdots, n)$ is an arbitrary distribution random variable that affects the robustness of bridge structures.

In theory, the return period can be obtained from Equation (3) by substituting Equation (5) into Equation (4):

$$\begin{aligned} P_F = P\{Z \leq 0\} = P\{g(X_1,\ X_2,\ \cdots,\ X_n) \leq 0\} \\ = \int \cdots \int_{g(X_1,\ X_2,\ \cdots,\ X_n)<0} f_{X_1X_2\cdots X_n}(x_1,\ x_2,\ \cdots,\ x_n)dx_1dx_2\cdots dx_n \end{aligned} \tag{6}$$

where $f_{X_1X_2\cdots X_n}(x_1,\ x_2,\ \cdots,\ x_n)$ is the joint probability density function of n fundamental variables.

In general, it is difficult to calculate the failure probability directly through Equation (6), and equivalent central or checking point methods used to be introduced in the calculation of failure probability.

2.4.1. Equivalent Normal Distribution

Since the random variable $X_i(i = 1, \cdots, n)$ may obey any type of distribution in the random function of safety domain $Z = g(X_1,\ X_2,\ \cdots,\ X_n)$, it needs to be transformed into a random variable with standard normal distribution if equivalent central or checking point methods are used [19]. The mean value and standard deviation of the equivalent normal variable need to be calculated at the calculating point $X = (X_1,\ X_2,\ \cdots,\ X_n)$:

$$\mu'_{X_i} = X'_i - \Phi^{-1}\left[F_{X_i}(X'_i)\right]\sigma'_{X_i} \tag{7}$$

$$\sigma'_{X_i} = \frac{\varphi\left\{\Phi^{-1}\left[F_{X_i}(X'_i)\right]\right\}}{f_{X_i}(X'_i)} \tag{8}$$

where $\Phi(\cdot)$ and $\varphi(\cdot)$ are the standard normal distribution function and standard normal distribution density function, respectively, and $F_{X_i}(\cdot)$ and $f_{X_i}(\cdot)$ are the distribution function and density function of fundamental variables.

2.4.2. Equivalent Central Point Method

For linear random function of safety domain Z and normally distributed fundamental variables, there is the following corresponding relationship between probability P_F and reliability index β:

$$P_F = \Phi(-\beta),\ \ \beta = \Phi^{-1}(P_F) \tag{9}$$

$$\beta = \frac{\mu_Z}{\sigma_Z} \tag{10}$$

where μ_Z and σ_Z are the mean value and standard deviation of Z, respectively; and $\Phi(\cdot)$ is the standard normal distribution function.

For the random function of the safety domain with arbitrary fundamental variables, its Taylor expansion formula at the point $(X_1,\ X_2,\ \cdots,\ X_n) = (\mu_1,\ \mu_2,\ \cdots,\ \mu_n)$ can be derived as:

$$Z = f(X) = f(X_1,\ X_2,\ \cdots,\ X_n) \cong f(\mu_1,\ \mu_2,\ \cdots,\ \mu_n) + \sum_{i=1}^{n} \frac{\partial f}{\partial X_i}(X_i - \mu_i) \tag{11}$$

where $\frac{\partial f}{\partial X_i}$ is calculated at the point $(\mu_1,\ \mu_2,\ \cdots,\ \mu_n)$.

From Equation (11), the approximate values of μ_Z and σ_Z can be expressed as:

$$\mu_Z \cong f(\mu_1,\ \mu_2,\ \cdots,\ \mu_n) \tag{12}$$

$$\sigma_Z^2 \cong \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial f}{\partial X_i}\frac{\partial f}{\partial X_j} Cov\left[X_i,\ X_j\right] \tag{13}$$

Assuming that the fundamental variables are independent of one another, the reliability index β based on the equivalent central point method can be calculated as:

$$\beta = \frac{f(\mu_1,\ \mu_2,\ \cdots,\ \mu_n)}{\sqrt{\sum_{i=1}^{n}\left(\frac{\partial f}{\partial X_i}\Big|_{\mu_X}\sigma_{X_i}\right)^2}} \tag{14}$$

2.4.3. Equivalent Checking Point Method

When the random function of the safety domain for fundamental variables is nonlinear, it needs to be expanded at the checking point $P^*\left(\mu'^*_{X_1},\ \mu'^*_{X_2},\ \cdots,\ \mu'^*_{X_n}\right)$:

$$Z = g(X) \cong g\left(\mu'^*_{X_1},\ \mu'^*_{X_2},\ \cdots,\ \mu'^*_{X_n}\right) + \sum_{i=1}^{n}\left(\frac{\partial g}{\partial X_i}\right)_{P^*}\left(X_i - \mu'^*_{x_i}\right) \tag{15}$$

From Equation (15), the approximate value of μ_Z can be expressed as:

$$\mu_Z \cong g\left(\mu'^*_{X_1},\ \mu'^*_{X_2},\ \cdots,\ \mu'^*_{X_n}\right) + \sum_{i=1}^{n}\left(\frac{\partial g}{\partial X_i}\right)_{P^*}\left(\mu_{X_i} - \mu'^*_{X_i}\right) = \sum_{i=1}^{n}\left(\frac{\partial g}{\partial X_i}\right)_{P^*}\left(\mu_{X_i} - \mu'^*_{X_i}\right) \tag{16}$$

Assuming that the fundamental variables are independent of one another, the approximate value of σ_Z can be expressed as:

$$\sigma_Z = \sqrt{\sum_{i=1}^{n}\left[\left(\frac{\partial g}{\partial X_i}\right)_{P^*}\sigma_{X_i}\right]^2} \tag{17}$$

Introducing the separation function formula, Equation (17) can be linearized as:

$$\sigma_Z = \sum_{i=1}^{n}\alpha_i\sigma_{X_i}\left(\frac{\partial g}{\partial X_i}\right)_{P^*} \tag{18}$$

where α_i refers to the relative influence of the random variable $X_i(i = 1, \cdots, n)$ on the whole standard deviation, which is called the sensitivity coefficient. It can be calculated using the following formula:

$$\alpha_i = \frac{\left(\frac{\partial g}{\partial X_i}\right)_{P^*}\sigma_{X_i}}{\sqrt{\sum_{i=1}^{n}\left[\left(\frac{\partial g}{\partial X_i}\right)_{P^*}\sigma_{X_i}\right]^2}} \tag{19}$$

where α_i can be completely determined by the checking point $P^*\left(\mu'^*_{X_1},\ \mu'^*_{X_2},\ \cdots,\ \mu'^*_{X_n}\right)$ when the deviation of the random variable is known.

Therefore, the reliability index β can be obtained:

$$\beta = \frac{\mu_Z}{\sigma_Z} = \frac{\sum_{i=1}^{n}\left(\frac{\partial g}{\partial X_i}\right)_{P^*}\left(\mu_{X_i} - \mu'^*_{X_i}\right)}{\sum_{i=1}^{n}\alpha_i\sigma_{X_i}\left(\frac{\partial g}{\partial X_i}\right)_{P^*}} \tag{20}$$

From Equation (20), the coordinates of the checking point can be calculated as:

$$\mu'^{*}_{X_i} = \mu_{X_i} - \alpha_i \sigma_{X_i} \beta \tag{21}$$

The checking point also needs to meet the following conditions:

$$g\left(\mu'^{*}_{X_1},\ \mu'^{*}_{X_2},\ \cdots,\ \mu'^{*}_{X_n}\right) = 0 \tag{22}$$

The reliability index β based on the equivalent checking point method can be obtained by iteratively solving the $(n+1)$ equations contained in Equations (21) and (22) [20].

3. Aerodynamic Flutter Stability Robustness Evaluation

3.1. Stochastic Evaluation Equation of Aerodynamic Flutter Stability

As part of the wind-resistant performance of bridges, flutter robustness can be calculated using Equations (3) and (4), as discussed above. Therefore, the random function of the safety domain for flutter robustness evaluation in Equation (4) needs to be defined.

In the evaluation of bridge flutter performance based on the allowable stress method and ultimate limit state method, the wind speed that the bridge can resist is generally expressed as the product of the flutter critical wind speed and the corresponding correction coefficient [7]:

$$U_{re} = C_f U_f \tag{23}$$

where both C_f and U_f are random variables, C_f is the corresponding correction coefficient, and U_f is the flutter critical wind speed.

The maximum design wind speed is given by the product of the reference wind speed and the corresponding correction coefficient [7]:

$$U_{ac} = C_b U_b \tag{24}$$

where both C_b and U_b are random variables, C_b is the corresponding correction coefficient, and U_b is the reference wind speed.

The random function of the safety domain for flutter robustness evaluation can be determined by the function of the four fundamental variables introduced above:

$$Z = g(X_1,\ X_2,\ X_3,\ X_4) = g\left(C_f,\ U_f,\ C_b,\ U_b\right) = \frac{C_f}{C_b} U_f - U_b \tag{25}$$

where $g(X)$ is the joint probability density function of the four fundamental variables.

3.2. Fundamental Variables of Aerodynamic Flutter Stability Robustness Evaluation

3.2.1. Flutter Critical Wind Speed U_f

Flutter critical wind speed U_f is mainly determined by the mass, stiffness, and damping of structures and is a random variable that can partially reflect the structural behavior of bridges. According to Ge [21], U_f can be assumed to follow lognormal distribution and the mean value and standard deviation can be conservatively defined as:

$$E\left[U_f\right] = \mu_{U_f} \tag{26}$$

$$\sigma\left[U_f\right] = \sigma_{U_f} = 0.075 \mu_{U_f} \tag{27}$$

where μ_{U_f} is the mean value of the flutter critical wind speed based on wind tunnel experiments or numerical analysis.

3.2.2. Correction Coefficient of Flutter Critical Wind Speed C_f

The correction coefficient of flutter critical wind speed U_f is largely influenced by the uncertain difference between the wind speed in experimental model tests and the wind speed in field measurements. A comparison of these two kinds of wind speed indicated that the correction coefficient has a large fluctuation [22]. In order to simplify the calculation process, the coefficient is assumed to distribute normally, with a mean value of 1.0 and standard deviation of 5% [21].

$$E\left[C_f\right] = \mu_{C_f} = 1.0 \tag{28}$$

$$\sigma\left[C_f\right] = \sigma_{C_f} = 0.05 \tag{29}$$

3.2.3. Bridge Reference Wind Speed U_b

The bridge reference wind speed is usually assumed to follow an extreme value distribution. In Chinese wind-resistant design specifications for highway bridges [23], it is stipulated that U_b follows extreme value distribution type I, and the probability distribution function is:

$$F(U_b) = \exp\left[-\exp\left(-\frac{U_b - b}{a}\right)\right] \tag{30}$$

where a and b are variables related to the deviation and location, respectively, and can be given as:

$$E(U_b) = \mu_{U_b} = 0.5772a + b \tag{31}$$

$$\sigma(U_b) = \sigma_{U_b} = \frac{\pi}{\sqrt{6}}a \tag{32}$$

3.2.4. Correction Coefficient of Design Wind Speed C_b

The correction coefficient of design wind speed C_b mainly concerns modification for the gust factor. According to Ge [21], C_b is assumed to distribute normally and its mean value and standard deviation are conservatively defined as:

$$E[C_b] = \mu_{C_b} \tag{33}$$

$$\sigma[C_b] = \sigma_{C_b} = 0.07\mu_{C_b} \tag{34}$$

where the value of μ_{C_b} is stipulated in Chinese wind-resistant design specifications for highway bridges [23].

3.3. Robustness Calibration of Aerodynamic Flutter Stability

3.3.1. Flutter Robustness Analysis of Eight Completed Bridges

Taking eight representative long-span suspension bridges as examples, including Nansha Bridge, Zhoushan Xihoumen Bridge, Runyang Yangtze River Bridge, Jiangyin Yangtze River Bridge, Tsing Ma Bridge, Huangpu Bridge, Humen Bridge, and Haicang Bridge, flutter robustness analysis and comparisons were carried out. The key design parameters and dynamic characteristics for the eight existing bridges are listed in Table 1. The mean values and standard deviations of the four fundamental random variables of these eight bridges are listed in Table 2.

Table 1. Key design parameters and fundamental frequencies of vertical bending and torsion modes for eight existing bridges.

Bridge	Main Span (m)	Girder Type	Width of Girder (m)	Depth of Girder (m)	Vertical (Hz)	Torsion (Hz)
Nansha Bridge	1688	Box	49.7	4.0	0.1018	0.2201
Xihoumen Bridge	1650	Twin box	36.0	3.5	0.1001	0.2323
Runyang Yangtze River Bridge	1490	Box	38.7	3.0	0.1241	0.2308
Jiangyin Yangtze River Bridge	1385	Box	36.9	3.0	0.1418	0.2625
Tsing Ma Bridge	1377	Truss	41.0	7.6	0.1050	0.2680
Huangpu Bridge	1108	Box	41.7	3.5	0.1502	0.3180
Humen Bridge	888	Box	35.6	3.0	0.1715	0.3612
Haicang Bridge	648	Box	36.6	3.0	0.1680	0.4570

Table 2. Mean values and standard deviations of fundamental random variables for eight existing bridges.

Bridge	μ_{C_f}	σ_{C_f}	μ_{U_f}	σ_{U_f}	μ_{C_b}	σ_{C_b}	μ_{U_b}	σ_{U_b}
Nansha Bridge	1	0.05	70.7	5.30	1.16	0.08	27.04	5.41
Xihoumen Bridge	1	0.05	95.0	7.13	1.16	0.08	33.59	6.72
Runyang Yangtze River Bridge	1	0.05	55.1	4.13	1.16	0.08	23.05	4.61
Jiangyin Yangtze River Bridge	1	0.05	61.0	4.58	1.16	0.08	22.17	4.43
Tsing Ma Bridge	1	0.05	99.0	7.43	1.16	0.08	40.56	8.11
Huangpu Bridge	1	0.05	87.2	6.54	1.16	0.08	28.93	5.79
Humen Bridge	1	0.05	79.3	5.95	1.16	0.08	29.29	5.86
Haicang Bridge	1	0.05	95.0	7.13	1.18	0.08	31.39	6.28

Based on the equivalent central point method and equivalent checking point method, the robustness evaluation of flutter stability for these eight bridges was conducted, and the calculation results are summarized in Tables 3 and 4. The return periods of the reference wind speed that the eight existing bridges could resist ranged from 584 years to 12,018 years, which were much longer than the service life. Therefore, these eight bridges all possessed the ability to resist the maximum wind speed that is unforeseen or unusual.

Table 3. Results of flutter robustness evaluation for eight existing bridges based on the equivalent central point method.

Bridge	The Equivalent Central Point Method		
	β	P_F	T_m (Year)
Nansha Bridge	3.2327	6.13×10^{-4}	1631
Xihoumen Bridge	3.4988	2.34×10^{-4}	4280
Runyang Yangtze River Bridge	2.9267	1.70×10^{-3}	584
Jiangyin Yangtze River Bridge	3.3951	3.43×10^{-4}	2915
Tsing Ma Bridge	2.9983	1.40×10^{-3}	737
Huangpu Bridge	3.7073	1.05×10^{-4}	9546
Humen Bridge	3.3288	4.36×10^{-4}	2293
Haicang Bridge	3.6729	1.20×10^{-4}	8341

Figure 1 shows the comparison of the reliability indices calculated using the equivalent central point method and equivalent checking point method. It can be seen from the figure that the relative calculation errors of the reliability index calculated using the equivalent central point method and equivalent checking point method were between 1.06% and 1.87%. The equivalent central point method could be adopted for the purpose of simplified calculations.

Table 4. Results of flutter robustness evaluation for eight existing bridges based on the equivalent checking point method.

Bridge	The Equivalent Checking Point Method		
	β	P_F	T_m (Year)
Nansha Bridge	3.2709	5.36×10^{-4}	1865
Xihoumen Bridge	3.5441	1.97×10^{-4}	5076
Runyang Yangtze River Bridge	2.9582	1.50×10^{-3}	646
Jiangyin Yangtze River Bridge	3.4504	2.80×10^{-4}	3572
Tsing Ma Bridge	3.0314	1.20×10^{-3}	822
Huangpu Bridge	3.7652	8.32×10^{-5}	12,018
Humen Bridge	3.3922	3.47×10^{-4}	2884
Haicang Bridge	3.7235	9.82×10^{-5}	10,181

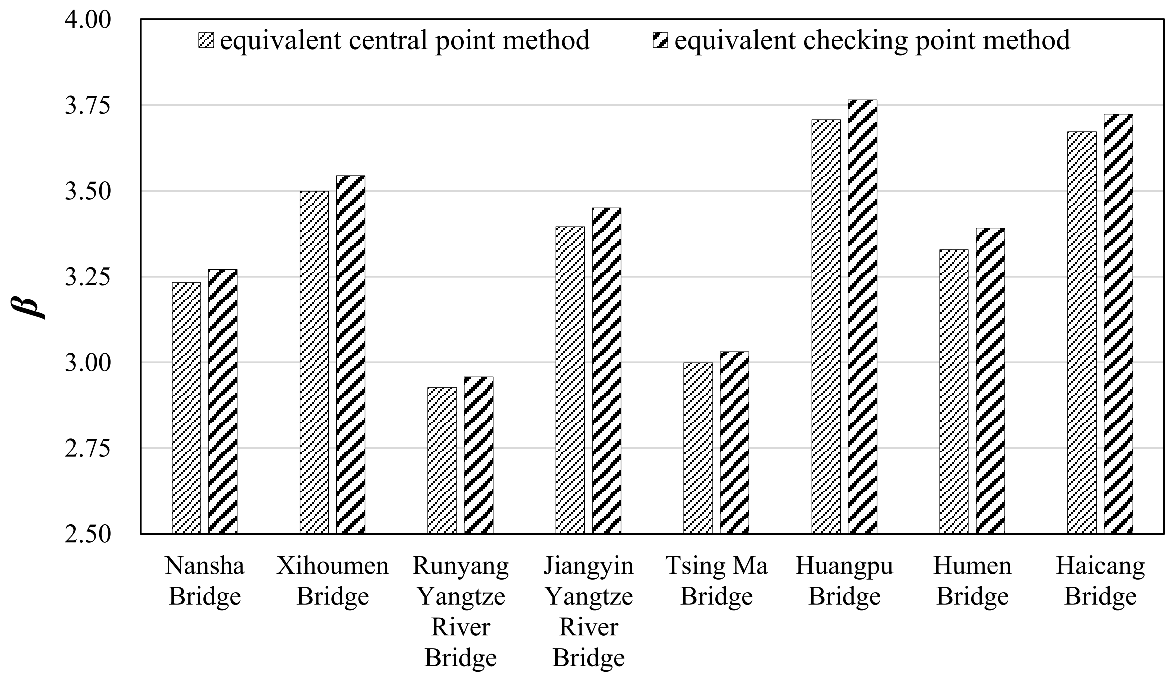

Figure 1. Comparison of the reliability indices calculated using the equivalent central point method and equivalent checking point method for eight existing bridges.

3.3.2. Flutter Robustness Analysis of Two Bridges to Be Built

Based on the analysis of the above eight completed long-span bridges, the robustness evaluation was conducted for two other long-span suspension bridges, Shuangyumen Bridge (with a 1768 m main span, under construction, China) and Sunda Strait Bridge (with a 2016 m main span, in design, Indonesia), to re-check the evaluation method of flutter robustness. The statistic characteristics of the fundamental random variables of these two bridges are listed in Table 5.

Table 5. Mean values and standard deviations of fundamental random variables for two bridges to be built.

Bridge	μ_{C_f}	σ_{C_f}	μ_{U_f}	σ_{U_f}	μ_{C_b}	σ_{C_b}	μ_{U_b}	σ_{U_b}
Shuangyumen Bridge	1	0.05	84.1	6.30	1.22	0.09	36.10	7.22
Sunda Strait Bridge	1	0.05	93.0	7.0	1.22	0.09	40.05	8.01

Based on the equivalent central point method and equivalent checking point method, the calculated failure probability, reliability index, and return period were compared and listed in Tables 6 and 7. The robustness evaluation results showed that the flutter robustness of these two bridges was relatively low, and the return periods of the maximum wind speed they could resist were between 239 and 247 years. Furthermore, flutter safety problems may occur if these two bridges encounter a strong wind with a return period of 300 years.

Table 6. Results of flutter robustness evaluation for two bridges to be built based on the equivalent central point method.

Bridge	The Equivalent Central Point Method		
	β	P_F	T_m (Year)
Shuangyumen Bridge	2.6481	4.05×10^{-3}	247
Sunda Strait Bridge	2.6367	4.19×10^{-3}	239

Table 7. Results of flutter robustness evaluation for two bridges to be built based on the equivalent checking point method.

Bridge	The Equivalent Checking Point Method		
	β	P_F	T_m (Year)
Shuangyumen Bridge	2.6793	3.69×10^{-3}	271
Sunda Strait Bridge	2.6668	3.83×10^{-3}	261

Figure 2 shows the comparison of the reliability indices calculated using the equivalent center point method and equivalent checking point method. The same conclusion as discussed for the eight existing bridges can be drawn from the figure.

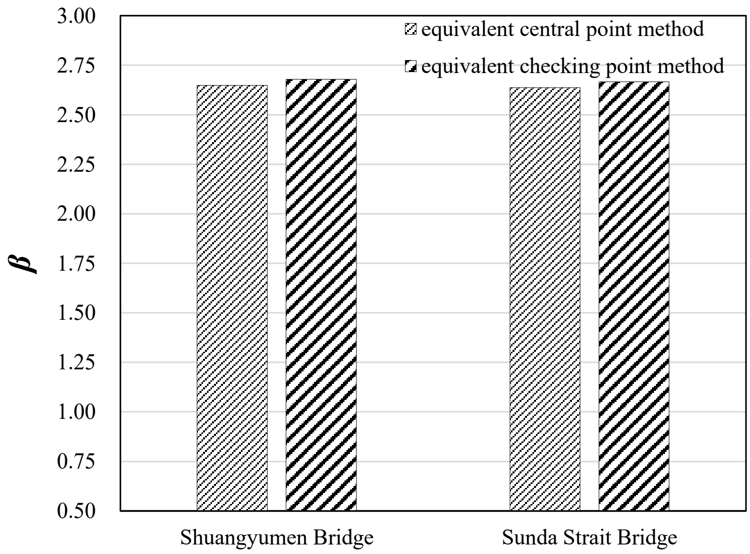

Figure 2. Comparison of the reliability indices calculated using the equivalent central point method and equivalent checking point method for two bridges to be built.

4. Aerostatic Torsional Stability Robustness Evaluation

4.1. Stochastic Evaluation Equation of Aerostatic Torsional Stability

In Chinese wind-resistant design specifications for highway bridges [24], it is required that the static wind stability test of the bridge meet the requirements of the following formula:

$$U_{td} \geq \gamma_{ai} U_d \tag{35}$$

where U_{td} is the critical wind speed of aerostatic instability, and U_d is the reference wind speed, with both being random variables. γ_{ai} is the partial coefficient of aerostatic torsional stability, which is determined by obtaining the critical wind speed of aerostatic instability.

The random function of the safety domain for aerostatic torsional stability robustness evaluation can be determined by the function of the two fundamental variables introduced above:

$$Z = g(X_1,\ X_2) = g(U_t,\ U_b) = U_t - \gamma_{ai} U_b \tag{36}$$

where $g(X)$ is the joint probability density function of the two fundamental variables.

4.2. Fundamental Variables of Aerostatic Torsional Stability Robustness Evaluation

4.2.1. Aerostatic Instability Critical Wind Speed U_{td}

The critical wind speed of aerostatic instability U_{td} is mainly determined by the mass, stiffness, and geometrical dimension of the structures and is a random variable that can partially reflect the structural behavior of bridges. U_{td} can be assumed to follow lognormal distribution and the mean value and standard deviation can be conservatively defined as:

$$E[U_{td}] = \mu_{U_{td}} \tag{37}$$

$$\sigma[U_{td}] = \sigma_{U_{td}} = \delta_t \mu_{U_{td}} \tag{38}$$

where U_{td} is the mean value of the critical wind speed of aerostatic instability obtained by wind tunnel tests or numerical simulations, and $\delta_t = \sigma_{U_{td}} / \mu_{U_{td}}$ is the coefficient of variation of the critical wind speed of aerostatic instability U_{td}. In order to analyze the influence of different coefficients of variation of U_{td} on the evaluation of aerostatic torsional stability robustness, the coefficients of variation were taken as 0.05, 0.075, and 0.1, respectively, and the reliability index, failure probability, and return period of the bridge under each value were calculated.

4.2.2. Bridge Reference Wind Speed U_b

The bridge reference wind speed U_b was the same as discussed in the flutter robustness evaluation. Through a comparative study on the return period coefficient of extreme wind speed in Chinese wind-resistant design specifications for highway bridges, Xu et al. [25] pointed out that when the coefficient of variation is 0.13, the calculated return period coefficient of extreme wind speed is consistent with that given in the specification. In order to analyze the influence of different coefficients of variation of U_b on the evaluation of aerostatic torsional stability robustness, the coefficients of variation δ_b were taken as 0.12, 0.14, 0.16, 0.18, and 0.2, respectively, and the reliability index, failure probability, and return period of the bridge under each value were calculated.

4.2.3. Partial Coefficient of Aerostatic Torsional Stability γ_{ai}

In the following cases of aerostatic torsional stability robustness evaluation, the critical wind speed of aerostatic instability was obtained using a three-dimensional calculation method considering aerodynamic nonlinearity and geometric nonlinearity or full-bridge aeroelastic model wind tunnel testing. According to Chinese wind-resistant design specifications for highway bridges [24], when the calculation method considering aerodynamic nonlinearity and geometric nonlinearity is used to analyze the critical wind speed of aerostatic instability, γ_{ai} should be taken as 1.60; when the critical wind speed of aerostatic instability is obtained by wind tunnel testing with a full-bridge aeroelastic model, γ_{ai} should be taken as 1.40.

4.3. Robustness Calibration of Aerostatic Torsional Stability

Taking existing long-span suspension bridges as examples, including Jiangyin Yangtze River Bridge, Xihoumen Bridge and Nansha Bridge, aerostatic torsional stability robustness analysis and comparisons were carried out. The critical wind speeds of aerostatic instability of Jiangyin Yangtze River Bridge were 113 m/s at 0° initial wind attack angle and 110 m/s at +3° initial wind attack angle [26,27]. The critical wind speeds of aerostatic instability of Xihoumen Bridge were 105 m/s at 0° initial wind attack angle and 95 m/s at +3° initial wind attack angle [28]. The critical wind speeds of aerostatic instability of Nansha Bridge were 114 m/s at 0° initial wind attack angle and 108 m/s at +3° initial wind attack angle [29].

Considering the different values of the coefficient of variation of the critical wind speed of aerostatic instability U_{td} and the design reference wind speed U_b, the statistical characteristics of these two fundamental random variables are given in Tables 8 and 9, respectively.

Table 8. Mean values and standard deviations of U_{td} for three existing bridges.

Bridge	Initial Wind Attack Angle	U_{td}	$\sigma_{U_{td}}$		
			δ_t = 0.05	δ_t = 0.075	δ_t = 0.1
Jiangyin Yangtze River Bridge	0°	113	5.65	8.48	11.30
	+3°	110	5.50	8.25	11.00
Xihoumen Bridge	0°	105	5.25	7.88	10.50
	+3°	95	4.75	7.13	9.50
Nansha Bridge	0°	114	5.70	8.55	11.40
	+3°	108	5.40	8.10	10.80

Table 9. Mean values and standard deviations of U_b for three existing bridges.

δ_b	Jiangyin Yangtze River Bridge		Xihoumen Bridge		Nansha Bridge	
	μ_{U_b}	σ_{U_b}	μ_{U_b}	σ_{U_b}	μ_{U_b}	σ_{U_b}
$\delta_b = 0.2$	23.78	4.76	33.11	6.62	27.24	5.45
$\delta_b = 0.18$	24.74	4.45	34.43	6.20	28.33	5.10
$\delta_b = 0.16$	25.77	4.12	35.87	5.74	29.52	4.72
$\delta_b = 0.14$	26.89	3.76	37.43	5.24	30.80	4.31
$\delta_b = 0.12$	28.12	3.37	39.14	4.70	32.21	3.87

4.3.1. Jiangyin Yangtze River Bridge

The calculated reliability indices and return periods at 0° initial wind attack angle and +3° wind attack angle are listed in Tables 10 and 11, respectively. At the initial wind attack angle of 0°, the maximum failure probability of Jiangyin Yangtze River Bridge was 8.66×10^{-6}, and the shortest return period of the reference wind speed that the bridge could resist was 115,471 years. Even at a relatively unfavorable initial wind attack angle of +3°, the maximum failure probability was 1.33×10^{-5}, and the shortest return period could still reach 75,162 years, which was far beyond the design service life. Therefore, Jiangyin Yangtze River Bridge possesses the ability to resist the maximum wind speed that is unforeseen or unusual. It can be known from the results listed in Tables 10 and 11 that in different combinations of the coefficients of variation of U_{td} and U_b, the relative errors of the reliability index were within 16.72% at the initial wind attack angle of 0° and 16.53% at the initial wind attack angle of +3°.

Table 10. Results of aerostatic torsional stability robustness evaluation of Jiangyin Yangtze River Bridge at 0° initial wind attack angle.

δ_b	δ_t = 0.05		δ_t = 0.075		δ_t = 0.1	
	β	T_m (Year)	β	T_m (Year)	β	T_m (Year)
$\delta_b = 0.12$	5.1593	8,068,588	4.9826	3,187,985	4.7643	1,055,351
$\delta_b = 0.14$	4.9500	2,694,928	4.7997	1,258,626	4.6124	502,454
$\delta_b = 0.16$	4.7832	1,159,243	4.6521	608,695	4.4875	277,532
$\delta_b = 0.18$	4.6491	599,904	4.5323	342,789	4.3850	172,435
$\delta_b = 0.20$	4.5361	349,016	4.4306	212,831	4.2969	115,471

Table 11. Results of aerostatic torsional stability robustness evaluation of Jiangyin Yangtze River Bridge at +3° initial wind attack angle.

δ_b	δ_t = 0.05		δ_t = 0.075		δ_t = 0.1	
	β	T_m (year)	β	T_m (year)	β	T_m (year)
$\delta_b = 0.12$	5.0324	4,129,013	4.8607	1,709,820	4.6472	594,405
$\delta_b = 0.14$	4.8311	1,472,963	4.6852	715,086	4.5022	297,382
$\delta_b = 0.16$	4.6707	666,278	4.5435	361,482	4.3829	170,780
$\delta_b = 0.18$	4.5418	358,578	4.4286	210,866	4.2849	109,396
$\delta_b = 0.20$	4.4331	215,314	4.3309	134,679	4.2007	75,162

4.3.2. Xihoumen Bridge

The calculated reliability indices and return periods of Xihoumen Bridge at 0° initial wind attack angle and +3° wind attack angle are listed in Tables 12 and 13, respectively. At the initial wind attack angle of 0°, the maximum failure probability of Xihoumen Bridge was 4.00×10^{-4}, and the shortest return period of the reference wind speed that the bridge could resist was 2368 years. At a relatively unfavorable initial wind attack angle of +3°, the maximum failure probability was 1.43×10^{-3}, and the shortest return period could still reach 700 years, which indicated that aerostatic instability problems may occur if Xihoumen Bridge encounters a strong wind with a return period of 1000 years. It can be known from the results listed in Tables 12 and 13 that in different combinations of the coefficients of variation of U_{td} and U_b, the relative errors of the reliability index were within 14.02% at the initial wind attack angle of 0° and 12.38% at the initial wind attack angle of +3°.

Table 12. Results of aerostatic torsional stability robustness evaluation of Xihoumen Bridge at 0° initial wind attack angle.

δ_b	δ_t = 0.05		δ_t = 0.075		δ_t = 0.1	
	β	T_m (Year)	β	T_m (Year)	β	T_m (Year)
$\delta_b = 0.12$	3.882	19,305	3.7448	11,078	3.5711	5626
$\delta_b = 0.14$	3.7596	11,752	3.6443	7459	3.4972	4254
$\delta_b = 0.16$	3.6615	7976	3.5619	5432	3.4341	3364
$\delta_b = 0.18$	3.5828	5884	3.4948	4216	3.3815	2774
$\delta_b = 0.20$	3.519	4617	3.4399	3437	3.3377	2368

Table 13. Results of aerostatic torsional stability robustness evaluation of Xihoumen Bridge at +3° initial wind attack angle.

δ_b	δ_t = 0.05		δ_t = 0.075		δ_t = 0.1	
	β	T_m (Year)	β	T_m (Year)	β	T_m (Year)
$\delta_b = 0.12$	3.4041	3013	3.2776	1910	3.1164	1092
$\delta_b = 0.14$	3.3177	2204	3.2121	1518	3.0765	955
$\delta_b = 0.16$	3.248	1721	3.1572	1256	3.04	845
$\delta_b = 0.18$	3.1922	1416	3.1124	1078	3.009	763
$\delta_b = 0.20$	3.1472	1213	3.0757	952	2.9827	700

4.3.3. Nansha Bridge

The calculated reliability indices and return periods of Nansha Bridge at 0° initial wind attack angle and +3° wind attack angle are listed in Tables 14 and 15, respectively. At the initial wind attack angle of 0°, the maximum failure probability of Nansha Bridge was 6.02×10^{-5}, and the shortest return period of the reference wind speed that the bridge could resist was 16,624 years. Even at a relatively unfavorable initial wind attack angle of +3°, the maximum failure probability was 1.29×10^{-4}, and the shortest return period could still reach 7749 years, which was far beyond the design service life. Therefore, Nansha

Bridge possesses the ability to resist the maximum wind speed that is unforeseen or unusual. It can be known from the results listed in Tables 14 and 15 that in different combinations of the coefficients of variation of U_{td} and U_b, the relative errors of the reliability index were within 15.59% at the initial wind attack angle of 0° and 15.07% at the initial wind attack angle of +3°.

Table 14. Results of aerostatic torsional stability robustness evaluation of Nansha Bridge at 0° initial wind attack angle.

δ_b	δ_t = 0.05		δ_t = 0.075		δ_t = 0.1	
	β	T_m (Year)	β	T_m (Year)	β	T_m (Year)
$\delta_b = 0.12$	4.5558	383,240	4.4004	185,097	4.2051	76,638
$\delta_b = 0.14$	4.3896	176,119	4.2577	96,833	4.0909	46,544
$\delta_b = 0.16$	4.2547	95,543	4.14	57,586	3.994	30,784
$\delta_b = 0.18$	4.1453	58,933	4.0436	37,996	3.9135	21,986
$\delta_b = 0.20$	4.055	39,893	3.9633	27,056	3.8455	16,624

Table 15. Results of aerostatic torsional stability robustness evaluation of Nansha Bridge at +3° initial wind attack angle.

δ_b	δ_t = 0.05		δ_t = 0.075		δ_t = 0.1	
	β	T_m (Year)	β	T_m (Year)	β	T_m (Year)
$\delta_b = 0.12$	4.3023	118,319	4.1544	61,323	3.9677	27,560
$\delta_b = 0.14$	4.153	60,949	4.0278	35,523	3.8687	18,279
$\delta_b = 0.16$	4.0316	36,102	3.923	22,869	3.7842	12,970
$\delta_b = 0.18$	3.9334	23,880	3.8372	16,071	3.7137	9792
$\delta_b = 0.20$	3.8523	17,092	3.7657	12,042	3.6541	7749

5. Conclusions

The wind-resistant robustness of bridges is defined as the ability of a bridge to resist the maximum wind speed that is unforeseen or unusual. The return period of the maximum wind speed that a bridge can resist was used to represent the wind-resistant robustness index.

Aiming at the problem of flutter robustness evaluation, a stochastic model of the flutter safety domain determined by four random variables was established. Flutter robustness analysis of eight existing long-span suspension bridges and two long-span suspension bridges to be built was carried out. The robustness evaluation results showed that the return periods of the reference wind speed that the eight existing bridges could resist ranged from 584 years to 12,018 years, which were much longer than the service life. The flutter robustness of the two bridges to be built was relatively low, and the return periods of the maximum wind speed they could resist were between 239 and 247 years. Flutter safety problems may occur if these two bridges encounter a strong wind with a return period of 300 years.

As for the problem of aerostatic torsional stability robustness evaluation, a stochastic model of the aerostatic torsional stability safety domain represented by two random variables was proposed. The coefficients of variation of these two random variables varied within a reasonable range. By comparing the calculation results of the aerostatic torsional stability robustness index under different coefficients of variation, the influence of the coefficient of variation on the evaluation of aerostatic torsional stability robustness was analyzed. The shortest return periods of the reference wind speed that Jiangyin Yangtze River Bridge and Nansha Bridge could resist, not only at the initial wind attack angle of 0° but also at a relatively unfavorable initial wind attack angle of +3°, were beyond 1000 years. These two bridges possess the ability to resist the maximum wind speed that is unforeseen or unusual. The shortest return period of the reference wind speed that Xihoumen Bridge could resist was 700 years, which indicated that aerostatic instability problems may occur if Xihoumen Bridge encounters a strong wind with a return period of 1000 years.

With the robustness evaluation of aerodynamic flutter and aerostatic torsional stability of long-span suspension bridges, the unforeseen and multi-disaster characteristics of wind hazards were effectively highlighted. Furthermore, the robustness evaluation method can possibly unify the design and evaluation procedures of bridge structures under the action of multiple disasters in future works.

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Nomenclature

U_{re}	the wind speed that the bridge can resist
U_{ac}	the maximum design wind speed
K_1	the partial coefficient for the wind speed that the bridge can resist
K_2	the partial coefficient for the maximum design wind speed
T_m	the return period of the design wind speed
P_F	the failure probability for the wind resistance of bridge structures
Z	the random function of the safety domain
$X_i(i = 1, \cdots, n)$	an arbitrary distribution random variable that affects the robustness of bridge structures
$f_{X_1X_2\cdots X_n}(x_1,\ x_2,\ \cdots,\ x_n)$	the joint probability density function of n fundamental variables
$\Phi(\cdot)$	standard normal distribution function
$\varphi(\cdot)$	standard normal distribution density function
$F_{X_i}(\cdot)$	the distribution function of fundamental variables
$f_{X_i}(\cdot)$	the density function of fundamental variables
μ_Z	the mean value of Z
σ_Z	the standard deviation of Z
α_i	the sensitivity coefficient
C_f	the corresponding correction coefficient
U_f	the flutter critical wind speed
C_b	the corresponding correction coefficient
U_b	the reference wind speed
$g(X)$	the joint probability density function of the four fundamental variables
U_{td}	the critical wind speed of aerostatic instability
U_d	the reference wind speed
γ_{ai}	the partial coefficient of aerostatic torsional stability
U_{td}	the mean value of the critical wind speed of aerostatic instability obtained by wind tunnel tests or numerical simulations
$\delta_t = \sigma_{U_{td}} / \mu_{U_{td}}$	the coefficient of variation of the critical wind speed of aerostatic instability U_{td}

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